MATHEMATICAL STATISTICS-III

Subject Code: 16SACMS2

Unit V

Test of significance

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UNIT - V.

Small Sample Test (t - test).

TYPE 1:

Test of significance of a single mean

Procedure:

1. Null hypothesis $H_0 : \mu$ = specified value.
2. Alternative hypothesis $H_1 : \mu$ = specified value.
3. if α is not given take $\alpha = 5\%$ and find t_α
4. $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ (or) $t = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

where,

s = same, $S.D = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}}$

S = population S.D =

If $|t| < t_\alpha$ H_0 is accepted H_1 is rejected.

Assumption for student t test :-

(i) A parent population from which a sample is drawn is normal.

(ii) a sample observation are independent

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1. Find the Student t - statistic for the following sample −6, −4, −1, −1, 0, 1, 1, 3, 4, 5 discuss the suggestion the mean of univers to be 0

solu:

$n = 10 < 30$ (small sample)

$$\bar{x} = \frac{\Sigma x}{n} = \frac{2}{10} = 0.2.$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
−6	−6.2	38.44
−4	−4.2	17.64
−1	−1.2	1.44
−1	−1.2	1.44
0	−0.2	0.04
1	0.8	0.64
1	0.8	0.64
3	2.8	7.84
4	3.8	14.44
5	4.8	23.04
		$\Sigma(x-\bar{x})^2 = 105.6$

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{105.6}{9}}$$

$$= 3.4254.$$

Null hypothesis : $H_0 : \mu = 0$. ③

Alternative hypothesis : $H_1 : \mu \neq 0$

Let $\alpha = 5\% = 0.05$ $d.f = n-1 = 9$.

$t_\alpha = 2.26$.

t - statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{0.2 - 0}{3.4254/\sqrt{10}} = 0.1846.$$

$|t| < t_\alpha \Rightarrow H_0$ is accepted.

2. A machanist is making a Engine parts with axis diameter of 0.700 inch. A random sample of 10 parts shows a mean diamoter of 0.742 inch with a standard deviation of 0.40 inch. Test wether the work is meeting the specification at 5% level.

sdu:

Given that

$\mu = 0.700$ inches   (less than t test.
greater

$n = 10 < 30$

$\bar{x} = 0.742$ inches

$s = 0.40$ inches

$\alpha = 5\%$.

Null hypothesis : H_0

$\mu = 0.7$.

Alternative hypothesis : H_1.

$\mu \neq 0.7$ (Two- tailed test)

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LOS :-

$\alpha = 5\%$ $df = n-1$ $t_\alpha = 2.26$

$\alpha = 0.05$ $= 9.$

Test statistic :-

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{(0.742 - 0.7)\sqrt{10}}{0.40}$$

$$= 0.3820$$

$|t| < t_\alpha$.

H_0 is accepted.

3.) A random sample of 16 values from a normal population shows a mean of 41.5 inches and sum of squares of deviation from this mean equal to 135 square inches show that a assumption of mean of 43.5 inches for the population is not reasonable.

Solu: Given that

$n = 16.$

$\bar{x} = 41.5$ inches

$\Sigma(x - \bar{x})^2 = 135$ square inches.

$\mu = 43.5$ inches.

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n-1}} = \sqrt{\frac{135}{15}} = \sqrt{9} = 3.$$

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i) Null hypothesis $H_0 : \mu = 43.5$ (5)

ii) Alternative hypothesis $H : \mu \neq 43.5$ (Two - tailed test)

iii) LOS : $\alpha = 5\%$ $df = n-1$
$= 15$ $t_\alpha = 2.13$.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{41.5 - 43.5}{3/\sqrt{16}}$$

$|t| = 2.6667$.

$|t| > t_\alpha$.

H_0 is rejected.

4.) The height of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64 inches is it reasonable to believe that the average height in (>) 64 inches. Test at 5% Level of significance.

Solu:

$n = 10$.

Null hypothesis $H_0 : \mu = 64$

Alternative hypothesis $H_1 : \mu_1 > 64$ (one tailed test)

LOS:

$\alpha = 5\%$ $df = n-1 = 9$ $t_\alpha = 1.83$

(one tailed test $- 2 \times \alpha$)

$\alpha = 0.10$.

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⑥

Test statistic :-

x	$x-\bar{x}$	$(x-\bar{x})^2$
70	4	16
67	1	1
62	-4	16
68	2	4
61	-5	25
68	2	4
70	4	16
64	-2	4
64	-2	4
66	0	0

$\Sigma(x-\bar{x})^2 = 90$.

$$s^2 = \frac{1}{n-1} \Sigma(x-\bar{x})^2$$

$$= \frac{1}{9}(90)$$

$$= \frac{90}{9}.$$

$$s^2 = 10.$$

$$s = \sqrt{10}.$$

$$t = \frac{\bar{x}-\mu}{s/\sqrt{10}} = \frac{66-64}{\sqrt{10}\sqrt{10}} = 2.$$

$|t| > t_\alpha$

H_0 is rejected, H_1 is accepted.

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Test of significance of consider two mean. 7

consider a independent sample of size n_1, n_2 taken from a normal population when their variance is are equal.

Let $\bar{x}_1$ and $\bar{x}_2$ be means of 2 samples and s_1^2 & s_2^2 be the sample variance.

Procedure :-

i) Null hypothesis $H_0 : \mu_1 = \mu_0$.

ii) Alternative hypothesis $H_1 : \mu_1 \neq \mu_2$.

iii) LOS :

$\mu_1 \neq \mu_2$ (two-tailed test)

$\left.\begin{array}{l}\mu_1 < \mu_2 \\ \mu_1 > \mu_2\end{array}\right\}$ one-tailed test.

Let $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} - \frac{1}{n_2}}}$ (or) $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

1) In a test examination given to two group of students the marks obtain where has follows

I group : 18 20 36 50 49 36 34 49 41

II group : 29 28 26 35 30 44 46.

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Examine the significance of difference b/w the average mark secured by the student of above 2 group. (8)

Solu:

$$\bar{x} = \frac{333}{9} = 37.$$

Calculation:

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
18	-19	361	29	-5	25
20	-17	289	28	-6	36
36	-1	1	26	-8	64
50	13	169	35	1	1
49	12	144	30	-4	16
36	-1	1	44	10	100
34	-3	9	46	12	144
49	12	144			$\Sigma(y-\bar{y})^2 = 386$
41	4	16			
		$\Sigma(x-\bar{x})^2 = 1134$			

Null hypothesis $H_0: \mu_1 = \mu_2$.

Alternative hypothesis $H_1: \mu_1 \neq \mu_2$ (Two tailed test)

Los:-

Let $\alpha = 5\%$. $df = n_1 + n_2 - 2$

$= 9 + 7 - 2$ $t_\alpha =$

$= 14$

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Test statistic :-

$$s^2 = \frac{1}{n_1+n_2-2}\left(\sum_i (x_i-\bar{x})^2 + \sum_i (y_i-\bar{y})^2\right)$$

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$$= \frac{1}{14}(1134 + 386)$$

$$= \frac{1520}{14}$$

$$s^2 = 108.5714.$$

$$s = 10.4198.$$

$$t = \frac{\bar{x}-\bar{y}}{s\Big/\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}.$$

$$= \frac{37-34}{10.4198\Big/\sqrt{\frac{1}{9}+\frac{1}{7}}} = \frac{3\times.}{10.4198\times0.5040}$$

$$= \frac{3}{5.2516} = 0.5713$$

Ho is accepted.

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2) samples of 2 types of electric tubes where tested for length of life and the following where obtain (10)

	Number	Mean	S.D
Type - I	10	1240 hrs	37 hrs.
Type - II	8	1042 hrs	40 hrs

Test the hypothesis that $\mu_0 : \mu_1 - \mu_2 = 8$ verses $\mu_1 : \mu_2 > 8$ at 5 % LOS.

Solu:

Given that.

	Number	Mean	S.D.
Type I	10 n_1	1340 hrs x_1	37 hrs (s_1)
Type II	8 n_2	1042 hrs x_2	40 hrs (s_2)

i) Null hypothesis H_0:

$$H_0 : \mu_1 - \mu_2 = 8$$

ii) Alternative hypothesis H_1:

$$H_1 : \mu_1 - \mu_2 > 8 \quad \text{(one tailed test)}$$

iii) LOS:

$\alpha = 5\% = 0.05 \times 2$

$= 0.10$

$df = n_1 + n_2 - 2$

$= 10 + 8 - 2$

$df = 10$

$t_\alpha = 1.75$

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iv) T · S :

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$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

$$= \frac{10(37)^2 + 8(40)^2}{10+8-2}$$

$$s^2 = 1655.625$$

$$s = \sqrt{1655.625}$$

$$s = 40.6894.$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} - \frac{1}{n_2}}}$$

$$= \frac{(1240 - 1042) - (8)}{40.6894\sqrt{\frac{1}{10} + \frac{1}{8}}} = \frac{190}{40.6894\sqrt{0.2250}}$$

$$= \frac{190}{19.3007}$$

$|t| = t_\alpha$.

$t = 9.8448$

H_1 is accepted.

Paired t-test for difference of mean :-

Let us consider the case when

(i) The sample size are equal (ie) $n_1 = n_2 = n$

(ii) The 2 sample are not independent but the sample observation are paired together.

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Procedure:

i.) Null hypothesis : H_0 $\mu_1 = \mu_2$.

ii.) Alternative hypothesis: $H_1 : \mu_1 \neq \mu_2$ (>, <)

iii.) L o s: df = n-1 and to find t_α

iv.) T - s:

$$t = \frac{\bar{d}}{s/\sqrt{n}}$$

Where,

$$d_i = x_i - y_i$$

$$\bar{d} = \frac{1}{n} \Sigma d_i .$$

$$s^2 = \frac{1}{n-1} \Sigma (d_i - \bar{d})^2$$

v.) If $|t| < t_\alpha$.

H_0 is accepted H_1 is rejected.

1.) A certain stimulus administtated two each of the 12 patient resulted in following increase of blood pressure 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6 canit conclude that the stimulus in general be accombined by an increase in BP

Solu:

i) Null hypothesis H_0:

$H_0 : \mu_1 = \mu_2$ (there is one significance diff in BP of the patient before and after the drug)

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(ii) Alternative hypothesis H_1:

$H_1: \mu_1 < \mu_2$

(iii) LOS:-

$\alpha = 5\%$; $df = n-1$ $\qquad t_\alpha = 1.80$

$= 0.05$ $\qquad = 12-1$

$= 1$

$d = x_i - y_i$	$d_i - d$	$(d_i - \bar{d})^2$
5	2.42	5.8564
2	-0.58	0.3364
8	5.42	29.3764
-1	-3.58	12.8164
3	0.42	0.1764
0	-2.58	6.6554
-2	-4.58	20.9764
1	-1.58	2.4964
5	2.42	5.8564
0	-2.58	6.6564
4	1.42	2.0164
6	8.42	11.6964
		104.9168

$\because \bar{d} = \frac{\Sigma d}{n}$

$= \frac{31}{12}$

$\boxed{\bar{d} = 2.58}$

$s^2 = \frac{1}{n-1} \Sigma (d_i - \bar{d})^2$

$= \frac{1}{11}(104.9168)$ (14)

$= 9.5379.$

$s = \sqrt{9.5379}$

$s = 3.0883.$

$$t = \frac{\bar{d}}{s/\sqrt{n}}$$

$$= \frac{2.58 / \sqrt{12}}{3.0883} = \frac{8.9374}{3.0883} = 2.8939$$

$|t| > t_\alpha.$

H_0 is rejected.

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chi-square test (or) χ^2.

If o_i $(i=1,2,\dots n)$ are set of observed (experimental) frequency and E_i $(i=1,2,\dots n)$ are the corresponding set of expected. (theoretical or hypothetical) frequency then, the statistic chi-square

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(o_i - E_i)^2}{E_t}\right]$$

and the degree of freedom (d.F) in the statistic is $v = n-1$.

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Conditions for applying χ^2 test.

i.) The sample observation should be independent

ii.) The consteraints on cell frequency must be linear 15

iii.) Total frequency $n > 50$

iv.) No theoretical frequency should be < 5 (less than 5)

Applications:

1.) To test the goodness of fit.

2.) To test the independence of attribute.

3.) To test the hypothetical value of the population ~~σ^2~~ workers σ^2.

4.) To test the homogenity of independent estimate of the population variance

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i) The number of automobile accidents per work in a certain commurity are as follows 13, 8, 20, 12, 14, 10, 15, 19, 9, 4 are these frequence in with the belief that accident conditions were the same during this 10 week period.

Solu:

1.) Null hypothesis :-

H_0 : The accidents conditions were the same during 10 week period.

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2.) Alternative hypothesis:-

H_1: The accidents conditions were not the same during to week period. 16

3.) LOS: $\alpha = 5\%$ $df = n-1 = 10-1 = 9$.

$= 0.05\%$. $x^2_\alpha = 16.919$.

4.) Test statistic:-

Expected frequency of accident each week

$= \frac{100}{10} = 10$.

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2$
12	10	2	4	0.4
8	10	-2	4	0.4
20	10	10	100	10
2	10	-8	64	6.4
14	10	4	16	1.6
10	10	0	0	0
15	10	5	25	2.5
6	10	-4	16	1.6
9	10	-1	1	0.1
4	10	-6	36	3.6
				26.6

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$= 26.6$.

$x^2 > x^2_\alpha$

Ho is rejected, The accident were not the same during ten week period.

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2) The theory predicts that the population of beans in the four group A, B, C, D should be 9 : 3 : 3 : 1 in an experiment among 1600 beans the number in 4 groups were 882, 313, 287, 118 does the experimental result support the theory.

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solu:

1) Null hypothesis H_0 :-

The experimental result support the theory.

2) Alternative hypothesis : H_1 :

The experimental result does not support the theory.

3) LOS :- $\alpha = 5\%$ $df = n-1$ $\chi^2_\alpha = 7.815$

$= 4-1$

$= 3$

4) Test statistic :-

$E_i(A) = \frac{9}{10} \times 1000 = 900$

$E_i(B) = \frac{3}{16} \times 1600 = 300$

$E_i(C) = \frac{3}{16} \times 1600 = 300$

$E_i(D) = \frac{1}{16} \times 1600 = 100$

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o_i	E_i	$o_i - E_i$	$(o_i - E_i)^2$	$(o_i - E_i)^2 / E_i$
882	900	-18	324	0.36
313	300	13	169	0.563
287	300	-13	169	0.563
118	100	18	324	3.24

(18)

$$\chi^2 = \frac{\Sigma (o_i - E_i)^2}{E_i}$$

$$\chi^2 = 4.726.$$

$$\chi^2 < \chi^2_\alpha.$$

H_0 is accepted.

Goodness of Fit.

Chi-square Test enable us to ascertain, how will the theoretical distribution such as binomial, poisson and so on fit Experimental distribution.

If calulate chi-square < Tabulated χ^2 is considered to be good and otherwise it i a poor.

Binomial distribution:

$$df = v = n-1$$

poisson distribution,

$$df = v = n-2$$

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Fitting Normal distribution:. (19)

$df = v = n - 3.$

1.) A survay of 320 families with 5 childrens yielded the following distribution.

No. of girls	0	1	2	3	4	5
No. of Boys	5	4	3	2	1	0
No. of Families	12	40	88	110	56	14.

Is this result consist with the hypothesis that male and Female birth are equally probable:

solu:

1.) Null hypothesis Ho:

Ho: The male and Female are equally probable $p = \frac{1}{2}$, $q = 1/2$.

2.) Alternative hypothesis H_1:

H_1: male and female birth are not equally probable.

3.) Los: $\alpha = 5\%$. $v = 6-1 = 5$ $\chi^2_\alpha = 11.07$.

In Binomial distribution

$P(x = r) = nC_r\, p^r q^{n-r}$

$E(x = r) = NP(x = r) = N\, nC_r p^r q^{n-r} = 320\, 5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$

$= 320(5C_r\, 1/32) = 10 \times 5C_r.$

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$E(X=0) = 10(5C_0) = 10$

$E(X=1) = 10(5C_1) = 50$.

$E(X=2) = 10(5C_2) = 100$.

$E(X=3) = 10(5C_3) = 100$

$E(X=4) = 10(5C_4) = 50$.

$E(X=4) = 10(5C_5) = 10$.

O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
12	10	2	4	0.4
40	50	-10	100	2
88	100	-12	144	1.44
110	100	10	100	1
56	50	6	36	0.72
14	10	4	16	1.6.

$$\chi^2 = \sum_i \left(\frac{(O_i - E_i)^2}{E_i} \right)$$

$$= 7.16.$$

$$\chi^2 < \chi^2_\alpha.$$

Ho is accepted.

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(x.) The number of defects per unit in a sample of 330 units of the manufactured products was found as follows. (21)

No. of units	: 0	1	2	3	4
No. of units	: 214	92	20	3	1.

Fit the poisson distribution.

Soln:

Null hypothesis:

Ho the fit is good.

Test statistic:

λ = mean

$$\lambda = \frac{\Sigma fx}{\Sigma x}$$

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

$$\lambda = \frac{145}{330} = 0.439.$$

~ 0.44.

x	f	fx
0	214	0
1	92	92
2	20	40
3	3	9
4	1	4
	330	145

$$P(X=r) = \frac{e^{-0.44}(0.44)^{r}}{r!}$$

$$E_i = NP(X=r) = \frac{330 \times 0.64\,(0.44)}{r!}$$

$$= \frac{211.2 \times (0.44)^{r}}{r}$$

(22)

x	o_i	E_i	$o_i - E_i$	$(o_i - E_i)^2$	$\frac{(o_i - E_i)^2}{E_i}$
0	214	211.2	2.8	4.84	0.03
1	92	92.9	0.9	0.81	0.0087
2	20	20.44	0.24	0.0576	0.0024
3	3 } 24	2.99	} 2.396		
4	1	0.33			

$$x^2 = \frac{\Sigma (o_i - E_i)^2}{E_i} = 0.0411.$$

Let $\alpha = 5\%$. $df = 3 - 2 = 1$.

$$x^2_\alpha = 3.841.$$

$x^2 < x_\alpha^2$ Ho is accepted

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