SWAMI DAYANANDA
COLLEGE OF ARTS & SCIENCE,
MANJAKKUDI-612610

DEPERTMENT OF MATHEMATICS

Mathematical Foundation for Computer Science(P16CS11)

Study Material

Class : I-M.Sc Computer Science

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CORE COURSE I

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

Objective :

To learn the basis of the mathematical applications for developing the program.

Unit I

Propositions - evaluation - precedence rules -tautologies - reasoning using equivalence transformation - laws of equivalence - substitution rules - a natural deduction system. Deductive proofs - inference rules - proofs - sub proofs.

Unit II

Introduction - Cryptography – Ceaser Cyphor Coding - Matrix encoding – scrambled codes - Hamming metric - Hamming distance - Error detecting capability of an encoding.

Unit III

Assignment problem and its solution by Hungarian method. Project Scheduling by PERT -CPM: Phases of project scheduling - Arrow diagram - Critical path method – Probability and Cost Considerations in project scheduling - Crahing of Networks.

Unit IV

Testing of hypothesis : Tests based on normal population - Applications of chi-square,Student's-t, F-distributions - chi-square Test - goodness of fit - Test based on mean,means, variance, correlation and regression of coefficients.

Unit V

Graph - Directed and undirected graphs - Subgraphs - Chains, Circuits, Paths, Cycles -Connectivity - Relations to partial ordering - adjacency and incidence matrices – Minimal paths - Elements of transport network - Trees - Applications.

Text Books

1. "The Science of Programming", David Gries. Narosa Publishing House, New Delhi,1993.

2. "Application Oriented Algebra", James L. Fisher, Dun Donnelly Publisher, 1977.

3. "Operation Research - An Introduction", Hamdy A.Taha, Macmillan Publishing Co.,4th Edn., 1987.

4. "Fundamentals of Mathematical Statistics", Gupta,S.C. and V.K.Kapoor, Sultan Chand & Sons, New Delhi, 8th Edn., 1983.

5. "Fundamentals of Applied Statistics", Gupta.S.C. and V.K.Kapoor, Sultan Chand & Sons, New Delhi, 2nd Edn., 1978.

References

1. "Discrete Mathematics", Seymour Lipschutz and Marc Laris Lipson, Second edition,Schuam's Outlines by Tata McGraw- Hill publishing Company Limited, New Delhi 1999.

2. "Operations Research", Kanti Swarup, P.K.Gupta and Man Mohan, Sultan Chand & Sons, New Delhi, 1994.

3. "Introductory Mathematical Statistics", Erwin Kryszig, John Wiley & Sons, New York, 1990.

4. "Probability and Statistics Engineering and Computer Science", Milton, J.S. and J.C.Arnold, McGraw Hill, New Delhi, 1986.

Mathematical foundation for Computer Science

Unit - I

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peg evalution & review technique

Unit - IV

Testing of hypothesis: Tests based on normal population - Applications of Chi-Square, Student's-t, F-distributions - Chi-Square Test - goodness of fit. Test based on mean, means, Variance, Correlation and Regression of co-efficients.

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Contingency → F T

Boolean Expressions

T	F
True	False
Truth	Falsity

Proposition :

Kind of boolean a logical expression

Propositions are similar to arithmetic expression.

These are operands, which represents the values T & F (instead of integers) and operators (AND, OR, instead of *, +)

And parentheses are used to determining the order of evaluation.

Propositions are formed according to the following rules :

As can be seen, parenthesis are required around each proposition that includes an operation

T & F are propositions

An identifiers is a proposition (An identifiers is a sequence of one or more digits and letters the first of which is a letter.

If b is a proposition, then is $(\sim b)$

If b and c are proposition then are $(b \wedge c)$, $(b \vee c)$, $(b \Rightarrow c)$ & $(b = c)$

Examples:

F, $(\sim T)$, $(b \vee xyz)$, $((\sim b) \wedge (c \Rightarrow d))$

$((ab (1 = id) \wedge (\sim d))$

the following are not proposition,

FF, $(b \vee cc)$, $(b) \wedge)$, $a+b$, $(1abc = 1d)$

Five Operation:

$\sim b$	nor b	negation
$(b \wedge c)$	b and c	conjunction
$(b \vee c)$	b or c	disjunction
$(b \Rightarrow c)$	b implies c	implication
$(b = c)$	b equals c	equality

Evaluation of constant propositions

Constant propositions - propositions that contain only constants as operands and we do this in 3 cases based on the structure of a proposition:

Case (i): e with no operators

The value of proposition T is T

The value of F is F.

Case (ii):

e with one operators

$(\sim b)$, $(b \wedge c)$, $(b \vee c)$; $(b \Rightarrow c)$; $(b = c)$ where

b and c are each one of the constant T and F

Ex:

b	c	$b \wedge c$	$b \vee c$	$b \Rightarrow c$	$b = c$	$\sim b$
T	T	T	T	T	T	F
T	F	F	T	F	F	F
F	T	F	T	T	F	T
F	F	F	F	T	T	T

Case (iii)

e with more than one operators

It is evaluated repeated by applying the single evaluation to a sub proposition and replacing the subproposition by its value, until the propositions is reduced to T or F.

Ex: $(T \wedge T) \Rightarrow (F)$

$(T \Rightarrow F) = F$

Evaluation $(b \Rightarrow c) = (\sim b \vee c)$

b	c	$\sim b$	$b \Rightarrow c$	$(\sim b \vee c)$	$(b \Rightarrow c) = (\sim b \vee c)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

State:

A state s is a function from a set of identifiers to the set of value T and F.

Ex: Let state s be the function defined by a set $s = \{(a, T), (bc, F), (ye, T)\}$ then

$s(a) = T$

$s(bc) = F$

$s(ye) = T$

Proposition e is well defined in state s if each identifiers in e is associated with T or F in a state s.

Let proposition e is well defined in a state s then s(e) is the value obtained by replacing all occurrences of identifiers b in e by their values s(b) and evaluating the resulting constant proposition according to the rules.

ex: $s((\sim b) \vee c)$ is evaluated in state s

$= \{(b, T), (c, F)\}$

$s((= \sim b) \vee c)) = ((\sim T) \vee F) = (F \vee F) = F$

Precedence rules for Operators:

Sequence of the same operators are evaluate from the left to right.

Ex: $b \wedge c \wedge d$ is equivalent to $((b \wedge c) \wedge d)$

The order of evaluation of different, adjacent operators is given by the list NOT, AND, OR,

Ex: $\sim b = b \wedge c$ is equivalent to $(\sim) = (b \wedge c)$

$b \vee \sim c \Rightarrow d$ is equivalent to $(b \vee (\sim c) \Rightarrow d$

$b \Rightarrow c \Rightarrow d \wedge e$ is equivalent to $(b \Rightarrow c) = (d \wedge e)$

Tautology:

A Tautology is a proposition that is true in every state in which it's well defined.

for Ex: proposition T is a Tautology

proposition F is not

b	$\sim b$	$b \vee \sim b$
T	F	T
F	T	T

for a proposition with n distinct identifiers there are 2^n case.

prove that $((b \wedge c) \wedge d) \to (d \to b)$ is a tautology

b	c	d	$(b \wedge c)$	$((b \wedge c) \wedge d))$	$d \to b$	$((b \wedge c) \wedge d \to (d \to b))$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	F	F	F	T
F	T	F	F	F	T	T
F	F	T	F	F	F	T
F	F	F	F	F	T	T

Disproving a Conjecture:

To prove a Conjecture, it is neccessary to prove it is true in all cases.

To disprove a conjecture, it is sufficient to find a single cases, where it is false.

Sometimes we conjecture that a proposition e is a tautology, but are unable to develope a proof of it, so we decide to try to disprove it

It possible to prove the Converse (ie) $\sim c$ is a tautology.

Proposition :

It represents or describes the set of States in which it is True. Conversely, for any set of states containing only identifiers associated with T or F.

We can derive a proposition that represent that state set. Thus, the empty set, the set containing no states, is represented by F because F is true is no state.

the set of all state is represented by proposition T because T is true in all state states.

Ex: the set of two states $\{(b,T), (c,T), (d,T)\}$

& $\{(b,F), (c,T) (d,T)\}$ is represented by the proposition $\{b \wedge c \wedge d) \vee (\sim b \wedge c \wedge \sim d)$

proposition b is weaker than c if $c \Rightarrow b$. Correspondenly c is said to be strong than b

A stronger propositions makes more reg on the combinations of value its identifiers can be associated with a weaker proposition makes fewer.

$c \Rightarrow b$ less restrictive.

the weakest proposition is T (or any tautology) because it represents the set of all states. the strongest is F, because it represents the set of no states.

Transforming English to propositional form

Words	identifiers
It rains	r
picnic is cancelled	pc
Be wet	wet
Stay at home	s

If it rains but I do stay at home, I won't be wet.

$(r \wedge s) \Rightarrow \sim wet$

I will be wet if it rains. $r \Rightarrow wet$

If it rains and the picnic is not cancelled.

(or)

I don't stay home, I will be wet

$((r \wedge \sim pc) \vee \sim s) \Rightarrow wet$

or

$(r \wedge (\sim pc \vee \sim s)) \Rightarrow wet$

whether or not the picnic is cancelled, I am staying at Rome it is rains.

$(pc \vee \sim pc) \wedge r \rightarrow s$

this reduces $r \Rightarrow s$

(Either) it doesnot rain or I am staying Rome

$\sim r \vee s$

Equivalent in proposition:

propositions E_1 & E_2 are equivalent. if $E_1 = E_2$ is a tautology. In this case $E_1 = E_2$ is an equivalent.

Thus ∴ an equivalence is an equality that is a tautology.

Below, we give a list of equivalence these are the basic equivalence from which all others will be derived. So we call than the laws of equivalence.

Actually they are schemas,

Commutative Laws.

$(E_1 \wedge E_2) = [E_2 \wedge E_1)$

$(E_1 \vee E_2) = (E_2 \vee E_1)$

$(E_1 = E_2) = (E_2 = E_1)$

2) Associative Laws:

$$E_1 \wedge (E_2 \wedge E_3) = (E_1 \wedge E_2) \wedge E_3$$

$$E_1 \vee (E_2 \vee E_3) = (E_1 \vee E_2) \vee E_3$$

3) Distributive Laws:

$$E_1 \vee (E_2 \wedge E_3) = (E_1 \vee E_2) \wedge (E_1 \vee E_3)$$

$$E_1 \wedge (E_2 \vee E_3) = (E_1 \wedge E_2) \vee (E_1 \wedge E_3)$$

4) De - Morgan's Laws

$$\sim (E_1 \wedge E_2) = (\sim E_1) \vee (\sim E_2)$$

$$\sim (E_1 \vee E_2) = (\sim E_1) \wedge (\sim E_2)$$

5) Laws of Negation:

$$\sim (\sim E_1) = E_1$$

6) Law of the Excluded middle:

$$E_1 \vee \sim E_1 = T$$

7) Laws of contradiction

$$E_1 \wedge \sim E_1 = F$$

8) Law of implification

$$E_1 \Rightarrow E_2 = \sim E_1 \vee E_2$$

9) Law of equality

$$(E_1 = E_2) = (E_1 \Rightarrow E_2) \wedge (E_2 \Rightarrow E_1)$$

10) Laws of Or - Simplification

$$E_1 \vee E_1 = E_1$$

$$E_1 \vee T = T$$

$E_1 \vee F = E_1$

$E_1 \vee (E_1 \wedge E_2) = E_1$

11) Laws of And simplification

$E_1 \wedge E_1 = E_1$

$E_1 \wedge T = E_1$

$E_1 \wedge F = F$

$E_1 \wedge (E_1 \vee E_2) = E_1$

12) Laws of identities

$E_1 = E_1$

Proving that the logical laws are equivalences:

We have stated, without proof that laws, 1 - 12 are equivalences. One way to prove this is ~~th~~ to build truth tables note that the laws are true in all states.

For Ex:

the truth law of Demorgon's law

$\sim (E_1 \wedge E_2) = \sim E_1 \vee \sim E_2$

E_1	E_2	$\sim E_1$	$\sim E_2$	$(E_1 \wedge E_2)$	$\sim (E_1 \wedge E_2)$	$\sim E_1 \vee \sim E_2$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Clearly, the laws is true in all state, so that is a tautology.

The rules of substraction & transitivity.

$$a + (c - a) = c$$

$$(a + (c - a)) * d = c * d$$

Rule of Substitution:

Let $e_1 = e_2$ be an equivalence & $E(p)$ be a proposition, written as a function of one of its identifier p.

Then $E(e_1) = E(e_2)$ & $E(e_2) = E(e_1)$ are also equivalence.

Ex: $(b \Rightarrow c) = (\sim b \vee c)$ is an equivalence

$$E(p) = d \vee p$$

$$E(e_1) = d \vee (b \Rightarrow c)$$

$$E(e_2) = d \vee (\sim b \vee c)$$

So that $d \vee (b \Rightarrow c) = d \vee (\sim b \vee c)$ is an equivalence.

Rule of Transitivity:

If $e_1 = e_2$, $e_2 = e_3$ are equivalences then, so is $e_1 = e_3$ (& hence e_1 is equivalent is to e_3).

Ex 1 $(b \Rightarrow c) = (\sim c \Rightarrow \sim b)$ is an equivalence.

b —

$= \sim b \vee c$ (implication)

$= c \vee \sim b$ (commutativity)

$= \sim(\sim c) \vee \sim b$ (negation)

$= \sim c \Rightarrow \sim b$ (Implication).

$(b \wedge (b \Rightarrow c)) \Rightarrow c$

$= (b \wedge (\sim b \vee c)) \Rightarrow$ (implication)

$= \sim(b \wedge (\sim b \vee c)) \vee c$ (Implication)

$= \sim b \vee \sim(\sim b \vee c) \vee c$ (Demorgan)

$= T$

Transforming an Implication:

Suppose we want to prove that, $E_1 \wedge E_2 \wedge E_3 \Rightarrow E$ is an equivalence. This proposition is transformed as follows (E denotes *)

$(E_1 \wedge E_2 \wedge E_3) \Rightarrow E$

$= \sim(E_1 \wedge E_2 \wedge E_3) \vee E$ (implication)

$= \sim E_1 \vee \sim E_2 \vee \sim E_3 \vee E$ (Demorgan)

The final proposition is true in any state in which atleast one of $\sim E_1$, $\sim E_2$, $\sim E_3$, E is true

Hence to prove * is a tautology, we need only prove that in any state in which three of them are false, the fourth is true.

And we can choose which three to assume false based on other form.

$$E1 \wedge E2 \wedge E3 \Rightarrow E$$

$$E1 \wedge E2 \wedge \sim E \Rightarrow \sim E3$$

$$E1 \wedge \sim E \wedge E3 \Rightarrow \sim E2$$

$$\sim E \wedge E2 \wedge E3 \Rightarrow \sim E1$$

$$\sim E1 \vee \sim E2 \vee \sim E3 \vee E.$$

Prove that $(\sim(b \Rightarrow c) \wedge \sim(\sim b \Rightarrow (c \wedge d))) \Rightarrow (\sim c \Rightarrow d)$ is a tautology.

$$(\sim(b \Rightarrow c) \wedge \sim(\sim b \Rightarrow (c \vee d))) \Rightarrow (\sim c \Rightarrow d)$$

Eliminate the main implication & use demorgan laws,

$$(\sim\sim(b \Rightarrow c) \vee \sim\sim(\sim b \Rightarrow (c \vee d)) \Rightarrow (\sim c \Rightarrow d)$$

$$\sim\sim(b \Rightarrow c) \vee \sim\sim(\sim b \Rightarrow (c \vee d)) \vee (\sim c \Rightarrow d)$$

$$= (b \Rightarrow c) \vee (\sim b \Rightarrow (c \vee d)) \vee (\sim c \Rightarrow d)$$

$$= (\sim b \vee c) \vee (b \vee c \vee d) \vee (c \vee d)$$

use the laws of Associativity, Commutativity, and Or-Simplification to arrive at

$b \vee \sim b \vee c \vee d$

which is true because of excluded middle

$b \vee \sim b = T$ & Or-Simplification T.

A formal System of Axioms & ~~Interface~~ inference rules.

Define the propositions that arise direct from laws 1-12 to be theorems.

These are called axioms (and the laws 1-12 are axiom schemas)

Axioms:

Any proposition that arises by substituting propositions for E_1, E_2 & E_3 in one of the laws 1-12 is called theorem.

Interference rule:

Define the propositions that arise by using the rules of substitution & transitivity and an already derived theorem to be a theorem.

In this content, the rules are often called inference rules, for they can be used to infer that proposition is a theorem.

The inference rule is often written in the form

$$\frac{E_1 \ldots E_n}{E} \quad \text{and} \quad \frac{E_1, E_2 \ldots E_n}{E, E_0}$$

where the E_i & E stands for arbitrary propositions

The inference rule has the following meaning.

If propositions $E_1, E_2 \ldots E_n$ are theorems, then so is proposition E. (& E_0 in the second case)

written in this form, the rules of substitution & transitivity are

$$\text{Rule of substitution} = \frac{e_1 = e_2}{E(e_1) = E(e_2),\ E(e_2) = E(e_1)}$$

$$\text{Rule of transitivity} = \frac{e_1 = e_2,\ e_2 = e_3}{e_1 = e_3}$$

A theorem of the formal system, then is either an axiom or a proposition that is derived from one of the inference rules (2.3.2) & (2.3.3)

Natural Deduction System:

Introduction to deductive proofs.

Consider the problem of proving that a conclusion follows from certain premises.

for, example, we might want to prove that $p \wedge (r \vee q)$ follows from $p \wedge q$

(i.e) $p \wedge (r \vee q)$ is true in every state in which $p \wedge q$ is true. This problem can be written as follows

premise : $p \wedge q$

Conclusion : $p \wedge (r \vee q)$

Since $p \wedge q$ is true (in state s). So is p and So is q. One property of OR is that, for any r, $r \vee q$ is true if q is, So $r \vee q$ is true, finally Since p & $r \vee q$ are both the true.

The properties of and allows us to conclude that $p \wedge (r \vee q)$ is true in S also

From	$p \wedge q$	Infer, $p \wedge (r \vee q)$
1	$p \wedge q$	premise
2	p	property of and, 1
3	q	property of and. 1
4	$r \vee q$	property of or, 3
5	$p \wedge (r \vee q)$	property of and, 2, 4.

Show that $\sim(p \wedge \sim q) \wedge (\sim q \vee r) \wedge \sim r \Rightarrow \sim p$

From	$\sim(p \wedge \sim q) \wedge (\sim q \vee r) \wedge \sim r \Rightarrow \sim p$	
1	$\sim(p \wedge \sim q) \wedge (\sim q \vee r) \wedge \sim r$	premise
2	$\sim(p \wedge \sim q)$	property of And, 1
3	$\sim q \vee r$	property of And, 1
4	$\sim r \vee$	property of and, 1
5	$\sim p \vee q$	property by demorgans law, 2
6	$q \Rightarrow r$	Implication law, 3
7	$p \Rightarrow q$	Implication law, 5
8	$p \Rightarrow r$	Rule of transitivity 6, 7
9	$\sim r \Rightarrow \sim p$	Negation law, 8
10	$\sim p$	from 4

Inference rules:

not - 2
and - 2
or - 2
imp - 2
equals - 2

10 interference rules in the natural deduction system.

The rule for introducing and is called $\wedge$ - I

The rule for eleminating and is called $\wedge$ - E

Inference rules:

$\wedge$-I, $\wedge$-E and $\vee$-I

$$\wedge\text{-I} : \frac{E_1, E_2, \ldots E_n}{E_1 \wedge E_2 \wedge \ldots E_n}$$

$$\wedge\text{-E} : \frac{E_1 \wedge E_2 \wedge E_3 \ldots \wedge E_n}{E_i}$$

$$\vee\text{-I} : \frac{E_i}{E_1 \vee \ldots \vee E_n}$$

for example, since $p \vee q$ and $\sim r$ are propositions, the following is an instance of $\wedge$-I

$$\wedge\text{-I} : \frac{E_1, E_2}{E_1 \wedge E_2}$$

$$\frac{p \vee q, \sim r}{(p \vee q) \wedge (\sim r)}$$

From	$p \wedge q$	infer $p \wedge (r \vee q)$
1	$p \wedge q$	pr 1
2	p	$\wedge$-E, 1
3	q	$\wedge$-E, 1
4	$r \vee q$	$\vee$-I, 3
5	$p \wedge (r \vee q)$	$\wedge$-I, 2, 4

Inference rule

$\vee$-E

$$\vee\text{-E} : \frac{E_1 \vee \ldots \vee E_n,\ E_1 \Rightarrow E, \ldots E_n \Rightarrow E}{E}$$

From $p \vee (q \wedge r)$; $p \Rightarrow s$, $(q \wedge r) \Rightarrow s$ infer $s \vee p$

1	$p \vee (q \wedge r)$	pr 1
2	$p \Rightarrow s$	pr 2
3	$(q \wedge r) \Rightarrow s$	pr 3
4	s	$\vee$-E, 1, 2, 3
5	$s \vee p$	$\vee$-I, 4

Inference rule $\Rightarrow$-E

$$\Rightarrow\text{-E} \quad \frac{E_1 \Rightarrow E_2,\ E_1}{E_2}$$

From $p \wedge q$, $p \Rightarrow r$ infer $r \vee (q \Rightarrow r)$

1	$p \wedge q$	pr 1
2	$p \Rightarrow r$	pr 2
3	p	$\wedge$-E, 1
4	r	$\Rightarrow$-E, 2, 3
5	$r \vee (q \Rightarrow r)$	$\vee$-I, 4

Inference rule:

$=$-I, $=$-E

$$=\text{-I}: \quad \frac{E_1 \Rightarrow E_2,\ E_2 \Rightarrow E_1}{E_1 = E_2}$$

$$=\text{-E}: \quad \frac{E_1 = E_2}{E_1 \Rightarrow E_2,\ E_2 \Rightarrow E_1}$$

From	$p,\ p = (q \Rightarrow r)$	$r \Rightarrow q$ infer $r = q$
1	p	Pr 1
2	$p = (q \Rightarrow r)$	Pr 2
3	$r \Rightarrow q$	Pr 3
4	$p \Rightarrow (q \Rightarrow r)$	$=$-E, Pr 2
5	$q \Rightarrow r$	$\Rightarrow$-E, Pr 1
6	$r = q$	$=$-I, Pr 3, 5

Proofs and Subproofs:

proofs:

A theorem of the form "From $e_1, \ldots e_n$ infer e" is interpreted as if $e_1, e_2 \ldots e_n$ are true in a state then so is e

If $e_1, e_2 \ldots e_n$ appear on lines of a proof which is interpreted to mean that they are assumed or proven true, then we should be able to write e on a line also.

Rule $\Rightarrow$-I give us permission to do so

$$\Rightarrow\text{-I}: \frac{\text{From } E_1, \ldots E_n \text{ infer } E}{(E_1 \wedge E_2 \ldots \wedge E_n) \Rightarrow E}$$

Infer $(p \wedge q) = (q \wedge p)$

1	$(p \wedge q) \Rightarrow (q \wedge p)$	$\Rightarrow$-I
2	$(q \wedge p) \Rightarrow (p \wedge q)$	$\Rightarrow$-I, 1
3	$(p \wedge q) = (q \wedge p)$	$=$-I, 1, 2

Rule $\Rightarrow$-I allows us to conclude $p \Rightarrow q$ if we have a proof of q given premise p on the otherhand, if we take $p \Rightarrow q$ as a premise, then rule $\Rightarrow$-E allows us to conclude that q holds when p is given

Deduction theorem:

"Infer $p \Rightarrow q$", is a theorem of the natural deduction system, which can be interpreted to mean that $p \Rightarrow q$ is a tautology, iff "from p infer q" is a theorem.

Subproofs:

Infer $(p \wedge q) = (q \wedge p)$

1	From $(p \wedge q)$ infer $(q \wedge p)$	
1.1	p	$\wedge$-E, pr 1
1.2	q	$\wedge$-E, pr 1
1.3	$q \wedge p$	$\wedge$-I, 1.2, 1.1
2	$(p \wedge q) \Rightarrow (q \wedge p)$	$\Rightarrow$-I, 1

3	$(q \wedge p) \Rightarrow (p \wedge q)$	$\Rightarrow$ I
4	$(p \wedge q) \equiv (q \wedge p)$	= - I, 2, 3.

Unit - II.

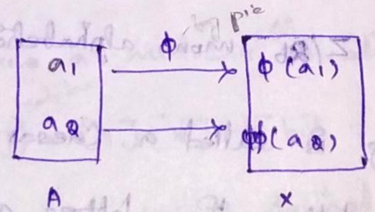
Cryptography:

Message $\longrightarrow$ Encoder $\xrightarrow{\text{encoded message}}$ Transmitter $\xrightarrow{\text{Transmitted message}}$ Receiver $\xrightarrow{\text{Received message}}$ decoder $\longrightarrow$ decoded message.

The study of encoding & decoding to ensure secrecy is called Cryptography.

Definition:

An encoding ϕ (pie) of a set A (called the base set) into set X is a one-to-one function ϕ from A into X



Caesar Cypher Coding:

ϕ	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T

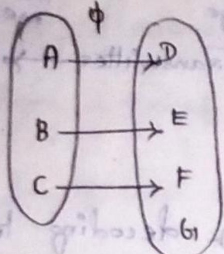
R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓
U	V	W	X	Y	Z	A	B	C

The encoding ϕ is extended to the set of all sequences $a_1, a_2 \dots a_n$ where a_i is contained in A, by

$$a_1, a_2 \dots a_n \longrightarrow \phi(a_1), \phi(a_2) \dots \phi(a_n)$$

the encoding of ϕ is defined to be the function ϕ^{-1} from $\phi(A)$ to A.

Decoding of sequences $x_1\ x_2 \dots x_n$ where x_i is in $\phi(A) = X$ by $x_1, x_2, \dots\dots x_n \to \phi^{-1}(x_1)\,\phi^{-1}(x_2) \dots \phi^{-1}(x_n)$



$\phi^{-1}(A)$

A　　　X

Z/K monoalphabetic

letter of the english alphabet, n tuples of letters of the english alphabet with n a small positive $Z/(K)$ with $K = 26, 29$, or 2.

$Z/26$ mono alphabetic

this encoding is called a Caesar cypher and ϕ (letter) is the letter three letters after a given letter

the ceaser encoding has a very simple mathematical explanation. If we associate with each letter of the alphabet one of integers from 0 to 25 take modulo 26

$\phi([x]) = [x] + [3]$ where the addition takes place in $Z/26$

A	B	C	D	E	F	G	H	I	J	K	L
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]

M N O P Q R S T U V W X Y Z
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
[13] [14] [15] [16] [17] [18] [19] [20] [21] [22] [23] [24] [25] [0]

thus encoding the message

ALL ARE GOOD

Can proceed as

1 12 12 1 18 5 7 15 15 4

then, + 3

4 15 15 4 21 8 10 18 18 7

and then

DOO DUH JRRG

A mathematical formulation however indicates an easy generalization of the above example, which is more difficult to decode.

If above integers with $\gcd\{a, 26\} = 1$

then define a modular encoding

$$\phi([x]) = [a][x] + [b]$$

the caesar cypher is defined to be a modular encoding with $a = 1$.

the $\gcd\{a, 26\}$ must equal 1, otherwise the modular encoding will not be one-one.

Infact $\phi(0) = [b]$ and $\phi(26/\gcd\{a, 26\}) = [b]$

thus, if the letter corresponding to $[b]$ were received it would be ambiguous as to whether this should be decoded as to letter corresponding to $[0]$ or $[26/\gcd\{a, 26\}]$

However, if the gcd $\{a, 26\} = 1$, then a^{-1} exists and there is decoding $\phi^{-1}(y) = [a^{-1}]\, y - [a]^{-1}[b]$

the element $[a]^{-1}$ can be determined by the Euclidean Algorithm

MILITARY
$\overset{24-13}{=11}$
XTWTELCJ is decoded as.

Hence we suppose that XTWTELCJ is decoded as 'milatary' and then the encoding is an example of a Caesar cypher in which the displacement is 11 letters.

$11 + \overset{inv}{15} = 26$. rem = 0.

$\phi^{-1}([y]) = [y] + [15]$, and the remainder of the message can be decoded.

<u>Matrix encoding:</u>

Mono alphabetic encoding is only one possibility among numerous methods of encoding. This section deals with a type of block encoding called matrix encoding

the alphabetic is again represented by the integer modulo 26.

the base set of this encoding is the vector space of m tuples with elements in the integers modulo 26.

(ie) A can be thought of as blocks of m letters

let m be an invertible m x m matrix with entries in z/26.

The encoding thus takes a block of m-letters, realizes this as a vector in $[\mathbb{Z}/26]^m$, multiplies this vector by M, and then produces the corresponding blocks of letters.

For example: if we encode block of two letters by multiplication by the matrix

$$M = \begin{bmatrix} 1 & 1 \\ 13 & 12 \end{bmatrix}$$

then "to" becomes

$$[20, 15] \times \begin{bmatrix} 1 & 1 \\ 13 & 12 \end{bmatrix} \qquad \begin{matrix} 20\times 1 + 15\times 13 = \\ 20\times 1 + 15\times 12 = \end{matrix} \quad \overline{26} = [20, 17]$$

$$= [7, 18] \text{ or } \text{"GR"}$$

The word "do" becomes

$$[4, 15] \begin{bmatrix} 1 & 1 \\ 13 & 12 \end{bmatrix} = [17, 12]$$

$$M^{-1} = \left(\begin{bmatrix} 1 & 1 \\ 13 & 12 \end{bmatrix} \right)^{-1}$$

$$\frac{1}{|M|} \operatorname{adj} M$$

$$|M| = 12 - 13 = -1$$

$$\operatorname{adj} M = \begin{bmatrix} 12 & -1 \\ -13 & 1 \end{bmatrix}$$

$$M^{-1} = \frac{1}{(-1)} \begin{bmatrix} 12 & -1 \\ -13 & 1 \end{bmatrix} \qquad \begin{bmatrix} -12 & 1 \\ 13 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 1 \\ 13 & 25 \end{bmatrix}$$

M^{-1} (GR)

$$\begin{bmatrix} 7 & 18 \end{bmatrix} \begin{bmatrix} 14 & 1 \\ 13 & 25 \end{bmatrix} = \begin{bmatrix} 20, & 15 \end{bmatrix}$$

"to"

This encoding is clearly not monoalphabetic, since in the first case the letter o was replaced by 'R'. in the ~~same~~ second case o was replaced by 'B'.

If the matrix M^{-1} is calculated, then the decoding it is simply multiplication by M^{-1}

$(VM)M^{-1} = V$ for each vector

multiplication by M^{-1} transforms GR back to "to" and OB back to "do".

LMRSX CCHDV SHBUS NBTGU RDKDY

WLUQE MQKYK YHWSA KYURG SPRYV

WQPTT UYM

OXJRM FURYK AGDVS HBUPH OBKYH

USAKY SBUVS FKVQT MEOTV M

The first message contains 58 characters, hence the block length is 2, 29, 58. Common no: 2 is block size

The second message contains 46 characters, hence the block length is 2, 23, 46.

Hence, the code is a matrix encoding then the block size is 2.

Indo china

In the first message the sequence KYHWSAKY occurs and furthermore, this sequence occurs in the second message.

Hence Assume $\phi^{-1}(KY) = IN$ $\phi^{-1}(HW) = DO$

this we have

$$[[11],[25]]\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = [[9],[14]]$$

$$[[8],[23]]\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = [[4],[15]]$$

where $\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ is the inverse of the encoding matrix. this yield the equations in $Z(26)$.

$11x_1 + 25x_2 = 9$, $11x_2 + 25x_4 = 14$

$8x_1 + 23x_3 = 4$, $8x_2 + 23x_4 = 15$

which have the unique solutions

$x_1 = 3$, $x_2 = 25$, $x_3 = 24$, $x_4 = 1$

To see if the code is actually a matrix encoding and the matrix $\begin{bmatrix} 3 & 25 \\ 24 & 1 \end{bmatrix}$

$$\phi^{-1}(LM) = \phi^{-1}[12,13] = [12,13]\begin{bmatrix} 3 & 25 \\ 24 & 1 \end{bmatrix}$$

$$= [10,1] = JA$$

$$\phi^{-1}(RS) = \phi^{-1}[18,19] = [18,19]\begin{bmatrix} 3 & 25 \\ 24 & 1 \end{bmatrix}$$

$$= [16,1] = PA$$

$= [24, 3]$

$$\phi^{-1}(xc) = \phi^{-1}[24, 3] \begin{bmatrix} 3 & 25 \\ 24 & 1 \end{bmatrix} = [14, 5] = NE$$

$$\phi^{-1}(CH) = \phi^{-1}[3, 8] = [3, 8]\begin{bmatrix} 3 & 25 \\ 24 & 1 \end{bmatrix} = [19, 5] = SE.$$

So that ϕ^{-1} (IMRSxCCH) = "Japanese".

Decoding the remainder of the encoded text gives the following messages.

Japanese troops currently stationed in Indochina will be withdrawn.

withdrawal of troops from Indochina is a smokescreen

Deciphering an encoded message encoded by matrix multiplication or vector of large block size is extremely difficult for blocksize 2 or 3

the problem is tractable

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

then the word 'prize' will be encoded as

$$\phi(PRIZE) = [16, 8, 9, 0, 5] \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [16, 8, 25, 16, 21] = PHYPU.$$

$$M^{-1} = \begin{bmatrix} 1 & 25 & 25 & 25 & 25 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

encode message PHYPU is Change to QHYPU. then the encoding of QHYPU will be

$$(17, 8, 25, 16, 21) M^{-1} = (17, 17, 8, 25, 4)$$

$$= QQHYD$$

Scrambled codes:

permutations are the basis for a type of Cryptographic code called a scrambled code.

If π is a permutation on $\{1, 2 \dots n\}$ then we define the encoding function π to be the map from $(a_1, a_2 \dots a_n)$ to $(a_{\pi(1)}, a_{\pi(2)}, \dots a_{\pi(n)})$

this is actually a very special type of matrix encoding. Since the mapping $\pi \begin{pmatrix} 1 & 3 & 2 & 5 & 4 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$

$(3, 5, 2, 1, 4)$

for example, Suppose $\pi = (1\ 3\ 2\ 5\ 4)$ and wish to use the encoding induced by this permutation to encode the message "the trial begins tomorrow". this message is divided into blocks of 5, and to encode a block the $\pi(1)$ letter is placed in the 1st position the $\pi(2)$ letter in the 2nd position, the $\pi(3)$ letter in the 3rd position

$\pi(4)$ in the 4^{th} position and $\pi(5)$ ~~lett~~ letter in the 5^{th} position.

the blocked message is

THETR IALTA BLEBE GINST OMORR

35214
THETR TALBE GINST OMORR OWZZZ OWZZZ
12345

which encoded as

ERHTT LEAIB NRIGS ORMOR ZZWOZ

Since $\pi(1) = 3$, $\pi(2) = 5$, $\pi(3) = 2$, $\pi(4) = 1$
$\pi(5) = 4$

To decode a permutation encoding, first makesure that the encoding is indeed a permutation encoding, Second determine the blocksize and third determine the permutation.

Block size is determined in the samemanner as the block size of matrix encoding.

If a sequence of letter is repeated then it can be assumed that the sameword or phrase was repeated in the original message, hence the blocksize is a divisor of the number of letters separating the successive occurrences of the repeated sequence. once the block size is determined, construct a table whose i^{th} row is i^{th} block

permuse the columns of the table to obtain a reasonable fit of pairs and continue until the message can be read from the table. Suppose the following encoded message interpreted.

SAIRL GIEINR EELOA MHDSE

YNAAA SSYSA IRLFE IREOC

WLSIP LLUBC LATMK OTAIL

ASPSZ

Since the frequency of occurrence of letters in the message so closely matches the ~~statements~~ standard count, we assume that the code is a permutation code, the block size must now be determined.

the sequence of 28 letters from the start of the first to the second occurrence.

thus the blocksize of the encoding is a divisor of 28, hence must be 2, 4, 7, 28.

Since the only permutations of two elements are the identity permutations and a permutation interchanging the two elements, a quick check shows that the block length is not 2. Suppose now that the block length is 4. If we write the message in a table with 4 columns, where row 1 is simply blocks of the message, then by permutating the columns, we will eventually, if the block size is 4, obtain the message, continue.

SAIR	LFEI
LGEI	RFOC
NREE	WLSI
LOAM	POLLU
HDSE	BCLA
YNAA	TMKO
ASSY	TAIL
SAIR	ASPS
	Z

and we can attempt to permute the columns to determine the message since column 1, column 2, donot seem to belong together, we can try col 1, col 3 but in this case we obtain the unusual combinations. It's from row 5 and Pk from row 14. If we try col 1 adjacent to col 4 there will be fewer, unusual two letters combinations, so we may adjacent. this gives

SR
LI
NE
LM
HE
YA
AY
SR
LI
RC
WI
PU
BA
TO
TL
AS

If column 2 or column 3 is placed before column 1, column 3 will yield slightly better, 3 letter sequences putting column 2 before the already determined portion yields are nonsense. but putting it after the determine portion yields

I SIR A

Israeli, general Moshe Dayan Says
Israeli force will pull back to milta pass

Unit - iii.

Assignment problem, Hungarian method.

Assignment problem:

The assignment problem is a special case of the transportation problem in which the objective is to assign a no. of origins to the equal number of destinations at a minimum cost (or) maximum profit.

Mathematical formulation of the Assignment problem.

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \& \quad \sum_{j=1}^{n} x_{ij} = 1, \quad x_{ij} = 0 \text{ or } 1$$

for all $i = 1, 2, \dots n$

$j = 1, 2 \dots n$

	J_1	J_2	J_3	...	J_n
M_1	C_{11}	C_{12}	C_{13}	...	C_{1n}
M_2	C_{21}	C_{22}	C_{23}	...	C_{2n}
M_3	C_{31}	C_{32}	C_{33}	...	C_{3n}
⋮					
M_n	C_{n1}	C_{n2}	C_{n3}	...	C_{nn}

Balanced Assignment Problem.

Assignment problem is said to be balanced if total no. of rows is equal (=) to total no. of columns.

Ex:

	D	E	F
A	1	2	3
B	4	5	6
C	7	8	9

Unbalanced assignment problem:

Total number of rows ≠ total number of columns.

In this case, we add dummy row or dummy column to balance the problem.

Ex:

	B_1	B_2	B_3	B_4
A_1	1	2	3	4
A_2	5	6	7	8

	B_1	B_2
A_1	9	10
A_2	11	12
A_3	13	14.

	B_1	B_2	B_3	B_4
A_1	1	2	3	4
A_2	5	6	7	8
A_3	0	0	0	0
A_4	0	0	0	0

	B_1	B_2	B_3
A_1	9	10	0
A_2	11	12	0
A_3	13	14	0

Q. Solve the assignment problem to minimize the total working hours.

	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

Total number rows = total number of columns

∴ the given Assignment problem is balanced.

Step 1

7	15	6	0
0	15	1	13
23	4	3	0
9	16	14	0

→ Subtract minimum value with other value. (row – clmn)

Row reduction

Step 2

7	11	5	0
0	11	0	13
23	0	2	0
9	12	3	0

Column reduction.

Step 3

Row scanning

7	11	5	$\boxed{0}$
0	11	0	13
23	$\boxed{0}$	2	$\cancel{0}$
9	12	3	$\cancel{0}$

$\boxed{0}$ — optimal assignment

Step 4

Column scanning

7	11	5	$\boxed{0}$
$\boxed{0}$	11	$\cancel{0}$	13
23	$\boxed{0}$	2	$\cancel{0}$
9	12	13	$\cancel{0}$

Step 5:

$$\begin{array}{cccc|l} 7 & 11 & 5 & \boxed{0} & \checkmark_3 \\ \boxed{0} & 11 & 0 & 13 & \\ 23 & \boxed{0} & 2 & 0 & \\ 9 & 12 & 13 & \cancel{0} & \checkmark_1 \\ \hline & & & \checkmark_2 & \end{array}$$

add minimum value with the ele[ment] intersect

1) unassigned row
2) zero column
3) assigned row

Unmarked row, marked column.

Step 6:

Smallest element

$$\begin{array}{c|cccc} & E & F & G & H \\ \hline A & 2 & 6 & \boxed{0} & \cancel{0} \\ B & \boxed{0} & 11 & \cancel{0} & 18 \\ C & 23 & \boxed{0} & 2 & 5 \\ D & 4 & 7 & 8 & \boxed{0} \end{array}$$

Optimum Solution is

$A \to G$ 17

$B \to E$ 13

$C \to F$ 19

$D \to H$ 10

Total = 17 + 13 + 19 + 10 = 59 //.

Find an optimum assignment schedule & maximum profit to the given assignment problems.

Zones

		A	B	C	D
Sales engineer	P	140	112	98	154
	Q	90	72	63	99
	R	110	88	77	121
	S	80	64	56	88

Step 1:

↳ we have to convert our given maximization problem into minimization problem.

↳ largest element = ~~max~~ 154

↳ Subtract all the elements from 154

↳ we get the following resultant matrix which is a minimization problem.

	A	B	C	D
P	14	42	56	0
Q	64	82	91	55
R	44	66	77	33
S	74	90	98	66

Number of rows = number of columns.

∴ the problem is balanced

Step 2:

(Row reduction)

14	42	56	0
9	27	36	0
11	33	44	0
8	24	32	0

Step 3:

Column reduction

6	18	24	0
1	3	4	0
3	9	12	0
0	0	0	0

Step 4:

row scanning

6	18	24	$\boxed{0}$
1	3	4	~~0~~
3	9	12	~~0~~
$\boxed{0}$	~~0~~	~~0~~	~~0~~

Step 5:

Column Scanning

6	18	24	$\boxed{0}$ ✓
1	3	4	0 ✓
3	9	12	~~0~~ ✓
$\boxed{0}$	~~0~~	~~0~~	~~0~~

Step 6:

Drawing minimum number of lines.

5	17	23	$\boxed{0}$
$\boxed{0}$	2	3	0
2	8	11	0
0	$\boxed{0}$	0	1

5	17	23	0 ✓
0	2	3	0
2	8	11	0 ✓
~~0~~	0	~~0~~	1

unassigned row
↓
zero column
↓
assigned row

Intersect value add with 0

Unmarked row &
marked column.

least element subtract with un
marked

3	15	[illegible]	$\boxed{0}$
$\boxed{0}$	2	3	2
$\cancel{0}$	6	9	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	0	3

	A	B	C	D
P	3	13	19	$\boxed{0}$
Q	$\cancel{0}$	$\boxed{0}$	1	2
R	$\boxed{0}$	4	7	0
S	2	$\cancel{0}$	$\boxed{0}$	5

Zones	profit
$P \to D$	= 154
$Q \to B$	= 72
$R \to A$	= 110
$S \to C$	= 56

392

	A	B	C	D
1	10	25	15	20
2	15	30	5	15
3	35	20	12	24
4.	17	25	24	20

No. of rows = 4

No. of columns = 4.

therefore no. of rows = no. of columns

the given assignment problem is balanced.

Step 1: Row reduction

0	15	5	10
10	25	0	10
13	8	0	12
0	8	7	3

Step 2: Column reduction

0	7	5	7
10	17	0	7
13	0	0	9
0	0	7	0

Step 3: Row reduction & column reduction

	A	B	c	d
1	$\boxed{0}$	7	5	7
2	10	17	$\boxed{0}$	7
3	13	$\boxed{0}$	~~0~~	9
4	~~0~~	~~0~~	7	$\boxed{0}$

Optimum solution is

$1 \rightarrow A$

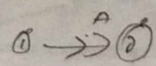
$2 \rightarrow c$

$3 \rightarrow B$

$4 \rightarrow d$

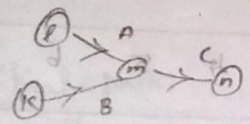
$10 + 5 + 20 + 20 = 55$ //.

Critical path method



Immediate predecessor

" Successor.

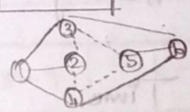


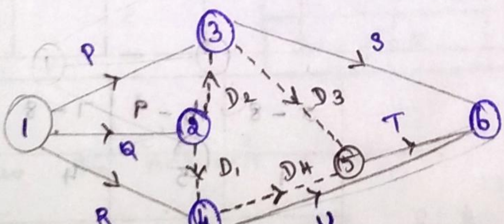
"A is a predecessor of C" is denoted as

A < C

(1) Draw the network for the project whose activities and their precedence relationships are given below.

Activity	P	Q	R	S	T	U
Predecessor	-	-	-	P, Q	P, R	Q, R

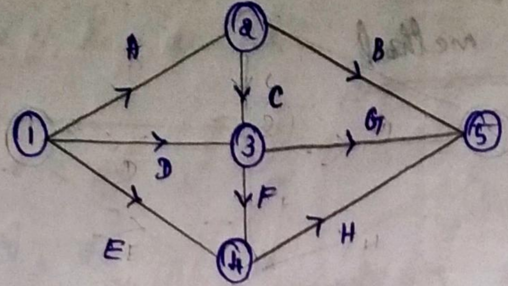




2) Construct the network for the project whose activities and their relationships are given below.

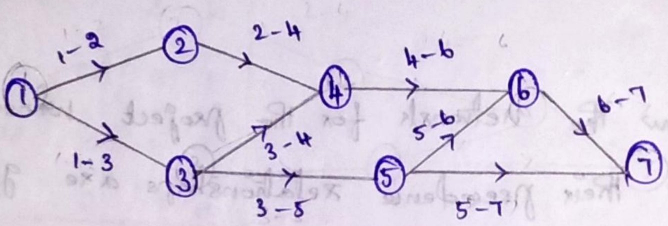
Activities A, D, E can start simultaneously

Activities B, C > A ; G, F > D, C; H > E, F



3) Draw the network.

Event no	1	2	3	4	5	6	7
Immediate predeccessors	-	1	1	2,3	3	4,5	5,6

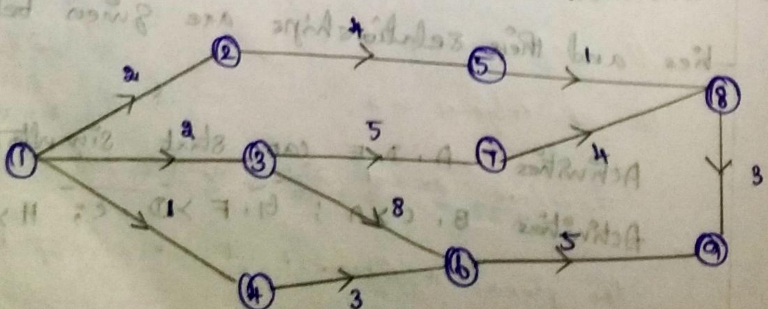


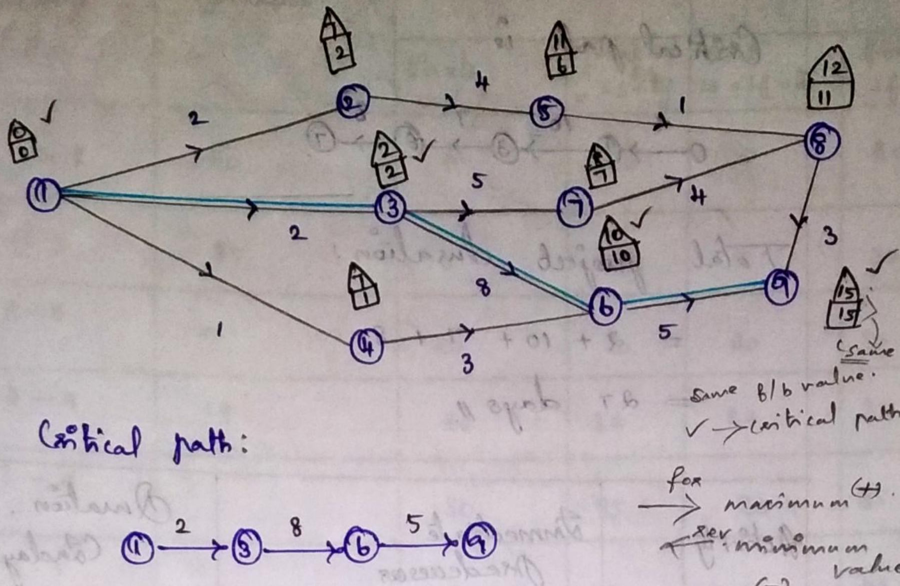
4)

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Time (duration)	2	2	1	4	8	5	3	1	5	4	3

i, Draw network diagram

ii, find the critical path & total project duration.





Critical path:

$① \xrightarrow{2} ③ \xrightarrow{8} ⑥ \xrightarrow{5} ⑨$

Total project duration

$2 + 8 + 5 = 15$ Weeks.

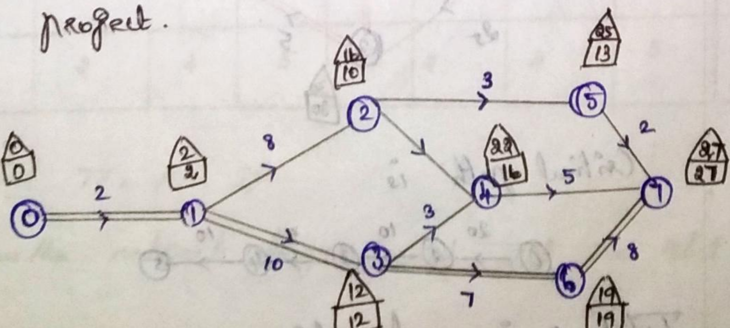
①

Activity	0-1	1-2	1-3	2-4	2-5	3-4
duration	2	8	10	6	3	3

3-6	4-7	5-7	6-7
7	5	2	8

Draw the network diagram.

Identify the critical path, find the total duration of the project.



Critical path is

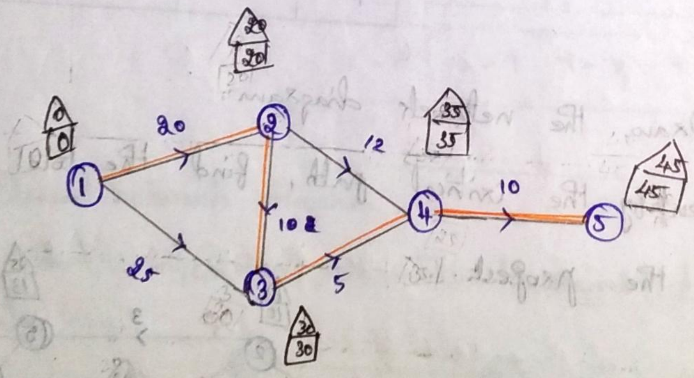
$0 \xrightarrow{2} 4 \xrightarrow{10} 3 \xrightarrow{7} 6 \xrightarrow{8} 7$

Total project duration:

$= 2 + 10 + 7 + 8$

$= 27$ days.

Activity	Immediate predecessor	Duration (In days)
1 – 2	–	20
1 – 3	–	25
2 – 3	1 – 2	10
2 – 4	1 – 2	12
3 – 4	1 – 3 & 2 – 3	5
4 – 5	2 – 4 & 3 – 4	10



Critical path is

$1 \xrightarrow{20} 2 \xrightarrow{10} 3 \xrightarrow{5} 4 \xrightarrow{10} 5$

Total project duration:

$20 + 10 + 5 + 10 = 45$ days.

Activity	Duration	Earliest		Latest	
		Starts Slack ES	Finish EF End EF = ES + t_{ij}	Start $LS = LF - t_{ij}$	finish LF
1 - 2	20	0	20	0	20
1 - 3	25	0	25	5	30
2 - 3	10	20	30	20	30
2 - 4	12	20	32	23	35
3 - 4	5	30	35	30	35
4 - 5	10	35	45	35	45

total flat = latest End - Earlier End. Slack - difference head event $\boxed{\frac{14}{12}} = 2$

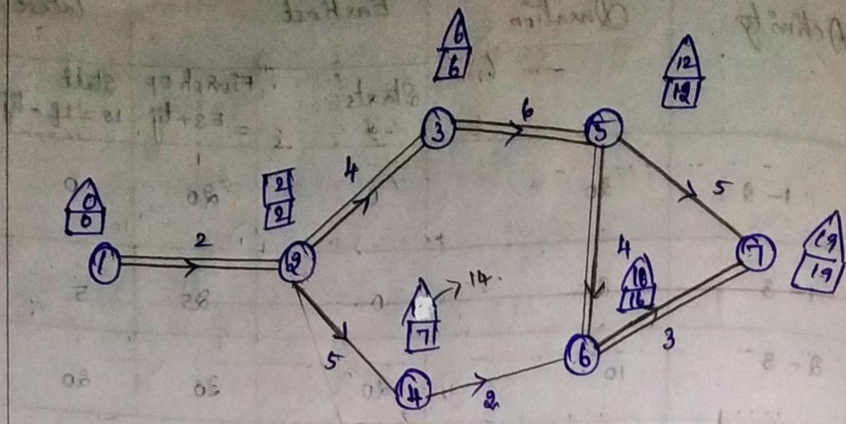
SHE STE

Total float (LF - EF)	Free float (total float - SHE)	Independent float (free float - STE)
0	0 - 0 = 0	0 - 0 = 0
5	5 - 0 = 5	5 - 10 = 5
0	0 - 0 = 0	0 - 0 = 0
3	3 - 0 = 3	3 - 0 = 3
0	0 - 0 = 0	0 - 0 = 0
0	0 - 0 = 0	0 - 0 = 0

Activity	1 - 2	2 - 3	2 - 4	3 - 5	4 - 6	5 - 6	5 - 7	6 - 7
Duration	2	4	5	6	2	4	5	3

i) find TF, FF, IF

ii) Draw the network diagram and find the total project time.



Activity	duration	Earliest time		Latest time		TF	FF	IF
		Start ES	End EF	LS	LF	(LF-EF)	TF-SHE	FF-
1-2	2	0	2	0	2	0	0-0=0	0-0=
2-3	4	2	6	2	6	0	0-0=0	0-0=
2-4	5	2	7	9	14	7	7-7=0	0-0=
3-5	6	6	12	6	12	0	0-0=0	0-0=
4-6	2	7		14		7	7-0=7	
5-6	4	12				0		
5-7	5	12	17	14	19		2-0=2	2-0=
6-7	3	16	19	16	19	0	6-0=0	0-0=

Critical path :

$$1 \xrightarrow{2} 2 \xrightarrow{4} 3 \xrightarrow{6} 5 \xrightarrow{4} 6 \xrightarrow{3} 7$$

Total project time = 19 hours.

Activity	Immediate predecessor	t_o	t_m	t_p
A	-	1	1	7
B	-	1	4	7
C	-	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D, E	3	6	15
H	F, G	1	2	3

t_e - expected time.
t_o - optimistic time.

i) Draw the PERT Network, findout the expected project completion time.

ii) What is the probability of completing the project in 14 days.

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

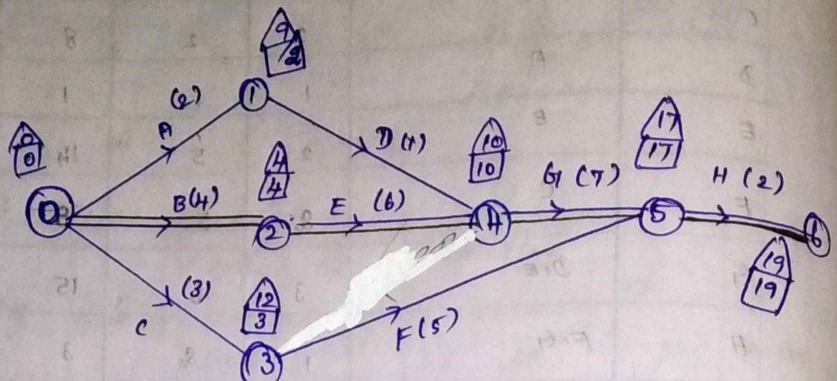
$$\delta^2 = \left(\frac{t_p - t_o}{6}\right)^2 \qquad \delta^2 e.$$

Variance

$$SD = \sqrt{\text{variance}} = \sqrt{\delta^2} = \delta$$

Activity	Immediate predecessor	t_o	t_m	t_p	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\delta^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A	-	1	1	7	$\frac{1+4+7}{6} = \frac{12}{6} = 2$	$\left(\frac{7-1}{6}\right)^2 = 1$
B	-	1	4	7	$\frac{1+16+7}{6} = \frac{24}{6} = 4$	$\left(\frac{7-1}{6}\right)^2 = 1$
C	-	2	2	8	$\frac{2+8+8}{6} = \frac{18}{6} = 3$	$\left(\frac{8-2}{6}\right)^2 = 1$
D	A	1	1	1	$\frac{1+4+1}{6} = 6/6 = 1$	0
E	B	2	5	14	$\frac{2+20+14}{6} = \frac{36}{6} = 6$	4
F	C	2	5	8	$\frac{2+4(5)+8}{6} = 5$	1

G	D, E	3	6	15	$\frac{3+4(6)+15}{6} = 7$	$\left(\frac{15-3}{6}\right)^2 = 4$
H	F, G	1	2	3	2	$1/9$



Critical path

$$ⓞ \xrightarrow{4} ② \xrightarrow{6} ④ \xrightarrow{7} ⑤ \xrightarrow{2} ⑥$$

Total project time = 19 days.

Variance $\qquad \sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

$$\sigma^2 e = 1 + 4 + 4 + 1/9$$

$$\sigma e^2 = \frac{9 + 36 + 36 + 1}{9} = \frac{82}{9} = 9.1 \quad \sqrt{9.1}$$

variance σ^2

SD σ

↓

standard deviation

$$S.D = \sigma e = 3.02.$$

In standard normal distribution

$$z = \frac{x - \mu}{\sigma}$$

let the project completing in 14 days

$x = 14, \quad \mu = 19 \quad \sigma = 3.02$

$$z = \frac{14 - 19}{3.02} = -1.6556$$

$P(x = 14) = P(z \le = z) = P(z \le -1.6556)$

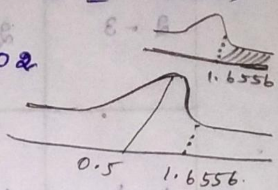
$= P(z \ge 1.6556)$

$= 1 - P(z \le 1.6556)$

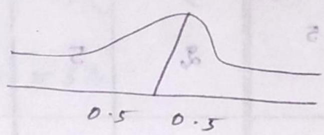
$= 0.5 - P(0 \le z \le 1.6556)$

$= 0.5 - 0.4502$

$= 0.0495$



thus the probability of completing the project in 14 days is 4.95%.

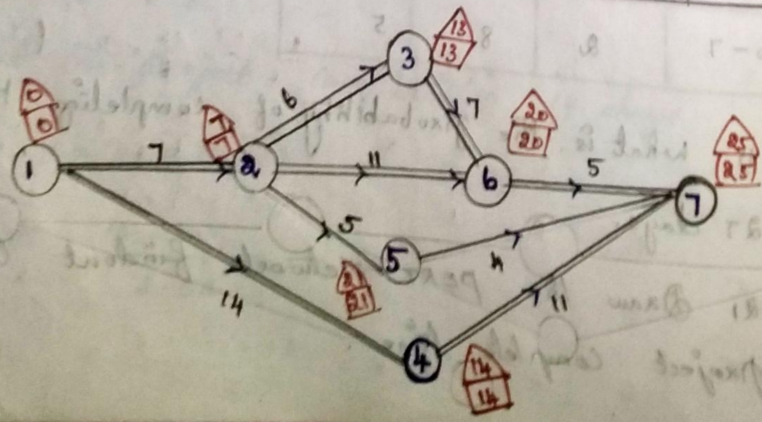


Task	to	tp	tm
1 - 2	3	15	6
2 - 3	2	14	5
1 - 4	6	30	12
2 - 5	2	8	5
2 - 6	5	17	11
3 - 6	3	15	6
4 - 7	3	27	9
5 - 7	1	7	4
6 - 7	2	8	5

1) what is the probability of completing the project in 27 days.

2) Draw the PERT Network, findout the expected project complete time.

task	t_o	t_m	t_p	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1 - 2	3	6	15	$\frac{3+4(6)+15}{6} = \frac{42}{6} = 7$	$\left(\frac{15-3}{6}\right)^2 = \left(\frac{12}{6}\right)^2 = 4$
2 - 3	2	5	14	$\frac{2+20+14}{6} = \frac{36}{6} = 6$	$\left(\frac{14-2}{6}\right)^2 = \left(\frac{12}{6}\right)^2 = 4$
1 - 4	6	12	30	$\frac{6+12\times4+30}{6} = \frac{84}{6} = 14$	$\left(\frac{30-6}{6}\right)^2 = 16$
2 - 5	2	5	8	$\frac{2+5\times4+8}{6} = \frac{30}{6} = 5$	$\left(\frac{8-2}{6}\right)^2 = 1$
2 - 6	5	11	17	$\frac{5+4\times11+17}{6} = \frac{66}{6} = 11$	$\left(\frac{17-5}{6}\right)^2 = 4$
3 - 6	3	6	15	$\frac{3+4\times6+15}{6} = 7$	$\left(\frac{15-3}{6}\right)^2 = 4$
4 - 7	3	9	27	$\frac{3+4\times9+27}{6} = \frac{66}{6} = 11$	$\left(\frac{27-3}{6}\right)^2 = 16$
5 - 7	1	4	7	$\frac{1+4\times4+7}{6} = \frac{24}{6} = 4$	$\left(\frac{7-1}{6}\right)^2 = 1$
6 - 7	2	5	8	$\frac{2+5\times4+8}{6} = \frac{30}{6} = 5$	$\left(\frac{8-2}{6}\right)^2 = 1$



$$① \xrightarrow{7} ② \xrightarrow{6} ③ \xrightarrow{7} ⑥ \xrightarrow{5} ⑦ \quad = 25$$

$$① \xrightarrow{7} ② \xrightarrow{11} ⑥ \xrightarrow{5} ⑦ \quad = 23$$

$$① \xrightarrow{14} ④ \xrightarrow{11} ⑦ \quad = 25$$

Total project time = 25 days.

Variance,

$$\sigma^2 e = \left(\frac{16 + 16}{6}\right)^2 = \left(\frac{32}{}\right)\left(\frac{30 - 27}{6}\right)$$

$$\sigma^2 e = 16 + 16 = 32$$

$$\sigma_e = 5.6568$$

In standard normal distribution

$$Z = \frac{x - \mu}{\sigma} = \frac{27 - 25}{5.6568} = \frac{2}{5.6568} = 0.3535$$

$$Z = 0.3535$$

$$p(x = 27) = p(z \le = z)$$

$$p(z \le 0.3535)$$

$$1 - p(0 \le z \le 0.3535$$

$$= 0.5 -$$

Free float: (FF) FF of an activity is time which can be used for rescheduling without affecting the position of the succeeding activity

T-F that activity without affecting the

$$\boxed{Z = \frac{T - T_e}{\sigma}}$$

FF = TF − (i, j) Slack of the head event

IF = TF − SHE − STE

FF − STE.

IF ≤ FF ≤ TF

$$1 \xrightarrow{7} 2 \xrightarrow{6} 3 \xrightarrow{7} 6 \xrightarrow{5} 7$$

$$1 \xrightarrow{7} 2 \xrightarrow{11} 6 \xrightarrow{5} 7$$

$$1 \xrightarrow{14} 4 \xrightarrow{11} 7$$

Total project time = 25 days.

Varience,

$$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

$$= 16 + 16 = 32$$

$$\sigma^2 = 32 \qquad \sigma = 5.6569$$

In standard normal distribution

$$Z = \frac{x - \mu}{\sigma}$$

$$Z = \frac{27 - 25}{5.6569}$$

$$= 0.3536$$

$$P(x = 27) = P(z \leq Z)$$

$$= P(Z \leq 0.3536)$$

$$= 0.5 + 0.1368$$

$$\therefore P(x = 27) = 0.6368$$

thus the probability of Completing the project in 27 days is 63.68 %.

Free float:

FF of an activity is that portion of the T.F which can be used for rescheduling that activity without affecting the succeeding activity.

$$\underset{(i,j)}{FF} = \underset{(i,j)}{TF} - \text{Slack of the head event}$$

$$I.F \leq F.F \leq T.F$$

Independent float:

IF of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities or that activity.

$$IF = FF - (STE)$$

of an activity (i, j)

STE → Slack Tail Event

$$IF \leq FF \leq TF$$

Graph theory:

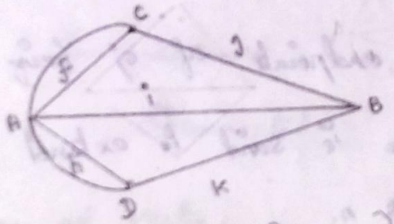
Graph:

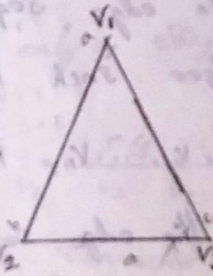
A graph G consists of a non empty set V of elements called vertices.

Set E elements called edges, and a function $\mathcal{E}$ from E to the set of undered pairs of elements.

the two vertrex in $\mathcal{E}(e)$ for an edge are called endpoints of e

$$G = \{V, E, \mathcal{E}\}$$





$\{a, b, c\}$

$\mathcal{E}(e) = \{A, C\}$

$\{v_1 v_2, v_2 v_3, v_3 v_1\}$

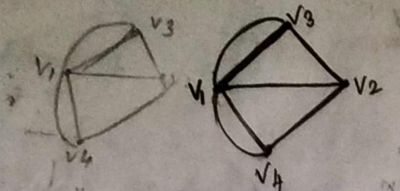
Adjincency Matrix:

Another method is to represent the graph as a matrix.

Label the vertices of the graph 1, 2, ..., n. the matrix $A = (a_{ij})_{n \times n}$ where i, j th entry equal to the number of edges with end points i, j

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

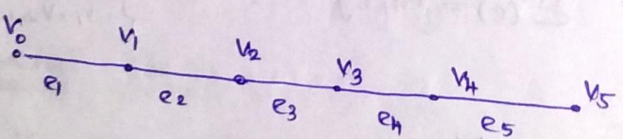
adjacent matrix



$$A = \begin{array}{c c} & \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} & \begin{pmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & \cancel{2}1 \\ 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Edge Sequence: Edge Sequence!

An edge sequence is a sequence $e_1, e_2, \dots e_n$ of edges such that there is a sequence of vertices $V_0, V_1, \dots V_n$, the endpoints of e_j being v_{j-1} & v_j. The edge sequence is said to extend from V_0 to V_n and to have length n.



length $= 5$

$V_0\ e_1\ V_1\ e_2\ V_2\ e_3\ V_3\ e_4\ V_4\ e_5\ V_5$

Chain Sequence!

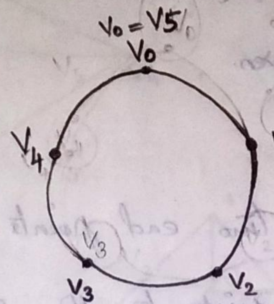
A chain sequence is an edge sequence in which there is no repetition of edges.

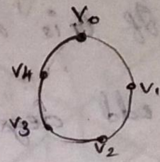
A cycyle sequence is an chain sequence that extends from V_0 to V_n.

$V_0 = V_n$

whether it extends from V_0 to V_n or V_n to V_0

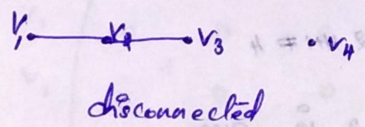
then the chain (cycle), sequence is called a chain (cycle).

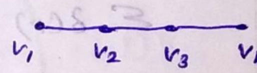




Connected graph:

A graph is said to be connected if there exists a chain between any two vertices of the graph or else if the graph contains only a single vertex.

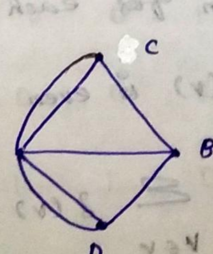




Degree of vertex:

The degree $\delta(v)$ of a vertex v of a graph G is the number of edges with endpoints v, with the convention that a loop with endpoints v and v contributes two to the degree of v

4 vertices
7 edges



$\delta(A) = 5$

$\delta(B) = 3$

$\delta(C) = 3$

$\delta(D) = 3$

Theorem:

If G is a finite graph, the number of vertices of odd degree is even.

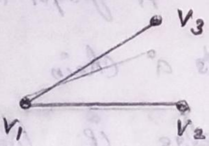
Proof:

Each edge has two end points and thus contributes two ~~end points~~ to the sum of degrees of the vertices.

$\delta(v_1) = 2$

$\delta(v_2) = 1$

$\delta(v_3) = 1$



$\sum \delta(v) = 2+1+1 = 4$

$\therefore \sum_{v \in V} \delta(v)$ is even

Let V_0 be the set of vertices with even degrees, V_1 be the set of vertices with odd degree.

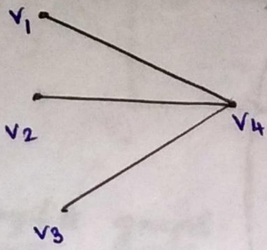
$$\text{thus} \sum_{v \in V} \delta(v) = \sum_{v \in V_0} \delta(v) + \sum_{v \in V_1} \delta(v)$$

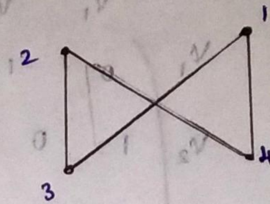
Since $\delta(v)$ is even for each $v \in V_0$

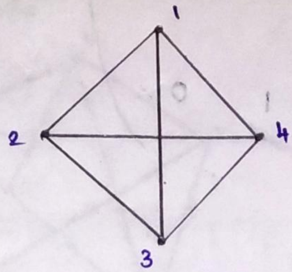
$$\Rightarrow \sum_{v \in V_0} \delta(v) \text{ is even}$$

& we know that $\sum_{v \in V} \delta(v)$ is even   even = ev + ev.

Write adjacency matrix of



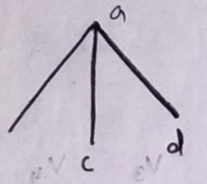




$$A = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 0 & 0 & 0 & 1 \\ V_2 & 0 & 0 & 0 & 1 \\ V_3 & 0 & 0 & 0 & 1 \\ V_4 & 1 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 0 & 0 & 1 & 1 \\ V_2 & 0 & 0 & 1 & 1 \\ V_3 & 1 & 1 & 0 & 0 \\ V_4 & 1 & 1 & 0 & 0 \end{array}$$

$$\begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$



$V(G) = \{a, b, c, d\} = 4$

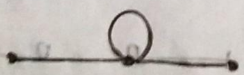
$X(G) = \{ab, ac, ad\} = 3$

$G = (4, 3)$ graph

A graph G with p vertices, q edges can be written as G = (p, q) graph.

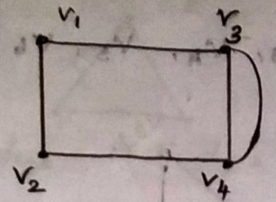
loop:

A line joining a point to itself is called a loop.

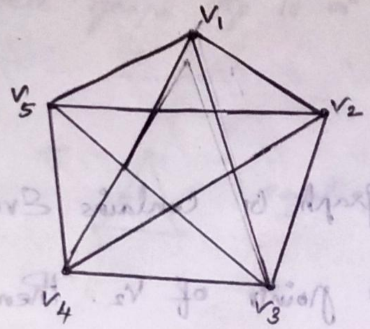


Multigraph:

If more than one line joining two vertices are allowed, the resulting object is called a multigraph. (multiple lines)

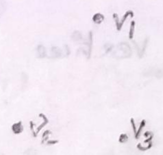


Complete graph:



A graph in which any two distinct points are adjacent is called a complete graph.

Totally disconnected graph:

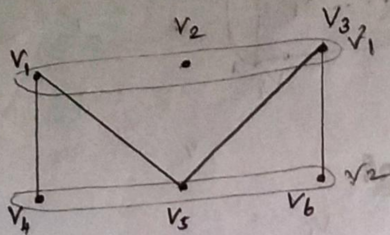


A graph whose edge set is empty is called a null graph or a totally disconnected graph.

bipartite graph: or bigraph.

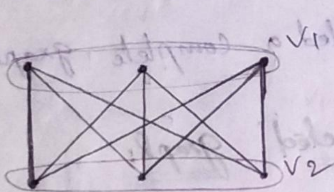
A Graph G is called a bipartite graph if V can be partitioned into two disjoint subsets V_1, V_2 such that every line of G joints a point of V_1 to a point of V_2. (V_1, V_2) is called a bipartion of G.

$V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$



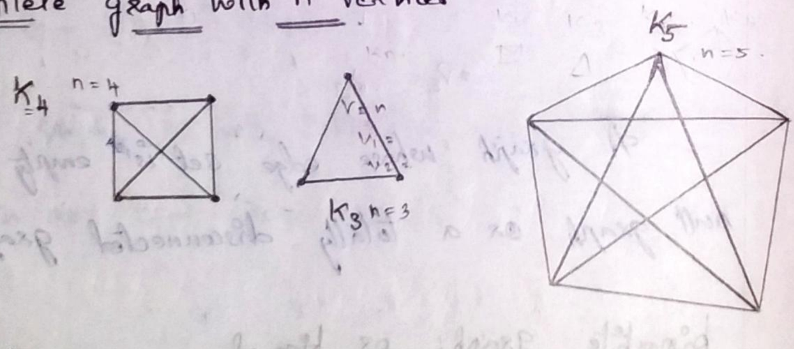
Complete bigraph:

If a bipartite graph G contains every line joining the point of V_1 to the points of V_2. Then G is called a complete bigraph.



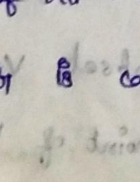
$K_{m,n}$.

Complete graph with n vertices



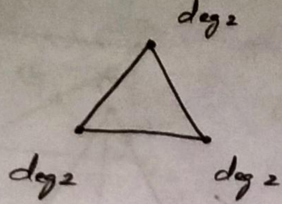
Regular graph:

If all the points of G have the same degree r then G is called a regular graph of degree r.



regular graph of degree 3 - cubic graph

$\deg(v) = 3 \quad \forall v \in V$ Regular graph of degree



deg (v) = 2

for all v ∈ V

regular of degree 2.

Complete graph K_p is a regular graph of degree $p-1$

K_3

regular graph of deg 2

K_4

regular graph of deg 3.

<u>Subgraphs:</u>

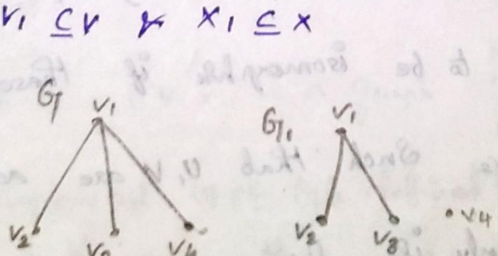
$G = (V, X)$ vertex, edge.

$G_1 = (V_1, X_1)$

$G_1 = (V_1, X_1)$ is said to be a subgraph of G if

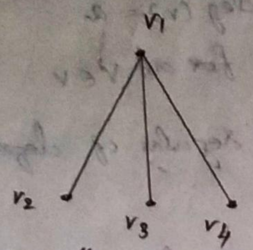
(V, X)

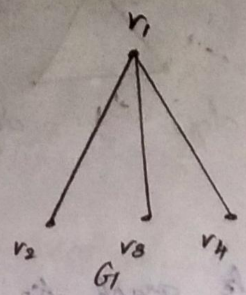
$V_1 \subseteq V$ & $X_1 \subseteq X$ subset.



If G_1 is a subgraph of G, then ~~we~~ we say that G is a supergraph of G_1.

Spanning Subgraph:

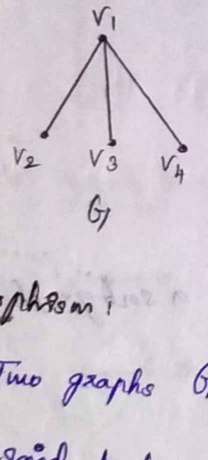


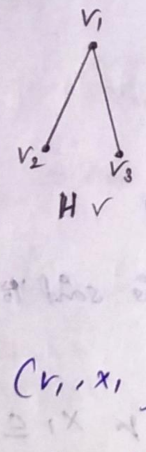


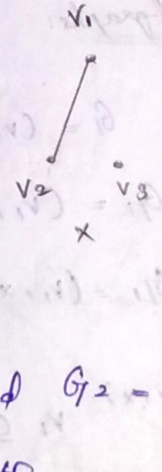
A subset is called a spanning subgraph of ~~graph~~ $G = (V, X)$ if $V_1 = V$. $H = (V_1, X_1)$

Induced Subgraph:

H is called an induced subgraph of G if, two points are adjacent in H iff they are adjacent in G.



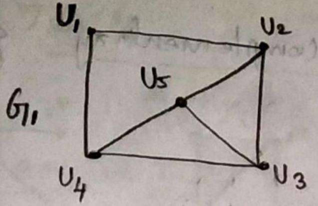


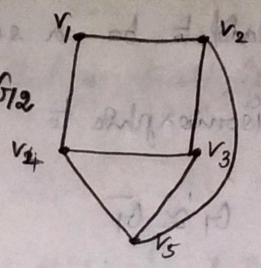


Isomorphism:

Two graphs $G_1 = (V_1, X_1)$ and $G_2 = (V_2, X_2)$ are said to be isomorphic if there exists a bijection $f : V_1 \to V_2$ such that u, v are adjacent in G_1 if and only if $f(u), f(v)$ are adjacent in G_2.

$G_1 \cong G_2$ the map f is called an isomorphism from G_1 ; G_2





$f : v_1 \to v_2$

$f(u_i) = v_i$

$f(u_1) = v_1$

$f(U_2) = V_2$

$f(U_3) = V_3$

$f(U_4) = V_4$

$f(U_5) = V_5$

Automorphism:

An isomorphism of a graph G onto itself is called an automorphism of G.

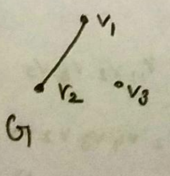
$\Gamma(G)$ ~~denote~~ can denote the set of all automorphisms of G.

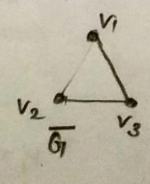
$f(v_i) = v_i$ - identity map. $f : V \to V$ automorphism of G.

Complement:

Let $G = (V, X)$ be a graph

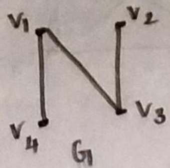
the complement $\overline{G}$ of G is defined to be the graph which has V vertices and two points are adjacent in $\overline{G}$ iff they are not a adjacent in G.

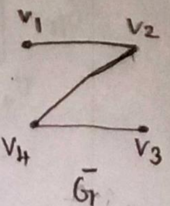


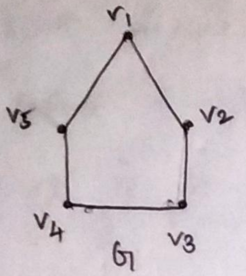


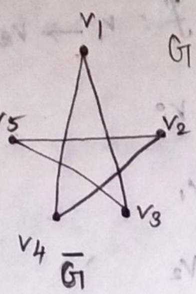
G is said to be a self complementary graph if G is isomorphic to $\bar{G}$

$G \cong \bar{G}$









$G \cong \bar{G}$

$f(v_i) = v_{i+1}$

$f(v_1) = v_2$

$f(v_2) = v_3$

$f(v_3) = v_4$

$f(v_4) = v_1$

Connectedness:

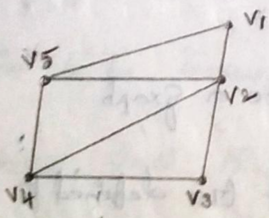
* walk
* path / Trail.

points are distinct.

i) **Walk:**

A walk of a graph G is an alternativelyng Sequence of points & lines $v_0 x_1 v_1 x_1 v_2 x_2 v_3 x_3 v_4 \ldots$

If $v_0 = v_n$ (Starting point = End point) — Initial point, terminal point — then the walk is said to closed.



$v_1\ v_2\ v_3\ v_4\ v_2\ v_1\ v_2\ v_5$

ii) Path:

A walk is called a path if all its points are distinct.

v_1, v_2, v_4, v_5

$v_1\ v_2\ v_4\ v_3\ v_2\ v_5$ — not a path

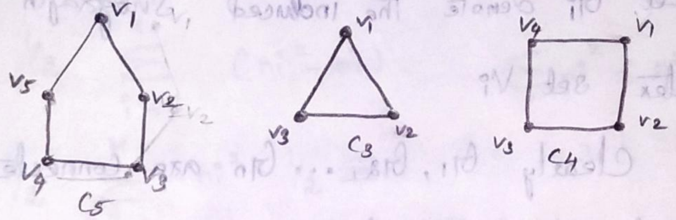
But its trail.

Trail:

A walk is called a trail if all its lines are distinct.

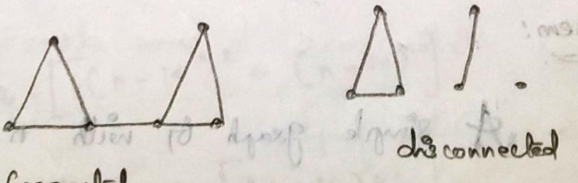
A $v_0 - v_n$ walk is called closed if $v_0 = v_n$.

A closed walk $v_0, v_1, v_2, \ldots v_n = v_0$ in which $n \geq 3$ and $v_0, v_1, \ldots v_{n-1}$ are distinct is called a cycle of length n (denoted by C_n)



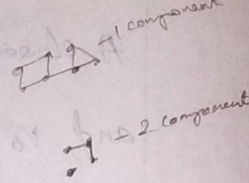
A graph G is said to be connected, if there exists a $u-v$ path between every pair of vertices u and v in G.

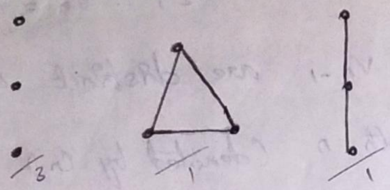
A graph which is not connected is said to be disconnected



Components:

V is partitioned into non empty subsets $V_1, V_2 \ldots V_n$ such that two vertices u and v are connected iff both u, v belong to the samset V_i



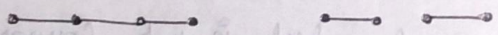


Let G_i denote the induced subgraph of G with vertex set V_i

Clearly $G_1, G_2, \ldots G_n$ are connected and are called the components of G.

G is connected iff it has only one component.

G is said to be disconnected if it has morethan one components.



Connected 1 - component

disconnected
2 components.

Theorem:

A simple graph G with n vertices, K components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges

$$\begin{array}{ccc} n_1 & n_2 & n_k \\ G_1 & G_2 & G_k \end{array}$$

$G = G_1 \cup G_2 \cup G_3 \ldots \cup G_k$

$n = n_1 + n_2 + \cdots + n_k \quad (\Sigma n_i)$

Let the number of vertices in each of the component of a graph be $n_1, n_2, \dots n_k$

$n_1 + n_2 + n_3 + \dots + n_k = n$ $n_i \geq 1$

Maximum number of edges in i^{th} component of a graph is $\frac{1}{2} n_i (n_i - 1)$

the maximum number edges in the graph G is

$$= \frac{1}{2} \sum_{i=1}^{k} n_i (n_i - 1)$$

$$= \frac{1}{2} \sum_{i=1}^{k} (n_i^2 - n_i) \qquad \sum n_i^2 \leq n^2 - (k-1)(2n-k)$$

$$= \frac{1}{2} \sum_{i=1}^{k} n_i^2 - \frac{1}{2} \sum n_i$$

$$= \frac{1}{2} \sum_{i=1}^{k} n_i^2 - n/2$$

$$\leq \frac{1}{2} \left[n^2 - (k-1)(2n-k) \right] - n/2$$

$$\leq \frac{1}{2} \left[n^2 - (2nk - k^2 - 2n + k) \right] - n/2$$

$$\leq \frac{1}{2} \left[n^2 - 2nk + k^2 + 2n - k \right] - n/2$$

$$\leq \frac{1}{2} \left[(n-k)^2 + (n-k) \right]$$

$$\leq \frac{1}{2} (n-k) \left[(n-k) + 1 \right]$$

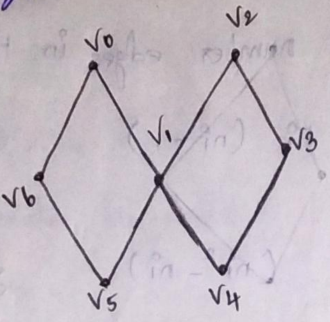
$$\leq \frac{(n-k)(n-k+1)}{2}$$

Hence prooved.

Eulerian Trail:

A closed trail containing all the points & lines is called an Eulerian trail.

A graph having an Eulerian trail is called an Eulerian graph.



Eulerian trail

$V_0\ V_1\ V_2\ V_3\ V_4\ V_1\ V_5\ V_6\ V_0$

Euler graph: Theorem:

*: A connected graph G is Euler graph iff all the vertices of G are of even degree.

Proof:

Direct part:

Let G be a connected graph assume that G is an Euler graph to prove $d(v)$ is even $\forall\ v \in V$

G is an Euler graph

G contains an Eulerian 'trial'

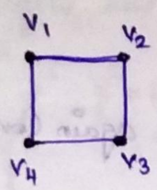
(G contains a closed walk containing all edges)

In a closed walk every time a walk meets a vertex v

It goes through two new edges incident with v (one we entered and the other exited)

this is true for all vertices in a closed walk.

∴ degree of every vertex is even



$v_1 v_2 v_3 v_4 v_1$

Converse part :

Assume that $d(v)$ is even $\forall v \in V$.

to prove G is an Euler Graph.

i.e to prove G contains atleast one Eulerian trail (closed walk contains all the edges)

Construct a closed walk starting at v and going through the edges of G such that no edge is repeated

Name the closed walk as h

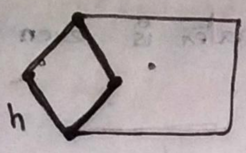
Case (i) If h covers all the edges. Then h becomes an Eulerian trail.

and hence G is an Eulerian graph.

Case (ii) : If h doesn't cover all the edges of G then remove all the edges of h from G and obtain the graph G'.

Every vertex of G' is also of an even degree.

Since G is connected, h will touch G' atleast one vertex v'



Starting from v', we can again construct a new walk h' in G'

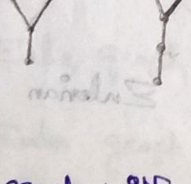
Now this walk h' combined with h forms a closed walk. Starts and ends at v and h as more edges than h. This process is repeated until we get a closed walk covering all the edges of G.

thus G is an Euler Graph.

Tree:

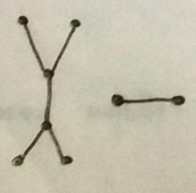
A connected graph with no cycles is called a tree.

A connected acyclic graph.



forest:

Any graph without cycles is also called a forest So the components of a forest are trees.



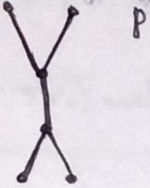
If G is a tree

then

1) Every points of G are joined by a unique path.

2) G is connected and $p = q + 1$

3) G is acyclic and $p = q + 1$



$p = 6$ - vertices

$q = 5$ - edges

---

(1) Every connected graph has a spanning tree:

Proof:

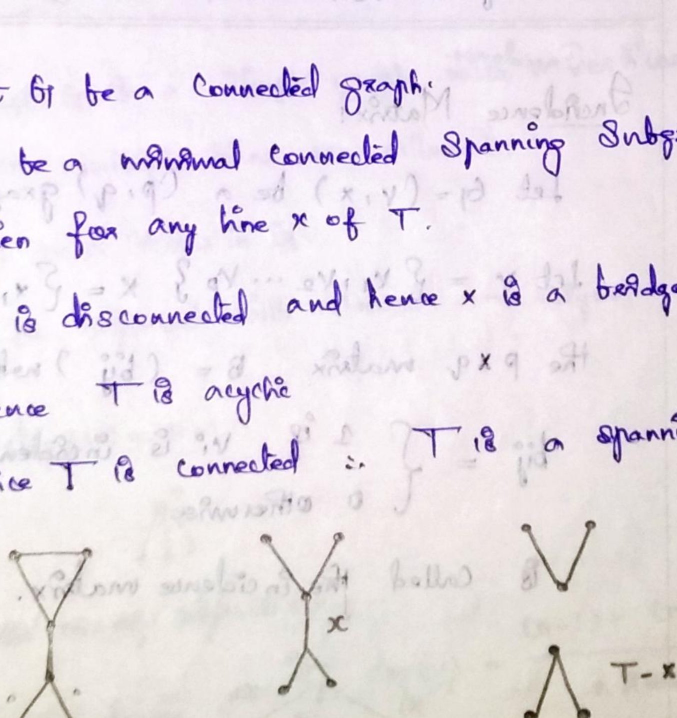
↳ Let G be a connected graph.

↳ Let T be a minimal connected spanning subgraph of G. Then for any line x of T.

↳ $T - x$ is disconnected and hence x is a bridge of T.

↳ Hence T is acyclic

↳ Since T is connected ∴ T is a spanning tree.



G T

(2) Sum of the degree of the vertices in a finite graph G is even.

Proof:

Let G be a finite graph with n vertices. Every line of G is incident with two points. Hence Everyline contributes 2 to the sum of the degrees of the points

Hence Sum of the degrees of the vertice in G $\Bigg\} = \sum_{i=1}^{n} \deg(v_i) = 2q$ = even

Incidence.

Incidence Matrix:

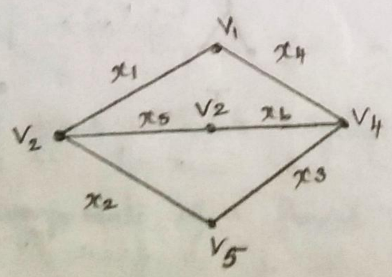
Let $G = (V, X)$ be a (p, q) graph

Let $V = \{v_1, v_2 \dots v_p\}$ $X = \{x_1, x_2, \dots x_q\}$

the $p \times q$ matrix $B = (b_{ij})$ where

$$b_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is incident with } x_j \\ 0 & \text{otherwise} \end{cases}$$

is called the incidence matrix.



$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

(5x6) matrix.

Q) Show that the maximum number of edges in a simple graph is $\frac{n(n-1)}{2}$

Proof:

Let G be a simple graph with n vertices.

(no loop or multiple lines are allowed)

Let G be a Vertex in G.

V can be adjacent with atmost $n-1$ edges

Maximum Value of $\deg(V) = n-1$

~~Maximum no. of edges in G~~

$$\text{Sum of the degree} = \sum_{i=1}^{n} \deg(v) = \underbrace{(n-1)+(n-1)+\dots+(n-1)}_{n \text{ times}}$$

$$= n(n-1)$$

$2E = n(n-1)$ (where E = number of edges)

$E = \frac{n(n-1)}{2}$

$\therefore$ Maximum no. of edges in a simple graph is $\frac{n(n-1)}{2}$

Directed graph (digraph)

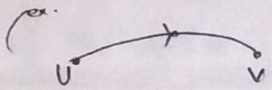
A directed graph D is a pair (V, A) where V is a vertex set & A is a arc set.

In particular A is a subset of $V \times V - \{(x,x) / x \in V\}$

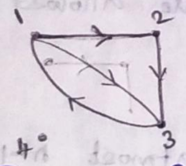
If $(u,v) \in A$ then the arc (u,v) is said to have u as its initial vertex (tail) & Terminal vertex v (head)

Also the arc (u,v) is said to join u to v.

ex.



(u,v) $(u,u) \notin V \times V - \{(x,x)\}$ ex.



$V = \{1, 2, 3, 4\}$ $A = \{(1,2), (2,3), (1,3), (3,1)\}$

$D = \{V, A\}$

Indegree:

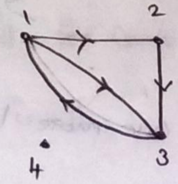
the indegree $d^-(v)$ of a vertex v in a digraph D is the number of arcs having v as its terminal vertex.

Out degree $d^+(v)$

$d^+(v)$ of v is the number of arcs having v as its initial vertex.

Degree pair:

the ordered pair $(d^+(v), d^-(v))$ is called the degree pair of v)



Degree pairs.

$(d^+(1), d^-(1)) = (2,1)$

$(d^+(2), d^-(2)) = (1,1)$

$(d^+(3), d^-(3)) = (1,2)$

$(d^+(4), d^-(4)) = (0,0)$

---

theorem

If G has morethan 2 vertices of odd degree than there can be no Euler path.

If G has exactly two vertices of odd degree. then there can be Euler path

proof:

Let G be a finite graph with exactly two vertices of odd degree call them u & v.

If G is connected, then the proof is trivial.

thus we can assume that G is disconnected

Let G_1 be the connected component of G such that

$u \in G_1$

Since G_1 is a graph $\rightarrow G_1$ must contain an even number of odd vertices.

Since $\sum_{w \in V(G_1)} \deg(w) = 2E(G_1)$ ↳ edges in G_1

G thus there is atleast one more vertex of odd degree, we must have $V \in G_1$

$\rightarrow$ this implies that there is a path from U_1 to V

$\rightarrow$ there is an Euler path in G_1.

Hence proved.

---

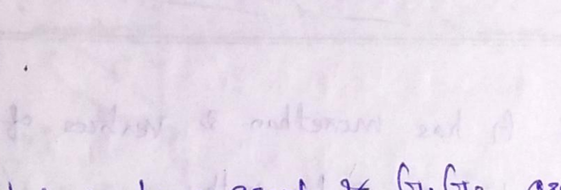
2) If G_1, G_2 are connected subgraph's of G with atleast one vertex in common then $G_1 \cup G_2$ is connected.

Proof:

Let G be a graph & G_1, G_2 are connected subgraphs of G to prove $G_1 \cup G_2$ is connected.

$$G_1 \cup G_2 = \{(V_1 \cup V_2), (X_1 \cup X_2)\}$$

where $G_1 = (V_1, X_1)$ $G_2 = (V_2, X_2)$



Let V_1 be a point in G_1

$V_1 \in V_1$

& $V_2 \in V_2$

U be the common point between G_1 & G_2.

Since G_1 is connected. therefore we can find a $V_1 - U$ path in G_1

Since G_2 is connected, therefore we can find a $U - V_2$ path in G_2

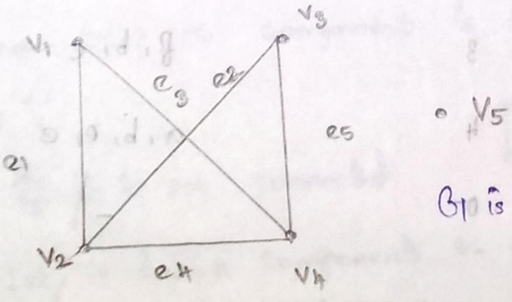
We can find a path from $V_1 - V_2$ along U by joining $V_1 - U$ & $U - V_2$ path

(Since $V_1 \in V_1$, $V_2 \in V_2$)

$\therefore$ $G_1 \cup G_2$ is connected.

(because we can find a path between every pair of vertices in $G_1 \cup G_2$).

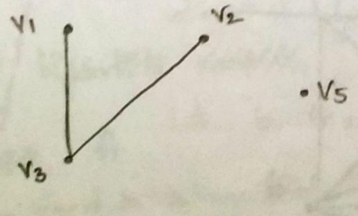
Let G_1 be the following graph.



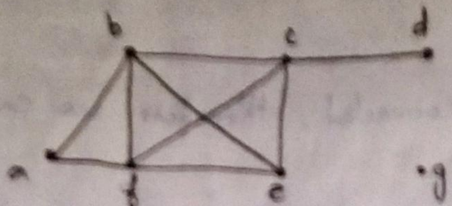
G_1 is a (5, 5) graph

G_1 has 2 components

G_1 is disconnected



what are the degrees & neighbourhoods of the vertices in the graph G.



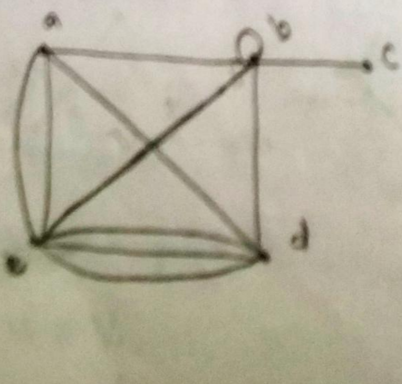
$V = \{a, b, c, d, e, f, g\}$

$E = \{(a,b), (b,c), (c,d) (a,f),$

degrees & neighbourhoods of Vertices.

Vertex	degree	Neighbourhoods.
a	2	b, f
b	4	a, f, e, c
c	4	b, f, e, d
d	1	c
e	3	f, b, c
f	4	a, b, c, e
g	0	

loop add 2 degree



$V = \{a, b, c, d, e\}$

$E = \{(a,b), (b,c), (a,e), (a,d), (e,b), (e,d), (b,d),$
$(b,b), (d,e), (e,a)\}$

Vertex	degree	neighbourhoods.
a	4	e, b
b	6	a, d, c
c	1	b
d	4	e, a, b
e	5	a, b, d.

Every component in a graph is bipartite then the graph is bipartite.

If G is connected, then it has only one component

we know that the component is bipartite → G is bipartite.

If G is not connected

Let G has n components & all are bipartite

$G_1, G_2, \dots G_n$ are bipartite graphs.

which is bipartite. the proof is trivial.

~~G_1 is bipartite~~ ~~G_1~~, ~~Has no odd cycles~~.

Let $G_1, G_2 \dots G_n$ are bipartite

~~proof~~ A

we need to show that $G = G_1 \cup G_2 \dots G_n$ is bipartite

Let $G_1, G_2, \ldots G_n$ are bipartite

We need to show that $G = G_1 \cup G_2 \ldots \cup G_n$ is bipartite

Since G_i is bipartite $\Rightarrow$ G_i has no odd cycles.

$\forall i \in \{1, 2, \ldots, n\}$

$\Rightarrow G = G_1 \cup G_2 \ldots \cup G_n$ has no odd cycles

(because $G_1 \cap G_2 \neq 0$)

$\Rightarrow$ G has no odd cycle (Since we know that graph is bipartite iff it has no add cycles).

$\Rightarrow$ G is bipartite

A graph G is bipartite iff all its cycles are of even length.

To prove:

Assume G is bipartite $\xRightarrow{\text{to prove}}$ G has all cycles are of even length.

G is bipartite $\Rightarrow$ V can be partitioned into (V_1, V_2) such that every line joins a point of V_1 to a point of V_2

Consider any cycle $v_0, v_1 \ldots v_n = v_0$ of length n

Suppose $v_0 \in V_1$

then $v_2, v_4, v_6 \ldots \in V_1$

& $v_1, v_3, v_5 \ldots \in V_2$

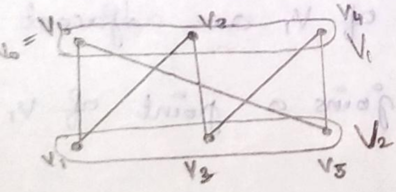
Since $v_n = v_0 \in V_1$ (possibilities of n are 2, 4, 6...)

$\Rightarrow$ n is even

$\Leftarrow$ converse part

Assume that G has all cycles of even length

(G has no odd cycles)



To prove G is bipartite

Let $v_1 \in V$

Define $V_1 = \{v \in V / d(v, v_1)$ is even$\}$

$V_2 = \{v \in V / d(v, v_1)$ is odd$\}$

Clearly $V_1 \cap V_2 = \phi$ & $V_1 \cup V_2 = V$

We have to prove V_1, V_2 be a bipartition of V.

to prove every line joins a point of V_1 to a point of V_2

Suppose two points in V_1 are adjacent,

Suppose $u, v \in V_1$ are adjacent

Let P be the shortest $v_1 - u$ path of length m

& Q be the shortest $v_1 - v$ path of length n.

Since $u, v \in V_1$, both m, n are even.

Now, let u_1 be the last common point between P & Q

then the $v_1 - u_1$ path along P & $v_1 - u_1$ path along Q and both shortest paths & have same length i

Now $u_1 - u$ path along P, then line uv followed by the $v - u_1$ path along Q form a cycle of length

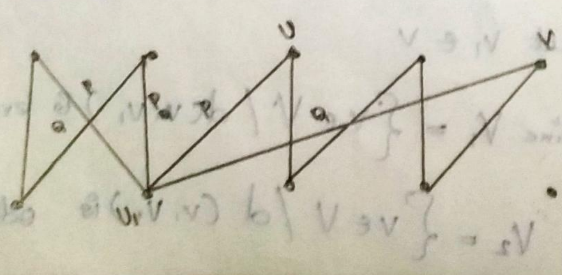
$(m-i) + 1 + (n-i) = m + n - 2i + 1$

which is odd & this is a contradiction.

thus no two points of V_1 are adjacent

$\Rightarrow$ every line joins a point of V_1 to a point of V_2.

$\Rightarrow$ G is bipartite.



$V_1 - u$ path P

$V_1 - v$ path Q

U_1 is a last common point b/w $P - Q$

$U_1 - v$ path along P, uv, $v - U_1$ along Q

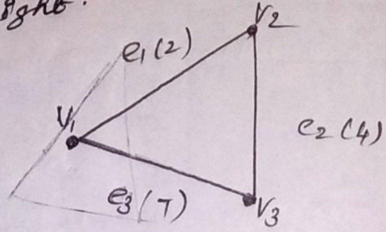
Weighted graph:

A graph G is called a weighted graph if there is a realnumber associated with each edge of G. The real number associated with each edge is called its weight.

$w(e_1) = 2$

$w(e_2) = 4$

$w(e_3) = 7$



Unit - IV

Hypothesis:

Find mean, median, mode for the following data

40, 50, 55, 78, 58, 60, 73, 35, 43, 58

To find

mean, median, mode

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

where n = number of observations

$$\text{Median } M = \text{Size of } \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

mode : Z = highest value

Here we have $n = 10$

i, mean

$$= \frac{\sum_{i=1}^{n} x_i}{n} = \frac{40 + 50 + 55 + 78 + 58 + 60 + 73 + 35 + 43 + 58}{10}$$

$$= \frac{550}{10}$$

$$\bar{x} = 55 \,//.$$

ii, Median:

$$M = \text{Size of } \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

$$= \left(\frac{10+1}{2}\right)^{th} \text{ item}$$

values	1	2	3	4	5	6	7	8	9	10
observation	35	40	43	50	55	58	58	60	73	78

$= {}^{11}/_{2} = 5.5$

$$= \frac{5^{th}\text{ item} + 6^{th}\text{ item}}{2} = \frac{55 + 58}{2}$$

$= 56.5$

Median $= 56.5$

iii, Mode.

Mode = highest value = 78.

2) Find mean, median, mode

x	75	100	120	150	100
$f(x)=y$	5	12	20	14	9

i, Mean $= \bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{N}$

where, $N = \sum_{i=1}^{n} f_i$

x_i	f_i	$f_i x_i$
75	5	375
100	12	1200
120	20	2400
150	14	2100
100	9	900

$N = \sum_{i=1}^{n} f_i = 60$ $\quad \sum_{i=1}^{n} f_i x_i = 6975$

$$\text{Mean} = \bar{x} = \frac{6975}{60}$$

$$= 116.25$$

ii, Median

$$M = \text{Size of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$\text{where } N = \sum_{i=1}^{n} f_i$$

Cummulative frequency.

x_i	f_i	c.f
75	5	5
100	12	17
120	20	37
150	14	51
100	9	60

$\sum f_i = 60$

$$\text{Median } M = \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$= \left(\frac{60+1}{2}\right)^{th} \text{ item}$$

$$= 30.5$$

$$\text{Median } M = 120.$$

iii, Mode:

x_i	f_i
75	5
100	12
120	20
150	14
100	9

Highest value in frequency = 20

Corresponding x value = 120

$\therefore$ Mode = 120

Standard Deviation.

$$S.D = \sigma = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}}$$

$$\text{Variance} = \frac{\Sigma (x-\bar{x})^2}{n}$$

$$S.D = \sqrt{\text{Variance}}$$

where $\Sigma (x-\bar{x})^2$ is the sum of the square of deviations from Arithmetic mean.

Coefficient of Standard deviation

$$= \frac{\sigma}{\bar{x}}$$

Coefficient of Variation (or) coefficient of Variability

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

Calculate the range & Standard deviation, coefficient of Variation for the following observations.

50 55 57 49 54 61 64 59 59 56

$$\text{Range} = \text{Highest value} - \text{lowest value}$$

$$= 64 - 49$$

$$= 15$$

$$\text{Mean} = \bar{x} = \frac{\Sigma x_i}{n} = \frac{50+55+49+57+54+61+64+59+59+56}{10}$$

$$= \frac{550}{10} = 55.$$

Calculation of Standard deviation.

x	$x-\bar{x}$ $x-55$	$(x-\bar{x})^2$ $(x-55)^2$
50	-5	25
55	0	0
57	2	4
49	-6	36
54	-1	1
61	6	36
64	9	81
59	4	16
59	4	16
64	1	1
	$\Sigma(x-\bar{x}) = 14$	$\Sigma(x-\bar{x})^2 = 216$

$$\text{Variance} = \frac{\Sigma(x-\bar{x})^2}{n}$$

$$= \frac{\Sigma(x-55)^2}{10} = \frac{216}{10} = 21.6$$

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} =$$

$$\text{Standard deviation} = \sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n} - \left(\frac{\Sigma(x-\bar{x})}{n}\right)^2}$$

$$= \sqrt{\frac{216}{10} - \left(\frac{14}{10}\right)^2}$$

$$= \sqrt{21.6 - (1.4)^2}$$

$$= \sqrt{21.6 - 1.96}$$

$$= \sqrt{19.64}$$

$$= 4.43$$

$$\bar{X} = \bar{x} + \frac{\Sigma(x-\bar{x})}{n}$$

Capo.

$$= 55 + 14/10 = 55 + 1.4 = 56.4$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.43}{56.4} \times 100$$

$$= 7.85\ \%$$

Find the S.D for the following data

45, 36, 40, 37, 39, 42, 45, 35, 40, 39

$n = 10$

$$\text{Arithmetic mean } \bar{x} = \frac{\Sigma(x_i)}{n} = \frac{45+36+40+37+39+42+45+35+40+39}{10}$$

$= \frac{398}{10} = 39.8 \approx 40$

$\bar{x} = 40.$

x	$d = (x - \bar{x}) = (x - 40)$	$d^2 = (x - \bar{x})^2 = (x - 40)^2$
45	5	25
36	-4	16
40	0	0
37	-3	9
39	-1	1
42	2	4
45	5	25
35	-5	25
40	0	0
39	-1	1

$\Sigma(x - \bar{x}) = -2$ $\qquad \Sigma(x - \bar{x})^2 = 106$

-17
$+\frac{12}{2}$

Standard deviation $= \delta = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n} - \left(\frac{\Sigma(x-\bar{x})}{n}\right)^2}$

$= \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$

$= \sqrt{\frac{106}{10} - \left(\frac{-2}{10}\right)^2}$

$= \sqrt{10.6 - 0.04} = \sqrt{10.56}$

$= 3.24961$

$= 3.25$

$\overline{X} = \bar{x} + \frac{\Sigma d}{n} = \bar{x} + \frac{\Sigma (x-\bar{x})}{n}$

$= 40 + \left(\frac{-2}{10}\right)$

$= 40 - 1/5 = \frac{200-1}{5} = \frac{199}{5}$

$= 39.8 \simeq 40$

$\overline{X} = 40$

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{3.25}{39.8} \times 100$

$= 8.16582 \%$

Correlation coefficient =

$$r = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$= \frac{\Sigma xy}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}}$$

Calculate the coefficient of correlation b/w x and y from the following table

x	1	3	5	8	9	10
y	3	4	8	10	12	11

$n = 6$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{1+3+5+8+9+10}{6}$$

$$= 36/6 = 6.$$

$$\bar{x} = 6.$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{3+4+8+10+12+11}{6}$$

$$= 48/6 = 8$$

$$\bar{y} = 8.$$

x	y	$(x-\bar{x})$ $=(x-6)$	$(y-\bar{y})$ $=(y-8)$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
1	3	-5	-5	25	25	25
3	4	-3	-4	9	16	12
5	8	-1	0	1	0	0
8	10	2	2	4	4	4
9	12	3	4	9	16	12
10	11	4	3	16	9	12
$\Sigma x = 36$	$\Sigma y = 48$	$\Sigma(x-\bar{x}) = 0$	$\Sigma(y-\bar{y}) = 0$	$\Sigma(x-\bar{x})^2 = 64$	$\Sigma(y-\bar{y})^2 = 70$	$\Sigma(x-\bar{x})(y-\bar{y}) = 65$

Correlation Coefficient $\gamma = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}} = \frac{65}{\sqrt{64}\sqrt{70}} = 0.9$

(1) Calculate the Coefficient of correlation b/w x and y from the following table:

x	1	2	3	4	5	6	7
y	2	4	5	3	8	6	7

$$\text{correlation coefficient} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2}\ \sqrt{\Sigma (y-\bar{y})^2}}$$

$n = 7$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{1+2+3+4+5+6+7}{7}$$

$$= \frac{28}{7} = 4$$

$\bar{x} = 4$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{2+4+5+3+8+6+7}{7}$$

22
13
35

$$= \frac{35}{7} = 5$$

$\bar{y} = 5$

x	y	$(x-\bar{x}) = x-4$	$(y-\bar{y}) = (y-5)$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	
1	2	-3	-3	9	9	9
2	4	-2	-1	4	1	2
3	5	-1	0	1	0	0
4	3	0	-2	0	4	0
5	8	1	3	1	9	3
6	6	2	1	4	1	2
7	7	3	2	9	4	6

Σx	Σy	$\Sigma(x-\bar{x})$	$\Sigma(y-\bar{y})$	$\Sigma(x-\bar{x})^2$	$\Sigma(y-\bar{y})^2$	$\Sigma(x-\bar{x})(y-\bar{y})$
$=28$	$=35$	$=0$	$=0$	$=28$	$=28$	$=22$

Correlation coefficient $\quad r = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$

$$= \frac{22}{\sqrt{28}\sqrt{28}}$$

$$= 0.79$$

---

(1) Calculate the coefficient of correlation b/w x and y from the following table.

x	5	10	5	11	12	4	3	2	7	1
y	1	6	2	8	5	1	4	6	5	2

$$\boxed{r = \frac{\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{\Sigma x^2 - \left(\frac{\Sigma x}{N}\right)^2}\sqrt{\Sigma y^2 - \left(\frac{\Sigma y}{N}\right)^2}}}$$ - formula

$n = 10.$

$$= \frac{288 - (60 \times 40)/10}{\sqrt{494 - \frac{(60)^2}{10}}\sqrt{212 - \frac{(40)^2}{10}}}$$

$$= \frac{288 - 240}{\sqrt{494 - 360}\sqrt{212 - 160}} = \frac{48}{\sqrt{134} \times \sqrt{52}}$$

$$= \frac{48}{83.475} = 0.58 //.$$

x	y	xy	x^2	y^2
5	1	5	25	1
10	6	60	100	36
5	2	10	25	4
11	8	88	121	64
12	5	60	144	25
4	1	4	16	1
3	4	12	9	16
2	6	12	4	36
7	5	35	49	25
1	2	2	1	4
$\Sigma x = 60$	$\Sigma y = 40$	$\Sigma xy = 288$	$\Sigma x^2 = 494$	$\Sigma y^2 = 212$

Hypothesis Testing:

A test of hypothesis is a two action decision problem after the experimental sample values have been obtained, the two actions being the acceptance or rejection of the hypothesis under consideration.

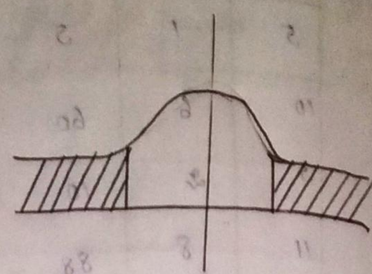
Hypothesis

null hypothesis, Ho → equal

Alternate hypothesis, H1 → not equal

Critical region

rejection region



One tail test

two tail test

Errors

$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

Type I, H_0 true, H_1 false
✗ ✓

$H_0 : = \mu_0$

Type II

H_0 false, H_1 true $H_1 : \mu < \mu_0$
✓ ✗

less than

---

level of significance (α)

the probability of type I error is known as the level of significance (or) size of critical region.

power of the test:

$1-\beta$ is called the power functions of the test hypothesis H_0 against the alternate hypothsis H_1

the value of the power function at a parameter point is called the power of the test at that point.

---

(1) A random sample of 200 tins of coconut oil gave an average weight of 4.95 kgs with a S.D of 0.21 kg. Do we accept the hypothesis of net

weight 5 kgs per tin at 1% level?

Sample Size = 200

Sample mean $\bar{x}$ = 4.95 kg

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Sample S.D = 0.21 kg

Population mean μ = 5 kg

We apply Z test

the test statistic is $Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$.

$$Z = \frac{4.95 - 5}{0.21/\sqrt{200}} = \frac{-0.05 \times \sqrt{200}}{0.21}$$

$$= -3.37$$

$$|Z| = 3.37$$

At 1% level of significance the tabulated value of Z is 2.58

Our calculated value of |Z| is greater than the tabulated value of Z.

Conclusion:

H_0 is rejected at 1% level. ~~the~~ ~~wag~~ therefore the next net weight of a tin is not equal to 5 kg.

Test for equality of two means:

Suppose two independent large samples of sizes n_1, n_2 drawn from two populations with means μ_1, μ_2 and st.D s_1, s_2.

We want to test whether the means are equal.

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (two tailed test)

$$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ sample, sample SD, size

①

	place A	place B
mean height	68.50	68.58
SD of heights	2.5	3.0
Sample size	1200	1500

Test at 5% level that the mean height is the Same for adults in 2 places (table values of z at 5% level for two tailed test is 1.96)

Mean of 1st Sample $\bar{x}_1 = 68.50$

S.D of 1st Sample $s_1 = 2.5$

Sample size of 1st sample $\}$ $n_1 = 1200$

Mean of 2nd Sample $\bar{x}_2 = 68.58$

S.D of 2nd sample $s_2 = 3.0$

Sample size of 2nd sample $\}$ $n_2 = 1500$

$H_0 : \mu_1 = \mu_2$ (mean height is the Same in the two places)

$H_1 : \mu_1 \neq \mu_2$

the test statistic $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

$$= \frac{68.5 - 68.58}{\sqrt{\frac{(2.5)^2}{1200} + \frac{(3.0)^2}{1500}}}$$

$$= \frac{-0.8}{\sqrt{0.00052 + 0.006}} = \frac{-0.08}{0.1058} = -0.76$$

$$\therefore |Z| = 0.76$$

Conclusion:

Ho is accepted at 5% level since the calculated value ~~for~~ of Z is less than the table value of Z. Hence the mean height is same.

---

(1) Electric bulbs manufactured by x, y, companies gave the following results.

	x company	y company.
No. of bulbs used	100	100
Mean life in hrs	1300	1248
S.D in hrs	82	93

(using std err of the diff b/w 1% level sign makes.) Question

$n_1 = 100$ $\quad n_2 = 100$

$\bar{x}_1 = 1300$ $\quad \bar{x}_2 = 1248$

$S_1 = 82$ $\quad S_2 = 93$

$H_0 : \mu_1 = \mu_2$ (there is no significant different between meanlife of the two makes.

$H_1 : \mu_1 \neq \mu_2$

Test statistic is
$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= \frac{1300 - 1248}{\sqrt{\frac{82^2}{100} + \frac{93^2}{100}}}$$

5% 1.96
1% 2.58

$$= \frac{52}{}$$

67.24 + 86.49

Table value of z at 1% level = 2.58

Ho is rejected at 1% level of significance since the calculated value of z is greater than the table value of z.

therefore there is significant difference in the mean life of two makes.

Test for equality of two S.D

$S_1 \cdot S_2$ - SD of the samples of size n_1, n_2 respectively from two populations with SD σ_1, σ_2 respectively.

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$

the test statistic is z =

$$z = \frac{S_1 - S_2}{\sqrt{\frac{S_1^2}{2n_1} + \frac{S_2^2}{2n_2}}}$$

two (2) two (2)

(1) In two random samples of sizes 150, 250 the S.D are calculated as 15.8, 18.8 can we conclude that the samples are drawn from the populations with same S.D?

Soln:

$n_1 = 150 \qquad n_2 = 250$

$s_1 = 15.3 \qquad s_2 = 13.8$

$H_0: \sigma_1 = \sigma_2$ (the sample belong to the populations with same S.D)

$H_1: \sigma_1 \neq \sigma_2$ (the samples belong to populations with Diff S.D)

Test statistic $z = \dfrac{s_1 - s_2}{\sqrt{\dfrac{s_1^2}{2n_1} + \dfrac{s_2^2}{2n_2}}}$

$= \dfrac{15.3 - 13.8}{\dfrac{(15.3)^2}{300} + \dfrac{(13.8)^2}{500}}$

$= \dfrac{1.5}{1.0770} = 1.34$

Since $|z| < 1.96$, H_0 is accepted at 5% level of significance. Hence, the samples belong to the populations with the same S.D.

Hypothesis testing

Large Sample	Small Samples
z test	t-test
Z, F	t-distributions
$n > 30$	$n < 30$

Small sample:

Sample mean $\bar{x}$ — population mean μ

Sample SD s — population SD $= \sigma$

Sample Var $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$ — population var $= \sigma^2$

t statistic is $t = \frac{\bar{x}-\mu}{SE(\bar{x})}$

$SE(\bar{x}) = \frac{s}{\sqrt{n-1}}$ (if sample SD is given)

$SE(\bar{x}) = \frac{s}{\sqrt{n}}$ (if population SD is given)

degrees of freedom $= \nu = n-1$

Type 1:

Test of hypothesis of a single mean

$H_0 : \mu =$ a specified value

or

$\mu > a$

$\mu < a$

test statistic $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$ (or) $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

(1) A soap manufacturing company was distributing a particular brand through a large number of retail shops. Before heavy advertisement campaign, the mean sales per shop was 140 dozens. after the campaign, a sample of 26 shop was taken, and the mean sales was found to be 147 dozens. with SD 16 can you considering the advertisements, effective an 5 % level of significant.

$H_0: \mu = 140$

$H_1: \mu > 140$ (one tailed test)

$n = 26, \quad \bar{x} = 147 \quad S = 16$

test statistic $t = \dfrac{\bar{x} - \mu}{\frac{S}{\sqrt{n-1}}}$

$= \dfrac{147 - 140}{\frac{16}{\sqrt{26-1}}} = \dfrac{7}{16/\sqrt{25}} = \dfrac{7}{\frac{16}{5}}$

$= \dfrac{7 \times 5}{16} = 2.19$

level of significant $\alpha = 0.05$

Critical value of t at 0.05, $v = 25$ is 1.708

$t_{\alpha, 25} = 1.708$

$|t| > t_{0.05, 25}$, H_0 is rejected.

H_1 is accepted.

Hence the advertisement is effective

($\mu > 140$ is accepted)

Type 5:

Difference of two means (small samples)

two independent samples of sizes n_1, n_2 with mean $\bar{x}_1, \bar{x}_2$ & the sample variance s_1^2 & s_2^2 test the diff. b/w the means of two samples is significant or not

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

caps

where, $$S = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

deg of freedom $v = n_1 + n_2 - 2$

(1) Two independent samples of 8 & 7 items respectively had the following values.

Sample I	9	13	11	11	15	9	12	14
II	10	12	10	14	9	8	10	

Is the diff b/w the mean of the samples significant

two tailed test

let $H_0: \mu_1 = \mu_2$ (two sample means are equal)

$H_1: \mu_1 \neq \mu_2$ (two sample means are not equal)

test statistic $$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

x_1	$x_1 - \bar{x}$ $x_1 - 11$	$(x_1 - \bar{x_1})^2$	$\bar{x_2}$	$(x_2 - \bar{x_2})$ $x_2 - 10$	$(x_2 - \bar{x_2})^2$
9	-2	4	10	0	0
13	2	4	12	2	4
11	0	0	10	0	0
11	0	0	14	4	16
15	4	16	9	-1	1
9	-2	4	8	-2	4
12	1	1	10	0	0
14	3	9			
	Σd_1 or $\Sigma x_1 - \bar{x}$ $= 6$	Σd_1^2 or $\Sigma(x_1 - \bar{x_1})^2$ $= 38$		Σd_2 or $\Sigma x_2 - \bar{x_2}$ $= 3$	Σd_2^2 or $\Sigma(x_2 - \bar{x_2})^2$ $= 25$

$$\bar{x_1} = A + \frac{\Sigma d_1}{n} \text{ or } A + \frac{\Sigma x_1 - \bar{x_1}}{n}$$

$$\bar{x_1} = 11 + \frac{6}{8} = \frac{88+6}{8} = \frac{94}{8} = 11.75$$

$$\bar{x_1} = 11.75$$

$$\bar{x_2} = 10 + \frac{\Sigma x_2 - \bar{x_2}}{n} = 10 + \frac{3}{7} = \frac{70+3}{7} = \frac{73}{7}$$

$$\bar{x_2} = 10.42$$

$\Sigma(x_1 - \bar{x})$

38

7

$$\Sigma(x_1 - \bar{x_1})^2 = \Sigma(x_1 - \bar{x})^2 - \frac{(\Sigma d_1)^2}{n_1} = 38 - \frac{36}{8}$$

$$= \frac{304 - 36}{8} = \frac{268}{8} = 33.5$$

$$\Sigma(x_2 - \bar{x_2})^2 = \Sigma(x_2 - \bar{x_2})^2 - \frac{(\Sigma d_2)^2}{n_2}$$

$$= 25 - \frac{9}{7} = \frac{175 - 9}{7} = \frac{166}{7} = 23.71$$

$$\Sigma(x_2 - \bar{x_2})^2 = 23.71$$

Two random samples of size 8 and 11, drawn from two normal populations, are characterised as follows.

	Sample Size	Sum of observations	Sum of squares of observations
Sample I	8	9.6	61.52
Sample II	11	16.5	73.26

you are to decide if two populations can be taken to have the same variance.

Let x, y be the observation for two samples.

$H_0 : \sigma_1^2 = \sigma_2^2 \ / \ H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$$S_1^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

$$= \frac{61.52}{8} - \left(\frac{9.6}{8}\right)^2$$

$$= 7.69 - (1.2)^2 = 6.285$$

$$S_1^2 = 6.285$$

$$S_2^2 = \frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2$$

$$= \frac{73.26}{11} - \left(\frac{16.5}{11}\right)^2$$

$$= 6.66 - 2.25$$

$$= 4.41$$

$$\hat{\sigma}_1^2 \ \mu_1 = \frac{n_1 S_1^2}{n_1 - 1} = \frac{8(6.25)}{8-1} = \frac{50}{7} = 7.143$$

$$= \frac{n_2 s_2^2}{n_2 - 1} = \frac{11\,(4.41)}{10} = \frac{48.51}{10} = 4.851$$

$$= \frac{\mu_1}{\mu_2} \Big/ \frac{\hat{s}_1^2}{\hat{s}_2^2} = \frac{7.143}{4.851} = 1.472$$

the table value of F at 5% level for degree of freedom $(7, 10)$ is 3.14

Conclusion:

Since computed value of F < table value of F, H_0 is accepted at 5% level.

$\therefore$ Variance of two populations may be a same.

Random samples are drawn from two populations and the following results are obtained.

Sample x:	16	17	18	19	20	21	22	24	26	27		
Sample y:	19	22	23	25	26	28	29	30	31	32	35	36

find the variance of two populations and test whether the two samples have same variance.

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

x	$(x-\bar{x}) = x-21$	$(x-\bar{x})^2 = (x-21)^2$	y	$(y-\bar{y}) = y-28$	$(y-\bar{y})^2 = (y-28)^2$
16	-5	25	19	-9	81
17	-4	16	22	-6	36
18	-3	9	23	-5	25
19	-2	4	25	-3	9
20	-1	1	26	-2	4
21	0	0	28	0	0
22	1	1	29	1	1
24	3	9	30	2	4
26	5	25	31	3	9
27	6	36	35 32	4 7	16 49
			36	8	64
$\Sigma x = 210$		$\Sigma (x-x)^2 = 126$	$\Sigma y = 336$		$\Sigma (y-\bar{y})^2 = 298$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{210}{10} = 21$$

$$\bar{x} = 21$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{336}{12} = 28$$

$$s_1^2 = \frac{\Sigma (x_1-\bar{x})^2}{n_1-1} = \frac{126}{9} = 14$$

$$s_2 = \frac{\Sigma (y_1-\bar{y})^2}{n_2-1} = \frac{298}{11} = 27.09$$

test statistic $F = \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2}$; $\hat{\sigma}_2^2 > \hat{\sigma}_1^2$ $N_2 > N_1$

$= \frac{27.09}{14} = 1.94$

the tabled value of F at 5% level for degrees of freedom $(\overset{N_1}{14}, \overset{N_2}{9})$ is 2.6458.

Conclusion:

The computed value of F < table value of F,

Ho is accepted. Since, the two samples have same variance.

30 - F test
< 29 - t test

Chi Square Test (χ^2 test) goodness of fit

O_i = observed frequency of i^{th} event

E_i = Expected frequency of i^{th} event

test statistic $\chi^2 = \sum_{i=1}^{n} \left(\frac{O_i - E_i}{E_i} \right)^2$

Degrees of freedom

$v = n-1$ where n is a number of items in the series.

for binomial distribution $v = n-1$

poisson distribution $v = n-2$

normal $v = n-3$

① A dice is thrown 264 times with the following results.

No. appeared on the die	1	2	3	4	5	6
frequency	40	32	28	58	54	60

Show that die is biased.

H_0: the die is not biased

H_1: the dice is ~~not~~ biased

test statistic $\phi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$

Expected (E) frequence $= \frac{264}{6} = 44.$

No. of the die	1	2	3	4	5	6
Observed frequence O_i	40	32	28	58	54	60
Expected freq E_i	44	44	44	44	44	44

test statistic $= \frac{(40-44)^2}{44} + \frac{(32-44)^2}{44} + \frac{(28-44)^2}{44} +$

$\frac{(58-44)^2}{44} + \frac{(54-44)^2}{44} + \frac{(60-44)^2}{44}$

$= \frac{1}{44}\left[16 + 144 + 256 + 196 + 100 + 250\right]$

$\boxed{= 22.}$

level of significance $\alpha = 0.05$ 6-1=5

Critical value of ϕ^2 at 0.05 for degree of freedom 5 $= 11.07$ df - degrees of freedom

Conclusion: Calculated value is greater than the table value of ψ^2

$\therefore$ H_0 is rejected

H_1 is accepted

$\therefore$ Hence, the die is biased.

Regression Equation:

(1) The following table gives the no. of aircraft accidents that occured during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tues	Wed	Thurs	Fri	Sat	total
no. of accidents	14	18	12	11	15	14	84

Let H_0: the accidents are uniformally distribution over the week

H_1 the accidents are not uniformally distributed over the

$$\psi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

expected frequency accidents on each day.

Days

No. of accidents

$$\psi^2 = \frac{(14-14)^2}{14} + \frac{(18-14)^2}{14} + \frac{(12-14)^2}{14} + \frac{(11-14)^2}{14} + \frac{(14-14)^2}{14} +$$

$= 2.14$

$\alpha = 0.05$

Critical value of ϕ^2 at 0.05 for the degree of freedom. $n-1$ $\qquad 6-1 = 5$

11.071

$\phi^2 = 0.05, 6$

Conclusion:

So, Ho is rejected calculated value greater than the ϕ^2

Chi 2 test

Correlation

Linear Regression

Large Sample - 10