Relation between roots & coefficient

In a complete equation of n^{th} degree S_r is given by the formula $\begin{pmatrix} A.P \\ G.P \\ H.P \end{pmatrix}$

$\boxed{S_r, r = 1, 2, 3}$

If $ax^3 + bx^2 + cx + d = 0$ in the form, then

S_1 = sum of the roots

$$= (-1)\frac{\text{co. of } x^2}{\text{co. of } x^3} = -b/a.$$

Nan
Std Sec
Sub
Date
AN1
AN2

S_2 = sum of the product of the roots

$$= \frac{\text{cof. of } x}{\text{co. of } x^3} = c/a$$

S_3 = product of the roots.

$$(-1)\frac{\text{constant term}}{\text{co. of } x^3} = -d/a$$

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Notes:

i) $a-d, a, a+d$ are the roots are when the equation is in a.p

ii) $a, ar, a/r$ are the roots are when the equation is in G.p.

Relation between roots & coefficient

In a complete equation of n^{th} degree

S_r is given by the formula $\begin{pmatrix} A \cdot P \\ G \cdot P \\ H \cdot P \end{pmatrix}$

$\boxed{S_r \; (r = 1, 2, 3)}$

If $ax^3 + bx^2 + cx + d = 0$ in the form, then

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Ans 1

Ans 2

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Notes:

i) $a-d, a, a+d$ are the roots are when the equation is in a.p

ii) $a, ar, a/r$ are the roots are when the equation is in G.p.

1. solve the equation $4x^3-24x^2+23x+18=0$ given that the roots are in a.p

Sol

Let the roots be $a-d, a, a+d$, then we have the relation

$\boxed{(\because (a+b)(a-b)=a^2-b^2)}$

$S_1 =$ sum of the roots $= -b/a$

$(a-d)+a+(a+d) = +(+24/4)$ 6

$3a = 6 \Rightarrow \boxed{a=2}$

$S_3 =$ product of the roots $= -d/a$

$(a-d)\cdot(a)\cdot(a+d) = -18/4$

$a(a^2-d^2) = -18/4$

$2(4-d^2) = -18/4$ 9

$4-d^2 = -9/4$.

$-d^2 = -9/4 - 4$

$-d^2 = \frac{-9-16}{4}$

$-d^2 = +25/4$

$d = \pm 5/2 \Rightarrow \boxed{d = 5/2, -5/2}$

Subs in the value of a and d, we get the roots

when $d = 5/2$	when $d = -5/2$
$a-d = 2-5/2 = \frac{4-5}{2} = -1/2$	$a-d = 2+5/2 = \frac{4+5}{2} = 9/2$
$a = 2$	$a = 2$
$a+d = 2+5/2 = \frac{4+5}{2} = 9/2$	$a+d = 2-5/2 = \frac{4-5}{2} = -1/2$

Solve $8x^3 - 14x^2 + 7x - 1 = 0$, given that its root are in G.P.

Let a/r, a, ar be the roots of the given equation

Then S_1 = sum of the roots.

$S_1 = -b/a$

$a/r + a + ar = +(+14/8)$

$a/r + a + ar = 7/4 \rightarrow (1)$

$S_3 = a/r \cdot a \cdot ar = -d/a$

$a^3 = \frac{-(-1)}{8}$

$\boxed{a = \frac{1}{2}}$

$(1) \rightarrow \frac{a + ar + ar^2}{r} = 7/4$

$a\left(\frac{1 + r + r^2}{r}\right) = 7/4$

$1 + r + r^2 = 7/4 \times r/a$

$= 7/4 \times r/1/2$

$= 7/4 \times 2r$

$1 + r + r^2 = 7r/2$

$1 + r + r^2 - 7r/2 = 0$

$\frac{2 + 2r + 2r^2 - 7r}{2} = 0$

$\cancel{2 + 2r + 2r^2 - 7r = 0}$

$\frac{2 - 5r + 2r^2}{2} = 0$

$2r^2 - 5r + 2 = 0$

$a=2,\ b=5,\ c=2$

$$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25-(4)(2)(2)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25-16}}{4}$$

$$= \frac{5 \pm \sqrt{9}}{4}$$

$$= \frac{5 \pm 3}{4}$$

$r = \frac{5+3}{4} = 8/4 = 2$ $\boxed{r = 2}$

$r = \frac{5-3}{4} = 2/4 = 1/2$ $\boxed{r = 1/2}$

If $a = 1/2,\ r = 2$

$a/r,\ a,\ ar$

$a/r = \frac{1/2}{2} = 1/4$

$a = 1/2$

$ar = (1/2)(2) = 1$

If $a = 1/2$ $r = 1/2$

$a/r,\ a,\ ar$

$a/r = 1/2 / 1/2 = 1$

$a = 1/2$

$ar = (1/2)(1/2) = 1/4$

2. Show that the roots of the equation $x^3+px^2+qx+r=0$ are in a.p if $2p^3-9pq+27r=0$. Show that the above condition is satisfied by the equation $x^3-6x^2+13x-10=0$.
Hence (or) otherwise solve the equation.

Soll

Let the roots of the equ. $x^3+px^2+qx+r=0$ be $a-d, a, a+d$ we have from the relation of the roots and co-efficient.

$$S_1 \Rightarrow a-d+a+a+d = -P/1 = -P \Rightarrow (1)$$

$$S_2 \Rightarrow (a-d)(a)+(a)(a+d)+(a+d)(a-d) = q/1 = q \Rightarrow (2)$$

$$S_3 \Rightarrow a(a-d)(a+d) = -r/1 = -r \Rightarrow (3)$$

from (1) $\Rightarrow$

$$3a = -P$$

$$a = -P/3$$

from (2) $\Rightarrow$

$$a^2-ad+a^2+ad+a^2-d^2 = q$$

$$3a^2-d^2 = q \Rightarrow (4)$$

from (3) $\Rightarrow$

$$a(a^2-d^2) = -r$$

$$a^3-ad^2 = -r \Rightarrow (5)$$

from (4) $\Rightarrow$

$$-d^2 = q-3a^2$$

$$d^2 = 3a^2-q$$

$$d^2 = 3(-P/3)^2-q$$

$= 3(p^2/9) - q$

$$\boxed{d^2 = p^2/3 - q}$$

subs this values in ⑤ we get,

frm ⑤ =>

$(-p/3)^3 - (-p/3)(p^2/3 - q) = -r$

$-p^3/27 + (p/3)(p^2/3 - q) = -r$

$-p^3/27 + p^3/9 - pq/3 = -r$

$$\frac{-p^3 + 3p^3 - 9pq}{27} = -r$$

$-p^3 + 3p^3 - 9pq = -27r$

$2p^3 - 9pq + 27r = 0$

In the equ, $x^3 - 6x^2 + 13x - 10 = 0$

$p = -6 \quad Q = 13 \quad R = -10$

$\therefore 2p^3 - 9pq + 27r = 2(-6)^3 - 9(-6)(13) + 27(-10)$

$= 0$

$\therefore$ The condition is statisfied and so the roots of the equation are in a.p.

In this case equ ①, ②, ③ becomes

$S_1 = a-d + a + a+d = -(+b/1)$

$3a = 6 \Rightarrow a = 2$

$S_2 = (a-d)(a) + (a)(a+d) + (a+d)(a-d) = 13$

$a^2 - ad + a^2 + ad + a^2 - d^2 = 13$

$3a^2 - d^2 = 13 \rightarrow ①$

put $a = 2$

$3(4) - d^2 = 13$

$12 - d^2 = 13$

$-d^2 = 1$

$d^2 = -1$

$d^2 = i^2$

$d = \pm i$

$a-d, a, a+d$

If $a=2, d=i$ then

$2-i, 2, 2+i$.

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(4) Find the condition that the root of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in G.P solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$ whose roots, are in G.P

Sol

Let the roots of the equ be $k/r, k, kr$

$k/r + k + kr = -3b/a$

$k(1/r + 1 + r) = -3b/a \rightarrow (1)$

$(k/r)(k) + k(kr) + (kr)(k/r) = 3c/a$

$k^2/r + k^2 r + k^2 = 3c/a$

$k^2(1/r + r + 1) = 3c/a \rightarrow (2)$

$\cancel{k^2(1/r + r + 1)} = \cdot$

$(k/r)(k)(kr) = -d/a$

$k^3 = -d/a \rightarrow (3)$

$\frac{(2)}{(1)} \Rightarrow \dfrac{k^2(1/r + r + 1) = 3c/a}{k(1/r + r + 1) = -3b/a}$

$k = 3c/a \times -a/3b$

$k = -c/b$

$(3) \Rightarrow (-c/b)^3 = -d/a$

$-c^3/b^3 = -d/a$

$ac^3 = db^3$

In the equation

$27x^3 + 42x^2 - 28x - 8 = 0$

$S_1 = k(1/r + 1 + r) = -42/27 \to (4)$

$S_2 = k^2(1/r + r + 1) = -28/27$

$S_3 = k^3 = -(-8/27) \Rightarrow k = 2/3$

$(4) \Rightarrow 2/3\,(1/r + 1 + r) = -42/27$

$\frac{1+r+r^2}{r} = -\frac{42}{27} \times 3/2$

$\frac{1+r+r^2}{r} = -21/9$

$1 + r + r^2 = -21/9\, r$

$1 + r(1 + 21/9) + r^2 = 0$

$1 + 10/3\, r + r^2 = 0$

$\frac{3 + 10r + 3r^2}{3} = 0$

$3r^2 + 10r + 3 = 0$

$a = 3 \quad b = 10 \quad c = 3$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-10 \pm \sqrt{100 - 4(3)(3)}}{6}$

$= \frac{-10 \pm \sqrt{100 - 36}}{6}$

$= \frac{-10 \pm \sqrt{64}}{6}$

$= \frac{-10 \pm 8}{6}$

$= \frac{-10 + 8}{6} \qquad = \frac{-10 - 8}{6}$

$= -2/6 \qquad = -18/6$

$r = -1/3 \qquad r = -3$

$1 + r + 21/9\, r + r^2 = 0$

$1 + r(1 + 21/9) + r^2 = 0$

$1 + r(30/9) + r^2$

$3 + 10r/3 + 3r^2$

$Kr, K, K/r$

$K = 2/3, r = -3$

$Kr = (2/3) \cdot -3 = -2$

$K = 2/3$

$K/r = 2/3 / -3 = -2/9$

Transform the equ $x^7+4x^6-3x^4+5x^3-8x+1=0$ into another whose roots shall be equal in magnitude, but opposite in sign to those of of the gn equ.

Sol

The gn equ. is incomplete equ but - it can made complete in the following.

$f(x) = x^7+4x^6+0.x^5-3x^4+5x^3+0.x^2-8x+1=0$

put $(x=-y)$

$\Rightarrow (-y)^7+4(-y)^6+0(-y)^5-3(-y)^4+5(-y)^3+0.(-y)^2$
$-8(-y)+1=0$

$\Rightarrow -y^7+4y^6-0.y^5+3y^4-5y^3-0.y^2+8y+1=0$

$(-)x = y^7-4y^6+0.y^5-3y^4+5y^3+0.y^2-8y-1=0$ //

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remainder theorem.

If $f(x)$ is a polynomial, then $f(a)$ is the remainder, when $f(x)$ is divided by $(x-a)$

Defn

In an equation with real co.eff, imaginary root occur in pairs). 1 form a rational cubic equ. which shall have for roots $1, 3-\sqrt{2}$. so since $3-\sqrt{2}$ is a root of the equation, $3+\sqrt{-2}$ is also a root.

So, we have to form an equ whose roots are

$1, 3-\sqrt{-2}, 3+\sqrt{-2}$.

Hence the required equ is

Solution

$x-1[x-(3-\sqrt{2})][x-(3+\sqrt{-2})]=0$

$x-1[x-3+\sqrt{-2}][x-3-\sqrt{-2}]=0$

$(x-1)[(x-3)^2-(\sqrt{-2})^2]=0$

$(x-1)[x^2-6x+9-(-2)]=0.$

$(x-1)[x^2-6x+9+2]=0$

$(x-1)[x^2-6x+11]=0$

$x^3-6x^2+11x-x^2+6x-11=0$ //

② Solve the equ. $x^4+4x^3+5x^2+2x-2=0$ of which one root is $-1+\sqrt{-1}$.

Sol

Imaginary roots occur in pair.

Hence $-1-\sqrt{-1}$ is also a root of the equation

∴ the expression of the left side of the equ has the factors,

$(x-a)(x-b)=0$

$[x-(-1+\sqrt{-1})][x-(-1-\sqrt{-1})]=0$

$(x+1-\sqrt{-1})(x+1+\sqrt{-1})=0$

$(x+1)^2-(\sqrt{-1})^2=0$

$x^2+2x+1-(-1)=0$

$x^2+2x+2=0$

dividing $x^4+4x^3+5x^2+2x-2=0$ by $x^2+2x+2=0$

$$
\begin{array}{r|l}
 & x^2+2x-1 \\
x^2+2x+2 & x^4+4x^3+5x^2+2x-2=0 \\
 & x^4+2x^3+2x^2 \\
 & \underline{-\quad -\quad -} \\
 & 2x^3+3x^2+2x \\
 & 2x^3+4x^2+4x \\
 & \underline{-\quad -\quad -} \\
 & -x^2-2x-2 \\
 & \underline{-x^2-2x-2} \\
 & 0
\end{array}
$$

∴ The quotient is x^2+2x-1

∴ $x^4+4x^3+5x^2+2x-2$

$= (x^2+2x+2)(x^2+2x-1)$

∴ the other roots are obtained from $x^2+2x-1=0$

$a=1 \quad b=2 \quad c=-1$

$$x = \frac{-2 \pm \sqrt{4-4(1)(-1)}}{2(1)}$$

$= \frac{-2 \pm \sqrt{4+4}}{2} \Rightarrow \frac{-2 \pm \sqrt{8}}{2} \Rightarrow \frac{-2+2\sqrt{2}}{2} \Rightarrow \frac{2(-1 \pm \sqrt{2})}{2} \Rightarrow -1 \pm \sqrt{2}$

∴ The root becomes $-1+\sqrt{-1}, -1-\sqrt{-1}, -1+\sqrt{2}, -1-\sqrt{2}$ //

Note:

In an equ with rational co.eff. irrational roots occur in pairs.

3. form an equ with rational co.eff, one of whose roots is $\sqrt{5}+\sqrt{2}$

Sol

Let the gu root is $\sqrt{5}+\sqrt{2}$

The other roots are $\sqrt{5}-\sqrt{2}, -\sqrt{5}+\sqrt{2}, -\sqrt{5}-\sqrt{2}$

Hence the required equ is

$[x-(\sqrt{5}+\sqrt{2})] \cdot [x-(\sqrt{5}-\sqrt{2})] \cdot [x-(-\sqrt{5}+\sqrt{2})] \cdot$

$[x-(-\sqrt{5}-\sqrt{2})]=0$

$\{(x-\sqrt{5}-\sqrt{2})(x-\sqrt{5}+\sqrt{2})\}\{(x+\sqrt{5}-\sqrt{2})(x+\sqrt{5}+\sqrt{2})\}=$

$[(x-\sqrt{5})^2-(\sqrt{2})^2][(x+\sqrt{5})^2-(\sqrt{2})^2]=0$

$[x^2-2x\sqrt{5}+5-2)(x^2+2x\sqrt{5}+5-2)=0$

$(x^2-2x\sqrt{5}+3)(x^2-2x\sqrt{5}+3)=0$

$x^4+\cancel{2x^3\sqrt{5}}+3x^2-\cancel{2x^3\sqrt{5}}-4x^2+5x-6x\sqrt{5}$

$+3x^2+6x\sqrt{5}+9=0$

$x^4-14x^2+9=0$ //

Transformation of equation.

If an equ is gn, It is possible to transform this equ. into another whose roots bear with of the original equ a gn relation.

1. roots with signs changed.
2. roots multiplied by a gn number
3. reciprocal equation.

Note:

To transform an equ into another whose roots are m times that of the gn equ. The equ becomes

$y^n + mp_1 y^{n-1} + m^2 p_2 y^{n-2} + \dots + m^n p_n = 0$

has the roots $m\alpha_1, m\alpha_2 \dots m\alpha_n$

Hence to effect this transformation we have to multiply the successive terms beginning with the second by, $m, m^2, m^3 \dots m^n$.

ange the equ to $2x^4 - 3x^3 + 3x^2 - x + 2 = 0$ into another co-eff. of whose highest term will be ty

multiply the roots by to

$\therefore m = 2$

Then the transformed equ becomes

$$2x^4-3\cdot 2x^3+3\cdot 2^2x^2-2^3\cdot x+2\cdot 2^4=0$$

$$2x^4-6x^3+12x^2-8x+32=0$$

$\div 2 \Rightarrow$

$$x^4-3x^3+6x^2-4x+16=0 \,//$$

2m

(4) remove the fractional co.eff from the equ.

$x^3-1/4\,x^2+1/3\,x-1=0$

Sol

$$x^3-1/4\,x^2+1/3\,x-1=0$$

$$12x^3-3x^2+4x-12=0$$

$$12x^3-3x^2+4x-12=0$$

$$m=2$$

$$12x^3-3\cdot 12x^2+4\cdot 12^2x-12\cdot 12^3=0$$

$\div 12 \Rightarrow$

$$x^3-3x^2+48x-1728=0 \,//$$

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(5) remove the fractional co.eff. from the equ ~~$x^3+1/4x$~~ $x^3+1/4\,x^2-1/16\,x+1/72=0$ to transform the equ into another whose roots are multiplied by m, we get.

Sol

$$x^3+1/4\,x^2-1/16\,x+1/72=0$$

$$x^3+1/4\,mx^2-1/16\,m^2x+1/72\,m^3=0$$

$$x^3+m/2^2\,x^2-m^2/2^4\,x+m^3/2^3\cdot 3^2=0$$

If $m=12$, the fraction $m/2^2$, $m^2/2^4$, $m^3/2^3\cdot 3^2$

Hence we have to multiply the root by 12

The equ becomes

$$x^3+12/2^4\,x^2-12^2/24\,x+12^3/2^3\cdot 3^2=0$$

$$x^3+3x^2-9x+24=0 \,//$$

Reciprocal root :

To transform an equ into another whose roots are the reciprocals of the roots of the equ.

Let $\alpha_1, \alpha_2 - - - \alpha_n$ be the roots of the equ.

$x^n + P_1 x^{n-1} + P_2 x^{n-2} + - - - + P_n = 0$

after some simplification, thes equ becomes

$P_n y^n + P_{n-1} y^{n-1} + - - - + P_1 y + 1 = 0$

has the roots $1/\alpha_1, 1/\alpha_2 - - - 1/\alpha_n$

reciprocal equation.

If an equ remains unalted when x is changed into its reciprocal, It is called a reciprocal equation.

Let $x^n + P_1 x^{n-1} + - - - + P_n = 0$ be a reciprocal equation.

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find the roots of the equ $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$

Sol

put $x = -1$, then $=$ $-1 + 4 - 3 + 3 - 4 + 1 = 0$

This is a reciprocal equ of odd degree with like sence.

$\therefore x + 1$ is a factor of $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 =$

the equ can be written as

$x^5 + x^4 + 3x^4 + 3x^3 + 3x^2 + 3x + x + 1 = 0$

$x^4(x+1) + 3x^3(x+1) + 3x(x+1) + (x+1) = 0$

$(x+1)\{x^4 + 3x^3 + 3x + 1\} = 0$

$x+1=0$
$x=-1$

$x^4+3x^3+3x+1=0$
$\div x^2$,
$x^2+3x+3/x+1/x^2=0$

$x^2+1/x^2+3(x+1/x)=0 \rightarrow (1)$

$x+1/x=0$

Squa on both sides

$(x+1/x)^2=z^2$

$x^2+1/x^2+2=z^2$

$x^2+1/x^2=z^2-2$

(1) $\Rightarrow$

$(z^2-2)+3z=0$

$z^2+3z-2=0$

$a=1, b=3, c=-2$

$$z=\frac{-3\pm\sqrt{9-4(1)(-2)}}{2(1)}$$

$$=\frac{-3\pm\sqrt{9+8}}{2}$$

$$=\frac{-3\pm\sqrt{17}}{2} \Rightarrow \frac{-3+\sqrt{17}}{2}, \frac{-3-\sqrt{17}}{2} \Rightarrow z=\frac{-3\pm\sqrt{17}}{2}$$

$x+1/x=-3\pm\sqrt{17}$

$$\frac{x^2+1}{x}=\frac{-3\pm\sqrt{17}}{2}$$

$2(x^2+1)=x(-3\pm\sqrt{17})$

$2x^2+2=x(-3\pm\sqrt{17})$

$2x^2+2=x(-3+\sqrt{17})$	$2x^2+2=x(-3-\sqrt{17})$
$2x^2-x(-3+\sqrt{17})+2=0$	$2x^2-x(-3-\sqrt{17})+2=0$
$2x^2+x(3-\sqrt{17})+2=0$ //	$2x^2+x(3+\sqrt{17})+2=0$ //

From this equ x can be found.

1-18 Daya

(c) Solve the equ $4x^3+20x^2-23x+6=0$, two of the roots being equal.

Sol

Let α, α, β be the roots of the gn equ

∵ two of the roots being equal.

$S_1 = \alpha+\alpha+\beta = -20/4$

$2\alpha+\beta = -5$

$\boxed{\beta = -5-2\alpha}$ → (1)

$S_2 = \alpha\cdot\alpha+\alpha\cdot\beta+\beta\cdot\alpha = -23/4$

$\alpha^2+2\alpha\beta = -23/4$ → (2)

$S_3 = \alpha\cdot\alpha\cdot\beta = -6/4$

$\alpha^2\cdot\beta = -3/2$ → (3)

(2) ⇒ $\alpha^2+2\alpha[-5-2\alpha] = -23/4$

$\alpha^2-10\alpha-4\alpha^2 = -23/4$

$-3\alpha^2-10\alpha = -23/4$

$-(3\alpha^2+10\alpha) = -23/4$

$3\alpha^2+10\alpha-23/4 = 0$

$\dfrac{12\alpha^2+40\alpha-23}{4} = 0$

$12\alpha^2+40\alpha-23=0$

$12\alpha^2+46\alpha-6\alpha-23=0$

$6\alpha[2\alpha-1]+23[2\alpha-1]=0$

$[2\alpha-1]$ $[6\alpha+23]=0$

$2\alpha-1=0$ | $6\alpha+23=0$

$2\alpha=1$ | $6\alpha=-23$

$\boxed{\alpha=1/2}$ | $\boxed{\alpha=-23/6}$

If $\alpha = 1/2$ then

$\beta = -5-2(1/2)$

$\beta = -6$

$\Rightarrow \beta = -5+2(+23/6)$

$= -5+23/3$

$= \frac{-15+23}{3}$

$\beta = 8/3$

$\alpha = 1/2$	$\beta = -6$
$\alpha = -23/6$	$\beta = 8/3$

(Q) Solve the equ $2x^3 - x^2 - 22x - 24 = 0$. two of the roots being in the ratio of 3:4

Sol

Let $3\alpha, 4\alpha, \beta$ be the roots of the gn equ,

then we have $S_1 = 3\alpha + 4\alpha + \beta = -(-1/2)$

$$7\alpha + \beta = 1/2$$

$$\beta = 1/2 - 7\alpha \rightarrow (1)$$

$$S_2 = (3\alpha)(4\alpha) + (4\alpha)(\beta) + (\beta)(3\alpha) = -22/2$$

$$12\alpha^2 + 4\alpha\beta + 3\alpha\beta = -11$$

$$12\alpha^2 + 7\alpha\beta = -11 \rightarrow (2)$$

$$S_3 = (3\alpha)(4\alpha)(\beta) = -(-24/2)$$

$$12\alpha^2\beta = 12$$

$$\alpha^2\beta = 1 \rightarrow (3)$$

$$12\alpha^2 + 7\alpha[1/2 - 7\alpha] = -11$$

$$12\alpha^2 + 7/2\alpha - 49\alpha^2 = -11$$

$$-37\alpha^2 + 7/2\alpha = -11$$

$$-37\alpha^2 + 7/2\alpha + 11 = 0$$

$$\frac{-74\alpha^2 + 7\alpha + 22}{2} = 0$$

$$-74\alpha^2+7\alpha+22=0$$

$$74\alpha^2-7\alpha-22=0$$

$$a=74,\ b=-7,\ c=-22$$

$$\alpha = \frac{-(-7)\pm\sqrt{49-4(74)(-22)}}{2(74)}$$

$$= \frac{7\pm\sqrt{49+6512}}{148}$$

$$= \frac{7\pm 81}{148}$$

$$\frac{7+81}{148},\ \frac{7-81}{148}$$

$$\alpha = 22/37,\ -1/2$$

If $\alpha = -1/2$ then

$$\beta = 1/2 - 7\alpha$$

$$\beta = 1/2 + 7(+1/2)$$

$$= \frac{1+7}{2}$$

$$\beta = 4$$

$3\alpha, 4\alpha, \beta$

Hence the roots are

$$3\alpha = 3(-1/2) = -3/2$$

$$4\alpha = \overset{2}{4}(-1/2) = -2$$

$$\beta = 4$$

(8) Solve the equ $18x^3+81x^2+121x+60=0$ the root being half the sum of the other two $\boxed{\therefore (\alpha = \frac{1}{2}(\beta+\gamma))}$

Sol.

Let α, β, γ be the roots of the gn equ.

Let $\alpha = \frac{\beta+\gamma}{2}$

$2\alpha = \beta+\gamma \quad \beta+\gamma = 2(-3/2) = \boxed{\beta+\gamma=-3}$

$S_1 = \alpha+\beta+\gamma = -(81/18)$

$\alpha+\beta+\gamma = -9/2$

$\alpha+2\alpha = -9/2$

$3\alpha = -\frac{9}{2}$

$\alpha = -3/2.$

$\therefore (a-b)^2 = (a+b)^2-4ab$

$a-b = \sqrt{(a+b)^2-4ab}$

$S_2 = \alpha.\beta.\gamma = -60/18$

$\alpha.\beta.\gamma = -10/3$

$\beta\gamma = -10/3 \times 1/\alpha$

$= +10/3 \times (-2/3)$

$= 20/9$

$\beta\gamma = 20/9$

$\beta-\gamma = \sqrt{(\beta+\gamma)^2-4\beta\gamma}$

$= \sqrt{(-3)^2-4(20/9)}$

$= \sqrt{9-80/9}$

$\beta-\gamma = \sqrt{\frac{81-80}{9}}$

$= \sqrt{1/9}$

$\beta-\gamma = 1/3$

Solve

$\beta-\gamma = 1/3$

$\beta+\gamma = -3$

$2\beta = 1/3 - 3$

$$= \frac{1 - 9/2}{3}$$

$$2\beta = -8/3$$

$$\beta = -4/3$$

$$\beta - \gamma - 1/3$$

$$-4/3 - \gamma = 1/3$$

$$-\gamma = 1/3 + 4/3$$

$$-\gamma = 5/3$$

$$\boxed{\gamma = -5/3}$$

2. solve the equ $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$

~~The~~ <u>Sol</u>

This is a reciprocal equ of odd degree with unlike signs.

Hence $x-1$ is a factor of the L.H.S

The equation can be written as for use.

$$6x^5 - 6x^4 + 5x^4 - 5x^3 - 38x^3 + 5x^2 + 38x^2 - 5x + 6x - 6 = 0$$

$$6x^4(x-1) + 5x^3(x-1) - 38x^2(x-1) + 5x(x-1) + 6(x-1) = 0$$

$$(x-1)\{6x^4 + 5x^3 - 38x^2 + 5x + 6\} = 0$$

$x - 1 = 0$ | $6x^4 + 5x^3 + 38x^2 + 5x + 6 = 0$

$x = 1$

We have to solve the equ.

$$6x^4 + 5x^3 - 38x^2 + 5x + 6 = 0$$

$\div\, x^2 = 4$

$$6x^2 + 5x - 38 + 5/x + 6/x^2 = 0$$

$$6(x^2 + 1/x^2) + 5(x + 1/x) \div 38 = 0$$

$$x + 1/x = z$$

$$x^2 + 1/x^2 = z^2 - 2$$

$6(z^2-2)+5(z)-38=0$

$6z^2-12+5z-38=0$

$6z^2+5z-50=0$

$a=6,\ b=5,\ \cancel{c=-38}\ c=-50$

$$z=\frac{-5\pm\sqrt{25-4(6)(-50)}}{2(6)}$$

$$=\frac{-5\pm\sqrt{25+1200}}{12}$$

$$=\frac{-5\pm\sqrt{1225}}{12}=\frac{-5\pm35}{12}$$

$\frac{-5+35}{12}$. $\frac{-5-35}{12}$

$30/12$ $-40/12$

$5/2$ $-10/3$

$z=5/2$ $z=-10/3$

$x+1/x=5/2$ $x+1/x=-10/3$

$\frac{x^2+1}{x}=5/2$ $\frac{x^2+1}{x}=-10/3$

$2(x^2+1)=5x$ $3(x^2+1)=-10x$

$2x^2-5x+2=0$ $3x^2+10x+3=0$

$2x^2-5x+2=0$

$a=2\ b=-5,\ c=2$

$$x=\frac{5\pm\sqrt{25-4(2)(2)}}{2(2)}$$

20

$= \frac{5 \pm \sqrt{25-16}}{4}$

$= \frac{5 \pm \sqrt{9}}{4}$

$= \frac{5 \pm 3}{4}$

$8/4 , 2/4$

$2 , 1/2.$

$3x^2 + 10x + 3 = 0$

$a = 3 \quad b = 10 \quad c = 3$

$x = \frac{-10 \pm \sqrt{100 - 4(3)(3)}}{2(3)}$

$= \frac{-10 \pm \sqrt{100-36}}{6}$

$= \frac{-10 \pm \sqrt{64}}{6}$

$= \frac{-10 \pm 8}{6}$

$= \frac{-10+8}{6} , \quad = \frac{-10-8}{6}$

$= -2/6 \quad = -18/6$

$= -1/3 \quad = -3$

∴ the roots of the equ. are 1, 1/2, 2, -

Form of the Quotient and remainder when a polynomial is divided by a binomial.

1) Find the Quotient and Remainder when $3x^3+8x^2+8x+12$ is divided by $x-4$

Sol

$$\begin{array}{r|rrrr} 4 & 3 & 8 & 8 & 12 \\ & 0 & 12 & 80 & 352 \\ \hline & 3 & 20 & 88 & \boxed{364} \end{array}$$

∴ The Quotient is $3x^2+20x+88=0$.

The remainder is 364.

) $2x^6+3x^5-15x^2+2x-4$ is divided by $x+5$.

$2x^6+3x^5+0x^4+0x^3-15x^2+2x-4=0$

$(x-(-5))$

$$\begin{array}{r|rrrrrrr} -5 & 2 & 3 & 0 & 0 & -15 & 2 & -4 \\ & 0 & -10 & 35 & -175 & 875 & -4300 & 21490 \\ \hline & 2 & -7 & 35 & -175 & 860 & -4298 & \boxed{21486} \end{array}$$

∴ The Quotient is $2x^5-7x^4+35x^3-175x^2+860x-4298=0$

The remainder is 21486

) Diminish the roots of $x^4-5x^3+7x^2-4x+5=0$ by 2.

2	1	-5	7	-4	5
	0	2	-6	2	-4
	1	-3	1	-2	1
	0	2	-2	-2	
	1	-1	-1	-4	
	0	2	2		
	1	1	1		
	0	2			
	1	3			

$\therefore$ The remainder is 3

The co-eff of the transformed equn. are 1, 3, 1, -4, 1

$\therefore$ The transformed equn. is $x^4+3x^3+x^2-4x$-

2m defn.

Rolle's theorem

Between two consecutive real roots 'a' & 'b' of the equ $f(x)=0$ where, $f(x)$ is a polynomial, there lies atleast one real root of the equation $f'(x)=$

(1) find the nature of the roots of the equ.

$4x^3-21x^2+18x+20=0$

Sol

Let us consider the function $f(x)=4x^3-21x^2+18x+20=0$

$f(x)=4x^3-21x^2+18x+20$

$f'(x)=12x^2-42x+18$.

$\Rightarrow 12x^2-42x+18=0$

$a=12 \quad b=-42 \quad c=18$

put $x=3$ in $f(x)$

$= 4(27)-21(9)+18(3)+20$

$= 108-189+54+20$

$= -7$

Therefore $f(x)=-7$.

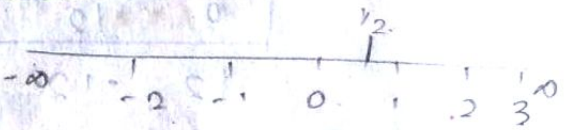
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{42\pm\sqrt{(42)^2-4(12)(18)}}{2(12)}$

$=\dfrac{42\pm\sqrt{1764-864}}{24}$

$=\dfrac{42\pm\sqrt{900}}{24}$

$=\dfrac{42\pm 30}{24}$



x	$-\infty$	$1/2$	3	∞
$f(x)$	$-$	$+$	$-$	$+$

$\dfrac{42+30}{24}, \dfrac{42-30}{24}$

$72/24, 12/24$

$3, 1/2$.

Hence the real roots of $f'(x)=0$ are $1/2, 3$

The root of $f(x)=0$, if any will be in the interval between $-\infty$ and $1/2$, $1/2$ and 3, 3 and ∞

$\therefore f(x)$ must vanish once in each of the above intervals,

Hence $f(x)=0$ has 3 real roots.

---

(2) Show that the equ. $3x^4-8x^3-6x^2+24x-7=0$ has one +ve, one -ve and two imaginary roots.

put $f'(x)$

$x=1$

$12-24-12+24=0$

$x=-1$

$12x^2-12x-24=0$

$12+12-24=0$

$f(x)=3x^4-8x^3-6x^2+24x-7=0$

$f'(x)=12x^3-24x^2-12x+24$

$f'(x)=0$

$\Rightarrow 12x^3-24x^2-12x+24=0$

Let $x-1=0$ is a root.

$\boxed{x=1}$

1	12	-24	-12	24
	0	12	-12	-24
	12	-12	-24	0

$12x^2-12x-24=0$

$\boxed{x+1}=0$ is a root

$\boxed{x=-1}$

$$\begin{array}{c|ccc} -1 & 12 & -12 & -24 \\ & 0 & -12 & 24 \\ \hline & 12 & -24 & \boxed{0} \end{array}$$

$-\infty \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad \infty$

$12x - 24 = 0$

$12x = 24$

$\boxed{x=2}$

H.W Discuss the reality of the roots
$x^4 + 4x^3 - 2x^2 - 12x + a = 0$
for all values of a

$\boxed{x = 1, -1, -3.}$ Ans

Hence the real roots of $f'(x)=0$ are $-1, +1, +2$.

x	$-\infty$	-1	$+1$	$+2$	∞
$f(x)$	$-$	$-$	$+$	$+$	$+$

$\therefore f(x)=0$ has a real root lies between -1 and 1, 1 and 2.

Hence the it is a (+)ve root.

The other real root lies between -1 and $-\infty$.
and so it is a (-)ve root.

$f(x) = x^4 + 4x^3 - 2x^2 - 12x + a = 0$

$f'(x) = 4x^3 + 12x^2 - 4x - 12$

$f'(x) = 0$

$4x^3 + 12x^2 - 4x - 12 = 0$

$x - 1 = 0$ $\qquad$ $x+1=0$

$x = 1$ $\qquad$ $x=-1$.

$4(1)^3 + 12(1)^2 - 4(1) - 12 = 0$ $\qquad$ $4(-1)^3 + 12(-1)^2 - 4(-1) - 12 = 0$

$\cancel{4} + 1\cancel{2} - \cancel{4} - \cancel{12} = 0$ $\qquad$ $-4 + 16 - 12 = 0$

$$\begin{array}{c|cccc} 1 & 4 & 12 & -4 & -12 \\ & 0 & 4 & 16 & 12 \\ \hline & 4 & 16 & 12 & \boxed{0} \end{array}$$

$4x^2 + 16x + 12 = 0$

$x = -1$

23.07

$$\begin{array}{r|rrr} -1 & 4 & 16 & 12 \\ & 0 & -4 & -12 \\ \hline & 4 & 12 & \boxed{0} \end{array}$$

$4x+12=0$ $\quad 4(-3)+12=0$

$-12+12=0$

$x=-3$

$$\begin{array}{r|rr} -3 & 4 & 12 \\ & 0 & 12 \\ \hline & 4 & \boxed{0} \end{array}$$

Hence the real roots of $f'(x)=0$ are $1, -1, -3$.

x	$-\infty$	-3	-1	1	∞
$f(x)$	$-$	$-$	$-$	$+$	$+$

$\therefore$ $f(x)=0$ has a real root lies between -3 and -1, -1 and 1

Hence, It was +ve root

The other real root lies between -1 and -3, -3 and $-\infty$ and it was $(-)$ve root.