I Bcom (BM)

Business TOOLS for Decision making

Unit IV. Interpolation.

Meaning :- Interpolation is the estimation. of the most likely estimate in given Conditions The technique of estimating a past figure from. The technique of estimating a Past figure from a given set of data is termed as inter-polation.

In business the data for the intermediate period may be needed for some ~~one~~. analysis. It is essential to use the available data to estimate the figure for the intermediate Period.

Method of of interpolation.

a, Graphic method. b) Algebraic method.

The following are some of the important and more popular method.

i, Binomial expansion method.
ii, Newton's Method
iii, Lagranges method.
iv, Methods of Parabolic Curve filing

Assumption of interpolation.

a, There should not be any sudden jump or fall in the Series from period to period.

b, The trend of changes is assumed to be Consistent, otherwise the values estimated will not accurately represent the data.

c, A good number of data should be available for Correct estimation.

d, The relation between the given variable and the variable to be estimated is assumed to be polynominal.

e, Rate of change should be in uniform' order from one period to another.

f, The gap between the values given are equal.

Importance of Interpolation.

In business and economic field. data Cannot be Compiled, and listed Continously. or at shorter intervals such. Compilation is time Consuming and expensive Any figure needed from intermediate time Points or Periods Can esily be estimated. through the interpolation.

The data relating to be records Lost or destroyed may be estimated by the interpolation. methods to Complete records..

Newton's method of interpolation :-

The newton's method Can be stated as

(a) Newton - Gregory forward interpolation. formula.

$$Yu = Yo + U\Delta 0^1 + \frac{U(U-1)^2}{2!}\Delta 0^2 \frac{U(U-1)(U-2)}{3}\Delta^3 yn + \ldots$$

where $U = \frac{X - X0}{h}$

x = The value of x varriable to be interpolated.

Xo = The value of x varriable at the orgin

h = difference between the adjoining values of x

b. The newton gregory. backward interpolation formula

$$Yv = Yn + V\Delta yn + \frac{V(V+1)}{2!}\Delta^2 yn + \frac{V(V+1)(V+2)}{3!}\Delta^3 yn +$$

where $V = \frac{x - (xo + nh)}{h}$

This formula is mainly helpful if we have to interpolate y_x for the value of x near the extreme end of a set of tabulated values..

The following table gives the expectation life (y) at age x Compute the expectation of life (y) at age x. Compute the expectation of life at age 12 by using Newton's forward interpolation formula

x	10	15	20	25	30	35
y	35.4	35.2.	29.1	26.0	23.1	20.4

The year of interpolation is in the begining of the table and the choice of Newton's forward. difference formula is appropriate.

The difference table constructed as fallows

Age	Expectation of Life	First	Second	Third	Fourth	Fifth
x	y	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
10 x_0	35.4 y_0					
15 x_1	32..2 y_1	-3.2	0.10	-0.10		
		3.1	0.00		0.30	
20 x_2	29.1 y_2	3-1		+0.20		-0.5
			0.20		-0.20	
25 x_3	26.0 y_3					
		2.9		0.00		
30 x_4	23.1 y_4		0.20			
		2.7				
35 x_5	20.4 y_5					

$$u = \frac{x - x_0}{h} = \frac{12 - 10}{5} = 0.4$$

$$y_{0.4} = 35.4 + 0.4 \times (-3.2) + \frac{0.4(0.4-1)}{1\times 2} \times (0.10)$$

$$+ \frac{0.4\,(0.4-1)\,(0.4-2)}{1\times 2\times 3} \times (-6.10)$$

$$+ \frac{0.4\,(.0.4-1)\,(0.4-2)\,(0.4-3)}{1\times 2\times 3\times 4} \times 0.3 +$$

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$$\frac{0.4\,(0.4-1)\,(0.4-2)\,(0.4-3)\,(0.4-4)}{1\times 2\times 3\times 4\times 5}$$

$= 35.4 - 1.28 + 0.012 - 0.0064 + 0.01248 - 0.014976$

$= 34.123104$.

Unit V : Index numbers

An Index number is a statistical measures designed to show changes in a variable or a group of related variables with respect to time geographic location or other charrecteristics such as income, profession etc.

Index numbers which shows changes in price or quantity in one time compared with another alone The year for which index number is calculated is called the current year. The year with which the current year is compared is called the base year methods.

- unweighted
  - Simple Aggregative method
  - Simple Average of Relative method
- weighted.
  - Weighted Aggregative Method
  - Weighted Average of Relatives Method

P_0 - Price of Commodity in the base year.

P_1 - Price of Commodity in the Current year

q_0 - quantity of Commodity in the base year

q_1 - quantity of Commodity in the Current year.

p - Price of Commodity.

q - quantity of Commodity

V (or) W - weight of a Commodity.

(4)

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Simple (or) unweighted Aggregative method.

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

Simple or unweighted Average of Relative method Price Index (P_{01}) (i) using AM, $P_{01} = \frac{\Sigma P}{N}$

(ii) using GM P_{01} = Antilog $\frac{\Sigma \log P}{N}$

Weighted Aggregative method.

Price index P_{01}

1. Laspeyre's Formula $P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$

2. Paasche's formula $P_{01}^{P} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$

3. Fisher's Formula $P_{01}^{F} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

4. Marshall. Edgeworth Formula $= \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$

5. Bowley's Formula $P_{01} = \frac{P_{01}^{L} + P_{01}^{P}}{2}$

6. Kelly's Formula $P_{01}^{K} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$

1. Compute Index numbers by (i) Laspeyre's (ii) Paasche's method, (iii) Bowley's method. (iv) Fisher's method and (v) Marshall Edgeworth method.

Commodities	1999 price	1999 quantity	1998 price	1998 quantity
A	4	6	2	8
B	6	5	5	10
C	5	10	4	14
D	2	13	2	19

Solution:-

Com	p_1	q_1	p_0	q_0	p_0q_0	p_1q_0	p_0q_1	p_1q_1
A	4	6	2	5	16	32	12	24
B	6	5	5	10	50	60	25	30
C	5	10	4	14	56	70	40	50
D	2	13	2	19	38	38	26	26.

$\Sigma p_0q_0 = \Sigma p_1q_0 \quad \Sigma p_0q_1 \quad \Sigma p_1q_1$

$160 = 200 = 103 = 130$

1) Laspreyres method. $P_{01} = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100 = \frac{200}{160} \times 100 = 125$

2. Paache's method $P_{01} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100 = \frac{130}{103} \times 100 = 126.21$

3. Bowley's method $P_{01} = \frac{P_{01}^L + P_{01}^P}{2} = \frac{125 + 126.21}{2} = 125.21$

4. Fisher's method $P_{01} = \sqrt{P_{01}^L \times P_{01}^P} = \sqrt{125 \times 126.21}$

$= 125.60$

5. Marshall - Edgeworth.

$$P_{01} = \frac{\Sigma p_1 + \Sigma p_1q_1}{\Sigma p_0 + \Sigma p_0q_1} \times 100 \quad \frac{200+130}{160+103} = 125.48$$

Test of Consistency

(i) Time Reversal Test.

$P_{01} \times P_{10} = 1$

(ii) Factor Reversal Test.

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}.$$

Compute Fisher's ideal index from the following data and show how it ~~statistics~~ satisfies Time Reversal Test and factor Reversal Test.

Commodity	Base year Price	Base year Qty	Current year Price	Current year Qty.
A	12	10	20	12
B	4	20	4	24
C	8	12	12	15
D	20	6	24	2

Commodity	P_0	q_0	P_1	q_1	P_0q_0	P_0q_1	P_0q_0	P_1q_0
A	12	10	20	12	12	120	144	240
B	4	20	4	24	24	80	96	96
C	8	12	12	15	15	96	120	180
D	20	6	24	2	2	120	40	48

$\Sigma p_0q_0 = \Sigma p_0q_1 \quad \Sigma p_1q_0 \quad \Sigma p_1q_1$

$416 = 400 \cdot 568 \quad 564$

Fisher ideal Index number

$$P_{01} = \sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0} \times \frac{\Sigma P_1q_1}{\Sigma P_0q_1}} \times 100$$

$$= \sqrt{\frac{568}{416} \times \frac{564}{400}} \times 100$$

$$= 138.75.$$

Time Reversal Test

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1 \times \Sigma p_0 q_0}{\Sigma p_1 q_1 \times \Sigma p_1 q_0}} = 0.7207$$

$P_{01} = 138.75$

Now $P_{01} \times P_{10} = 1.3875 \times 0.7201 = 0.9999$
$= 1$

Factor Reversal Test

$$P_{01} \times q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} =$$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \quad \frac{568}{416} \times \frac{564}{400}$$

$$q_{01} = \sqrt{\frac{q_1 p_0}{q_0 p_0} \times \frac{q_1 p_1}{q_0 p_1}} = \frac{400}{416} \times \frac{564}{568}$$

$$P_{01} \times q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$\frac{\cancel{568}}{416} \times \frac{564}{\cancel{400}} \times \frac{\cancel{400}}{416} \times \frac{564}{\cancel{568}} = \frac{564}{416}$$

$$P_{01} \times q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} = \frac{564}{416}.$$

Cost of Living index :- Cost of Living index number shows the impact of changes in the prices of a number of Commodeties and Services on a Particular class of the people in the Current year in Compassion with the base year. Cost of Living index number is also known as Consumer Price Index number.

Construct Cost of Index
1., Aggregate Expenditure method or weighted Aggregatives method.
(ii) Family Budget method or weighted Averages of Relative method.

Cost of Living Index number $= \frac{\Sigma wp}{\Sigma w}$.

Constructs the Price index number for 1982 on the basis of 1981 from the fallowing data using.

i, Aggregate expenditure method.
ii, Family budget Method.

Commodity.	quantity Consumed	Price 1981	Price 1982
A	6	5.75	6.00
B	6	5.00	8.00
C	1	6.00	9.00
D	6	8.00	10.00
E	4	2.00	1.50
F	1	20.00	15.00

Commodity.	q_0	P_0	P_1	P_1q_0	$V=P_0q_0$	$P=\frac{P_1}{P_0}\times 100$	PV
A	6	5.75	6.00	36.00	34.50	104.35	3600
B	6	5.00	8.00	48.00	30.00	1600.00	4800
C	1	6.00	9.00	9.00	6.00	150.00	900
D	6	8.00	10.00	60.00	48.00	125.00	6000
E	4	2.00	1.50	6.00	8.00	75.00	600
F	1	20.00	15.00	15.00	20.00	75.00	1500
				174.00	146.50		174.00

By. Aggregate expenditure method.

$$P_{10} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100 = \frac{174.00}{146.50} \times 100 = 118.77$$

Family Badget. method

$$= \frac{\Sigma PV}{\Sigma V} = 118.77$$

Compute Cost of Living index numbers

Items	Food	Fuel	clothing	Rent	others
Indexnumbers	852	220	230	160	190
Weights	48	10	8	12	15

solution.

Items	index Numbers (P)	weights W	WP
Food	352	48	16896
Fuel	220	10	2200
Clothing	230	8	1840
Rent	160	12	1920
others	190	15	2850
Total	-	ΣW = 93	ΣWP = 25706

Cost of Living Index number

$$P_{01} = \frac{\Sigma WP}{\Sigma W}$$

$$= \frac{25706}{93}$$

$$= 276.41.$$

Calculate Price index number using Kelly's Formula.

Commodity	A	B	C	D	E
Quantity	10	15	15	20	5
Price in 2000(Rs.)	100	15	70	20	5
Price in 2002 (Rs)	120	20	60	30	7

solution:

Com.	Qty	Price in Rs.			
	q	2000 (P_0)	2002 (P_1)	P_0q	P_1q
A	10	100	120	1000	1200
B	15	15	20	225	300
C	15	70	60	1050	900
D	20	20	30	400	600
E	5	5	7	25	35
Total	-	-	-	$\Sigma P_0q = 2700$	$\Sigma P_1q = 3035$

By Kelly's Formula

$$P_{01} = \frac{\Sigma P_1 q}{\Sigma P_0 q} \times 100$$

$$= \frac{3035}{2700} \times 10$$

$$= 112.41.$$