PROPERTIES OF MATTER AND ACOUSTICS
(16SCCPH1)

(Brief notes for reference)

BY

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Note: This material is helpful for the students those who are in exam point of view only and creates the idea(especially for the students 2016-2021). For more materials students are advised to refer the prescribed text and other references. This material is not enough. Students are instructed to refer book for study and reference respectively for further elaborate points as prescribed by the University.

Date : 02.07.18

properties of Matter & Accounstics

unit - I -> Elasticity

Hooke's law = stress - strain diagram - factors affecting elasticity - Different moduli of elasticity - Relation between moduli - poisson's ratio - Twisting couple on a cylinder - Determination of rigidity modulus by static torsion - work done in twisting a wire - Torsional oscillations - Torsion pendulum - Rigidity modulus - M.I.

unit - II -> Bending of Beams

Bending of beam - expression for Bending moment - cantilever - expression for depression of the loaded end of a cantilever - young's modulus by measuring the tilt in a loaded cantilever -

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oscillation of a cantilever - Non-uniform bending. Expression for depression - uniform bending - Expression for elevation - pin & Microscope (young's Modulus) - Koenig's method.

Unit - 3 -) surface Tension

Definition - Molecular forces - Explanation of surface tension on kinetic theory - surface energy - work done on increasing the area of the surface - Angle of contact - Neumman's triangle - Excess pressure inside a liquid drop, soap bubble, excess pressure inside a curved surface - force between 2 plates seperated by a thin layer of a liquid - surface tension - Jaegen's method - Drop weight method - capilary rise method - Temperature variation.

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unit - 4 → viscosity

Newton's law of viscous flow - streamlined and turbulent motion - Reynold's number - poiseuille's formula for the flow of a liquid through a horizontal capillary tube - Experimental determination of co-efficient of a liquid by poiseuille's method - ostwald's viscometer - Terminal velocity and stoke's formula - viscosity of gases - Meyer's formula - Rankine's Method - variation of viscosity with temperature & pressure - Lubricants.

Equations of continuity of flow - Euler's equation for unidirectional flow - Bernoulli's theorem - filter pump and wings of aeroplanes - Torricelli's theorem - pitot tube.

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unit - 5 -> Acoustics

Newton's formula for velocity of sound - Effect of temperature, pressure, humidity, density of medium and wind - musical sound and noise - speech - characteristics of musical sound. intensity of sound - measurement of intensity of sound - decibel and phon - Bel

Reverberation - Sabines reverberation formula - factors affecting the acoustics of buildings - sound distribution in an auditorium - Requisites - good acoustics - ultrasonics - production and deduction - Medical applications of ultrasonic waves - Acoustics grating.

Date 07.07.18

Surface Tension

2m/ Define surface Tension ?

The tension associated with the liquid surface which acts parallel to the surface is called surface tension. The unit of surface tension is N/m.

5m/ Nuclear forces:

The attraction between molecules of a substance is called cohesion while that of different substances is called adhesion. The cohesive forces in solid and liquids have the order of 10^{-7} cm. This is the radius of the sphere of influence. Beyond this the cohesive forces is ineffective

Explanation:

* Consider a molecule 'A' completely within the liquid. It will be attracted on an average equally by the surrounding molecule. withia a range of forces and

hence there will be no force acting on it.

* At the free surface, for a molecule like B or c there will be a net inward force of neighbouring molecules of a liquid. There are no molecules above the surface to counter act these force.

* There is a tendancy for molecules close to the liquid surface to be pulled towards the main mass of the liquid. This has the consequence of rendering the surface area as small as possible. So that the surface behaves a force which is dependent like stretched membrane.

* If a molecule is moved from a interior of the liquid to the surface work must be done against the downward force. Thus a molecule on or near the liquid surface posses

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greater potential energy than interior molecules.

2m
Angle of contact:

The angle between tangent to the liquid surface and the downward vertical at the point of contact is called Angle of contact. The angle of contact depends upon the nature of the liquid and solid. For mercury $\theta = 140^\circ$, for kerosene $\theta = 26^\circ$. If the angle of contact is greater than 90°, then in the case of mercury or water and paraffin wax, the liquid does not wet the glass.

2m
capillary rise:

If a glass tube of fine bore of radius 'r' be dipped in water. Then it is observed that the column of water rises up the tube of few cm. above that outside level. This phenomenon

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is called capillary rise.

5m/ What is surface film and surface energy?

If a plane parallel to the surface of a liquid and a distance equal to the molecular range it, the layer of the liquid lying between the surface and the plane is called the surface film.

The film tends to have the least square area in order that the number of molecules in it may be a minimum. The potential energy per unit area of the surface film is called surface energy.

Work done in Bglowing a bubble

If for the sake of a simplicity we neglect the cooling produced when a film is stritched, the work done is calculated as surface area of the film x

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surface tension. Therefore the work done in blowing a bubble is equal to $8\pi r^2 \times T$.

5m pressure difference across a liquid surface

* Suppose, the free surface of the liquid is plane, then the resultant force due to surface tension on a molecule on its surface is zero. And the cohessive pressure is negligible.

* If the free surface of the liquid is concave, the resultant force on a molecule on the surface would be upwards.

* If the surface is convex, the resultant force due to surface tension on a molecule is downwards. Here, the cohessive pressure is increased.

cases:

* Excess pressure inside a liquid drop

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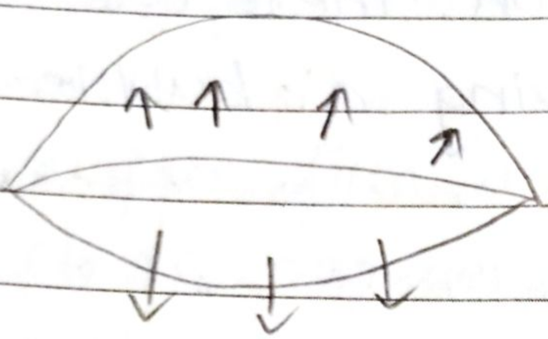

(i) The molecules near the surface of a drop experience a resultant pull inwards.

(ii) The pressure is greater inside the drop than outside.

(iii) 'p' is the pressure inside the drop, the radius of the drop is 'r' the surface tension is 'T'

(iv) The upper hemisphere of the drop with upward thrust ABCD due to excess pressure is $p\pi r^2$.

(v) under equilibrium, $p\pi r^2 = T \cdot 2\pi r$

$$\therefore T = \frac{p\pi r^2}{2\pi r}$$

$$T = \frac{pr}{2} \quad \text{(or)} \quad p = \frac{2T}{r}$$

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* Instead of liquid drop in a soap bubble it has two surfaces. Therefore it seems to be a sphere.

Therefore, $p\pi r^2 = 2 \times 2\pi r . T$

$$T = \frac{p\pi r^2}{4\pi r}$$

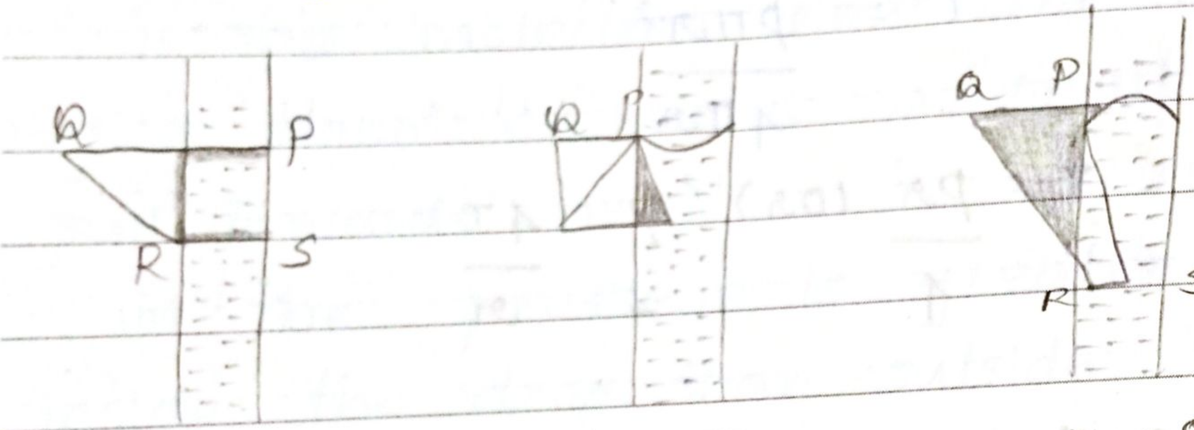
$$T = \frac{pr}{4} \quad (\text{or}) \quad p = \frac{4T}{r}$$

5M Shape of liquid meniscus in a capillary tube:

* Let a capillary tube of a glass is dipped in a liquid. The surface is 'p' then a liquid molecule at P in contact with the tube, there will be an attraction by solid molecules. and this is due to adhesion. This is an outward force.

* If it is inward, then the molecules of the liquid is under cohesive.

* The resultant of adhescent is at right angle at P, the resultant force of cohesive acts at 45°. The two forces acting on a molecule at an angle 135°.



plane

45°

135°

plane (i) If PQ/PS be equal to $1/\sqrt{2}$, (ie) the resultant PR will be vertical.

concave (ii) If $PS < \sqrt{2}.PQ$ then the resultant will lie outside the liquid.

convex (iii) If $PS > \sqrt{2}.PQ$ then the resultant will lie inside the liquid.

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Angle of contact & Neumann's triangle:

If incase of two liquids in contact with each other and with air then,

case (i) :

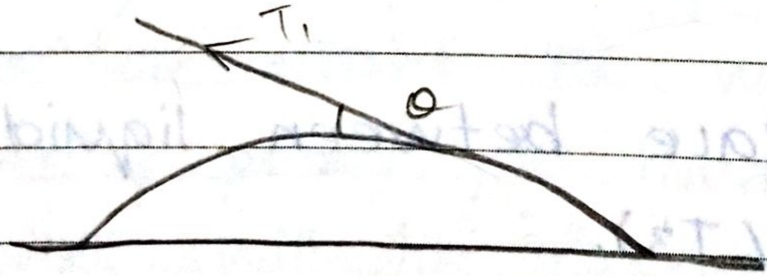
If these are not miscible with each other be brought into contact at O. Both being with air contact. Three surface tensions are to be taken.

* Surface between air and liquid (T_1).
* surface between air and liquid (T_2).
* Surface between liquid 1 and liquid 2 (T_3).
* For equilibrium, T_1, T_2 and T_3 should be represented by three sides of triangle known as Neumann's triangle. one of the surface tensions being always greater than the other two.

so that equilibrium condition is never attained.

For example, water, Mecury and air are pure This is so because the surface tension of Mercury is about 550 dymes / cm. But water has 75 dymes / cm. If the mercury is contaminated with greaze, then surface tension decreases and some water drops may stay on it. This is possible for Neumann's triangle.

* θ is the angle of contact with solid and liquid as shown.



For both cases, acute and obtuse under equilibrium

$$T_3 + T_1 \cos\theta = T_2$$

$$(\text{or}) \quad \cos\theta = \frac{(T_2 - T_3)}{T_1}$$

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10m // Rise of liquid in a capillary tube:

capillarity:

To raise a liquid in a capillary tube, by dipping it in a beaker containing a liquid.

constructions:

* when a capillary tube is dipped in a liquid like water, it can wet the surface.

* The angle of contact may be zero. If the tube is fine then the miniscus may be spherical or concave.

* Let r be the radius of the tube as shown in figure. B. The mark for raising level. practically the same as the radius of the concave miniscus based on the excess pressure above the minisus. Below the miniscus both depending on the atmosphereic pressure is $2T/r$.

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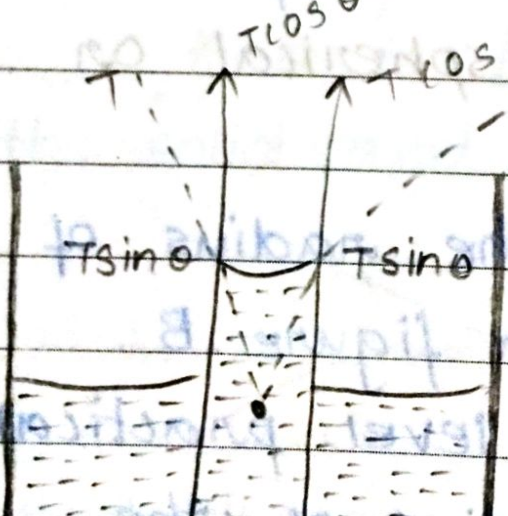
* Since the pressure on the liquid surface outside the tube is atmospheric. The liquid flows inside the tube until the hydrostatic pressure equals the excess pressure. If the liquid raises to the height h, the hydrostatic pressure due to liquid column in the tube on the surface will be h.p.g. Therefore surface tension $T = \frac{r.h.\rho.g}{2}$



* As shown in figure, there is a force inwardly inside the tube. under Newton's III law, this reaction is ressolved into two components. (i) T cos θ (ii) T sin θ

* The total upward force on the liquid = $2\pi r \cos\theta$

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* The weight of total volume of liquid $= (\pi r^2 . h + V)$

$\therefore$ Balancing the equations,

$$T = \frac{\pi r^2 h + V}{2\pi r \cos\theta} \rho . g$$

$$T = \frac{r h \rho g}{2 \cos\theta}$$

This equation is called Jurin's equation.

* Here V is negligible (narrow tube). The volume of liquid = volume of cylinder. $\therefore V = \frac{1}{3} \pi r^3$.

$\therefore$ Surface tension $T = \frac{r (h + r/3)}{2} \rho g$

conditions:

* If $\theta = 0$, then the above result is the surface tension value.

* If $\theta > 90°$, then the liquid level inside the tube is below the beaker level.

* If $\theta < 90°$, then the liquid level inside the tube is above the beaker level.

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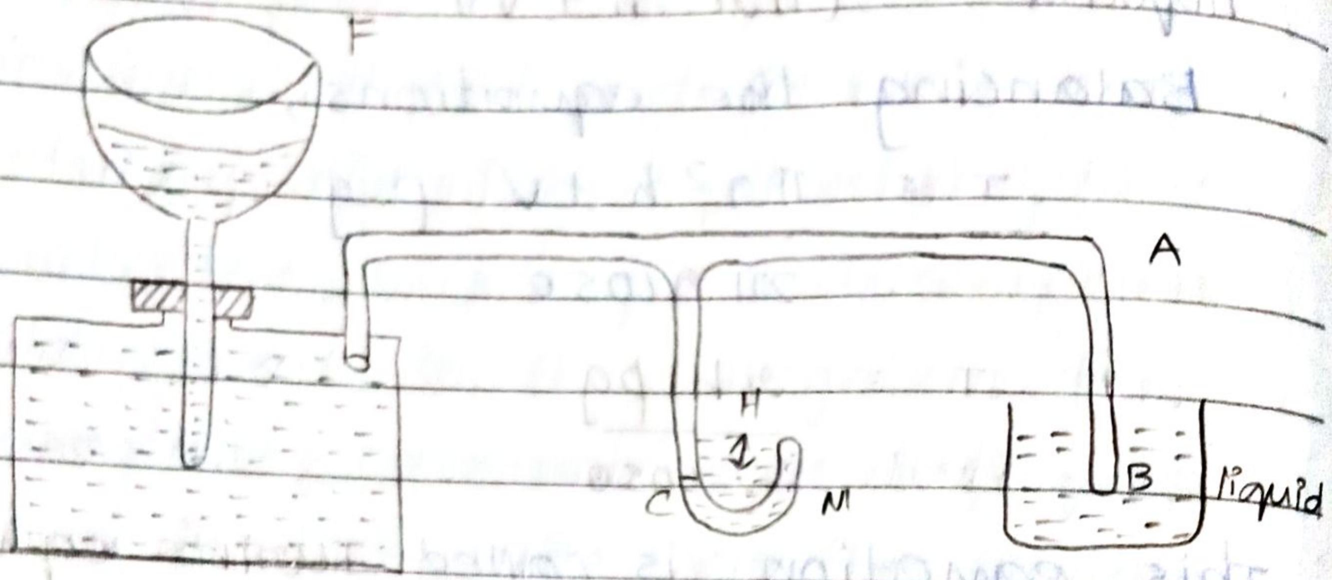
10m

Jaeger's method:



principle:

TO measure the excess pressure, Jaegar introduced a simple method for an air bubble in a liquid.

construction:

the apparatus consists of a long thin glass tube AB. Its lower portion is of 0.5 mm of diameter. This tube is dipped in a liquid which is connected with a

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beaker. It is then connected with manometer (M) and a woulff's bottle fitted with a funnel (F).

Experiment:

Due to capillary action. Some liquid rises up into tube AB. Some air is now forced into the tube by dropping water into the bottle. The liquid column AB slowly moves down until it reaches B. bubble is formed now. The radius of curvature of the bubble gradually decreases. with increase in pressure inside it. H shows the maximum level of pressure in the manometer.

The bubble is unstable which has increase in radius and decrease in pressure. This is due to the surface tension. Therefore the equilibrium between internal and external pressure destroys.

The excess pressure inside the bubble can be written as

$$(P + H\rho g) - (p + h\rho g) = g\,(H\rho \cdot hd)$$

Draw back:

* There is no absolute certainity as the radius of the bubble when it gets detached from the tube.
* It may not be hemispherical.

D/b moduli

Jaeger's method

Rise of liquid in capillary tube

Neumon's triangle

Drop weight method

Surface film & energy (or) measure diff across sur lin

shape of miniscus in c.t (or) D/b elastic behavior of ber

Neumon's triangle (or) excess press. inside a liquid drop

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4. How can distinguish a pressure in a liquid and a soap bubble (or) write about molecular theory of liquids.

3. Explain stress, strain curve (or) write short note on capillary tube.

5M Drop weight method:

principle:

To determine the surface tension of a liquid by considering the vertical force that keep a small drop of liquid in equilibrium, Just before it gets detached from the end of a vertical glass tube (circular aperture)

construction:

A burrette is fitted vertically in a suitable clamp and a thin clean dry tube of glass is attached to the nozzle by a piece of rubber tube

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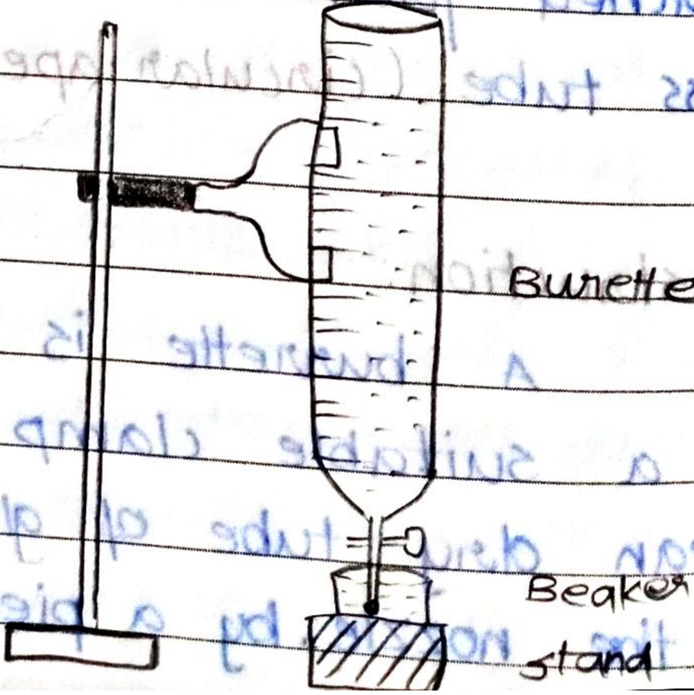
carying a pinch cock. the Burette is filled with liquid and small drops of liquid is regulated by pinch cock. the rate of detachment of liquid from the tube one per minute. The drops are collected in a beaker and the mass of the drop is calculated. the radius of the tube is measured using screw Guage or Travelling microscope. the surface tension of the liquid is calculated by m, mass of the liquid drop, r, radius of the tube and g gravity.

$$T = \frac{mg}{3.8\,r}$$



The downward force of the drop is $\pi r^2 . T/r + mg$.

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2m viscosity :

the property of a liqmid by virtue of its opposses relative motion between its different layers is known as viscosity.

2m co-efficient of viscosity:

The tangential force required per unit area to maintain unit relative velocity between two layers unit distance apart. unit : $Nm^{-2}s$

Dimension: $ML^{-1}T^{-1}$ Formula: $\frac{\text{Force}}{\text{area x velocity gradient}}$

$\eta = FL^{-2}T$

5m

2m What is stream lined flow?

The liquid in which the flow is having uniformity with the definite direction of the line at a point in a same path and same velocity is

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refered to as stream lined flow.

2m What is turbulent flow?

The flow of liquid in which the discrete impact of the molecules in the layer with indefinite direction of the line at a point with different velocity is refered to as turbulent flow.

Momentum $p = mv$

2m Critical velocity :

$$V_c = K\eta / \rho . r$$

V_c = critical velocity

K = Reynold's number

η = viscosity of the liquid

ρ = Density

r = radius of the tube.

critical velocity is directly proportional to viscosity of the liquid and inversely proportional to the density of the liquid and radius of the tube.

2m what is Reynold's number? 5m Write its significance.

$$K = \frac{V_c \rho r}{\eta}$$

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k = Reynold's number
V_c = critical velocity
ρ = Density
r = radius of the tube
η = viscosity of the liquid.

* It determines the process taking place inside a critical tube. During the flow of liquid.
* Reynold's number leads to the law of similarity.
* For stream lined flow, the number is same.

2M Define Terminal velocity.

The opposing force during the flow of liquid increases with the velocity of the body until in the case of small bodies just equal to the motive or driving force and the body then attains a constant value in velocity is called Terminal velocity, $v = S/t$

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where,
s is the displacement and
t is the time.

5m stoke's law:

principle:
The coefficient of viscosity is determined by the flow of liquid using stoke's method.

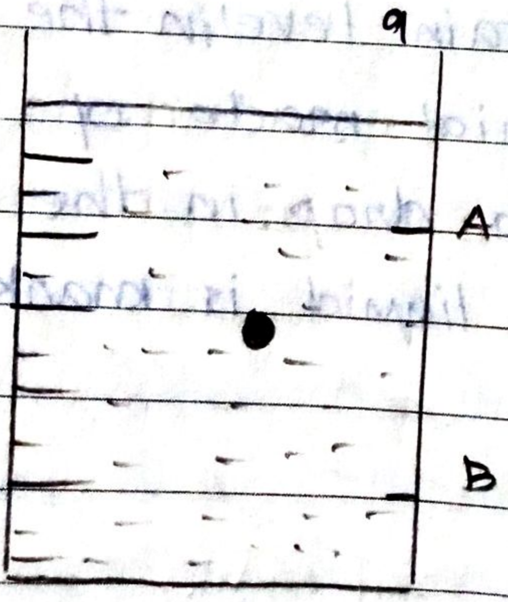
construction:
In a beaker, which is cylindrical like tube is arranged such that the liquid is filled upto certain levelin the tube like beaker. The material made of steel ball is allowed to drop in the beaker. various level of liquid is marked in the beaker.

procedure:

Highly viscus liquid is filled in the beaker, steel ball is droped in the beaker. the ball falls rapidly in the liquid initially and then suddenly velocity of the ball ~~dec~~reduces, this is due to terminal velocity of the liquid. The ball got its uniformity to penetrate in the liquid and such displacement is marked with respect to time.

It is due to the viscus drag. and the coefficient of viscosity is calculated using the formula

$$\eta = \frac{2}{9} \frac{r^2 g(\rho-\sigma)t}{s}$$



This method is known as stoke's method.

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Equation of continuty:

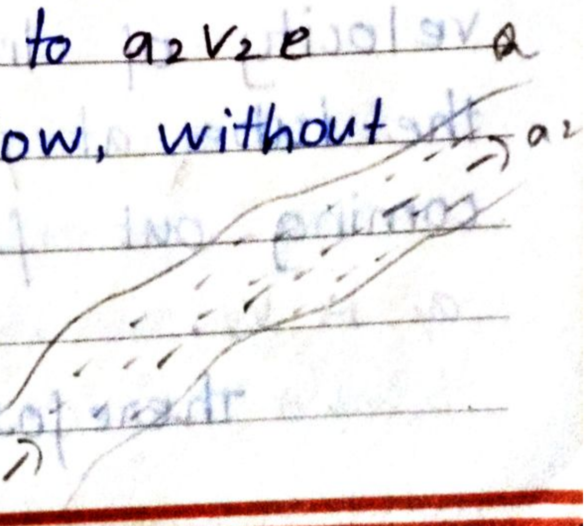
p.Q is a tube where uniformly flowing liquid without viscosity is considered. In this tube a_1 and a_2 are the cross sectional areas such that v_1 and v_2 is the velocity for the liquid flowing through a_1 and a_2 respectively. Therefore, in one second, the volume of a liquid is equal to $a_1 v_1$ (a_1 a +p). Mass of the liquid at p in one second is equal to $a_1 v_1 e$.

Therefore in one second, the volume of a liquid is equal to $a_2 v_2$ (a_2 at Q)

Mass of the liquid at p in one second is equal to $a_2 v_2 e$

In uniform flow, without loss $a_1 v_1 e = a_2 v_2 e$

av = constant.



10m

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Bernoulli's theorem:

Theorem statement:

According to Bernoulli's theorem a non-viscous liquid which is having low pressure and flowing through uniform tube, the sum of pressure energy, Kinetic energy and potential energy are equal to the unit mass constant.

$$P/\rho + v^2/2 + gh = \text{constant}.$$

proof:

As shown in figure, pq is a tube which has a_1 and a_2 cross sectional area respectively. The velocity of liquid going inside the tube at p is v_1, the liquid coming out from the tube through q is v_2.

Therefore, the pressure

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p_1 is greater than p_2. such that the position of q is high when compared to p. This is due to the gravity. By equation of continuivity

$$a_1 v_1 \cancel{\rho} = a_2 v_2 \cancel{\rho} = m$$

Here, a_1 is greater than a_2 $(a_1 > a_2)$ v_1 is less than v_2 $(v_1 < v_2)$. In p,

$F = ~~v_1~~ a_1$ in q, $F = v_2 a_2$.

The work done at p is $p_1 V$ And the work done at q is $p_2 V$. Therefore the total work done at one second is equal to $p_1 V - p_2 V$

The potential energy increases at one second is equal to $mgh_2 - mgh_1$ The increase in kinetic energy is $\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$. Therefore, by work energy theory, the work done by the pressure energy is equal to the sum of the kinetic energy and potential energy in one second.

$$p_1 V - p_2 V_1 = (mgh_2 - mgh_1) + (\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2)$$

$$p_1 v + mgh_1 + \frac{1}{2} mv_1^2 = p_2 v + mgh_2 + \frac{1}{2} mv_2^2$$

therefore, dividing the above equation by m,

$$\frac{p_1 v}{m} + \frac{mgh_1}{m} + \frac{\frac{1}{2} mv_1^2}{m} = \frac{p_2 v}{m} + \frac{mgh_2}{m} + \frac{\frac{1}{2} mv_2^2}{m}$$

$$\frac{p_1 v}{m} + gh_1 + \frac{1}{2} v_1^2 = \frac{p_2 v}{m} + gh_2 + \frac{1}{2} v_2^2$$

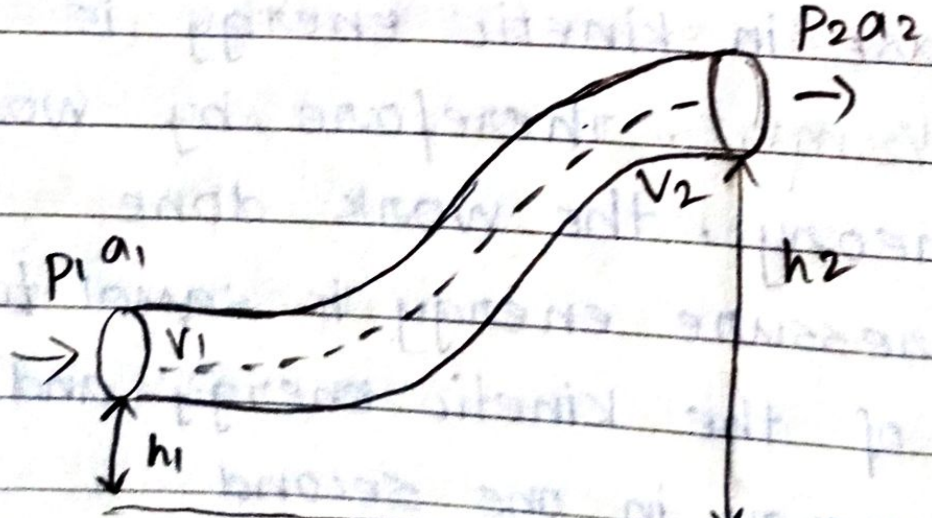
$$\left[\because \rho = \frac{m}{v}\right]$$

$$\frac{p_1}{\rho} + gh_1 + \frac{1}{2} v_1^2 = \frac{p_2}{\rho} + gh_2 + \frac{1}{2} v_2^2$$

Special cases:

If the tube is horizontal, then there will be no potential energy.

$\therefore P/\rho + v^2/2 = \text{constant}$



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10 m

Applications of Bernoulli's theorem:

(i) Torricelli's theorem
(velocity of efflux of liquid)

Let the surface of the liquid at a height h above the liquid level of the circular and sharp edge orifice in a tank. If the tank is wide the velocity at the liquid surface may be taken to be zero. The pressure is atmospheric. Therefore the liquid emerges it plays no part in the flow of the liquid. If v, the velocity of the liquid level considering a tube flow starting and ending. Therefore,

Total energy = pressure energy + potential energy + kinetic energy.

∴ At A, potential energy = gh

Kinetic energy = 0, pressure energy = 0

therefore at 0,

pressure energy = 0

kinetic energy = $\frac{1}{2} v^2$

potential energy = 0

Therefore,

Total energy => K.E = P.E

$$\frac{1}{2} v^2 = hg$$

$$v^2 = 2hg$$

$$v = \sqrt{2hg}$$

This is called velocity of efflux of liquid known as Torricelli's theorem

2m
ii) vena contracta :

* The whole liquid entering the orifice does not move perpendicularly into the tube.

* The liquid coming out from the sites of the vessel as it enters the orifice has a lateral jet until due to the inertia and continues to move inwards the centre of

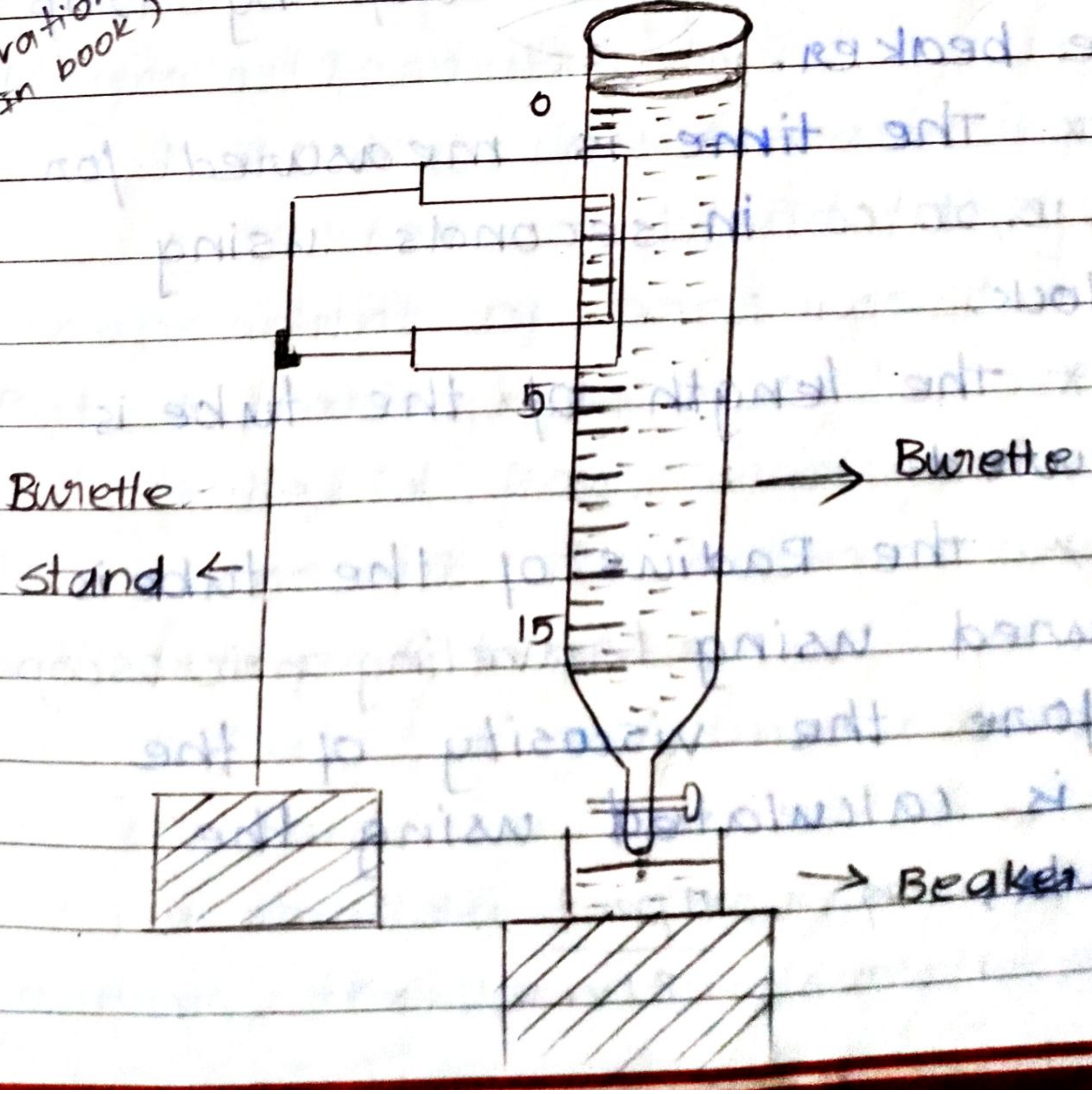
Date

cross section of the Jet. (ie) the liquid jet thus contracts from the mouth of the orifice. upto a distance above half the diameter of the orifice. It forms a neck called vena contracta or contracted vein.

10m

poiseuilli's method (Experiment)

Derivation (in book)



Explanation:

* A liquid is taken in a burette such that the burette level from 0, 5, 10, 15, 20, .. are measured according to the height of the levels with respect to the table.

* The liquid level is adjusted so as to collect the drop by drop in the beaker.

* The time is measured for 0, 5, 10, 15, 20 cc in seconds using stop clock.

* The length of the tube is measured.

* The Radius of the tube is measured using traveling microscope. Therefore the viscosity of the liquid is calculated using the formula, $\eta = \frac{\pi p r^4}{8 l V}$

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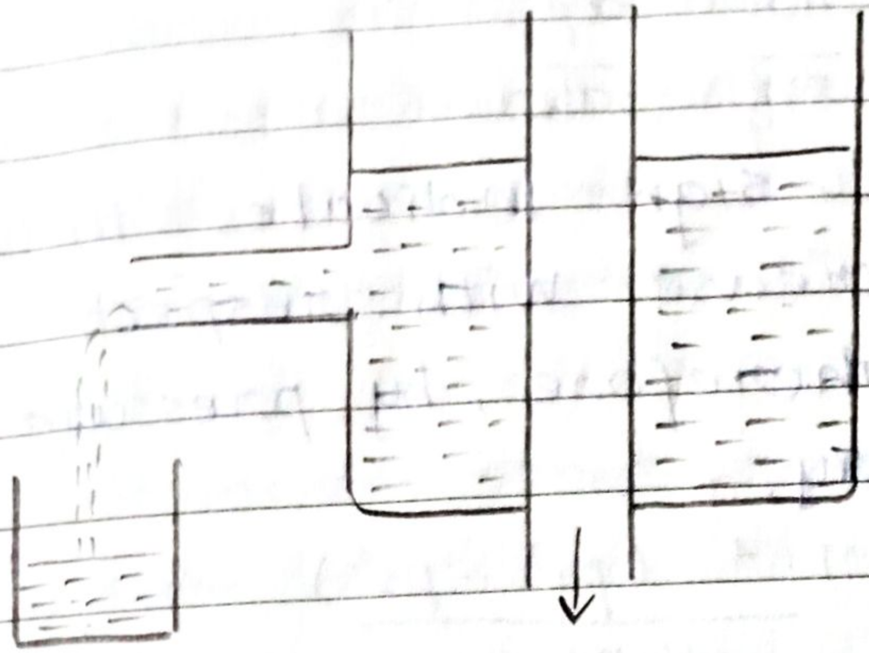

5m

Mayer's Formula:

* Mostly, liquids are incompressible
* In case of liquid, they are independent of pressure for density. But, it varies for gas.
* liquid flow through the tube is a constant. The volume is constant with respect to time.
* But, for gas, mass alone is constant.
* consider using poiseuilli's formula, the volume of gas is

$$v = \frac{-\pi r^4}{8\eta} \frac{dp}{dx}$$

The negative sign indicates the pressure decrease with respect to distance. Therefore, by pressure volume theory,

$$p_1 v_1 = \frac{\pi r^4 (p_1^2 - p_2^2)}{16 \eta l}$$

This is the Meyer's formula for gas flow in a tube.

Since, $p_1 v_1 l = \frac{\pi r^4}{8\eta} \frac{(p_1^2 - p_2^2)}{2}$

2m

1. capillary rise
2. work done in glowing a bubble
3. Angle of contact
4. capillarity
5. coefficient of viscosity
6. critical velocity
7. Terminal velocity
8. Define Reynold's number

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5m
1. Drop weight method
2. D/b turbulent & stream lined flow
3. stoke's theorem
4. ostwald's viscometer. 5. Meyer's formula.

10m
1. state and prove Torricelli's theorem
2. state and prove Bernouli's theorem
3. Torsional pendulum
4. Twisting of couple

5m
Ostwald's viscometer:

principle:

The comparision of two liquids is calculated using ostwald's viscometer.

Description:

ostwald's viscometer is a glass tube in U shape. The apparatus contain two glass bulbs connected by

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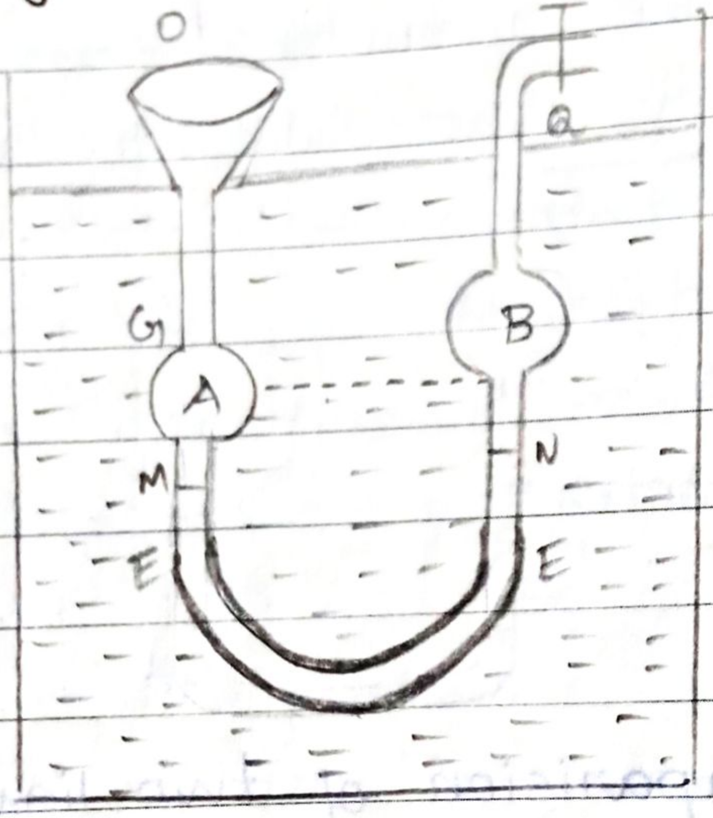
capillary tube like. The liquids which are taken in the tube should have different viscosities as shown in figure

Diagram:



procedure:

A beaker which contains liquid A is filled in the viscometer through O. A stop clock is fixed to note the level of liquid in the apparatus from points M E N Q

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from bulb A to B. The time t_1 is noted. The apparatus is dipped in a container which contains water at room temperature. The experiment is repeated for different temperatures and the readings are noted.

The another liquid B is taken in a apparatus and MENA levels are noted. The time t_2 is noted at different temperatures. The ratio between the two liquids are calculated using the formula $\eta = \frac{\rho_1}{\rho_2} \cdot \frac{t_1}{t_2}$. since $p \propto \rho$

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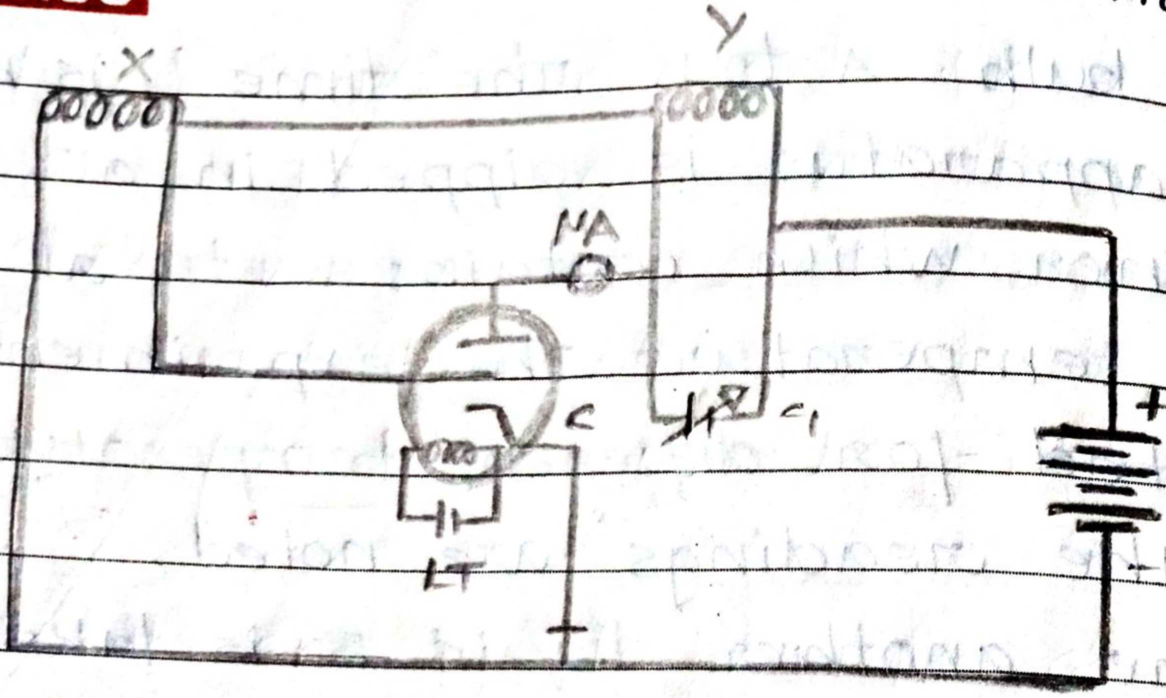

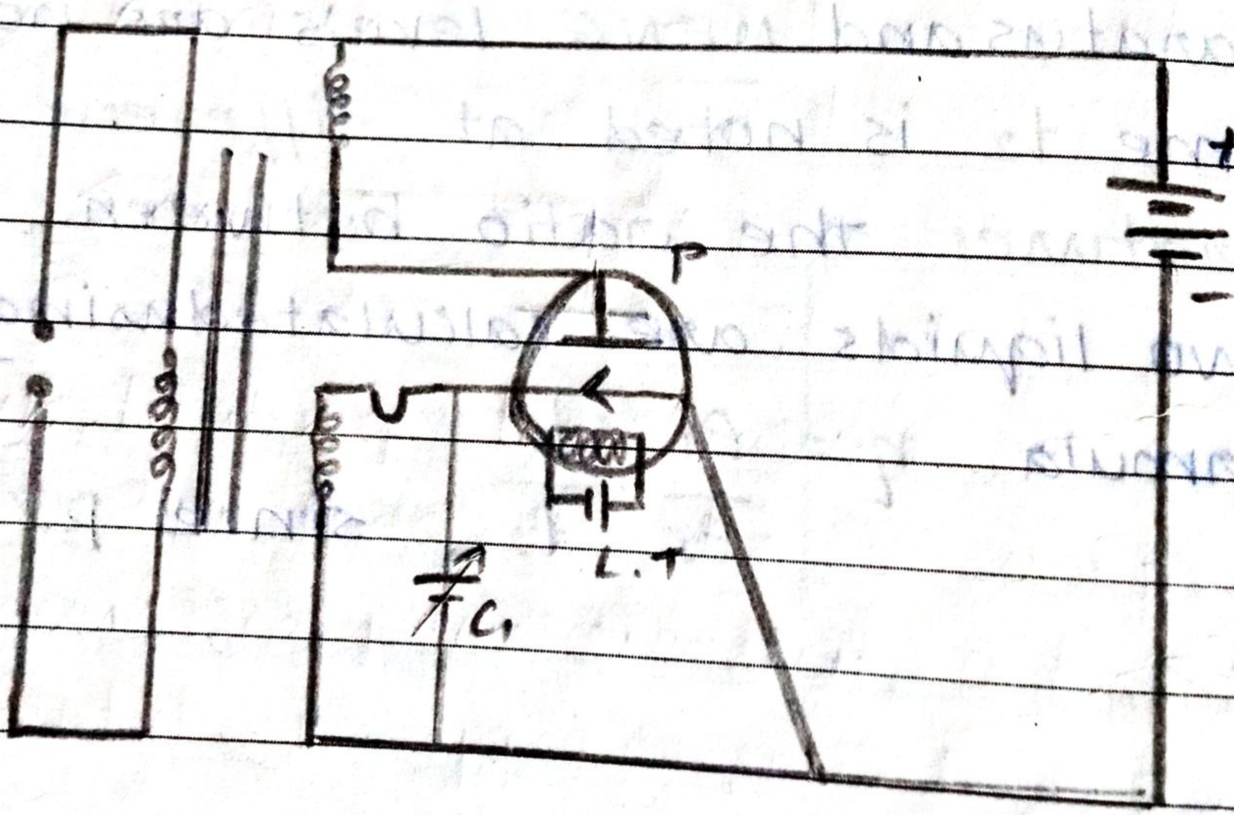

LT - Lower tension

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10m

Sabine formula

principle:

Sabine formulated reverberation formula based on the raise and fall of sound in auditorium.

Assumptions:

(i) The average energy per unit volume is uniform.

(ii) The energy is not lost in the auditorium. It's due to absorption effect of wall materials and so on.

Theory:

Consider σ is the energy confined in a unit volume. It depends on the single angle of $d\phi$.

The energy $= \frac{\sigma d\phi}{4\pi}$

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If a sound incident on a wall, at an angle θ with velocity v.

Total energy falling per second with respect to surface area of the wall $= \left(\frac{\sigma\, d\phi}{4\pi}\right)(\cos\theta)v \rightarrow ①$

Total energy falling per second with in a hemisphere $= \frac{\sigma v}{4\pi}\int \cos\theta\, d\phi \rightarrow ②$

Since $\phi = 2\pi(1-\cos\theta)$

$d\phi = 2\pi \sin\theta\, d\theta$

Substituting the values in eqn. ② and Applying the limits as per the acoustical calculation 0 to $\pi/2$,

Total energy per second $= \frac{\sigma v}{4}$

$$\Rightarrow \frac{\sigma v}{4\pi}\int_0^{\pi/2} 2\pi \sin\theta \cos\theta\, d\theta$$

$$= \frac{\sigma v}{2}\left[-\frac{\cos^2\theta}{2}\right]_0^{\pi/2}$$

$$= \sigma\frac{v}{4}$$

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Due to absorption, total energy absorbance

$= A \sigma \alpha \frac{v}{4}$

If volume of the auditorium $= V$

$\therefore$ Rate increase of energy $= Q - \frac{A\alpha\sigma V}{4} \rightarrow (3)$

$\frac{d(V\sigma)}{dt} = Q - K\sigma \rightarrow (4) \quad [\because K = \frac{A\alpha V}{4}]$

put $+\sigma = B + be^{-\beta t}$

on simplifying the equation after substitution, Average energy per unit time $\sigma = \frac{4\alpha}{A \times V}\left[1 - e^{\frac{-A\alpha v t}{4V}}\right]$

$\therefore$ Maximum value average energy per volume $= \frac{4Q}{A\alpha V}$

Decay of average energy per unit value $\sigma = \sigma_{max} e^{\frac{-A\alpha V}{\frac{t}{4V}}}$

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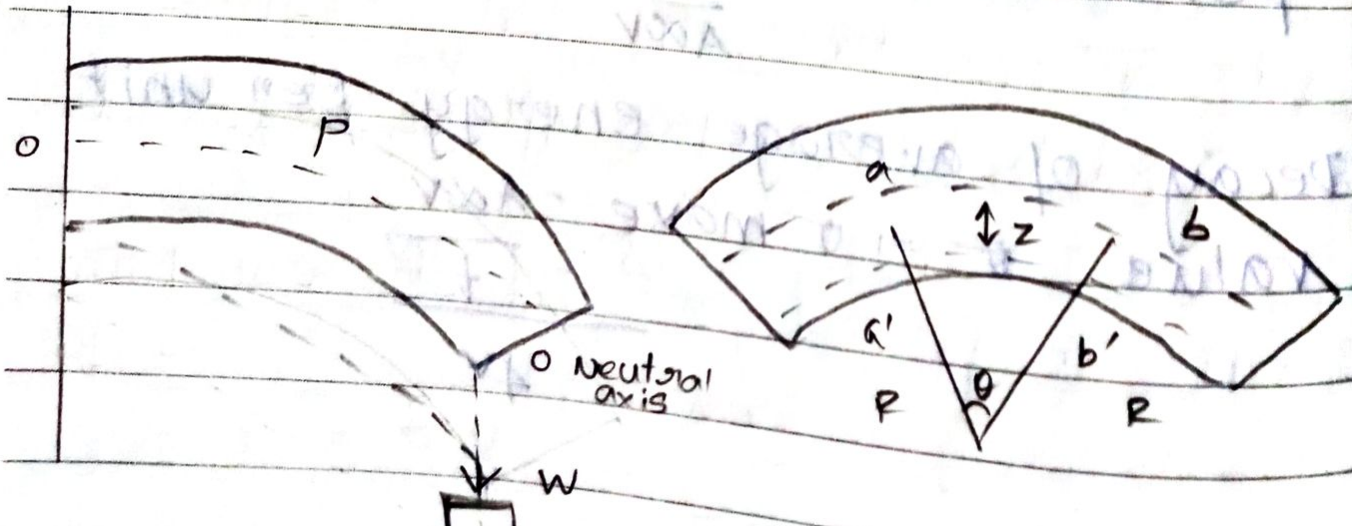
Expression for Bending moment:

consider a beam having one end fixed and another end is free. The free end is loaded by some weight w. As shown in figure, OO' represents the Neutral axis and R represents the radius of curvature.

when weight is suspended the original length varies due to the expansion and contraction. The angle subtended at this moment is θ. Due to the elastic property of the beam, we get tensile stress and tensile strain.



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Here as shown a' b' represents the variation in the extended length. z is the difference between two positions the tensile strain is equal to increase in length to the original length. The modulus of the material is represented by $\frac{\text{tensile stress}}{\text{tensile strain}}$.

Theory:

the original length = $R\theta$

the extended length = $(R+z)\theta$

Therefore tensile strain can be calculated by the increase in length with respect to original length

(ie) $a'b' - ab$

$\Rightarrow (R+z)\theta - R\theta$

$= R\theta + z\theta - R\theta$

$= z\theta$

Increase in length $= z\theta$

Tensile strain $= \frac{z\phi}{R\phi}$

$= z/R$

Now the beam having the rectangular cross section, the young's modulus of the beam is the ratio between the tensile stress and tensile strain. consider, the a surface area with respect to the filament regarding the neutral axis is δA. Therefore

$$Y = \frac{\delta A}{(z/R)}$$

From the diagram the force on the area $= \left(\frac{Vz}{R}\right)\delta A$.

therefore the moment of inertia geometrically represented by $\Sigma \delta A z^2$. From this, the quantity of $\Sigma \delta A z^2$ represents the second formation of inertial sequence.

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Therefore total moment of all the forces about the bending moment is

$$M = \frac{Y I_g}{R}$$

For a beam of rectangular cross section, bending moment

$$M = \frac{Y bd^3}{12R} \qquad \left[\because I_g = \frac{bd^3}{12} \right]$$

For a beam of circular cross section, bending moment

$$M = \frac{Y \pi R^4}{4R} \qquad \left[\because I_g = \frac{\pi R^4}{4} \right]$$

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Applications of Bernoulli's theorem to liquids

(i) Torricelli's theorem:

An ideal liquid of density ρ be filled in a vessel A. The bottom region of the vessel A provided with a narrow orifice B. The liquid escapes through B. Let p be the atmospheric pressure and h the orifice depth below the free surface of the liquid on the free surface becames zero. the velocity of efflux through B increases as height of the liquid increases. By Bernoulli's theorem. At A, v = 0

$$\frac{P}{\rho g} + h = \text{constant} \quad \rightarrow (1)$$

At B, h = 0

$$\frac{P}{\rho g} + \frac{v^2}{2g} = \text{constant} \quad \rightarrow (2)$$

From (1) & (2)

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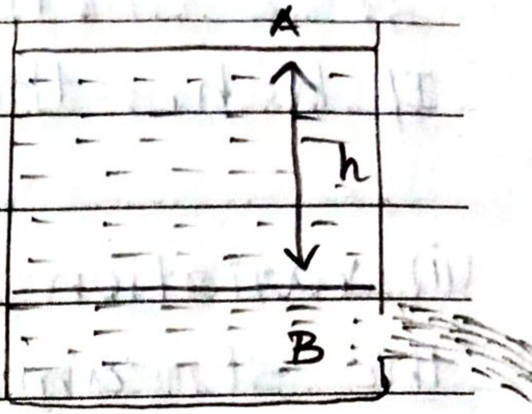
$$\frac{P}{\rho g} + h = \frac{P}{\rho g} + \frac{v^2}{2g}$$

$$h = \frac{v^2}{2g}$$

$$v = \sqrt{2gh} \rightarrow (3)$$



(ie) V represents the velocity of efflux at the orifice. The velocity of efflux of a liquid through an orifice is the same as that acquired by a freely falling body from the liquid surface to the orifice. This theorem is known as Torricelli's theorem. The liquid with parabolic path can reach the bottom with time $t = \sqrt{2h_1/g}$. The escaping liquid will strike the horizontal plane through the bottom of the vessel at a distance H, where $H = v \times t$

From (3), $H = \frac{2h_1}{g} \times \sqrt{2gh}$

$$= \sqrt{2gh} \times \sqrt{\frac{2h_1}{g}}$$

$$= \sqrt{\frac{4ghh_1}{g}}$$

$H = 2\sqrt{hh_1}$

If $h = h_1$, the range H is maximum

(ii) variation of pressure and velocity in the streamline flow of fluid through a horizontal pipe having a construction:

Let the horizontal pipe AB with water flow such that A and B are the extreme ends connected by a small neck tube C such that the end A contains a_1 and end B contains a_2 as cross sections and the velocity of liquid through A is v_1 and the velocity of liquid through B is v_2 pressure P_1 at A and P_2 at B respectively. By Bernoulli's theorem,

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + \frac{v_2^2}{2}$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_1^2 - v_2^2) \quad \text{---} (1)$$

Since $v_1 < v_2$, $P_2 < P_1$ the pressure of the liquid in C is smaller than that in the main pipe (AB). To measure

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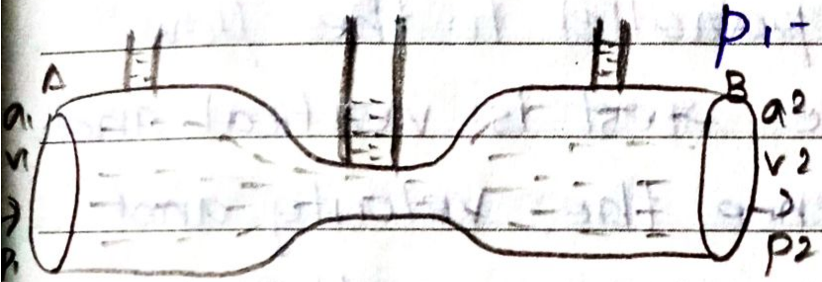
pressure difference $p_1 - p_2$ the vertical tube is attached with the main tube. If the fluid is a gas, $p_1 - p_2$ is measured by a liquid manometer. If Q be the rate of discharge of the fluid through the pipe then $v_1 = \frac{Q}{a_1}$ $v_2 = \frac{Q}{a_2}$ → ②

substitute ② in ①,

$$p_1 - p_2 = \frac{\rho}{2}\left(\frac{Q^2}{a_2^2} - \frac{Q^2}{a_1^2}\right)$$



$$Q^2 = \frac{2a_1^3 a_2^2}{\rho}\left[\frac{p_1 - p_2}{a_1^2 - a_2^2}\right]$$

$$Q = a_1 a_2 \sqrt{\frac{2(p_1 - p_2)}{\rho(a_1^2 - a_2^2)}}$$

(iii) venturimeter

(iv) pitot tube :

principle :

This device is used to measure the velocity of flow of liquids and gases through pipes when the flow is streamlined. It work under Bernoulli's principle

construction and working:

It consists of a manometric tube ending in a narrow apperture at the open ends A and B. The aperture plane of former end is horizontal and parallel to the flow direction and the rest is vertical and perpendicular. Hence, the velocity and pressure at A is same as that at every other point in the pipe. But at B the velocity of the liquid Quickly falls to zero. The pressure be P_1 and P_2 for the planes A and B end respectively. The pressure at B is rapidly raised to the maximum value. By Bernoulli's theorem,

$$\frac{P}{\rho} + \frac{v^2}{2} = 0$$

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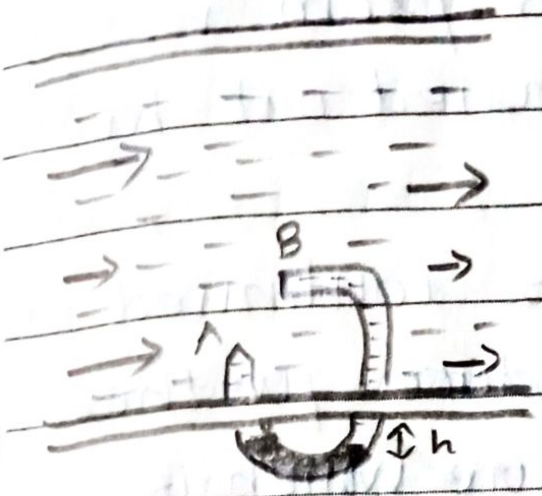


$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + 0$$

$$v_1^2 = \frac{2(P_2 - P_1)}{\rho}$$

(or) $v_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$

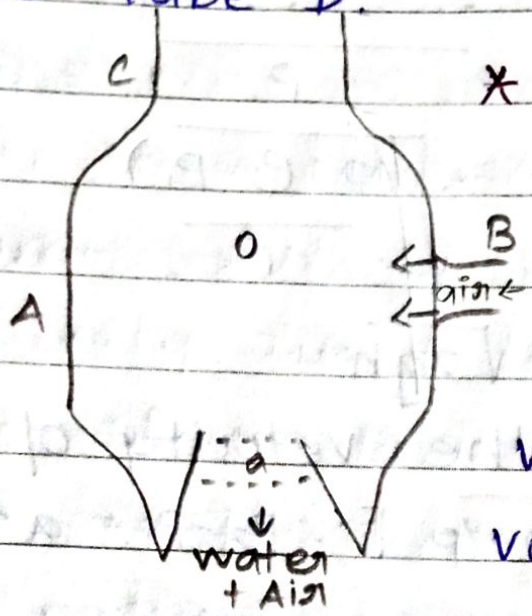
$$v_1 = \sqrt{2gh}$$

Note: To measure the velocity of gas through pipe $v_1 = \sqrt{2h\sigma g/\rho}$ [$\because P_2 - P_1 = a\sigma g$)

(v) Filter pump:

* This device is used to reduce the pressure inside the vessel.
* Let v the vessel such that B is a side tube connected with A.
* Through c water is allowed to flow to A through orifice at o. The velocity of water is high.
* The pressure low and will be further decreased below atmospheric pressure.

* Hence air rushes from v towards o, this is carried out by the tube D.



* This is continued to reduce further pressure which is slightly above the vapour pressure of water vapour in the vessel v.

(vi) vena contracta:

when a liquid from a vessel is made to flow out through an orifice of a side wall the flow is not exactly streamlined. Because flow is from all directions with varying velocities. After leaving the orifice the flow is towards the jet's centre section due to inertia. The velocity increases as the tube becomes narrow. outside the orifice the jet contracts to a neck called vena

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contracta. At this stage, the velocity equal to that given by Torricelli's theorem. The area of the vena contracta is about 0.62 times that of the orifice. The ratio between the area of the vena contracta and the area of orifice is called the coefficient of contraction. For a circular orifice the value is 0.62 ($\simeq$)

Applications of Bernoulli's theorem to gases:

(i) The orientation of the wing relative to the flow direction in air craft causes the flow lines to crowd together above the wing. This corresponds to increased velocity in this region and hence the pressure is reduced. But below the wing, the pressure is nearly equal to the atmospheric pressure. As a result, the upward force on the underside of the wing is greater than the downward force on the topside. Thus there is a net upward force or lift.

(ii) In a bunsen burner, the gas comes out of the nozzle with high velocity. Due to this the pressure in the Steam of the burner decreases. So, air from the atmosphere rushes into the burner.

(iii) During a storm, the roofs of huts are blown of without damaging hut. The blowing wind creates a low pressure on the top of the roof. A high pressure is united the roof. Due to the pressure difference, the roof

(iv) If two ping pong balls are suspended with a small seperation

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and if a vertical jet of air is sent between them, the pressure between them is reduced and the higher pressure on the outer edges of the balls causes them to move towards each other.

(v) In the paint sprayers, air will be forced through a jet with a high velocity near the mouth of the jet, a vertical tube dipped in a paint. when a lower pressure is created near the nozzle, the atmospheric pressure acting on the liquid forces it through the vertical tube in the form of a spray.

(vi) when two boats seperated by a small distance row parallel to each other along the same direction, the velocity of water between the boats becomes very large compared to that on the outer sides. (ie) the pressure in between the

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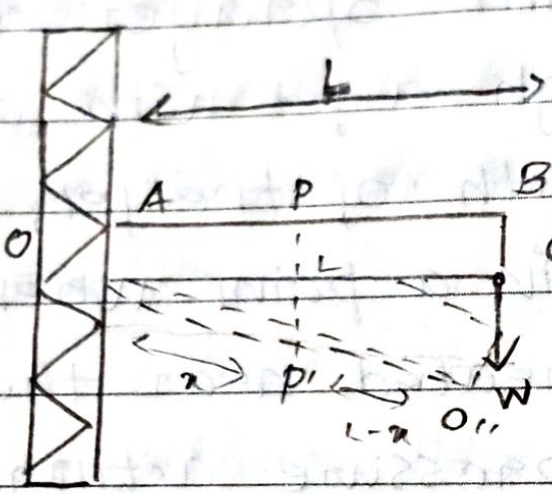
two boats gets reduced. The high pressure on the outer side pushes the boats inwards and come closer and may even collide.

cantilever



consider a cantilever in which the weight W is suspended as shown. Here we have three cases.

(i) when the weight of the cantilever is ineffective,

(ii) The weight of the cantilever is effective.

(iii) The weight of the cantilever is uniformly loaded.

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(i)

* consider AB cantilever which contains fixed end A and free end B with many sequential layers OO'.

* The length of the cantilever is L. The w weight loaded at its free end.

* Here the neutral axis OO' of the beam is changed to a new position O".

Due to the external force at a point P in which the change of depression appears. The external couple acting on the area due to the load is represented by the product of weight W and the point from which the change appears. Therefore,

The external couple $= W \times P'O''$

$= W \times (L-x) \rightarrow (1)$

Due to the equillibrium position, by the bending of moment with respect to the moment of inertia, we have,

$$W \times (L-x) = \frac{Y I_g}{R} \rightarrow (2)$$

then the curvature as we know,

$$\frac{1}{R} = \frac{(\partial^2 y/\partial x^2)}{(1+(\partial y/\partial x)^2)^{3/2}} \rightarrow ③$$

Due to the condition of cantilever under ineffective stage, the change appears is negligible. (i.e) dy/dx is small.

$$\therefore \frac{1}{R} = \frac{\partial^2 y}{\partial x^2} \rightarrow ④$$

Substituting eqn. ④ in ②, we get

$$w(L-x) = \frac{YIg}{1/(\partial^2 y/\partial x^2)} \qquad w(L-x) = \frac{YIg}{\frac{\partial x^2}{\partial^2 y}}$$

$$w(L-X) = \frac{YIg}{\partial x^2/\partial^2 y} \qquad w(L-x) = \frac{\partial^2 y}{\partial x^2} \cdot YIg$$

condition:

$$w(L-x) = \frac{\partial^2 y}{\partial x^2} YIg \qquad \frac{\partial^2 y}{\partial x^2} = \frac{w(L-x)}{YIg}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{w(L-X)}{YIg} \rightarrow ⑤$$

$$\therefore \frac{dy}{dx} = \int \frac{d^2 y}{\partial x^2} = \frac{w}{YIg}\int (L-x)$$

$$= \frac{w}{YIg}\left[Lx - \frac{x^2}{2}\right] + c \rightarrow ⑥$$

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condition:

Since, A is a fixed end and $dy/dx = 0$ at $x = 0$, integrating again

$$\frac{W}{YI_g}\left[\frac{Lx^2}{2} - \frac{x^3}{6}\right] + C_2 \rightarrow (7)$$

Therefore depression for lower end $(x = L)$ is,

$$\frac{W}{YI_g}\left[\frac{L^3}{2} - \frac{L^3}{6}\right] + C_2 \rightarrow (8)$$

where C_2 is the integral constant, therefore,

$$= \frac{W}{YI_g}\left[\frac{2L^3}{6}\right]$$

$$\frac{YI_g}{W}\left[\frac{L^3}{L^3}\right] = \frac{dx}{dy}$$

$$= \frac{W}{YI_g}\left[\frac{L^3}{3}\right]$$

$$\frac{dy}{dx}\ \frac{I_g}{W}\left[\frac{3}{L^3}\right]\ \frac{dx}{dy}\ y$$

$$y\ \frac{dx}{dx} = \frac{W}{I_g}\left[\frac{L^3}{3}\right]\left(\frac{dx}{dx}\right)$$

$$\frac{dy}{dx} = \frac{W}{YI_g}\left[\frac{L^3}{3}\right]$$

$$y = \frac{W}{I_g}\left[\frac{L^3}{3}\right]\left(\frac{dy}{dx}\right)$$

$$y = \frac{W^{1/Y}}{I_g}\left(\frac{L^3}{3}\right) \Rightarrow \frac{W}{YI_g}\left(\frac{L^3}{3}\right) \rightarrow (9)$$

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case (ii)

weight of the cantilever if effective, consider again the same case such that the load w acting at B which is equal to $L-x$. Here we are having additional weight $w(L-x)$ acting at a distance $L-x/2$ from p. Therefore the external couple applied due to depression

$$= W(L-x) + w/2\,(L-x)^2$$

$$w(L-x) + w/2\,(L-x)^2 = YIg/R$$

$$YIg\int\frac{dy^2}{dx^2} = w\int(L-x)\,dx + \frac{w}{2}\int(L-x)^2 dx$$

$$YIg\frac{dy}{dx} = w\left(Lx - \frac{x^2}{2}\right) + \frac{w}{2}\left(L x^2 - 2\frac{Lx^2}{2} + \frac{x^3}{3}\right)$$

$$YIg\int_0^y dy = w\int_0^L \left(Lx - \frac{x^2}{2}\right)dx + \frac{w}{2}\int_0^L \left(L^2 x - 2L\frac{x^2}{2} + \frac{x^3}{3}\right) dx$$

$$YIgy = \frac{WL^3}{3} + \frac{WL^4}{8}$$

(since $wL = W_0$ (weight of the beam))

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$$YI_g y = \frac{WL^3}{3} + \frac{W_0 L^3}{8}$$

$$y = \frac{L^3}{YI_g}\left(\frac{W}{3} + \frac{W_0}{8}\right)$$

case - III

$$w(L-x)\left(\frac{L-x}{2}\right) = \frac{w}{2}(L-x)^2 = YI_g \frac{dy}{dx}$$

$$YI_g\, dy = \frac{w}{2}\left(L^2x - \frac{2Lx^2}{2} + \frac{x^3}{3}\right)dx$$

$$YI_g \int dy = \frac{w}{2}\int_0^L \left(L^2x - Lx^2 + \frac{x^3}{3}\right)dx$$

$$= \frac{w}{2}\left[\frac{L^4}{2} - \frac{L^4}{3} + \frac{L^4}{12}\right]$$

$$= \frac{wL^4}{2}\left[\frac{6-4+1}{12}\right]$$

$$= \frac{wL^4}{2}\left[\frac{3}{12}\right]$$

$$\therefore YI_g y = \frac{wL}{2}\left(\frac{L^3}{4}\right)$$

Since $wL = W$

$$y = \frac{WL^3}{8YI_g}$$

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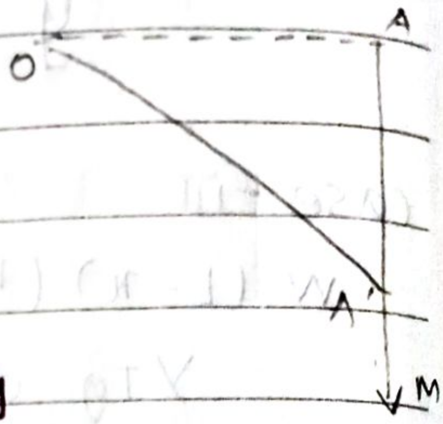
Oscillation of a cantilever

To find the period of oscillation

$$y = \frac{Wl^3}{3EAK^2}$$



(or) $$W = \frac{3EAK^2}{l^3} y$$

$$\text{Elastic reaction} = M.\frac{d^2y}{dt^2}$$

Here, M is the mass of the weight w, $\frac{d^2y}{dt^2}$ is the acceleration but $\frac{3EAK^2}{Ml^3}$ (A constant)

If therefore, excutes a S.MM of time period

$$T = 2\pi\sqrt{\frac{ML^3}{3EAK^2}}$$

$$T = 2\pi\sqrt{\frac{(M+1/3m)L^3}{3EAK^2}}$$

the period of oscillation T_1 is calculated:

$$T_1 = 2\pi\sqrt{\frac{(m+1/3m)l^3}{3EAK^2}}$$

$$T_1^2 = 4\pi^2\frac{(m+1/3m)l^3}{3EAK^2} \rightarrow ①$$

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similarly, the period, T_2 with a load M_2 is found.

$$T_2^2 = \frac{4\pi^2 (m_2 + 1/3 m) l^3}{3EAK^2} \rightarrow (2)$$

(2) - (1)

$$T_2^2 - T_1^2 = \frac{4\pi^2 (M_2 - M_1) l^3}{3EAK^2}$$

$$E = \frac{4\pi^2 (M_2 - M_1) l^3}{3AK^2 (T_2^2 - T_1^2)}$$

$$\therefore AK^2 = \frac{bd^3}{12}, \quad E = \frac{4\pi^2 (M_2 - M_1) l^3}{3 (bd^3/12) T_2^2 - T_1^2}$$

$$E = \frac{16\pi^2 l^3 (M_2 - M_1)}{bd^3 (T_2^2 - T_1^2)}$$

Rigidity Modulus - Torsion pendulum:

$$T_1 = 2\pi \sqrt{I_1/L}$$

$$T_1^2 = \frac{4\pi^2 I_1}{C}$$

$I_1 = I_0 + 2i + 2md_1^2$ (parallel axis theorem)

$$T_1^2 = \frac{4\pi^2}{C} [I_0 + 2i + 2md_1^2] \rightarrow (1)$$

$$T_2^2 = \frac{4\pi^2}{c}\left[I_0 + 2i + 2md_2^2\right] \rightarrow (2)$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{c} 2m\left(d_2^2 - d_1^2\right) \rightarrow (3)$$

$$\left[\quad c = \frac{4\pi^2 \, 2m(d_2^2 - d_1^2)}{T_2^2 - T_1^2} \quad\right]$$

$$c = \frac{\pi G a^4}{2L}$$

$$T_2^2 - T_1^2 = \frac{4\pi^2 \, 2m\left(d_2^2 - d_1^2\right) 2L}{\pi G a^4}$$

$$G = \frac{16\pi mL\left(d_2^2 - d_1^2\right)}{a^4\left(T_2^2 - T_1^2\right)}$$

Moment of inertia of the disc

$$T_0^2 = \frac{4\pi^2 I_0}{c}$$

$$I_0 = \frac{cT_0^2}{4\pi^2} \rightarrow (5)$$

From eqn. (3), in (5),

$$I_0 = \frac{4\pi^2 \, 2m\left(d_2^2 - d_1^2\right)}{\left(T_2^2 - T_1^2\right)} \frac{T_0^2}{4\pi^2}$$

$$I_0 = \frac{2m\left(d_2^2 - d_1^2\right) T_0^2}{\left(T_2^2 - T_1^2\right)}$$

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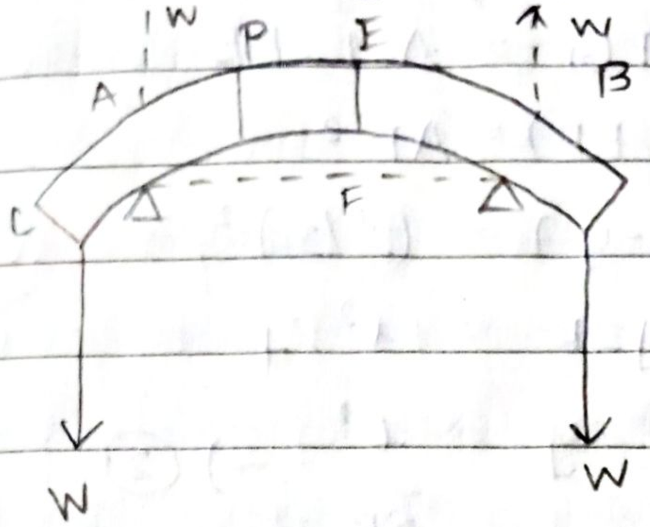
uniform bending - Expression for elevation:



$AC = BD = a$

$AB = l$

Elevation of mid point $EF = y$

The external bending moment with respect to p,

$= W.CP - W.AP$

$= W(CP - AP)$

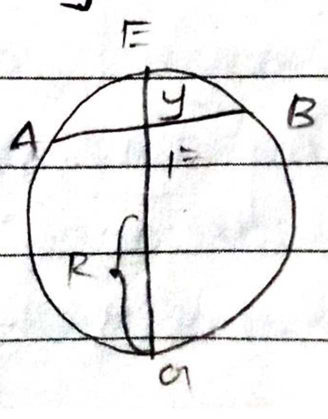
$= W(AC)$

$= Wc$

Internal bending moment EI_g/R.

$$\frac{EI_g}{R} = W/a$$

$$\frac{1}{R} = \frac{Wa}{EI_g} \rightarrow (1)$$



From properties of circle,

$$E.F.FG = A.F.FB$$

$$EF(2R - EF) = AF^2$$

$$y(2R - y) = (l/2)^2$$

$$y2R = l^2/4$$

$$y = \frac{l^2}{8R} \rightarrow (2)$$

sub. 1/R from (1),

$$\frac{8y}{l^2} = \frac{Wa}{EIg}$$

$$y = \frac{Wal^2}{8EIg}$$

For rectangular beam,

$$Ig = \frac{bd^3}{12}$$

$$y = \frac{mgal^2}{8E\frac{bd^3}{12}} = \frac{12mgal^2}{8Ebl^3}$$

$$y = \frac{3mgal^2}{2Ebd^3}$$

$$E = \frac{3mgal^2}{2bd^3y}$$

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Lubrication:

A lubricant is a substance which is ~~re~~dused to reduce friction. The lubricant forms a thin layer between the two surfaces in contact. It also fills the depressions in the surface of contacts and surface of contacts and reduces friction considerably.

The knowledge of viscosity has wide applications in the field of lubrications. The knowledge of viscosity and its with temperature help us to use a suitable lubricant. For a certain machine liquids moderate as good lubricants for light.

Machinery such as bicycles and serving machines thin oils leg clock oil with low viscosity are used. In heavy and fast moving machinery

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solids or thick highly viscous oils leg gease on oils are used

5m/ Work done in stretching a wire

Let a force f act on a wire of length and area of cross section A. the increase in length.

young's modulus = $E = \frac{Fl}{A\lambda}$

(or) $F = \frac{EA\lambda}{L}$

work done in producing a stretching

$d\omega = F.d\lambda = \frac{EA\lambda d\lambda}{L}$

Total work done to produce a stretching of the wire from 0 to l

$W = \int_0^l F dl$

$\Rightarrow \int_0^l \frac{EA\lambda}{L} dl = \frac{EA}{L}\left[\frac{\lambda^2}{2}\right]_0^l = \frac{1}{2}\frac{EAl^2}{L}$

$= \frac{1}{2}\frac{FAl}{L} = \frac{1}{2}Fl$

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$= \frac{1}{2} \times$ stretching force x Elongation period produced.

work done in producing a total work done to produce stretching to the wire from o to l $\Bigg\} = W = \int_0^l F\,dl$

$= \int_0^l \frac{EAl}{L}\,dl = \frac{EA}{L}\left[\frac{l^2}{2}\right]_0^l$

$= \frac{1}{2}\frac{EAl^2}{L} = \frac{1}{2}\frac{EAl}{L}\,l$

$= \frac{1}{2} Fl$

$= \frac{1}{2}$ stretching force x Elongation produced.

Now, volume of the wire $= A.L$

Hence, work done per unit volume of the wire $= \frac{1}{2} Fl$

$= 1/2\ F/A\ l/L$

$= 1/2$ stress x strain.

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Twisting torque on the whole cylinder $C = \int_0^a \frac{2\pi G\theta}{l} x^3 \, dx$

$$C = \frac{\pi G a^4 \theta}{2L}$$

the torque per unit twist $= C = \frac{\pi G a^4}{2L}$