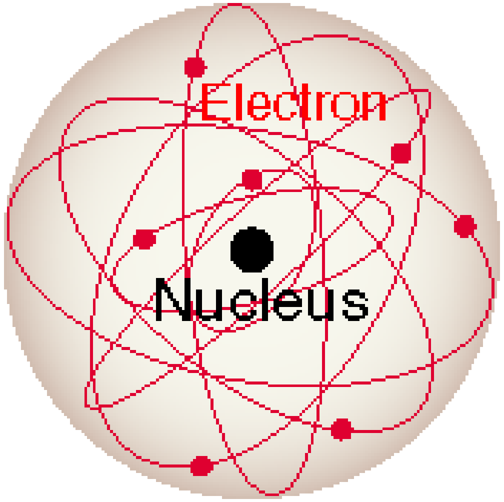
NUCLEAR PHYSICS



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UNIT I

General Properties of Nuclei and Nuclear Models

Constituents of nuclei - Classification of nuclei - Nuclear mass and binding energy - Binding energy and stability of nucleus, Mass defect and Packing fraction, Binding fraction Vs Mass number curve - Nuclear size – Nuclear spin-nuclear energy levels - Nuclear magnetic moment --Parity of nuclei - Nuclear forces - Yukawa’s model of nuclear force.

Nuclear Models - Liquid drop model, Semi-empirical mass formula – Shell model- Salient features of shell model.

INTRODUCTION

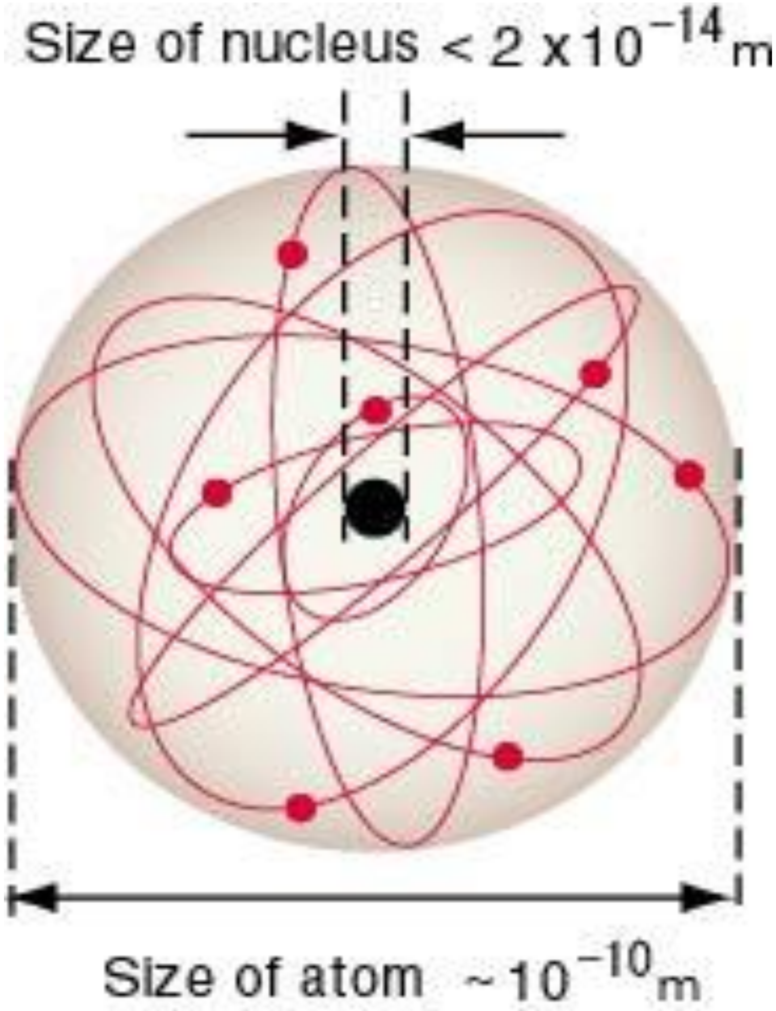
- E. Rutherford (UK, 1871-1937) investigated whether the experimental results of the alpha particle scattering could be explained well by the Rutherford atomic model.

- He derived the famous Rutherford scattering formula standing on the viewpoint of the Rutherford model and he found that the results of this formula fit well to the experimental data (1911).

Rutherford assumed that the total positive charge in an atom, $+Ze$, concentrates on the central point of the atom, i.e., the **nucleus**, and the incident alpha particle is scattered with a **repulsive Coulomb force** exerted by this nuclear point charge.

From this Rutherford's analysis, it was made clear that the almost total mass of the atom is carried by the nucleus. On the other hand, the size of the nucleus is extremely small compared with the whole size of the atom; it is smaller than 1/10000 of size of the whole atom.

This is a schematic image of the Rutherford atomic model.



The size of the nucleus is extremely small compared with the whole atom.

Nucleus of the Hydrogen Atom = Proton

Rutherford thought that the nucleus of the lightest atom, i.e. **hydrogen nucleus**, is one of the fundamental particles in **the Nuclear World**, i.e. the World of the Atomic Nucleus.

He named this particle **"proton"**, because Greek *proton* means "first". Namely, the proton is none other than the hydrogen ion, so that its electric charge is +*e*.

It was made clear by Rutherford's experiment of the artificial transmutation of elements that the proton is one of the fundamental constituents of atomic nuclei (1919).

The Constituents of Nuclei

- Up to the 1930s, people had thought that the atomic nucleus consists of protons and electrons. For example, they considered that the nitrogen 14 of the atomic number $Z = 7$ consists of 14 protons and 7 electrons.

- Just after the discovery of the neutron by Chadwick, D. Ivanenko (Soviet Union) and W. K. Heisenberg (Germany, 1901-76) independently proposed an idea that "the atomic nucleus consists of protons and neutrons" (1932).

- This was nothing else than the first step of Nuclear Physics. Hereafter this idea has been established; namely, a nucleus of an atomic number Z is a composite many-particle system consisting of Z protons and N neutrons.

- Accordingly, for example, the nitrogen 14 nucleus of the atomic number 7 consists of 7 protons and 7 neutrons. And we can easily understand that there exist some nuclei (isotopes) of the same atomic number Z but different neutron numbers N.

The Nucleon

- The proton and the neutron have almost the same mass. Different is their electric charges.
- The properties are listed in the following **Table**.

Properties	Proton	Neutron
Charge(e)	$+1$	0
Mass (MeV/c^2)	938.2723	939.5656
Spin ($\hbar$)	1/2	1/2
Life Time	$> 10^{31}$ year	887 s

From these properties, **Heisenberg** and **E. P. Wigner** thought that these two kinds of particles are of two different **states** of the same particle, called **"nucleon"** today.

We can therefore say, in a word, that the constituent of the atomic nucleus is **nucleon**.

The nucleus consists of the elementary particles, protons and neutrons which are known as nucleons. A proton has positive charge of the same magnitude as that of electron and its rest mass is about 1836 times the mass of an electron.

- A nucleus of an element is represented as $_{Z}X^{A}$, where X is the chemical symbol of the element. Z represents the atomic number which is equal to the number of protons and A, the mass number which is equal to the total number of protons and neutrons.

Classification of nuclei

(i) Isotopes

(ii) Isobars

(iii) Isotones

(iv) Isomers

(v) Mirror nuclei

(i) Isotopes

- Isotopes are atoms of the same element having the same atomic number Z but different mass number A.
- The nuclei $_1H^1$, $_1H^2$ and $_1H^3$ are the isotopes of hydrogen.
- In other words isotopes of an element contain the same number of protons but different number of neutrons.
- As the atoms of isotopes have identical electronic structure, they have identical chemical properties and placed in the same location in the periodic table.

(ii) Isobars

- **Isobars are atoms of different elements having the same mass number A, but different atomic number Z.**
- **The nuclei $_8O^{16}$ and $_7N^{16}$ represent two isobars. Since isobars are atoms of different elements, they have different physical and chemical properties.**

(iii) Isotones

- **Isotones are atoms of different elements having the same number of neutrons.**

- **$_6C^{14}$ and $_8O^{16}$ are some examples of isotones.**

(iv) Isomers

- **There are atoms, which have the same Z and same A, but differ from one another in their nuclear energy states and exhibit differences in their internal structure.**

- **These nuclei are distinguished by their different life times. Such nuclei are called isomeric nuclei or isomers.**

(v) Mirror nuclei

- **Nuclei, having the same mass number A, but with the proton and neutron number interchanged (that is, the number protons in one is equal to the number of neutrons in the other are called mirror nuclei.**

General properties of nucleus

Nuclear size

- According to Rutherford's $\propto$−particle scattering experiment, the distance of the closest approach of $\propto$ − particle to the nucleus was taken as a measure of nuclear radius, which is approximately 10^{-15}m.
- If the nucleus is assumed to be spherical, an empirical relation is found to hold good between the radius of the nucleus R and its mass number A.

  It is given by

- $R \propto A^{1/3}$
- $R = r_o A^{1/3}$
- where r_o is the constant of proportionality and is equal to 1.3 F (1 Fermi, F = 10^{-15} m)

Nuclear density

- The nuclear density ρ_N can be calculated from the mass and size of the nucleus.
- ρ_N = Nuclear mass / Nuclear volume
- Nuclear mass = Am_N
- where, A = mass number
- and m_N = mass of one nucleon and is approximately equal to 1.67×10^{-27} kg
- Nuclear volume = $4/3\ \pi R^3$
- $\rho_N = m_N / (\ 4/3\ \pi r_0^3)$
- Substituting the known values, the nuclear density is calculated as 1.816×10^{17} kg m^{-3} which is almost a constant for all the nuclei irrespective of its size.

- The high value of the nuclear density shows that the nuclear matter is in an extremely compressed state.

Nuclear charge

- **The charge of a nucleus is due to the protons present in it. Each proton has a positive charge equal to 1.6 x 10^{-19} C.**

- **The nuclear charge = Ze,**

  where Z is the atomic number.

Nuclear mass

- As the nucleus contains protons and neutrons, the mass of the nucleus is assumed to be the mass of its constituents.
- Assumed nuclear mass = $Zm_P + Nm_n$,

- where m_p and m_n are the mass of a proton and a neutron respectively. However, from the measurement of mass by mass spectrometers, it is found that the mass of a stable nucleus (m) is less than the total mass of the nucleons.

- i.e mass of a nucleus, $m < (Zm_p + Nm_n)$ $Zm_p + Nm_N - m = \Delta m$

- where Δm is the mass defect

- Thus, the difference in the total mass of the nucleons and the actual mass of the nucleus is known as the mass defect.

- Note : In any mass spectrometer, it is possible to determine only the mass of the atom, which includes the mass of Z electrons. If M represents the mass of the atom, then the mass defect can be written as

- $\Delta m = Zm_P + Nm_n + Zm_e - M$
- $= Zm_H + Nm_n - M$
- where m_H represents the mass of one hydrogen atom

Atomic mass unit

- It is convenient to express the mass of a nucleus in atomic mass unit (amu), though the unit of mass is kg. One atomic mass unit is considered as one twelfth of the mass of carbon atom $_6C^{12}$. Carbon of atomic number 6 and mass number 12 has mass equal to 12 amu.

- 1 amu = 1.66×10^{-27} kg
- The mass of a proton, $m_p = 1.007276$ amu

- This is equal to the difference in mass of the hydrogen atom which is 1.007825 amu and the mass of electron.
- The mass of a neutron, $m_n = 1.008665$ amu

- The energy equivalence of one amu can be calculated in electron-volt

- Einstein's mass energy relation is, $E = mc^2$ Here, m = 1 amu = 1.66×10^{-27} kg
- $c = 3 \times 10^8\ ms^{-1}$

- $E = 1.66 \times 10^{-27}\ kg\ (3 \times 10^8)^2\ J$

- One electron-volt (eV) is defined as the energy of an electron when it is accelerated through a potential difference of 1 volt.

- 1 eV = 1.6×10^{-19} joule
- Hence, $E = 1.66 \times 10^{-27} \times (3 \times 10^{8})^2 / 1.6 \times 10^{-19}$ eV
- = = 931 million electronvolt = 931 MeV
- Thus, energy equivalent of 1 amu = 931 MeV

NUCLEAR BINDINGN ENERGY

The neutrons and protons within a nucleus are held together by strong attractive forces among the nucleons.

In order to break a nucleus into its constituents, one has to apply some energy against the attractive forces.

This energy is known as the BINDING ENERGY of the nucleus and it is defined as the "the energy required to decompose a nucleus into its constituents".

Binding energy

- **When the protons and neutrons combine to form a nucleus, the mass that disappears (mass defect, Δm) is converted into an equivalent amount of energy (Δmc^2). This energy is called the binding energy of the nucleus.**

$$\text{Binding energy} = [Zm_P + Nm_n - m]\, c^2$$

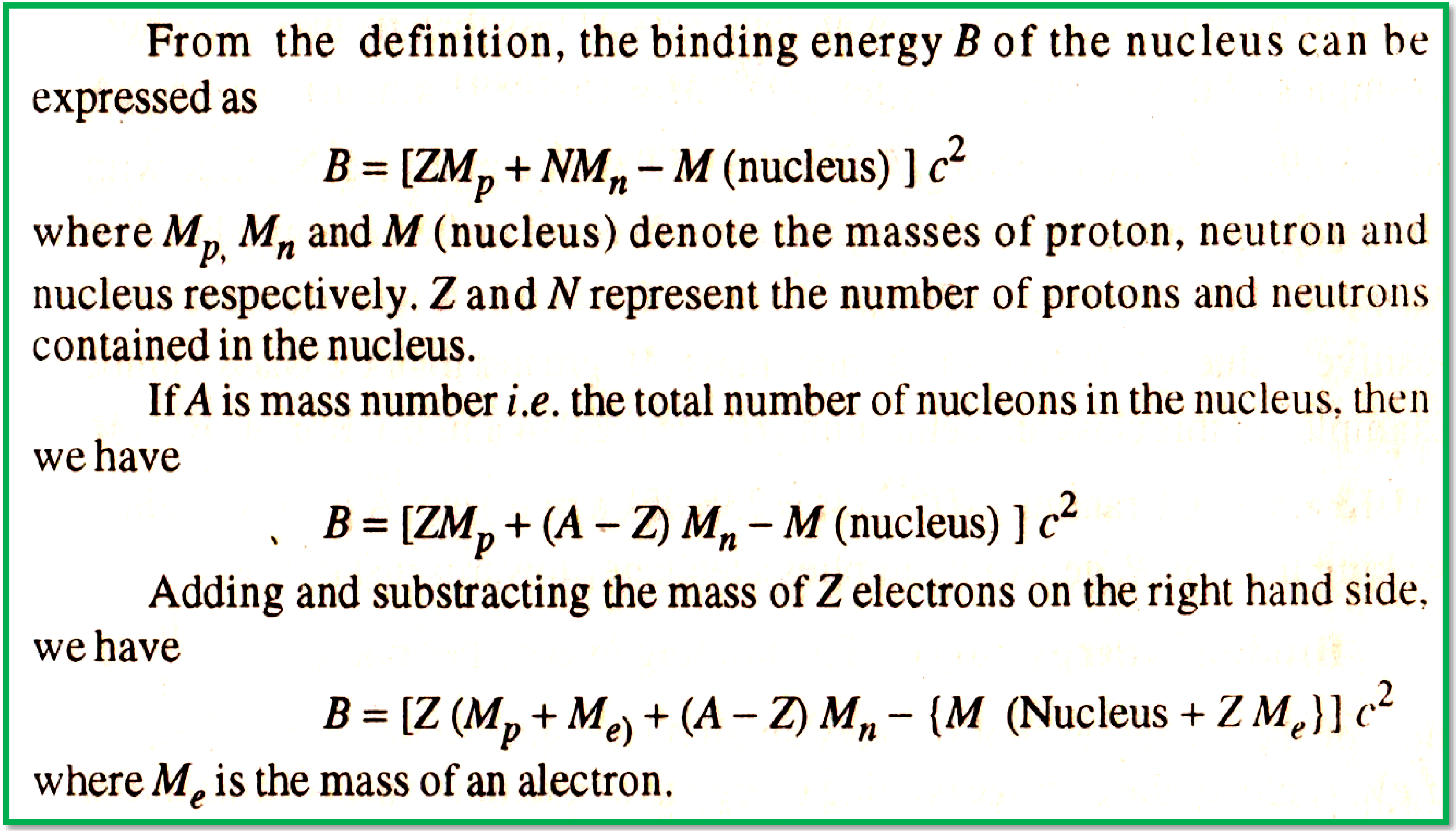
$$= \Delta m\, c^2$$

- **The binding energy of a nucleus determines its stability against disintegration. In other words, if the binding energy is large, the nucleus is stable and vice versa.**

The binding energy per nucleon is

***BE/ A =* Binding energy of the nucleus / Total number of nucleons**

- **It is found that the binding energy per nucleon varies from element to element.**



From the definition, the binding energy B of the nucleus can be expressed as

$$B = [ZM_p + NM_n - M\,(\text{nucleus})\,]\,c^2$$

where M_p, M_n and M (nucleus) denote the masses of proton, neutron and nucleus respectively. Z and N represent the number of protons and neutrons contained in the nucleus.

If A is mass number *i.e.* the total number of nucleons in the nucleus, then we have

$$B = [ZM_p + (A - Z)\,M_n - M\,(\text{nucleus})\,]\,c^2$$

Adding and substracting the mass of Z electrons on the right hand side, we have

$$B = [Z\,(M_p + M_e) + (A - Z)\,M_n - \{M\ \ (\text{Nucleus} + Z\,M_e\}]\,c^2$$

where M_e is the mass of an alectron.

But $(M_p + M_e) = M_H$ (mass of hydrogen atom) and

$$\{M\,(\text{nucleus}) + Z\,M_e\} = M$$

where M is the atomic mass. Hence the expression for binding energy becomes

$$B = [ZM_H + (A - Z)\,M_n - M\,]\,c^2$$

or
$$B = [ZM_H + AM_n - ZM_n - M\,]\,c^2$$

or
$$B = [AM_n - Z\,(M_n - M_H) - M\,]\,c^2$$

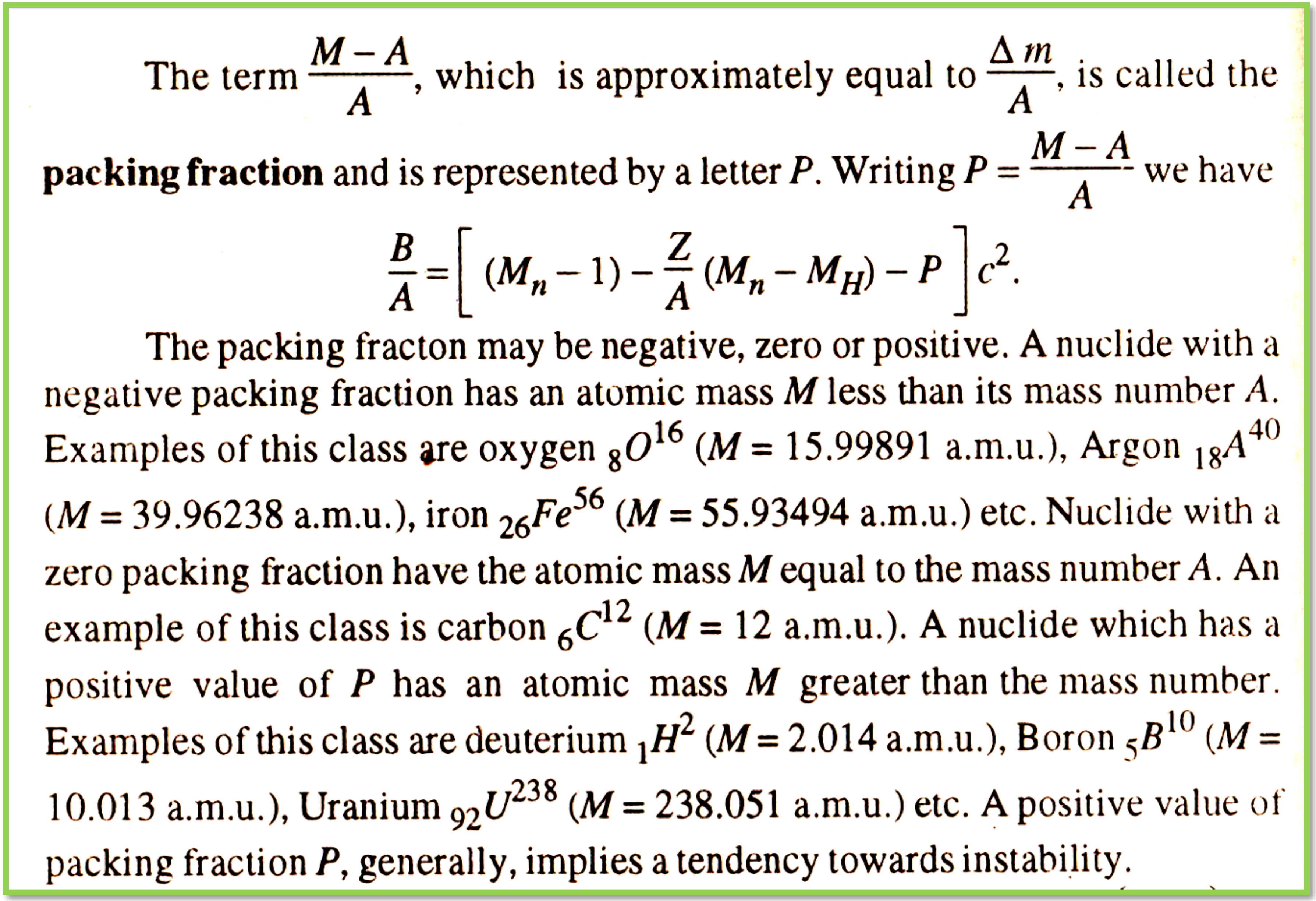
Dividing both sides by mass number A, we get the average binding energy per nucleon

$$\frac{B}{A} = \left[\,M_n - \frac{Z}{A}\,(M_n - M_H) - \frac{M}{A}\,\right] c^2$$

Let us now add and substract one on the right hand side of this equation. So that

$$\frac{B}{A} = \left[\,(M_n - 1) - \frac{Z}{A}\,(M_n - M_H) - \left(\frac{M}{A} - 1\right)\right] c^2$$

or
$$\frac{B}{A} = \left[\,(M_n - 1) - \frac{Z}{A}\,(M_n - M_H) - \left(\frac{M - A}{A}\right)\right] c^2$$

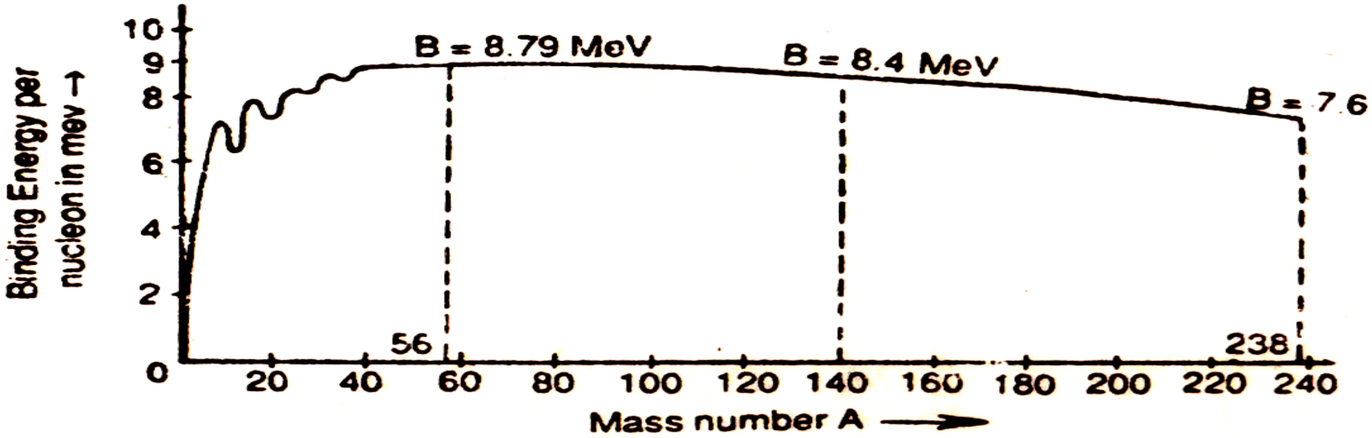


The term $\dfrac{M-A}{A}$, which is approximately equal to $\dfrac{\Delta m}{A}$, is called the **packing fraction** and is represented by a letter P. Writing $P = \dfrac{M-A}{A}$ we have

$$\frac{B}{A} = \left[(M_n - 1) - \frac{Z}{A}(M_n - M_H) - P \right] c^2.$$

The packing fracton may be negative, zero or positive. A nuclide with a negative packing fraction has an atomic mass M less than its mass number A. Examples of this class are oxygen ${}_8O^{16}$ (M = 15.99891 a.m.u.), Argon ${}_{18}A^{40}$ (M = 39.96238 a.m.u.), iron ${}_{26}Fe^{56}$ (M = 55.93494 a.m.u.) etc. Nuclide with a zero packing fraction have the atomic mass M equal to the mass number A. An example of this class is carbon ${}_6C^{12}$ (M = 12 a.m.u.). A nuclide which has a positive value of P has an atomic mass M greater than the mass number. Examples of this class are deuterium ${}_1H^2$ (M = 2.014 a.m.u.), Boron ${}_5B^{10}$ (M = 10.013 a.m.u.), Uranium ${}_{92}U^{238}$ (M = 238.051 a.m.u.) etc. A positive value of packing fraction P, generally, implies a tendency towards instability.

Binding energy curve : The binding energy per nucleon $\left(=\frac{B}{A}\right)$ for most of the nuclei (except for very small A) is found to be nearly equal to 8 $M\,eV$. A plot of the average binding energy as a function of mass number A has been shown in fig. 13.1. As it can be seen, the binding energy curve at first rises steeply. It shows certain peaks for $A = 4$, 8, 12 and 16 corresponding to ${}_2He^4$, ${}_3Li^7$, ${}_6C^{12}$ and ${}_8O^{16}$. Afterward it attains a maximum value equal to 8.79 MeV for $A = 56$ *i.e.* for the iron nucleus ${}_{26}Fe^{56}$. The curve then shows a dropping for higher values of mass numbers becoming 7.6 $M\,eV$ for $A = 238$ (uranium). It follows, that the nuclei having a mass number 56 or near about are the most stable. Both the light nuclei for which $A << 56$ or the heavy nuclei for which $A >> 56$ are not so stable. That is why energy is liberated when one combines two lighter nuclei into a heavier one (nuclear fusion) or when one wishes to split a heavy nucleus into two smaller fragments (nuclear fission).



NUCLEAR MODELS

A nucleus contains the protons and the neutrons. The problem of interaction between these nucleons was studied by considering the nucleus as a two body system. But the results obtained could not explain the behavior of a system consisting of so many nucleons. Therefore, a number of nuclear models have been developed, on the basis of which the nuclear properties are explain.

LIQUID DROP MODEL

The liquid drop model was proposed by Neils Bohr. Because of a number of similarities between the nucleus and a liquid drop, the model considers a nucleus to be identical to a liquid drop.

Some of the similarities are

(i) A liquid drop is spherical in shape and a nucleus can also be considered to be spherical.

(ii) The volume of a nucleus for the spherical symmetry can be written as

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} R_0{}^3 A (\because R = R_0 A^{1/3})$$

Thus the volume of a nucleus is proportional to the total number of nucleons. Hence the density of a nucleus can be assumed to be fairly constant.

A liquid drop has also a constant density for the liquid. However, while the density of nuclei of different elements is found to be nearly the same, the density of different liquid is different.

(iii) Both the nucleus and the liquid drop display surface tension effects.

(iv) A liquid drop has strong cohesive forces, which are identical to short range nuclear forces in a nucleus.

(v) If a drop is made to oscillate, it breaks into two parts. Similarly the capture of neutrons by the nuclei of heavy elements leads to the process of nuclear fission, where heavy nucleus is found to split into two almost equal fragments.

(vi) Sharing of excitation energy in a nucleus is a random process. There is always a definite probability that a large amount of energy may be concentrated on single nucleon, which then be able to escape from the nucleus. This is similar to the boiling of a liquid.

The binding energy per nucleon on the basis of liquid drop model is given by the expression

$$\frac{B}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{4/3}} - a_4 \frac{(a-2Z)^2}{A^2} \pm \frac{a_5}{A^{7/4}}$$

where a_1, a_2, a_3, a_4 and a_5 are constants.

The various terms on the right hand side correspond to the volume binding energy, surface binding energy, the coulomb energy, the asymmetry energy and the pairing energy, respectively.

The liquid drop model explains successfully the stability of a nucleus by comparing the short range nuclear forces to the cohesive forces in a liquid drop.

And also explains the phenomena of artificial radioactivity and the nuclear fission.
However, this model fails to explain for the measured spins and magnetic moments of nuclei.

The Shell Model and Magic numbers

The shell model of electrons in any atom satisfactorily explains the frequencies of spectral lines, resonance and ionization potentials etc.

A similar structure can be assigned to the nucleus, where we can assume the nucleons to be grouped in quantum controlled nuclear shells. These shells are filled when they contain a specific number of nucleons and a nucleus with filled shells is more stable than that with unfilled shells.

Nuclei having certain peculiar numbers of neutrons and / or protons are found to have greater stability.

The Shell Model and Magic numbers

The following evidences speak in favour of this statement:

(a) Helium 4 (Z=2, N=2) and Oxygen 16 (Z=8, N=8) are particularly stable as it can be seen from the Binding Energy Curve. Thus the numbers Z= 2 and 8 represent a peculiar stability.

(b) It has been observed the $_{50}$Sn (Tin) has 10 stable isotopes while $_{20}$Ca (Calcium) has six stable isotopes. Therefore, the elements with Z=50 and Z=20 are more stable than any other element.

(c) All the three radioactive series (ie. Uranium, Actinium and Thorium series) decay to $_{82}Pb^{208}$ having Z=82 and N=126 correspond to an unusual stability.

(d) Experimentally, it is found that when some isotopes are excited about their nucleon binding energy by a preceding beta decay, they spontaneously emit the neutrons.

Examples of such isotopes are ${}_{8}O^{17}$, ${}_{36}Kr^{87}$ and ${}_{54}Xe^{137}$.

Clearly the number of neutrons N for these isotopes is equal to 9, 51, 83, respectively. Theses numbers can be written as 8+1, 50+1 and 82+1.

If we consider the loosely bound neutron as a valency neutron, the remaining numbers, 2, 8, 20, 50, 82 and 126 for either Z or N represent an unusual nuclear stability. These numbers are known as the **MAGIC NUMBERS** and the nuclei for which Z or N has these particular values are termed as **MAGIC NUCLEI.**

The magic numbers indicate that the nucleons in a nucleus exist in pairs. The magic numbers correspond to completed energy levels or shells in a nucleus. As soon as a particular shell is filled, the resulting configuration is particularly stable and has an unusually low energy.

In the shell mode, it is assumed that each nucleon moves n its orbit within the nucleus, independently of all other nucleons. It is acted upon by an average field produced by the action of other nucleons. This model is also known as the single particle shell model, has been successfully used to explain nuclear stability, spin, magnetic moment etc.

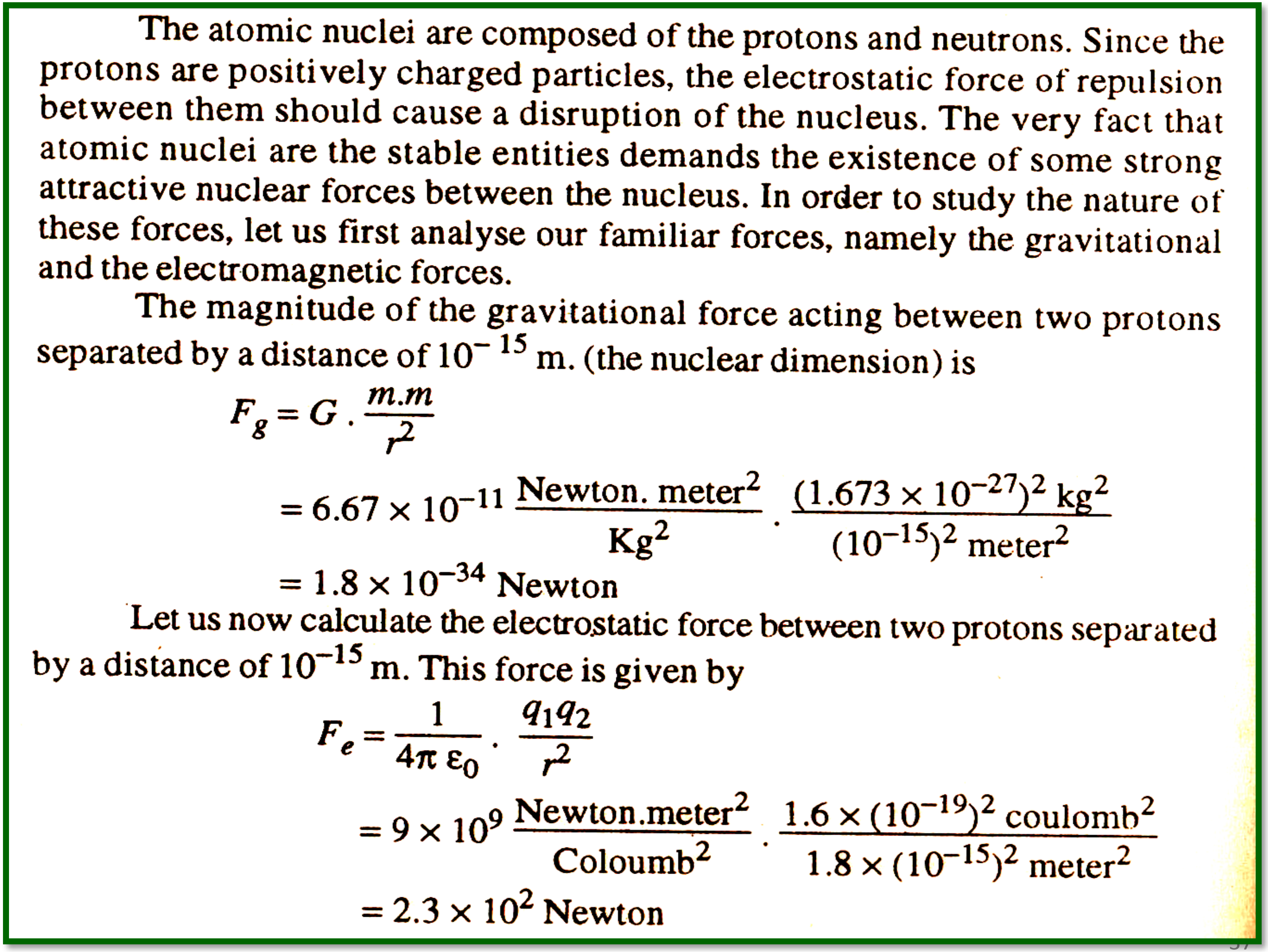
Limitations of Shell Model

(a) It does not explain the observed first excited states in even-even nuclei at energies much lower than those expected from single particle excitation.

(b) The model does not explain the observed large quadrupole moment in the case of odd A nuclei, particularly those which have their nucleon number far away from the magic numbers.

(c) The four stable nuclei ${}_1H^2$, ${}_3Li^6$, ${}_5B^{10}$ and ${}_7N^{14}$ are not covered under the scheme of this model.

Nuclear Forces



The atomic nuclei are composed of the protons and neutrons. Since the protons are positively charged particles, the electrostatic force of repulsion between them should cause a disruption of the nucleus. The very fact that atomic nuclei are the stable entities demands the existence of some strong attractive nuclear forces between the nucleus. In order to study the nature of these forces, let us first analyse our familiar forces, namely the gravitational and the electromagnetic forces.

The magnitude of the gravitational force acting between two protons separated by a distance of 10^{-15} m. (the nuclear dimension) is

$$F_g = G \cdot \frac{m.m}{r^2}$$

$$= 6.67 \times 10^{-11} \frac{\text{Newton. meter}^2}{\text{Kg}^2} \cdot \frac{(1.673 \times 10^{-27})^2 \text{ kg}^2}{(10^{-15})^2 \text{ meter}^2}$$

$$= 1.8 \times 10^{-34} \text{ Newton}$$

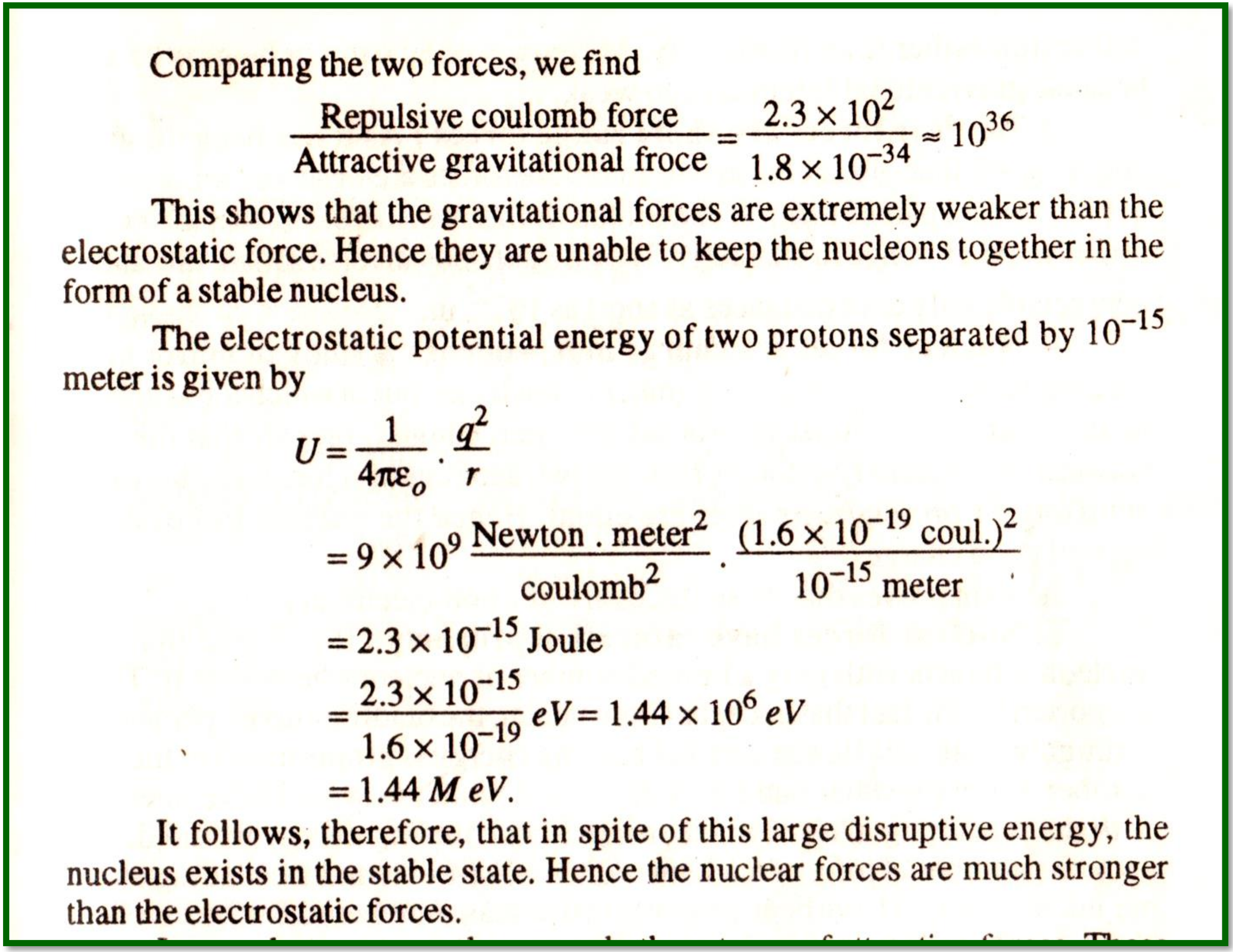
Let us now calculate the electrostatic force between two protons separated by a distance of 10^{-15} m. This force is given by

$$F_e = \frac{1}{4\pi \, \varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$= 9 \times 10^9 \frac{\text{Newton.meter}^2}{\text{Coloumb}^2} \cdot \frac{1.6 \times (10^{-19})^2 \text{ coulomb}^2}{1.8 \times (10^{-15})^2 \text{ meter}^2}$$

$$= 2.3 \times 10^2 \text{ Newton}$$

Nuclear Forces



Comparing the two forces, we find

$$\frac{\text{Repulsive coulomb force}}{\text{Attractive gravitational froce}} = \frac{2.3 \times 10^{2}}{1.8 \times 10^{-34}} \approx 10^{36}$$

This shows that the gravitational forces are extremely weaker than the electrostatic force. Hence they are unable to keep the nucleons together in the form of a stable nucleus.

The electrostatic potential energy of two protons separated by 10^{-15} meter is given by

$$U = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q^2}{r}$$

$$= 9 \times 10^{9} \frac{\text{Newton . meter}^2}{\text{coulomb}^2} \cdot \frac{(1.6 \times 10^{-19} \text{ coul.})^2}{10^{-15} \text{ meter}}$$

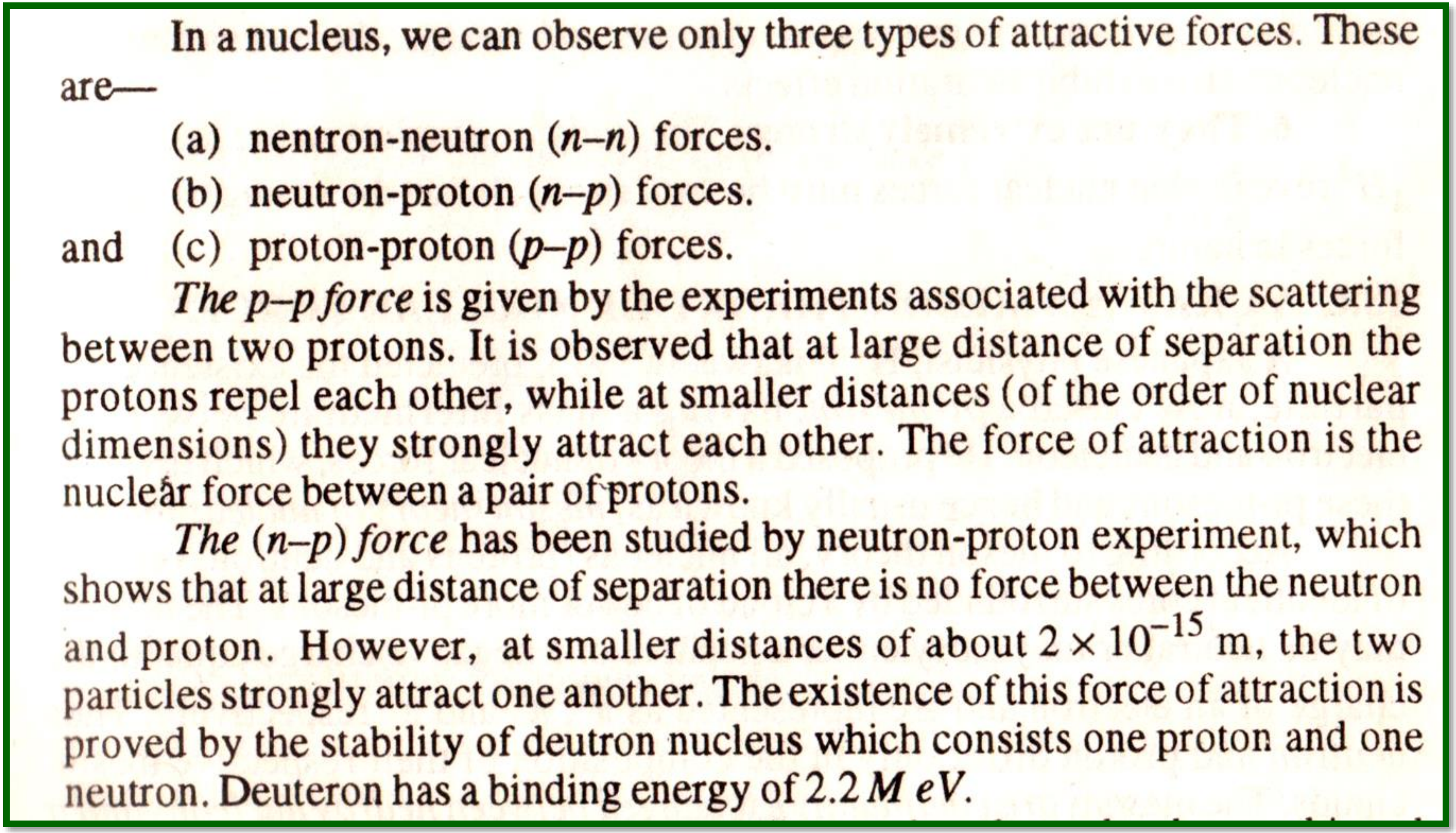
$$= 2.3 \times 10^{-15} \text{ Joule}$$

$$= \frac{2.3 \times 10^{-15}}{1.6 \times 10^{-19}} eV = 1.44 \times 10^{6} \, eV$$

$$= 1.44 \, M \, eV.$$

It follows, therefore, that in spite of this large disruptive energy, the nucleus exists in the stable state. Hence the nuclear forces are much stronger than the electrostatic forces.

Nuclear Forces



In a nucleus, we can observe only three types of attractive forces. These are—

(a) nentron-neutron (n–n) forces.
(b) neutron-proton (n–p) forces.
and (c) proton-proton (p–p) forces.

The p–p force is given by the experiments associated with the scattering between two protons. It is observed that at large distance of separation the protons repel each other, while at smaller distances (of the order of nuclear dimensions) they strongly attract each other. The force of attraction is the nuclear force between a pair of protons.

The (n–p) *force* has been studied by neutron-proton experiment, which shows that at large distance of separation there is no force between the neutron and proton. However, at smaller distances of about 2×10^{-15} m, the two particles strongly attract one another. The existence of this force of attraction is proved by the stability of deutron nucleus which consists one proton and one neutron. Deuteron has a binding energy of 2.2 MeV.

YUKAWA’S MESON THEORY OF NUCLEAR FORCES

Yukawa predicted the existence of a particle, is called pi-meson, having a mass intermediate between an electron and a nucleon.

The theory of nuclear forces, which involves these pi-mesons and hence usually known as meson theory of nuclear forces.

According to meson theory, all nucleons (protons and neutrons) consist of identical cores surrounded by a cloud of one or more pi-mesons.

YUKAWA’S MESON THEORY OF NUCLEAR FORCES

The mesons may be neutral or may carry either a positive or a negative charge equal to the charge of an electron and are represented as π^0, π^+, and π^-, respectively.

Thus neutron and proton differ only in the composition of their respective meson clouds.

The mesons are continually exchanged between nearby nucleons and it produces a force between a neutron and a proton.

In similarly with the emission of radiations by an accelerated charge, Yukawa proposed that in producing the exchange force the mesonic charge oscillates between the proton and neutron.

A proton is converted into a neutron and vice versa when π^+,a meson jumps from a proton to a neutron:

YUKAWA'S MESON THEORY OF NUCLEAR FORCES

$$\begin{aligned} p - \pi^{+} &\longrightarrow n \\ n + \pi^{+} &\longrightarrow p \end{aligned}$$

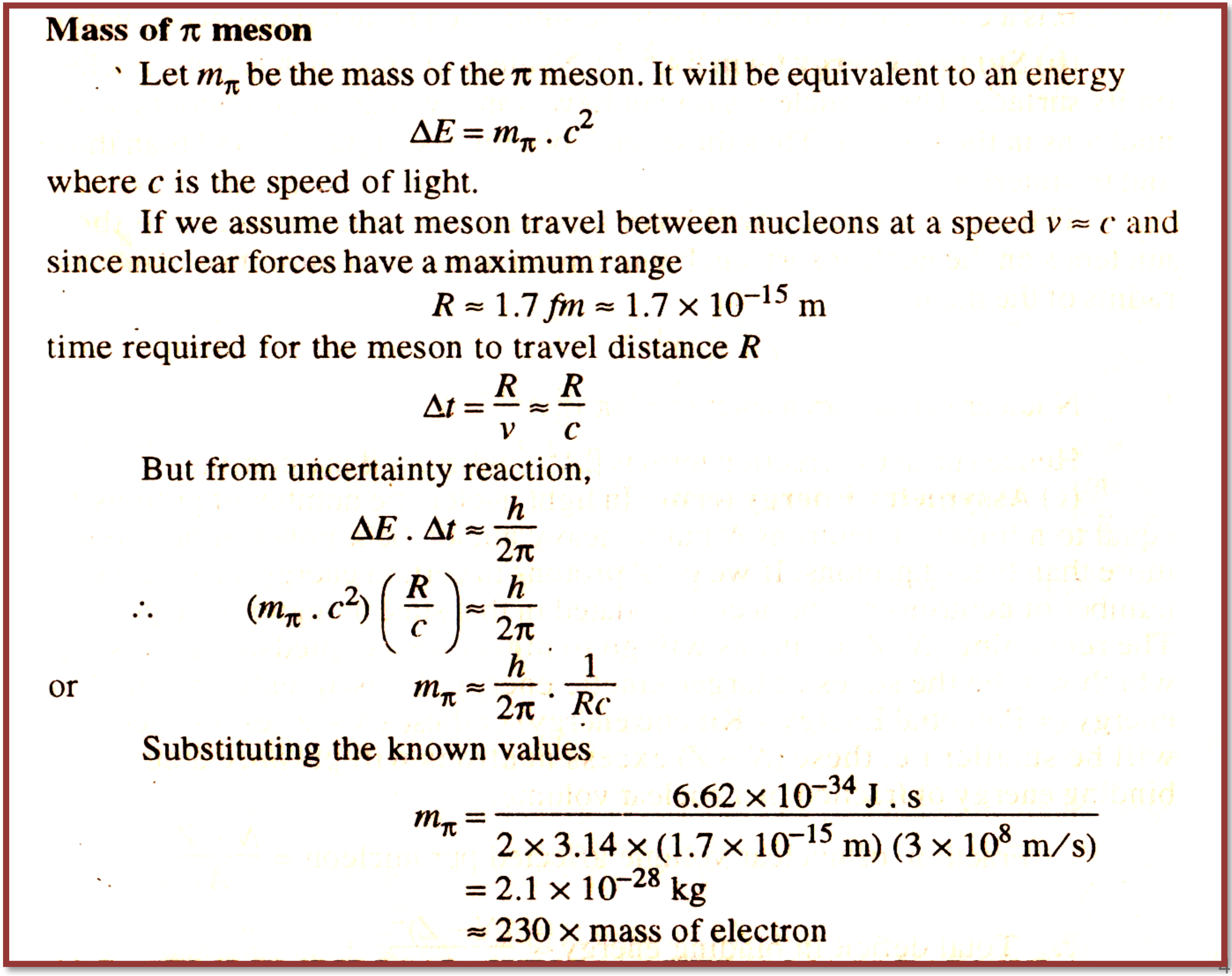
Conversely, a neutron is converted to a proton and vice versa when a π^- meson jumps from a neutron to a proton:

$$\begin{aligned} n - \pi^{-} &\longrightarrow p \\ p + \pi^{-} &\longrightarrow n \end{aligned}$$

Similarly, the forces between neutron-neutron and proton-proton pairs arise by the exchange of neutral meson (π^0) between them.

Since mesons are constantly running between the nucleons, i.e. the nucleons are constantly emitting and absorbing mesons, no change in the mass of the nucleons may be detected.

YUKAWA'S MESON THEORY OF NUCLEAR FORCES



Mass of π meson

Let m_π be the mass of the π meson. It will be equivalent to an energy

$$\Delta E = m_\pi \cdot c^2$$

where c is the speed of light.

If we assume that meson travel between nucleons at a speed $v \approx c$ and since nuclear forces have a maximum range

$$R \approx 1.7\ fm \approx 1.7 \times 10^{-15}\ \text{m}$$

time required for the meson to travel distance R

$$\Delta t = \frac{R}{v} \approx \frac{R}{c}$$

But from uncertainty reaction,

$$\Delta E \cdot \Delta t \approx \frac{h}{2\pi}$$

$$\therefore \qquad (m_\pi \cdot c^2)\left(\frac{R}{c}\right) \approx \frac{h}{2\pi}$$

or

$$m_\pi \approx \frac{h}{2\pi} \cdot \frac{1}{Rc}$$

Substituting the known values

$$m_\pi = \frac{6.62 \times 10^{-34}\ \text{J . s}}{2 \times 3.14 \times (1.7 \times 10^{-15}\ \text{m})\,(3 \times 10^{8}\ \text{m/s})}$$

$$= 2.1 \times 10^{-28}\ \text{kg}$$

$$\approx 230 \times \text{mass of electron}$$