Objective Function:

$$z = c_1 x_1 + c_2 x_2 + c_3 x_3 \; \cdots\cdots \; c_n x_n$$

which is to be minimized or maximized is called Objective Function of the general LPP.

Constraints:

The inequations (b) are called the constraints of the general LPP.

The std form of general LPP:

Optimize (Maximize or Minimize):

$$z = c_1 x_1 + c_2 x_2 + \cdots\cdots \; c_n x_n \quad \text{(objective function)}$$

Subject to linear constraints:

$$\begin{aligned}
a_{11} x_1 + a_{12} x_2 + &\cdots\cdots \; a_{1n} x_n \; (\leq, =, \geq) \, b_1 \\
a_{21} x_1 + a_{22} x_2 + &\cdots\cdots \; a_{2n} x_n \; (\leq, =, \geq) \, b_2 \\
\vdots \qquad \quad \vdots \qquad &\qquad \vdots \\
a_{m1} x_1 + a_{m2} x_2 + &\cdots\cdots \; a_{mn} x_n \; (\leq, =, \geq) \, b_n
\end{aligned}$$

$x_1, x_2 \; \cdots\cdots \; x_n \geq 0$ (non-negativity restriction)

Components of LPP:

(i) objective Function
(ii) Decision Variable or Activity variable
- unrestricted Variables
- Non-negativity condition
- co-efficients

(iii) constraints.

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Simplex Method

1. Use simplex method to slove bollowing LPP.

$$\text{Max } z = x_1 + 2x_2$$

Sub to constraints:

$$-x_1 + 2x_2 \leq 8$$

$$x_1 + 2x_2 \leq 12$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Introducing the slack variable in the given constraints $x_3 \geq 0$, $x_4 \geq 0$, $x_5 \geq 0$.

The given LPP becomes

$$z = x_1 + 2x_2 + 0x_3 + 0x_4 + 0x_5$$

Sub to constraints

$$-x_1 + 2x_2 + x_3 = 8$$

$$x_1 + 2x_2 + x_4 = 12$$

$$x_1 - x_2 + x_5 = 3$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

Procedure :

(i) Find z_j

(ii) $z_j - c_j$

(iii) From $z_j - c_j$. Find most (-)ve element

(iv) From most (-)ve element row we have

to find Pivot element

x_B / most (-) ve element row

In this minimum Number is called Pivot element.

(v) Next, Pivot element should be 1, other element should be 0 .

(vi) Again we have to find z_j and $z_j - c_j$.

(vii) continue the above said procedure again & again.

(viii) Finally $z_j - c_j$ should be Positive.

$$Z = x_1 + 2x_2 + 0x_3 + 0x_4 + 0x_5$$

$$-x_1 + 2x_2 + x_3 = 8$$

$$x_1 + 2x_2 + x_4 = 12$$

$$x_1 - x_2 + x_5 = 3$$

Table : 1

		c_j	1	2	0	0	0	
C_B	Y_B	X_B	X_1	X_2	X_3	X_4	X_5	
0	x_3	8	-1	2 * pivot element	1	0	0	← $8/2 = 4$ ✓ min
0	x_4	12	1	2	0	1	0	$12/2 = 6$
0	x_5	3	1	-1	0	0	1	
	z_j		0	0 ↑	0	0	0	
	$z_j - c_j$ =		-1	-2 most (-)ve	0	0	0	

∴ Pivot element = 2

x_2 enter the basis. Then, x_3 leave the basis

STEP : 1

		c_j	1	2	0	0	0	
C_B	Y_B	X_B	X_1	X_2	X_3	X_4	X_5	
2	x_2	4	$-1/2$	1	$1/2$	0	0	
0	x_4	4	2*	0	-1	1	0	$4/2 = 2$ ✓ min
0	x_5	7	$1/2$	0	$1/2$	0	1	$7 \times 2 = 14$
	z_j =		-1	2	1	0	0	
	$z_j - c_j$ =		-2 most (-)ve	0	1	0	0	

STEP : 1 - Workings

(No change) Old - New (change)

	12	1	2	0	1	0
(−)×2	(−) 8	(−) −1	(−) 2	(−) 1	(−) 0	(−) 0
	4	2	0	−1	1	0

(see Table 1)

	3	1	−1	0	0	1
(+)	4	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	0
	7	$\frac{1}{2}$	0	$\frac{1}{2}$	0	1

$1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

STEP : 2

C_B	Y_B	X_B	1 X_1	2 X_2	0 X_3	0 X_4	0 X_5
2	x_2	5	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0
1	x_1	2	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0
0	x_5	6	0	0	$\frac{3}{2}$	$-\frac{1}{2}$	1
	z_j =		1	2	$\frac{1}{2}$	$\frac{3}{2}$	0
	$z_j - c_j$ =		0	0	$\frac{1}{2}$	$\frac{3}{2}$	0

Since all $z_j - c_j$ are non-negative. The optimum solution has been obtained.

$x_1 = 2, \quad x_2 = 5$

$\text{Max } z = x_1 + 2x_2 = 2 + 2 \times 5 = 2 + 10 = 12$

STEP: 2 workings:

old →	4	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	0
New $\times \frac{1}{3}$	1	$\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{1}{2}$	0
	5	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0

	7	$\frac{1}{2}$	0	$\frac{1}{2}$	0	1
New $\times -\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{4}$	$-\frac{1}{2}$	0
	6	0	0	$\frac{3}{2}$	$-\frac{1}{2}$	1

---

2. Use simplex method to slove the following LPP.

$$Z = 3x_1 + 5x_2$$

(practice).

Sub to:

$$3x_1 + 2x_2 \le 18$$

$$0 \le x_1 \le 4$$

$$0 \le x_2 \le 6$$

$$x_1, x_2 \ge 0.$$