unit-II

Matrices

Defn.

2m. A matrix is a rectangular array (or) arrangement of entrys. (or) element displayed in rows and columns put within a square bracket (or) paranthesis.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Note

The order or size of a matrix is the number of rows and no. of columns that are present in a matrix

Singular matrices.

2m. A square matrix A is said to be singular if ~~modulus~~ $|A|=0$. Otherwise non-singular.

for Example.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix} \quad |A| = 1(6-2)-0(3-2)-1(2-4) = 6$$

Inverse of a non-singular matrix using adj method.

Inverse of a matrix.

Let A be a non-singular square matrix of order n. If there exists another square matrix B of the same order such that $AB = I = BA$, Then B is called the inverse of A and is denoted By A^{-1}.

The inverse of the matrix A is gen by

$$A^{-1} = \frac{1}{|A|}(\operatorname{adj} A), \quad |A| \neq 0$$

1) find A^{-1}, $A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$

Sol

$$A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= -2 - 4$$

$$= -6$$

$\text{adj} A = \begin{bmatrix} 1 & -2 \\ -2 & -2 \end{bmatrix}$

$A^{-1} = 1/-6 \begin{bmatrix} 1 & -2 \\ -2 & -2 \end{bmatrix}$

$= 1/+6 \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$ //

Note: find adj of A in m/n

1. co-factor of a_{ij}, ~~a_{12} a_{13}~~

2. $\text{adj} A = [a_{ij}]^T$

(2) If $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$, find A^{-1}

5m

$|A| = 1(6-2) - 0(3-2) - 1(2-4)$

$= 1(4) - 0(0) - 1(-2)$

$= 4 + 2$

$= 6$ //

$|A| \neq 0$

$\therefore A^{-1}$ is exist.

$$a_{ij} = \begin{bmatrix} +\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} \\ -\begin{vmatrix} 0 & -1 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} \\ +\begin{vmatrix} 0 & -1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} +(6-2) & -(3-2) & +(2-4) \\ -(0+2) & +(3+2) & -(2-0) \\ +(0+2) & -(1+1) & +(2-0) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & -2 \\ -2 & 5 & -2 \\ 2 & -2 & 2 \end{bmatrix}$$

$$\underline{\operatorname{adj} A} = a_{ij}^{T} = \begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$= \frac{1}{6} \begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$$

3) find the inverse of $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 8 & 2 \\ 4 & 9 & -1 \end{bmatrix}$

$|A| = 1(-8-18) - 2(-3-8) - 1(27-32)$

$= 1(-26) - 2(-11) - 1(-5)$

$= -26 + \cancel{22} + 5 = 4 - 26 + 27$

$= \overset{1}{\cancel{10}}$/1.

$|A| \neq 0$

$\therefore A^{-1}$ is ~~not~~ exist.

$$a_{ij} = \begin{bmatrix} +\begin{vmatrix} 8 & 2 \\ 9 & -1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} & +\begin{vmatrix} 3 & 8 \\ 4 & 9 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ 9 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 4 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 4 & 9 \end{vmatrix} \\ +\begin{vmatrix} 2 & -1 \\ 8 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 3 & 8 \end{vmatrix} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ \cancel{3} & \cancel{8} & 2 \\ 4 & 9 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} +(-8-18) & -(-3-8) & +(27-32) \\ -(-2+9) & +(-4+4) & -(9-8) \\ +(4+8) & -(2+3) & +(8-6) \end{bmatrix}$$

$$= \begin{bmatrix} -26 & 11 & -5 \\ -7 & 3 & -1 \\ 12 & -5 & 2 \end{bmatrix}$$

$$\operatorname{adj} A = a_{ij}^{T} = \begin{bmatrix} -26 & -7 & 12 \\ 11 & 3 & -5 \\ -5 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$= \frac{1}{+1} \begin{bmatrix} -26 & -7 & 12 \\ 11 & 3 & -5 \\ -5 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -26 & -7 & +12 \\ +11 & +3 & -5 \\ -5 & -1 & +2 \end{bmatrix} //$$

(4.) given $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ find A^{-1}.

$$|A| = 2(3-0) - 0(15-0) - 1(5-0)$$
$$= 2(3) - 0(15) - 1(5)$$
$$= 6 - 5$$
$$= 1 \qquad |A| \neq 0$$

$\therefore A^{-1}$ is exists.

$$a_{ij} = \begin{bmatrix} +\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} & +\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix} \\ -\begin{vmatrix} 0 & -1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \\ +\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} & +\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} +(3-0) - (15-0) + (5-0) \\ -(0+1) + (6-0) - (2-0) \\ +(0+1) - (0+5) + (2-0) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$$

$$\text{adj } A = a_{ij}^T = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A.$$

$$= \cancel{\tfrac{1}{2}}$$

$$= \frac{1}{1} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

2m.

Charateristic equation

If X is a eigen vector corresponding to an eigen value λ of A, then

$$AX = \lambda X$$

$$AX = \lambda X = 0$$

$$(A - \lambda I) X = 0$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{21}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn}-\lambda \end{bmatrix} = 0$$

$\therefore |A-\lambda I| = 0$ is called the characteristic equation.

Method for finding the characteristic equation.

1) for 2×2 matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\Rightarrow \lambda^2 - (a_{11}+a_{22})\lambda + (a_{11}a_{22} - a_{21}a_{12}) = 0$$

$$\Rightarrow \lambda^2 - C_1\lambda + C_2 = 0$$

where C_1 = sum of the leading diagnol elements.

$C_2 = |A|$.

2). for 3×3 matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$$

wher C_1 = sum of the leading diagnol elements

C_2 = sum of minors of leading diagnol elements.

$C_3 = |A|$

Obtain the characteristic polynomial of matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Let $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

The characteristic equ is,

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

c_1 = sum of the leading diagnol element.

$= 8+7+3$

$c_1 = 18$

c_2 = sum of the minors of leading diagnol elem

$= (21-16)+(24-4)+(56-36)$

$= 5+20+20$

$c_2 = 45$

$$c_3 = \left|\begin{matrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{matrix}\right|$$

$= 8(21-16) + 6(-18+8) + 2(24-14$

$= 8(5) + 6(-10) + 2(10)$

$= 40 - 60 + 20$

$c_3 = 0$

$\therefore$ the equ becomes.

$$\lambda^3 - (18)\lambda^2 + (45)\lambda - 0 = 0$$

$$\lambda^2 - 18\lambda^2 + 45\lambda = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$ find characteristic equ

Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

The characteristic equ is,

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

c_1 = sum of the leading diagonal element.

$= 1 + 2 + 3 = 6.$

c_2 = sum of the minors of leading diagonal elements.

$= (6-2) + (3+6) + (2-6)$

$\boxed{c_2 = 11} = 4 + 5 + 2 = 11.$

$$c_3 = |A| = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix}$$

$$= 1(6-2) - 0 - 1(2-4)$$

$$= 4 + 2$$

$$c_3 = 6$$

The char eqn becoms

$\lambda^3 - (6)\lambda^2 + (11)\lambda - 6 = 0.$

$$\boxed{\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0}$$

Rank of a matrix

Define -

(The Rank of a matrix A is r, If all the minors of order (r+1) (or) more of the matrix A are 0 but atleast one minor of order of r is not zero.)

In other words, The Rank of a matrix is the largest of orders of all the non-vanishing minors of the matrix.

i. ~~row of~~

$$\therefore \rho(A) = \text{minmum}\{m, n\}$$

1. If $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$ Then find the rank of the matrix.

2m

Sol.

Let $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$, the order of the matrix is 2.

$\Rightarrow \begin{vmatrix} 4 & -1 \\ 2 & 1 \end{vmatrix} = 4+2 = 6 \neq 0$

$\boxed{\therefore \rho(A) = 2.}$

2) $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$, find rank of the matrix.

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$.

minor of A becomes.

$$\begin{vmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{vmatrix}$$

$= 1(-4+6) + 2(2+30) + 3(-2-20)$

$= 2 + 64 - 66$

$= 0$

The minor of 3rd order matrix is $= 0$

$\therefore \rho(A) \neq 3$

$A = \begin{vmatrix} 4 & -6 \\ 1 & -1 \end{vmatrix} = -4+6 = 2.$

$|A| = 2 \quad \rho(A) \neq 0$

$\therefore$ second order minor is non-zero.

$\therefore \rho(A) = 2.$

Echelon form of a matrix.

1. $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ find rank of a matrix.

Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -5 & 6 \\ 0 & -5 & 6 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{matrix}$

$$\begin{array}{ccc} \cancel{2} & -3 & 4 \\ \cancel{2} & -2 & +2 \\ \hline 0 & -5 & 6 \end{array}$$

$$\begin{array}{ccc} \cancel{3} & -2 & 3 \\ \cancel{3} & -3 & +3 \\ \hline & -5 & 6 \end{array}$$

$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -5 & 6 \\ 0 & 0 & 0 \end{bmatrix} R_3 - R_2$

$$\begin{array}{cc} -\cancel{5} & \cancel{6} \\ +5 & -6 \\ \hline & 0 \end{array}$$

∴ the last equivalent matrix is in the form of accesnding order. It has 2 non rows.

$\boxed{\therefore \rho(A) = 2}$.

2. $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 7 \end{bmatrix}$ find the rank of matrix.

$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 7 \end{bmatrix}$

Let $A = \sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 4 & 3 & 6 & 7 \\ 0 & 1 & 2 & 7 \end{bmatrix} C_1 \leftrightarrow C_3$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & -5 & -10 & -5 \\ 0 & 1 & 2 & 7 \end{bmatrix} R_2 - 4R_1 \qquad \begin{array}{cccc} 4 & 3 & 6 & 7 \\ 4 & 8 & 16 & 12 \\ \hline & -5 & -10 & -5 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & -5 & -10 & -5 \\ 0 & 5 & 10 & 35 \end{bmatrix} 5R_3 \rightarrow \cancel{R_2}$$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & -5 & 10 & -5 \\ 0 & 0 & 0 & 30 \end{bmatrix} R_3 + R_2 \qquad \begin{array}{ccc} -5 & -10 & -15 \\ 5 & 10 & 35 \\ \hline & & 20 \end{array}$$

Crammer Rule

Let $a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

$$\Delta = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$\Delta x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}$$

$$\Delta y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}$$

$$\Delta z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

case (i)

$\Delta \neq 0$ ∴ It is consistent apply cramers rule.

case ii)

$\Delta = 0$, It has four sub cases

ii) (a)

$\Delta = 0$, any one of $\Delta x, \Delta y, \Delta z \neq 0$.

It has no solution.

ii) (b)

$\Delta = 0$, $\Delta x = \Delta y = \Delta z = 0$.

2 x 2 minor is non-zero

∴ It is consistent.

It has infinetely ~~no. of~~ many solution

ii) (c)

$\Delta = 0$, $\Delta x = \Delta y = \Delta z = 0$

All 2x2 minor is 0

but One element in Δ is non-zero.

. It has consisten. It has infinetely many solutio

ii) (d)

$\frac{12\times0}{108}$ $\frac{18\times4}{72}$ $\frac{18\times6}{108}$ $\frac{12\times9}{108}$

$\Delta = 0$, $\Delta x = \Delta y = \Delta z = 0$ 2×2 of Δ's minor is zero but Δx or Δy or Δz 2×2 is non-zero..

It is consistent.

1. Solve. $x + 2y + 3z = 6$,
$2x + 4y + 6z = 12$, use cramer rule.
$3x + 6y + 9z = 18$

Let.

$$\Delta = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} = 1(36-36) - 2(18-18) + 3(12-12)$$
$$= 0.$$

$$\Delta x = \begin{bmatrix} 6 & 2 & 3 \\ 12 & 4 & 6 \\ 18 & 6 & 9 \end{bmatrix} = 6(36-36) - 2(108-108) + 3(72-72)$$
$$= 0.$$

$$\Delta y = \begin{vmatrix} 1 & 6 & 3 \\ 2 & 12 & 6 \\ 3 & 18 & 9 \end{vmatrix} = 1(108-108) - 6(18-18) + 3(36-36)$$
$$= 0$$

$$\Delta z = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 4 & 12 \\ 3 & 6 & 18 \end{vmatrix} = 1(72-72) - 2(36-36) + 6(12$$
$$= 0$$

$\therefore \Delta = 0$, $\Delta x = \Delta y = \Delta z = 0$

$\therefore$ All the 2×2 minor is 0

in Δ one element is non zero

$\therefore$ It has consisten, infinetly many solutio

$x+2y+3z=6 \to (1)$

put $y=s$ $z=t$

$(1) \Rightarrow x=6-2y-3z$

$x=6-2s-3t$

$(x,y,z)=(6-2s-3t, s, t)$.

2. Solve $x+2y+z=7,$ Use cramer rule

$2x-y+2z=4,$

$x+y-2z=-1.$

$$\Delta=\begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & -2 \end{vmatrix} = 1(2-2)-2(-4-2)+1(2+1)$$

$$=1(0)-2(-6)+1(3)$$

$$=0+12+3$$

$$=15$$

$\Delta \neq 0$

$\therefore$ It has only one solution.

$$\Delta x=\begin{vmatrix} 7 & 2 & 1 \\ 4 & -1 & 2 \\ -1 & 1 & -2 \end{vmatrix}$$

$$=7(2-2)-2(-8+2)+1(4-1)$$

$$=7(0)-2(-6)+1(3)$$

$$=12+3$$

$$=15 \qquad \Delta x=15$$

$$\Delta y=\begin{vmatrix} 1 & 7 & 1 \\ 2 & 4 & 2 \\ 1 & -1 & -2 \end{vmatrix} = 1(-8+2)-7(-4-2)+1(-2-4)$$

$$=1(-6)-7(-6)+1(-6)$$

$$=-6+42-6$$

$$\Delta y=30$$

$$\Delta z = \begin{vmatrix} 1 & 2 & 7 \\ 2 & -1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

$= 1(1-4) - 2(-2-4) + 7(2+1)$

$= 1(-3) - 2(-6) + 7(3)$

$= -3 + 12 + 21$

$= 33 - 3$

$\Delta z = 30$

By Cramer rule.

$x = \frac{\Delta x}{\Delta} = 15/15 = 1$

$y = \frac{\Delta y}{\Delta} = 30/15 = 2$

$z = \frac{\Delta z}{\Delta} = 30/15 = 2$

$\therefore (x, y, z) = (1, 2, 2)$

5. solve. $x + y + 2z = 4,$
$2x + 2y + 4z = 8,$
$3x + 3y + 6z = 12$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{vmatrix} = 1(12-12) - 1(12-12) + 2(6-6)$$
$= 0$

$$\Delta x = \begin{vmatrix} 4 & 1 & 2 \\ 8 & 2 & 4 \\ 12 & 3 & 6 \end{vmatrix} = 4(12-12) - 1(48-48) + 2(24-24)$$
$= 0$

$$\Delta y = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ 3 & 12 & 6 \end{vmatrix} = 1(48-48) - 4(12-12) + 2(24-24$$
$= 0$

$$\Delta z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 2 & 8 \\ 3 & 3 & 12 \end{vmatrix} = 1(24-24) - 1(24-24) + 4(6-6)$$
$$= 0$$

$\therefore \Delta = 0$ & $\Delta x = \Delta y = \Delta z = 0$

All the minors of 2×2 matrix in $\Delta, \Delta x, \Delta y, \Delta z$ are zero

$\therefore$ It is consistent.

It has infinitely many solution.

put $x = s$ $y = t$.

$$z = \frac{1}{2}(4 - x - y)$$

$$z = \frac{1}{2}(4 - s - t)$$

$$\therefore (x, y, z) = \left(s, t, \frac{4-s-t}{2}\right)$$

4. $x + 2y + 3z = 6$,
$2x + 4y + 6z = 12$,
$3x + 6y + 9z = 24$.

$\frac{24\times4}{96}$ $\frac{18\times4}{6}$

$\frac{12\times9}{8}$ $\frac{12\times6}{72}$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix} = 1(36-36) - 2(18-18) - 3(12-12)$$
$$= 0$$

$\frac{24\times6}{144}$

$$\Delta x = \begin{vmatrix} 6 & 2 & 3 \\ 12 & 4 & 6 \\ 24 & 6 & 9 \end{vmatrix} = 6(36-36) - 2(108-144) - 3(72-96)$$
$$= -2(-36) + 3(-24)$$
$$= 72 - 72$$

$$\Delta y = \begin{vmatrix} 1 & 6 & 3 \\ 2 & 12 & 6 \\ 3 & 24 & 9 \end{vmatrix} = 1(108-144)-6(18-18) + 3(48-36)$$
$$= -36+36$$
$$= 0$$

$$\Delta z = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 4 & 12 \\ 3 & 6 & 24 \end{vmatrix} = 1(96-72)-2(48-36)+6(12-12)$$
$$= 24-24$$
$$= 0$$

All the 2×2 minors of Δ is 0.

but 2×2 minors of Δx or Δy ~~or Δz~~ is

non-zero $\left(\begin{vmatrix} 12 & 6 \\ 24 & 6 \end{vmatrix} \neq 0 \right)$

$\therefore$ It is ~~not~~ inconsistent.

It has no solution.

Solve in equ. By Rank method.

Let $AX=B$ be the matrix equ.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

The ~~aug~~ augmented matrix becomes

$$[A,B] \sim \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_n \end{bmatrix}$$

Steps

i) convert the equ to $AX = B$ form change into augumented matrix form

ii) (i.e) $[A, B]$

iii) if we see the rank of A and $[A, B]$ apply changes in row only

results

i) rank of $A = [A,B] = n$

where, n is the no. of variables and minor of A is non-zero. Then the equation form is consistent It has only one solution

ii) rank of $A \neq [A,B]$

The equ form is ~~con~~ inconsistent. It has no solution.

iii) rank of $A = [A,B] < n$.

The form is consistent It has infinitely man

Solution.

1) Solve $2x+5y+7z=52$,
$x+y+z=9$, by rank method
$2x+y-z=0$

$$\begin{bmatrix} 2 & 5 & 7 \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 52 \\ 9 \\ 0 \end{bmatrix}$$

$A \quad X = B$

The augmented matrix.

$$\sim \begin{bmatrix} 2 & 5 & 7 & 52 \\ 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \end{bmatrix}$$

$$\begin{array}{cccc} 2 & 5 & 7 & 52 \\ -2 & -2 & -2 & -18 \\ \hline & 3 & 5 & 34 \end{array}$$

$$\begin{array}{cccc} 2 & 1 & -1 & 0 \\ -2 & -2 & -2 & -18 \\ \hline & -1 & -3 & -18 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{bmatrix} R_1 \leftrightarrow R_2.$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{bmatrix} \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 2R_1 \end{array}$$

$$\begin{array}{cccc} -3 & -9 & -54 \\ 3 & 5 & 34 \\ \hline & -4 & -20 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{bmatrix} R_3 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{bmatrix} R_3 \to R_3 + 3R_2$$

the last equivalent matrix is in the form according

It has 3 non-zero rows.

$\therefore \rho[A,B] = 3$

$\rho[A] = 3$

$$A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & -4 \end{bmatrix}$$

It has 3 non-zero rows

$\therefore$

$\rho[A] = \rho[A,B] = 3$

$\therefore$ It is consistent and it has only one solution

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -18 \\ -20 \end{bmatrix}$$

$x + y + z = 9$ —(1)

$-y - 3z = -18$ —(2)

$-4z = -20$ —(3)

$+4z = +20^{5}$

$z = 5$

subs in (2)

$-y-3(5)=-18$

$-y-15=-18$

$-y=-3$

$y=3$

subs value of y & z in (1)

$x+3+5=9$

$x+8=9$

$x=1$

$$\boxed{\therefore (x, y, z) = (1, 3, 5)}$$

2) Solve $2x-3y+7z=5$,
$3x+y-3z=13$, by rank method
$2x+19y-47z=32$,

Let $AX=B$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

The augmented matrix

$[A|B]$

$$\sim \begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix}$$

$$\begin{array}{cccc} 3 & 1 & -3 & 13 \\ 3 & +9/2 & 21/2 & 15/2 \\ \hline & 11/2 & -27/2 & 11/2 \end{array}$$

$$\sim \begin{bmatrix} 1 & -3/2 & 7/2 & 5/2 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} R_1 \to R_1 \div 2$$

$$\sim \begin{bmatrix} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 22 & -54 & 27 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$R_3 \to R_3 - 4R_2$

$$\sim \begin{bmatrix} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

The last equivalent matrix is in the form of ascending order.

It has 3 non-zero rows.

$\therefore \rho[A, B] = 3$

$$A \sim \begin{bmatrix} 1 & -3/2 & 7/2 \\ 0 & 11/2 & -27/2 \\ 0 & 0 & 0 \end{bmatrix}$$

It has 2 non-zero rows

$\therefore \rho[A] = 2$

$\rho(A) \neq \rho[A, B]$

It is inconsistent

It has no solution.

$x+y+z=4$,
$x+2y+3z=18$ by rank method.
$y+2z=6$

Let $AX=B$

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}4\\ 18\\ 6\end{bmatrix}$$

The augmented matrix

$$\sim\begin{bmatrix}1 & 1 & 1 & 4\\ 1 & 2 & 3 & 18\\ 0 & 1 & 2 & 6\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 1 & 1 & 4\\ 0 & 1 & 2 & 11\\ 0 & 1 & 2 & 6\end{bmatrix} R_2-R_1$$

$$\sim\begin{bmatrix}1 & 1 & 1 & 4\\ 0 & 1 & 2 & 11\\ 0 & 0 & 0 & -5\end{bmatrix} R_3-R_2.$$

The last equivalent matrix is in the form of ascending order

It has 3 non-zero rows

$\therefore \rho[A,B]=3$

$$\sim\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 2\\ 0 & 0 & 0\end{bmatrix}$$

It has 2 non-zero rows
$\therefore \rho[A]=2$
$\rho[A]\neq\rho[A,B]$
It is inconsistent
has no solution.

Eigen values and
Eigen vectors.

(1) find the eigen values and eigen vectors of

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}_{3\times 3}$$

Sol

To find.

The characteristic equ.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

The char equ becomes

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

$c_1 = 1+2+3$

$c_1 = 6$

$$c_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$$

$= (6-2) + (3+2) + (2-0)$

$= 4 + 5 + 2$

$c_2 = 11$

$c_3 = |A|$

$= 1(6-2) - 0(0) - 1(2-4)$

$= 4 - (-2)$

$c_3 = 6$

the char equ is

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\therefore \lambda - 1 = 0$ is a root

$\lambda = 1$

using synthetic division

$1 - 6 + 11 - 6$

$12 - 12 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$\lambda^2 - 5\lambda + 6 = 0$

-5

-3 -2

6

$(\lambda - 3)(\lambda - 2) = 0$

$\lambda - 3 = 0 \quad | \quad \lambda - 2 = 0$

$\lambda = 3 \quad | \quad \lambda = 2$

$\therefore$ Eigen Values are $1, 2, 3$

To find.

Eigen vectors.

formula $(A - \lambda I)X = 0$

$$\begin{pmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Case 1:

If $\lambda = 1$

$$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$0x_1 + 0x_2 - x_3 = 0 \rightarrow (1)$

$x_1 + x_2 + x_3 = 0 \rightarrow (2)$

$2x_1 + 2x_2 + 2x_3 = 0 \Rightarrow \div 2 \; (x_1 + x_2 + x_3 = 0) \rightarrow (3)$

$(1) \Rightarrow \; -x_3 = 0$

$x_3 = 0$

(2) & (3) are equal.

$(2) \Rightarrow x_1 + x_2 + x_3 = 0$

put $x_3 = 0$ in (2)

$\Rightarrow x_1 + x_2 = 0$

$x_1 = -x_2$

$\frac{x_1}{1} = \frac{x_2}{-1}$

$x = 1, \; x_2 = -1$

$\therefore$ Eigen vectors

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Case 2:

If $\lambda = 2$

$$\begin{pmatrix} 1-2 & 0 & -1 \\ 1 & 2-2 & 1 \\ 2 & 2 & 3-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-x_1 + 0x_2 - x_3 = 0 \to (1) \div (-)\ x_1 + 0x_2 + x_3 = 0$

$x_1 + 0x_2 + x_3 = 0 \to (2)$

$2x_1 + 2x_2 + x_3 = 0 \to (3)$

(1) & (2) ~~becomes~~ are equal.

(2) & (3) becomes

$x_1 + 0x_2 + x_3 = 0$

$2x_1 + 2x_2 + x_3 = 0$

b c o b

	x_1	x_2	x_3
0	1	1	0
2	1	2	2

$\frac{x_1}{0-2} = \frac{x_2}{2-1} = \frac{x_3}{2-0}$

$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{2}$

$\therefore$ E.V $X_2 \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

Case 3:

If $\lambda = 3$

$$\begin{pmatrix} 1-3 & 0 & -1 \\ 1 & 2-3 & 1 \\ 2 & 2 & 3-3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-2x_1 + 0x_2 - x_3 = 0 \rightarrow ①$

$x_1 - x_2 + x_3 = 0 \rightarrow ②$

$2x_1 + 2x_2 + 0x_3 = 0 \rightarrow ③$

using ② & ③ and apply cross multiplication.

$$\begin{matrix} -1 & 1 & 1 & -1 \\ 2 & 0 & 2 & 2 \end{matrix}$$

$$\frac{x_1}{0-2} = \frac{x_2}{2-0} = \frac{x_3}{2+2}$$

$x_1/-2 = x_2/2 = x_3/4$

$\div 2$

$x_1/-1 = x_2/1 = x_3/2$

$\therefore X_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ //

If the product of two eigenvalues of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is 2. find the 3^{rd} eigenvalue.

sol

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ ...(a-1) let E. Vs. of A are $\lambda_1, \lambda_2, \lambda_3$.

we have ~~$\lambda_1 \times \lambda_2 \times$~~ $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |A|$

$$2\lambda_2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{vmatrix} = 1(9-1) - 0 - 0$$

$$= 8.$$

$$2\lambda_2 = 8$$

$$\lambda_2 = 4.$$

8\. Find the eigen values and corresponding E. $\vec{V}$ of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

Sol

E. Vs are $3, 2, 5$

To find E. $\vec{V}$

$$(A - \lambda I) X = 0$$

$$\begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(3-\lambda)x_1 + x_2 + 4x_3 = 0 \quad \text{--} (1)$

$0x_1 + (2-\lambda)x_2 + 6x_3 = 0 \quad \text{--} (2)$

$0x_1 + 0x_2 + (5-\lambda)x_3 = 0 \quad \text{--} (3)$

Case i) If $\lambda = 3$

$0x_1 + x_2 + 4x_3 = 0 \quad \text{--} (1)$

$0x_1 - x_2 + 6x_3 = 0 \quad \text{--} (2)$

$0x_1 + 0x_2 + 2x_3 = 0$

$2x_3 = 0$

$x_3 = 0$

put $x_3 = 0$ in (5)

$-x_2 + 6(0) = 0$

$-x_2 = 0$

$x_2 = 0$

$\therefore$ x_1 is arbitary.

$\therefore x_1 = 0$

$\therefore$ E. $\vec{V}$

$\therefore X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

case (ii) If $\lambda = 2$

$x_1 + x_2 + 4x_3 = 0 \rightarrow (1)$

$0x_1 + 0x_2 + 6x_3 = 0 \rightarrow (2)$

$0x_1 + 0x_2 + 3x_3 = 0$

$3x_3 = 0$

$x_3 = 0$

put $x_3 = 0$ in (1)

$x_1 + x_2 = 0$

$x_1 = -x_2$

$\frac{x_1}{1} = \frac{x_2}{-1}$

$\therefore$ E. $\vec{V}$ $\therefore X_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Case iii) If $\lambda = 5$.

$-2x_1 + x_2 + 4x_3 = 0 \to (1)$

$0x_1 + -3x_2 + 6x_3 = 0 \to (2)$

$0x_1 + 0x_2 + 0x_3 = 0$

$(2) \quad -3x_2 + 6x_3 = 0$

$3x_2 = 6x_3$

$\frac{x_2}{2} = \frac{x_3}{1}$

$x_2 = 2, \; x_3 = 1$

Subs in (1).

$-2x_1 + 2 + 4 = 0$

$-2x_1 + 6 = 0$

$2x_1 = 6$

$x_1 = 3$

$\therefore X_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

4) Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ find E.Vs, $E.\vec{V}$.

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

To find

E.V's

The char equ becomes

$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

$c_1 = 6+3+3$

$c_1 = 12$

$c_2 = \begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix}$

$= (9-1) + (18-4) + (18-4)$

$= 8 + 14 + 14$

$c_2 = 36$

$c_3 = |A|$

$= \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$

$= 6(9-1) + 2(-6+2) + 2(2-6)$

$= 6(8) + 2(-4) + 2(-4)$

$= 48 - 8 - 8$

$= 48 - 16$

$c_3 = 32$

The char equ is

$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$

$\lambda - 2 = 0$ is a root ~~arbitrary~~

$\lambda - 2 = 0$

$\boxed{\lambda = 2}$

$\lambda = 1$

$1 - 12(1) + 36(1) - 32$

$1 - 12 + 36 - 32$

$-44 + 37$

$\boxed{\lambda = 2}$

$8 - 12(3) + 36(2)$

$8 - 48 + 72 - 32$

$= 80 - 80$

$= 0$

Cayley - Hamilton Theorem

Stmt.

every square matrix satisfies its own characteristic equation.

1) using C-H theorem Find A^{-1} when $A=\begin{bmatrix}1&0&3\\2&1&-1\\1&-1&1\end{bmatrix}$

Sol:

$$A=\begin{bmatrix}1&0&3\\2&1&-1\\1&-1&1\end{bmatrix}$$

char equ. of A is

$$\lambda^3-c_1\lambda^2+c_2\lambda-c_3=0 \rightarrow (1)$$

$$c_1=1+1+1=3$$

$$c_2=\begin{vmatrix}1&-1\\-1&1\end{vmatrix}+\begin{vmatrix}1&3\\1&1\end{vmatrix}+\begin{vmatrix}1&0\\2&1\end{vmatrix}$$

$$(1-1)+(1-3)+(1) = -2+1 = -1$$

$$c_2=-1$$

$$c_3=|A|.$$

$$=1(1-1)-0(2+1)+3(-2-1)$$

$$c_3=-9$$

(1) $\Rightarrow$

$$\lambda^3-3\lambda^2+(-1)\lambda-(-9)=0$$

$$\lambda^3-3\lambda^2-\lambda+9=0$$

C-H Theorem

$$A^3 - 3A^2 - A + 9I = 0$$

Multiply A^{-1} of both sides.

$$A^{-1}(A^3 - 3A^2 - A + 9I) = 0$$

$$A^2 - 3A - I + 9A^{-1} = 0$$

$$9A^{-1} = -A^2 + 3A + I$$

$$A^{-1} = -\frac{1}{9}\left[A^2 - 3A - I\right] \rightarrow (2)$$

$$A^2 = A \cdot A.$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+3 & 0+0-3 & 3+0+3 \\ 2+2-1 & 0++1+1 & 6-1-1 \\ 1-2+1 & 0-1-1 & 3+1+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 9 \\ 6 & 3 & -3 \\ 3 & -3 & 3 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(2) \Rightarrow A^{-1} = -\frac{1}{9}\left\{\begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 9 \\ 6 & 3 & -3 \\ 3 & -3 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\}$$

$$= -\frac{1}{9}\left\{\begin{bmatrix} 1 & -3 & -5 \\ -3 & -1 & 7 \\ -3 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\}$$

$$= -\frac{1}{9}\begin{bmatrix} 0 & -3 & -5 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 0 & 5 & 5 \\ 5 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}_{//}$$

Find A^{-1} if $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & -2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$, using C-H theorem.

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$$

Char equ of A is

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

$c_1 = 1 + 2 - 1$

$= 3 - 1$

$c_1 = 2$

$$c_2 = \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 4 \\ 2 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}$$

$= (-2 + 1) + (-1 - 8) + (2 + 3)$

$= (-1) + (-9) + (5)$

$= -1 - 9 + 5$

$= -10 + 5$

$c_2 = -5$

$c_3 = |A|$

$= 1(-2+1) + 1(-3+2) + 4(3-4)$

$= 1(-1) + 1(-1) + 4(-1)$

$= -1 - 1 - 4$

$c_3 = -6$

$\therefore$ The char equ becomes.

$(1) \Rightarrow \lambda^3 - 2\lambda^2 + (-5)\lambda - (-6) = 0$

$\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$

By $\underline{C-H}$ theorem

$A^3 - 2A^2 - 5A + 6I = 0$

Multiply by A^{-1} on both sides,

$A^{-1}[A^3 - 2A^2 - 5A + 6I] = A^{-1}(0)$

$A^2 - 2A - 5I + 6A^{-1} = 0$

$6A^{-1} = -A^2 + 2A + 5I$

$A^{-1} = -\frac{1}{6}[A^2 - 2A - 5I] \Rightarrow (2)$

$A^2 = A \cdot A$

$$= \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 3 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-3+8 & -1-2+4 & 4+1-4 \\ 3+6-2 & -3+4-1 & 12-8+1 \\ 2+3-2 & -2+2-1 & 8-1+1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 6 & 1 & 1 \\ 7 & 0 & 11 \\ 3 & -1 & 8 \end{pmatrix}$$

$$2A = 2\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$$

$$2A = \begin{pmatrix} 2 & -2 & 8 \\ 6 & 4 & -2 \\ 4 & 2 & -2 \end{pmatrix}$$

$$5I = 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$5I = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

(2) =>

$$A^{-1} = -1/6 \left\{ \begin{pmatrix} 6 & 1 & 1 \\ 7 & 0 & 11 \\ 3 & -1 & 8 \end{pmatrix} - \begin{pmatrix} 2 & -2 & 8 \\ 6 & 4 & -2 \\ 4 & 2 & -2 \end{pmatrix} - \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \right\}$$

$$= -1/6 \left\{ \begin{pmatrix} 4 & 3 & -7 \\ 1 & -4 & 13 \\ -1 & -3 & 10 \end{pmatrix} - \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \right\}$$

~~= -1/6 { (~~

$$= -1/6 \begin{pmatrix} -1 & 3 & -7 \\ 1 & -9 & 13 \\ -1 & -3 & 5 \end{pmatrix}$$

$$A^{-1} = 1/6 \begin{pmatrix} 1 & -3 & 7 \\ -1 & 9 & -13 \\ 1 & 3 & -5 \end{pmatrix}$$

Find A^4 and A^{-1} If $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ using C-H theorem

Sol

$$A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

The char equ becomes

$$\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$$

$$C_1 = 2+2+2$$

$$C_1 = 6$$

$$C_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$= (4-1)+(4-2)+(4-1)$

$= 3+2+3$

$C_2 = 8$

$C_3 = |A|$

$$= \begin{vmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$= 2(4-1)+1(-2+1)+2(1-2)$

$= 2(3)+1(-1)+2(-1)$

$= 6-1-2$

$C_3 = 3$

$(1) \Rightarrow \lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0$

By C-H Theorem

$A^3 - 6A^2 + 8A - 3I = 0 \rightarrow (2)$

Multiply by 'A' on both sides

$A(A^3 - 6A^2 + 8A - 3I) = A(0)$

$A^4 - 6A^3 + 8A^2 - 3A = 0$

$A^4 = 6A^3 - 8A^2 + 3A \rightarrow (3)$

$A^3 = A^2 \cdot A$

$A^2 = A \cdot A$

$$= \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{pmatrix}$$

$A^3 = A^2 \cdot A$

$$A^3 = \begin{pmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{pmatrix}$$

$$6A^3 = \begin{pmatrix} 174 & -168 & 228 \\ -132 & 138 & -168 \\ 132 & -132 & 174 \end{pmatrix}$$

$$8A^3 = \begin{pmatrix} 56 & -48 & 72 \\ -40 & 48 & -48 \\ 40 & -40 & 56 \end{pmatrix}$$

$$3A = \begin{pmatrix} 6 & -3 & 6 \\ -3 & 6 & -3 \\ 3 & -3 & 6 \end{pmatrix}$$

$(3) \Rightarrow$

$$A^4 = \left\{ \begin{pmatrix} 174 & -168 & 228 \\ -132 & 138 & -168 \\ 132 & -132 & 174 \end{pmatrix} - \begin{pmatrix} 56 & -48 & 72 \\ -40 & 48 & -48 \\ 40 & -40 & 56 \end{pmatrix} + \begin{pmatrix} 6 & -3 & 6 \\ -3 & 6 & -3 \\ 3 & -3 & 6 \end{pmatrix} \right\}$$

$$A^4 = \left\{ \begin{pmatrix} 118 & -120 & 156 \\ -92 & 90 & -120 \\ 92 & -92 & 118 \end{pmatrix} + \begin{pmatrix} 6 & -3 & 6 \\ -3 & 6 & -3 \\ 3 & -3 & 6 \end{pmatrix} \right\}$$

$$A^4 = \begin{pmatrix} 124 & -123 & 162 \\ -95 & 96 & -123 \\ 95 & -95 & 124 \end{pmatrix}$$

$(2) \Rightarrow$

$$A^3 - 6A^2 + 8A - 3I = 0$$

Multiply by A^{-1} on both sides

$\underline{\underline{2}}$

$A^{-1}(A^3 - 6A^2 + 8A - 3I) = A^{-1}(0)$

$A^2 - 6A + 8I - 3A^{-1} = 0$

$-3A^{-1} = -A^2 + 6A - 8I$

$A^{-1} = 1/3\,[A^2 - 6A + 8I] \rightarrow (4)$

$$6A = \begin{pmatrix} 12 & -6 & 12 \\ 6 & 12 & -6 \\ 6 & -6 & 12 \end{pmatrix}$$

$$8I = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$(2) \Rightarrow$

$$A^{-1} = 1/3 \left\{ \begin{pmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ -5 & -5 & 7 \end{pmatrix} - \begin{pmatrix} 12 & -6 & 12 \\ 6 & 12 & -6 \\ 6 & -6 & 12 \end{pmatrix} + \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \right\}$$

$$= 1/3 \left\{ \begin{pmatrix} -5 & 0 & -3 \\ -11 & -6 & 0 \\ -1 & 1 & -5 \end{pmatrix} + \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \right\}$$

$$= 1/3 \begin{pmatrix} 3 & 0 & -3 \\ -11 & 2 & 0 \\ -1 & 1 & 3 \end{pmatrix}$$