Definition (Graphical Method)

Linear programming problem involving two decision variables can easily be solved by graphical method.

1. The objective function is maximum $z = 22x_1 + 18x_2$ subject to the constrains. 36

$$360x_1 + 240x_2 \leq 5760$$
$$x_1 + x_2 \leq 20$$

where, $x_1 + x_2 \geq 0$

draw the graph and find the optimum solution.

Soln

The mathematical formulation the objective function is.

$$\max z = 22x_1 + 18x_2$$

Subject to the constrains

$$360x_1 + 240x_2 \leq 5760$$
$$x_1 + x_2 \leq 20$$

Where. $x_1, x_2 \geq 0$

To Graphical method.

$$360x_1 + 240x_2 = 5760 \longrightarrow (1)$$
$$x_1 + x_2 = 20 \longrightarrow (2)$$

Put

$x_1 = 0$ in equn ①,

$0 + 240x_2 = 5760.$

$x_2 = 5760/240$

$x_2 = 24$

$\therefore A = (0, 24)$

Put

$x_2 = 0$ in equn ①

$360x_1 + 0 = 5760$

$x_1 = 16$

$B = (16, 0)$

Put $x_1 = 0$ in equn ②

$x_2 = 20$

$C = (0, 20)$

Put $x_2 = 0$ in equn (2)

$x_1 = 20$

$D = (20, 0)$

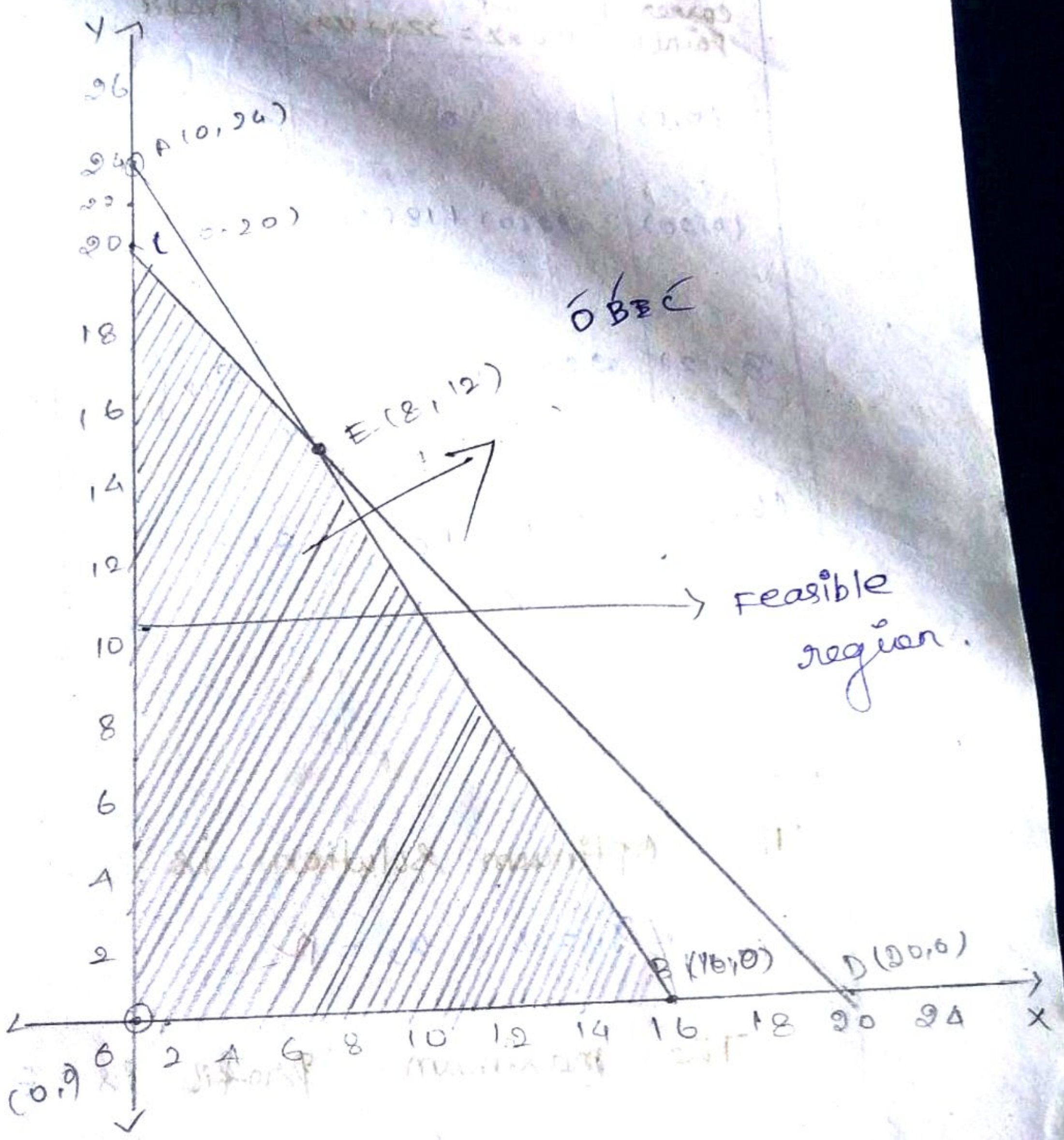
$\therefore$ the Points are

$A = (0, 24)$

$B = (16, 0)$

$C = (0, 20)$

$D = (20, 0)$



① ⟹ $360x_1 + 240x_2 = 5760$

② ⟹ $360x_1 + 360x_2 = 7200$

(−) (−) (−)

$-120x_2 = -1440 \Rightarrow x_2 = 12$

Put $x_2 = 12$ in eqn ②.

$x_1 + 12 = 20$

$x_1 = 8$

$E = (8, 12)$

Corner points	$max\ z = 22x_1 + 18x_2$	Profit
(0,0)	0	0
(0,20)	22(0) + 18(20)	360
(8,12)	22(8) + 18(12)	392
(16,0)	22(16) + 0	352.

The optimum solution is.

$x_1 = 8 ;\ x_2 = 12$

The maximum profit is 392

Solve the graphical method $max\ z = 4x_1 + 14x_2$ subject to the constrains $2x_1 + 7x_2 \leq 21$.

$$2x_1 + 7x_2 \leq 21$$
$$7x_1 + 2x_2 \leq 21$$

Max = 42

$x_1 = 7/3$

where, $x_1, x_2 \geq 0$

Sol

The mathematical formulation the objective function is

$$max\ z = 4x_1 + 14x_2$$

subject to the constrains

$360 \cdot 2x_1 + 7x_2 \leq 21$

$7x_1 + 2x_2 \leq 21$

where, $x_1, x_2 \geq 0$

To Graphical method.

$2x_1 + 7x_2 \leq 21 \rightarrow ①$

$7x_1 + 2x_2 = 21 \rightarrow ②$

put $x_1 = 0$ in equn ①

$0 + 7x_2 = 21$

$x_2 = 3$

$\therefore A = (0, 3)$

put $x_2 = 0$ in equn ①.

$2x_1 + 0 = 21$

$x_1 = 10.5$

$B = (10.5, 0)$

put $x_1 = 0$ in equn ②

$0 + 2x_2 \leq 21$

$x_2 = 10.5$

$C = (0, 10.5)$

Put $x_2 = 0$ in equn ②

$7x_1 + 0 = 21$

$x_1 = 3$

$\boxed{D = (3, 0)}$

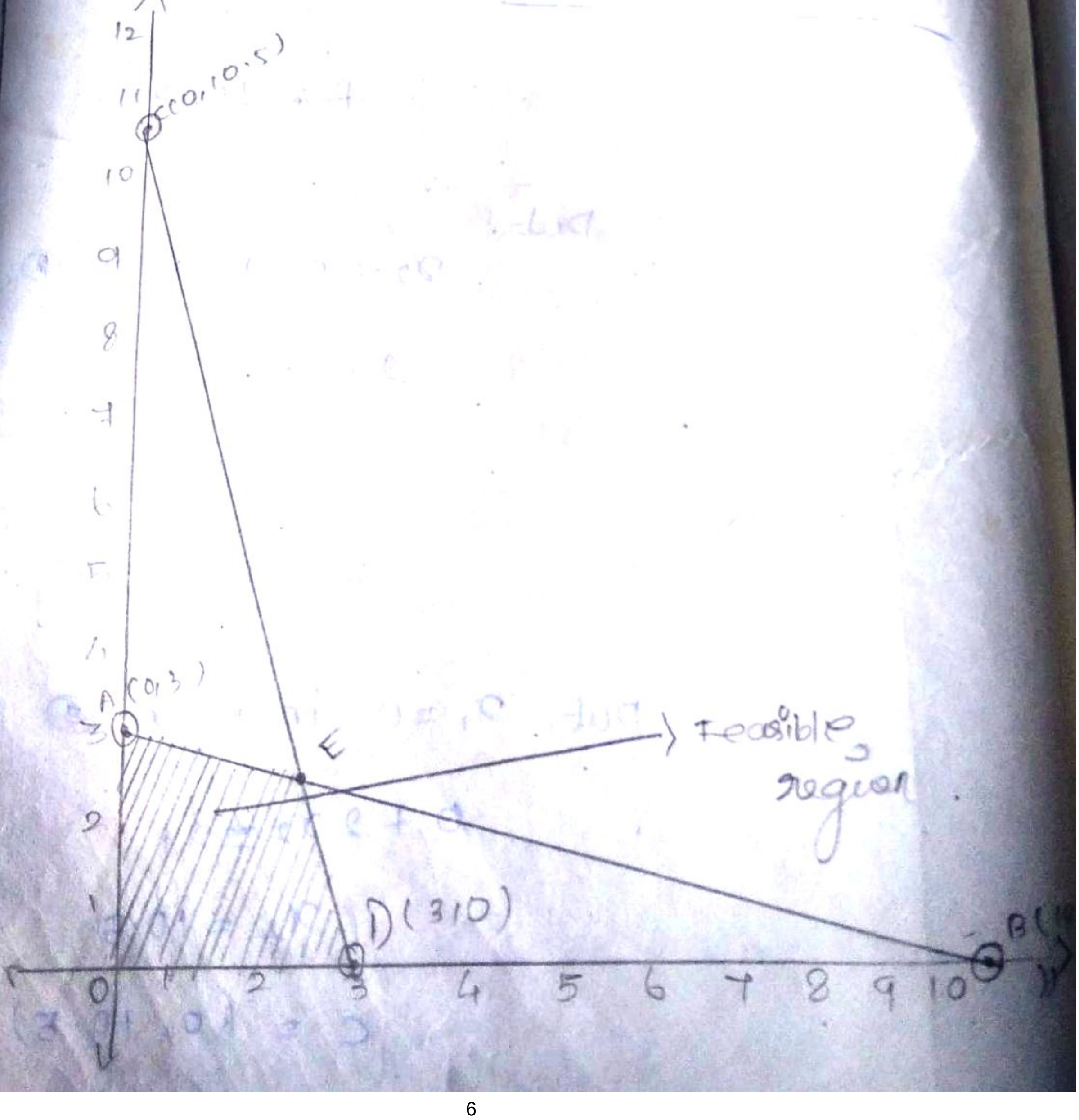
The points are

$A = (0, 3)$

$B = (10.5, 0)$

$C = (0, 10.5)$

$D = (3, 0)$



$2x_1 + 7x_2 = 21 \longrightarrow (1)$

$7x_1 + 2x_2 = 21 \longrightarrow (2)$

$(1) \times 7 \Rightarrow 14x_1 + 49x_2 = 147$

$(2) \times 2 \Rightarrow 14x_1 + 4x_2 = 42$

$45x_2 = 105$

$$x_2 = \frac{7}{3}$$

$x_2 = 7/3$ in equn $\rightarrow (1)$

$7x_1 + 2(7/3) = 21$

$7x_1 + 14/3 = 21$

$7x_1 = 21 - 14/3$

$$7x_1 = \frac{63 - 14}{3}$$ $\left(7x_1 = \frac{49}{3}\right)$

$x_1 = 7/3$

$x_2 = (7/3, 7/3)$

corner points	$\max z = 4x_1 + 14x_2$	profit.
(0, 0)	0	0
(0, 3)	$4(0) + 14(3)$	42
(7/3, 7/3)	$4(7/3) + 14(7/3)$	42
(3, 0)	$4(3) + 14(0)$	12.

The optimum solution is

$x_1 = 0$; $x_2 = 3$

The maximum Profit is 42.

Solve the following LPP by Graphical method.

3. max $z = 3x_1 + 2x_2$.

subject to constraints $= 3x_1 + 2x_2$.

$$3x_1 + 2x_2 \leq 6$$

$$2x_1 + 3x_2 \leq 6.$$

Sol

The mathematical formulation is

max $z = 3x_1 + 2x_2$.

Subject to the constrains

$$3x_1 + 2x_2 \leq 6$$

$$2x_1 + 3x_2 \leq 6$$

where, $x_1, x_2 \geq 0$

To Graphical method.

$$3x_1 + 2x_2 = 6 \rightarrow (1)$$

$$2x_1 + 3x_2 = 6 \rightarrow (2)$$

Put $x_1 = 0$ in equn (1)

$2x_2 = 6$

$x_2 = 3$

$A = (0, 3)$

put $x_2 = 0$ in equn ①

$$2x_1 + 0 = 6$$

$$x_1 = 2$$

$$\boxed{B = (2, 0)}$$

put $x_1 = 0$ in equn ②

$$2(0) + 3x_2 = 6$$

$$x_2 = 2$$

$$\boxed{C = (0, 2)}$$

put $x_2 = 0$ in equn ②.

$$2x_1 + 0 = 6$$

$$x_1 = 3$$

$$\boxed{D = (3, 0)}$$

The points are

$A = (0, 3)$

$B = (2, 0)$

$C = (0, 2)$

$D = (3, 0)$

$6x_1 + 4x_2 = 12$

$6x_1 + 4\left(\frac{6}{5}\right) = 12$

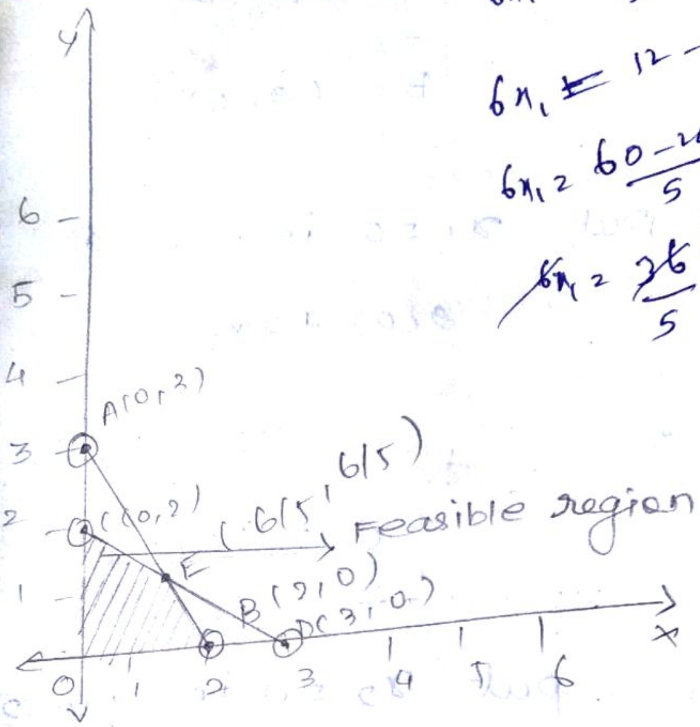
$6x_1 + \frac{24}{5} = 12$

$6x_1 = 12 - \frac{24}{5}$

$6x_1 = \frac{60-24}{5}$

$6x_1 = \frac{36}{5}$

$\boxed{x_1 = 6/5}$



$3x_1 + 2x_2 = 6 \rightarrow (1)$

$2x_1 + 3x_2 = 6 \rightarrow (2)$

$(1) \times 2 \Rightarrow 6x_1 + 4x_2 = 12$

$(2) \times 3 \Rightarrow 6x_1 + 9x_2 = 18$

$(-) \quad (-)$

$-5x_2 = -6$

$x_2 = 6/5$

Put $x_2 = 6/5$ in eqn (1)

$3x_1 + 2x_2 = 6$

$3x_1 + 2(6/5) = 6$

$6x_1 = 12 - 24/5$

$6x_1 = \frac{60-24}{5}$

$6x_1 = \frac{36}{5}$

$x_1 = \frac{36}{30}$

$x_1 = 6/5$

$3x_1 + 2x_2 = 6$

$3x_1 + 2\left(\frac{6}{5}\right) = 6$

$3x_1 + \frac{12}{5} = 6$

$3x_1 = 6 - \frac{12}{5}$

$= \frac{30-12}{5}$

$3x_1 = \frac{18}{5}$

$x_1 = 6/5$

$D = (6/5, 6/5)$

Corner points	$max = 3x_1 + 2x_2$	Profit
(0, 2)	3(0)+2(2)	4
(6/5, 6/5)	3(6/5)+2(6/5)	6
(2, 0)	3(2)+2(0)	6
(0, 0)	3(0)+2(0)	0

The optimum solution is

$x_1 = 6/5$ & $x_2 = 6/5$

Maximum profit is 6.

1. Draw ... Graph $\max z = 2x_1 + x_2$

Subject to the constraint

$x_1 + x_2 \leq 10$

$x_1 \leq 20$.

where $x_1, x_2 \geq 0$

The mathematical formulation is

$\max z = 2x_1 + x_2$

Subject to the constrains

$x_1 - x_2 \leq 10$

$x_1 \leq 20$.

where, $x_1, x_2 \geq 0$

To Graphical method,

$x_1 - x_2 \leq 10$

$x_1 \leq 20$.

$x_1 - x_2 = 10 \rightarrow (1)$

$x_1 = 20. \rightarrow (2)$

Put $x_1 = 0$ in Equn (1)

$0 - x_2 = 10$

$-x_2 = 10$

$x_2 = -10$

$A = (0, -10)$

Put $x_2 = 0$ in equn ①

$x_1 - 0 = 10$

$\boxed{x_1 = 10}$

$B = (10, 0)$

$C = 120$

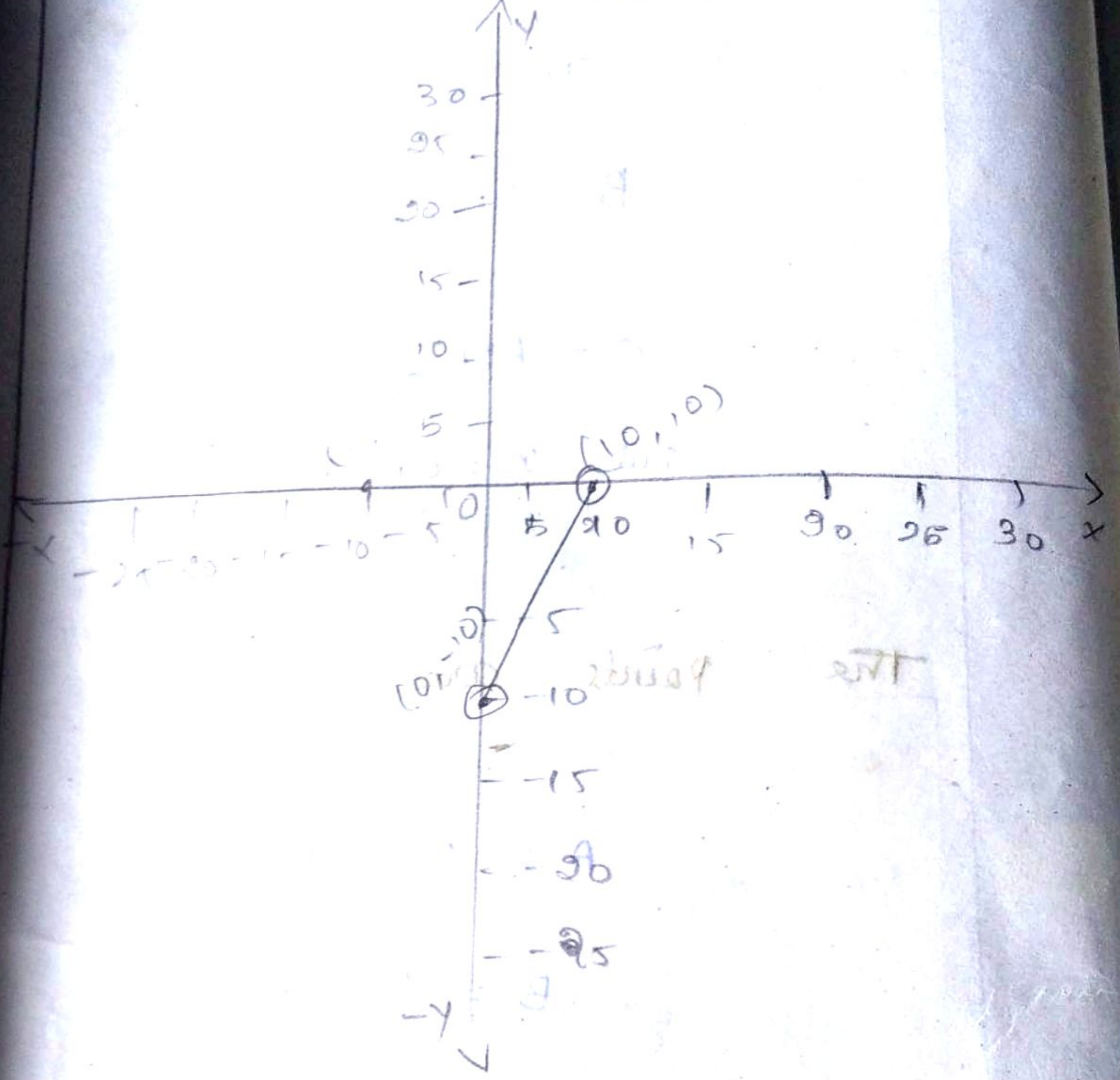
put $x_1 = 0$ in equn ①.

The points are.

$A = (0, -10)$

$B = (10, 0)$

$C = 20$.



we cannot find the common region, so there is no infeasible solution.

) Draw the Graph max $z = 3x_1 + 2x_2$
subject to the Constrains

$$-2x_1 + 3x_2 \leq 9$$

$$x_1 - 5x_2 \geq -20.$$

Where $x_1, x_2 \geq 0$.

Sol

The mathematical formulation is

$$\max z = 3x_1 + 2x_2$$

subject to the constrains

$$-2x_1 + 3x_2 \leq 9$$

$$x_1 - 5x_2 \geq -20.$$

where $x_1, x_2 \geq 0$

Graphical method

$$-2x_1 + 3x_2 = 9 \longrightarrow (1)$$

$$x_1 - 5x_2 = -20 \longrightarrow (2)$$

Put $x_1 = 0$ in equn (1).

$$-2(0) + 3x_2 = 9$$

$$3x_2 = 9$$

$$x_2 = 3$$

A = (0, 3)

put $x_2 = 0$ in equn ①

$$-2x_1 + 0 = 9$$

$$x_1 = -9/2$$

$$\boxed{B = (-9/2, 0)}$$

put $x_1 = 0$ in equn ②

$$0 - 5x_2 = -20$$

$$x_2 = 4$$

$$\boxed{C = (0, 4)}$$

put $x_2 = 0$ in equn ②

$$x_1 - 0 = -20$$

$$x_1 = -20$$

$$\boxed{D = (-20, 0)}$$

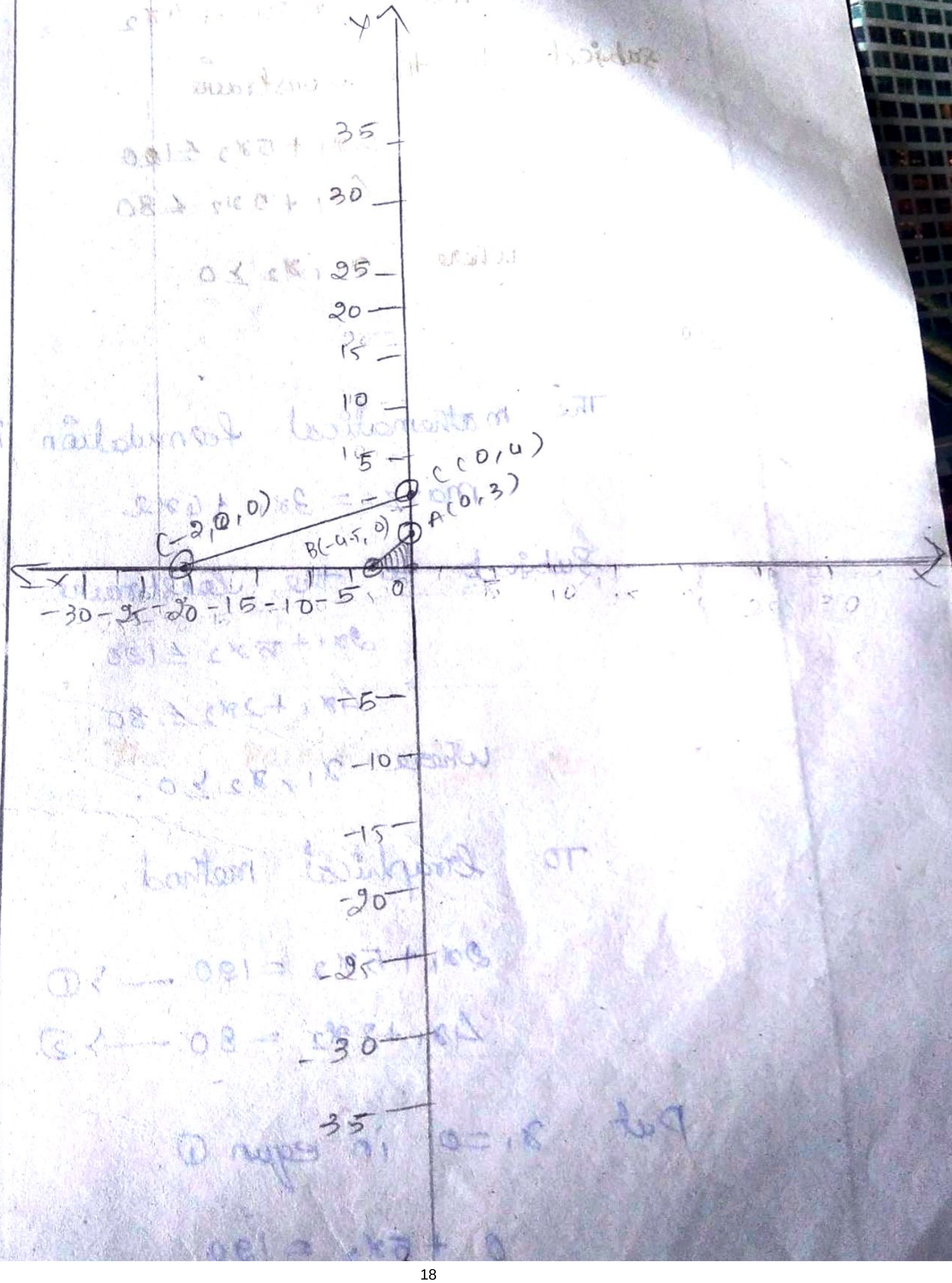
The points are.

$A = (0, 3)$

$B = (-9/2, 0)$ [-4.5]

$C = (0, 4)$

$D = (-20, 0)$


Y
C (0,4)
A (0,3)
B(-4.5,0)
0
35
30
25
20
15
10
5
-5
-10
-15
-20
-25
-30
-35

1. Solve the eqn by using simplex method.

$\max z = 4x_1 + 7x_2$

Subject to the constrains

$4x_1 + 3x_2 \leq 12$

$3x_1 + 4x_2 \leq 12$

Where $x_1, x_2 \geq 0$.

Sol

G.T

$\max z = 4x_1 + 7x_2$

Subject to the constrains

$4x_1 + 3x_2 \leq 12$

$3x_1 + 4x_2 \leq 12$

Where $x_1, x_2 \geq 0$

$4x_1 + 3x_2 + S_1 = 12 \rightarrow (1)$

$3x_1 + 4x_2 + S_2 = 12 \rightarrow (2)$

The new objective function

$\max z = 4x_1 + 7x_2 + 0S_1 + 0S_2$

Table : 1

CB	C_j Basis	4 x_1	7 x_2	0 S_1	0 S_2	RHS	Ratio
0	S_1	4	3	1	0	12	$12/3 = 4$
0	S_2	3	4*	0	1	12	$12/4 = 3$ (min)
	$Z_j - C_j$	-4	-7↑	0	0		

(max E)

$Z_j = CB \times x_1 + CB \times x_1$

$= 0 - 4 = -4$

∴ The leaving element = S_2

The entering element = x_2

Pivotal element = 4.

Table : 2

CB	C_j Basis	4 x_1	7 x_2	0 S_1	0 S_2	RHS	Ratio
0	S_1	$7/4$	0	1	$3/4$	3	
7	x_2	$3/4$	1	0	$1/4$	3	
	$Z_j - C_j$	$5/4$	0	0	$7/4$		

$Z_i - C_j$:

$x_2 = O.R - N.R \times P.C.E$

$3 - (\times 3) = 3 - 3 = 0$

$x_1 = 4 - (3/4 \times 3) = \frac{16-9}{4} = 7/4$

$S_1 = 1 - 0 \times 3$
$= 1 - 0 = 1$

$S_2 = 0 - (1/4 \times 3) = -3/4$

$RHS = 12 - (3 \times 3) = 12 - 9 = 3$

$z_i - c_j = x_1 = 0 + \frac{21}{4} - 4$

$= \frac{21 - 16}{4}$

$= 5/4$

$x_2 = 0 + 7 - 7 = 0$

$S_1 = 0 - 0 = 0$

$S_2 = 0 + 7/4 - 0 = 7/4$

Since all $z_j - c_j \geq 0$
we have optimum soln. is

$x_1 = 0$

$x_2 = 3$

$\therefore \max z = 4x_1 + 7x_2$
$= 4(0) + 7(3)$

$\max z = 21$

2. Solve the equn by using simplex method.

$max z = x_1 + 4x_2 + 5x_3$

Subject to the constriane.

$3x_1 + 3x_2 \leq 22$

$x_1 + 2x_2 + 3x_3 \leq 14$

$x_1 + 2x_2 \leq 14$

where, $x_1, x_2, x_3 \geq 0$.

leave = S_2
enter = x_3
pivot = 3

last $x_1 = 0$
$x_2 = 1$
$x_3 = 0$

28

enter = x_2
level = x_3
pivot = $2/3$

Sol

G.T

$max z = x_1 + 4x_2 + 5x_3$

Subject to the constrains

$3x_1 + 3x_2 \leq 22$

$x_1 + 2x_2 + 3x_3 \leq 14$

$x_1 + 2x_2 \leq 14$

where, $x_1, x_2, x_3 \geq 0$.

$3x_1 + 3x_2 + S_1 = 22 \rightarrow (1)$

$x_1 + 2x_2 + 3x_3 + S_2 = 14 \rightarrow (2)$

$x_1 + 2x_2 + S_3 = 14 \rightarrow (3)$

The new objective function

$max z = x_1 + 4x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$

Table : 1

CB	c_j Basis	2 x_1	4 x_2	5 x_3	0 s_1	0 s_2	0 s_3	RHS	Ratio
0	s_1	3	3	0	1	0	0	22	22/0 = ∞
0	s_2	1	2	3*	1	0	0	14	14/3 = 4.6
0	s_3	1	2	0	0	0	1	14	14/0 = ∞
	$z_j - c_j$	−4	−4	−5	0	0	0		

$z_j = CB \times x_1 + CB \times x_1$

$= 0 + 0 + 0 - 4$

∴ The leaving element = s_2

The entering element = x_3

pivotal element = 3

Table : 2

CB	c_j Basis	1 x_1	4 x_2	5 x_3	0 s_1	0 s_2	0 s_3	RHS	Ratio
0	s_1	3	3	0	1	0	0	22	0
5	x_3	1/3	[2/3]	1	1/3	0	0	= 14/3 4.7	4.6 7.3
0	s_3	1	2	0	0	0	1	14	
	$z_j - c_j$	2/3	−2/3	0	5/3	0	0	70/3	

$x_3 = O.R - (N.R \times PCE)$

$= 0 - (3 \times (0)) = 0$

$x_2 = O.R - (N.R \times PCE)$

$= 3 - (2/3 \cdot 0) = 3$

$x_1 = O.R - (N.R \times P(E))$

$= 3 - (1/3 \cdot 0) = 3$

$Z_j - C_j = CB \times x_1 + CB \times x_1 + CB \times x_1$

$= 0 \times 3 + 5 \times 2/3 + 0 \times 1 - 1$

$= 0 + 5/3 - 1$

$= 2/3$

$Z_j - C_j = CB \times x_2 + CB + x_2 * CB \times x_2$

$= 0 \times 3 + 5 \times 2/3 + 0 \times 2 - 4$

$= 5 \times 2/3 - 4$

$= 10/3 - 4$

$= -2/3$

$Z_j - C_j = CB \times x_3 + CB \times x_3 * CB \times x_3$

$= 0 * 0 + 5 \times 1 + 0 \times 0 - 5$

$= 5 - 5$

$= 0$

The leaving element $= x_3$

The entering element $= x_2$

The Pivoted element $= 2/3$.

	c_j	1	4	5	0	0	0		
	Basis	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
0	s_1	$3/2$	0	$-9/2$	$+1/2$	$-3/2$	0	1	
4	x_2	$1/2$	1	$3/2$	0	$1/2$	0	7	
4	s_3	$1/2$	0	0	0	0	1	0	
	$z_j - c_j$	1	0	1	0	2	0	28	

$z_j - c_j = x_1 = 2 - 1 = 1$

$x_2 = 4 - 4 = 0$

$x_3 = 0 + 6 - 5 - 5 = 1$

$s_1 = 0 + 0 + 0 = 0$.

$3 - 1/2(3)$

$3 - 3/2$

3

Since all

$z_j - c_j \geq 0$.

we have optimum solution is

$x_1 = 0$; $x_2 = 7$; $x_3 = 0$.

$max = x_1 + 4$

$\max Z = x_1 + 4x_2 + 5x_3$

$= 0 + 4(7) + 5(0)$

$\boxed{\max Z = 28.}$

3. $Max\ z = x_1 + 2x_2 + x_3$

subject to the constrains

$$2x_1 + x_2 - x_3 \le 2$$

$$2x_1 + x_2 - 5x_3 \ge 6$$

$$4x_1 + x_2 + x_3 \le 6$$

where : $x_1, x_2, x_3 \ge 0$.

Sol

OR.T

$$max\ z = x_1 + 2x_2 + x_3$$

subject to the constrains

$$2x_1 + x_2 - x_3 \le 2$$

$$2x_1 + x_2 - 5x_3 \ge 6$$

$$4x_1 + x_2 + x_3 \ge 6.$$

Where $x_1, x_2, x_3 \ge 0$.

$$2x_1 + x_2 - x_3 + s_1 = 2 \longrightarrow (1)$$

$$-2x_1 - x_2 - 5x_3 + s_2 = -6 \longrightarrow (2)$$

$$4x_1 + x_2 + x_3 + s_3 = 6 \longrightarrow (3)$$

The new objective function is

$$max\ Z = x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$$

Table : 1

CB	C_j / Basis	1 x_1	2 x_2	1 x_3	0 s_1	0 s_2	0 s_3	RHS	Ratio
0	s_1	2	1*	-1	+1	0	0	2	2 →
0	s_2	-2	-1	5	0	1	0	-6	6
0	s_3	4	+1	1	0	0	1	6	6
	$z_j - c_j$	-1	-2 ↑	-1	0	0	0	0	

∴ The leaving element = s_1

The entering element = x_2

Pivotal element = 1

Table : 2

CB	C_j / Basis	1 x_1	2 x_2	1 x_3	0 s_1	0 s_2	0 s_3	RHS	Ratio
2	x_2	2	1	-1	1	0	0	2	-2
0	s_2	0	0	4	1	1	0	-4	-1
0	s_3	2	0	2*	-1	0	1	4	2 →
	$z_j - c_j$	3	0	-3 ↑	2	0	0		

∴ The leaving element = s_3

The entering element = x_3

Pivotal element = 2

CB	c_j Basis	1 x_1	2 x_2	1 x_3	0 s_1	0 s_2	0 s_3	RHS	Ratio.
2	x_2	3	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	4	
0	s_2	-4	0	0	3	1	-2	-12	
1	x_3	1	0	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	2	
	$z_j - c_j$	6	0	0	$+\frac{1}{2}$	0	$\frac{3}{2}$	10	

since all

$$z_j - c_j \geq 0$$

we have optimum solution is

$x_1 = 0$: $x_2 = 4$: $x_3 = 2$

$$\max z = 4x_1 + 2x_2 + x_3$$

$$= 4(0) + 2(4) + 2$$

$$= 8 + 2$$

$$\max z = 10$$

4. $\min z = -x_1 + 2x_2$

subject to the constrains

$$-x_1 + 3x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

where $x_1, x_2 \geq 0$.

<u>Sol</u>

$\min z = -x_1 + 2x_2$

subject to the constrains

$$-x_1 + 3x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

where $x_1, x_2 \geq 0$.

$$-x_1 + 3x_2 + S_1 = 10 \longrightarrow (1)$$

$$x_1 + x_2 + S_2 = 6 \longrightarrow (2)$$

$$x_1 - x_2 + S_3 = 2 \longrightarrow (3)$$

∴ The new objective function is

$\max z = x_1 - 2x_2 + 0S_1 + 0S_2 + 0S_3$.

able : 1

CB	C_j / Basis	1 x_1	-2 x_2	0 s_1	0 s_2	0 s_3	RHS	Ratio
0	S_1	-1	3	1	0	0	10	-10
0	S_2	3	1	0	1	0	6	6
0	S_3	1	-1	0	0	1	2	2
	$Z_j - C_j$	-1	2	0	0	0		

The leaving element $= S_3$

The entering element $= S_1$

Pivotal element $= 1$

able : 2

CB	C_j / Basis	1 x_1	-2 x_2	0 s_1	0 s_2	0 s_3	RHS	ratio
0	S_1	0	2	1	0	1	12	
0	S_2	0	2	0	1	-1	4	
1	x_1	1	-1	0	0	1	2	
	$Z_j - C_j$	0	1	0	0	1	2	

$\therefore$ All $Z_j - C_j \geq 0$ optimum solution

$x_1 = 2$; $x_2 = 0$; $\max Z = 2$

$\min z = -\max z$

$\boxed{\min z = -2}$

verification :

$\min z = -2 + 0$

$\boxed{\min z = -2}$

5. $\min z = 12x_1 + 20x_2$

subject to the constrains

$6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

where, $x_1, x_2 \geq 0$

Sol

$\min z = 12x_1 + 20x_2$

subject to the constrains

$6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$-6x_1 - 8x_2 + S_1 = -100 \rightarrow (1)$

$-7x_1 - 12x_2 + S_2 = -120 \rightarrow (2)$

$\max z = -12x_1 - 20x_2 + 0S_1 + 0S_2$

Table : 1

	C_j	-12	-20	0	0		
CB	Basis	x_1	x_2	S_1	S_2	RHS	Ratio
0	S_1	-6	-8	1	0	-100	
0	S_2	-7	-12	0	1	-120	
	$Z_j - C_j$	12	20	0	0		

since all $Z_j - C_j \geq 0$ but the basis table appears slack variable there is infeasible