ABSTRACT ALGEBRA

Subject Code: 16SCCMM12

Unit V

Important questions

Prepared by,

Mrs. H. SABITHA BEGUM, M.Sc., B.Ed., M.Phil., SET.,

Assistant Professor,

Department of Mathematics,

AIMAN College of Arts and Science for Women,

Trichy 21.

Unit - 5

IMPORTANT QUESTIONS.

1 Define.

is Maximal Ideal.

If a maximal ideal in z. Let U be an ideal Peoperly contain. U contains an odd integer say, 2n+1 $1 = (2n+1) = 2n \in U$ U = zThus thue is no peoper ideal of z peopaly. Containing is a Maximal ideal of z.

in Prime I deal. Abolis to impognat Lader (

Let p be any peime. The (p) is maximal ideal in z.

Let U be aby ideal of Z S.T. (P) CU

every ideal of z is a peincipal ideal U = cn, $n \in z$ $P \in CP$ $\subseteq U \Rightarrow P \in U = cn$. $P \notin nm$ for some indeger m $P \Leftrightarrow prime = either , n = 1, or n = p$ $n \leq 1$ Then U = zn = P " U = (p).

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in way have a

US Unique factorization domain. Let R be commutative sing, Let a, b e R and a to. noe say that a dividus b and white a/b if there exist an element cer such that b = ac. It a/b we say that a is a divisor of a factor of b.

The seal was a company

civil Eulidean domain. Let R be a commentative sing with zerodivisors R is called an Euclidean domain 2 Euclidean sing if for every non-zero element is defined a non-Regative integer d(a)... I will be defined a non-Regative integer d(a)... I will be defined a non-Regative integer d(a)... I will be defined a non-Regative integer d(a)... I will it is be defined a non-Regative integer d(a)... I will be defined a non-Regative integer d(a)...

For any two non-zero elements $a, b \in \mathbb{R}$, there exist $a, r \in \mathbb{R}$ such that a = a/b + r either r = 0 of d(r) < d(b).

From the As Hartory C de

(V) Quotiens stops. Let R be a sing. Let (1, +) be subgraup $\Theta(R, +)$. Since addition is commutative $\sin R, 1$ is a not mal subgroup $\Theta(R, +)$. $R/I = (I + a/a \in R)$ is a group under the opposition (I + a) + (1 + b) = 2 + (a + b). (1 + a) (1 + b) = 2 + (a + b).

(2) Let R be a commutative ring with Identity.
An ideal M of R is maximal iff R/m is a

a interes a maximal ideal in R. a interes a maximal ideal in R.

Since R is a commutative ring with identity $M \neq R$, R/n is also a commutative ring with identity.

Let M + a be an non-zero element in R/M 80 that a ∉ M. M+a has a nultiplicative inverse in R/M.

Let U = gra +m/r ER and mery g

 $(7_1 a + m_1) - (7_2 a + m_2) = (r_1 - r_2) a + (m_1 - m_2) \in U$ JANELJM LJEI $\tau(\tau_1 \alpha + m_1) = (\tau_{\tau_1}) \alpha + \tau_{m_1} \in U + (\sigma_{\sigma_1})$ U is an ideal of R Let ment. Then m=DatmEU . MCU a=1a+0 EU and a & MIN and in () M # U W I WIT A be suchi in ad it is V is an I deal of R properly containing M 149 M is a maximal ideal of R. have all all as at 1 tel U=R. Hance 1EU all interview since material in & 1 = bat m for some be R. rubi shin you with surmas on M11=Mtbatm=Mtba = (M+b) (M+a) MAD is the inverse of MAA. 3 and 30 R/M is a field and to the act 31910 9 3 NH96 Let U be any ideal of R Peoperly Containgry. March And and There exist an element a E U Such that a & M and in 1 . M/a is a non-zero element of RIM 2 ali 9 since RIM is a field MAA has an inverse we who and a way that a start of the 89 x (0+17 (0+9) -Mtb. : (M+a) (M+b) = M+1 MAAD = M + I = 1 to young the 1 - ab em

+ " = LOBE ANT J + CARE ANT J 1=(1-ab)+ab Eu Summer of a the company 16.0 U=R. This There is no proper Ideal of R properly containing M. M is a maximal ideal ine. 3 Let R be any commutative ring with identity Let P be an ideal of R. Then P is a pine ideal (s R/P is an integral domain. Put I be had a horrison to all 14 Let P be a peime ideal N1214 - 21 R is commutative ving with identity R/P Las mrail is commatative sing with identity M - ALCANE ME LENA (Pta) (P+b) = P+0 3 Ptab=P service with an citized =) ab EP =) A EP of DEP with a av M/H >> Pta = Pol Ptb=p is a be any ideal of K Pice; why leavening, R/p has no zero divisors . . RIP is integral domain o historic at 1 manual IN OTH ! R is a prime ideal of R. Let abep. Then Prab=p and Mile Unit -. (Pta) (Ptb) = p der 12 = (of y) (21 49) · PHQ = P of Ptb=P

-. Pis a prima ideal of R. (2) Fundamental thim of Homomorphism. water world in c Let f: Or -> G' be an epimalphism Let k be the kernel of f. Cilk & Ci'. P241 annexed in & Crisipalis Define of: ON | K -> C' by of (ka) = 1 (a). e cond) peralandad p Step is to is well defined. Let kb = ka. Then be ka. Hence b= ka Where kek. f(b) = f(ka) = f(k) f(a) = o'f(a) = f(a):. $\phi(kb) = f(b) = f(a) = \phi(ka)$ Hence $\phi(ka) = d(kb)$ Stup (ii) & is 1-1 $\phi(ka) = \phi(kb) \Rightarrow f(a) = f(b)$ => 5 (a) [+(b)] = e => + (ab) = e = in 10 40 trance ming to late => ablex de dauber / -> a e kb 1. 12 1. 10 =) ka = kb the an here a as a man crist - cash the 2-11 an Mathematic

Step (in) & is onto

1

Let a' e u' 3 is ordo, mere exists a E CI S.T flas = a manuful in a la a la la la Hence & Char = 3 (a) = a' with all with all a sup 180.9 Step (iv) of is homomosphism ϕ (kakb) = ϕ (kab) = f (ab) = f (ab) = O(Ka) O(Kb) - p in all a is an isomarphism from Gi/k NON DE Kay What is NO. M onto ci GALL GIR SCHLADE - CANE - CANE CANE (ma) & = coit = (d) + = (d) + ... 3 Any Euclidean domain R is a U.F.D DI- CARI & DORES 1 - 1 is to wingst? First we shall p. T any climent of in R is either a unit of can be expressed as the Product of a finite number of prime dement Of R. 12 3 N 6induction of dead Et dead = dead then a is a unit in R.

assertion is the.

Assume that, the resultant is bus for all REP. S.T. d. (21) 2 d. (2), and prove that the result interest for a.

55 a is prime there is nothing to prove 53 not, a=bc where neither box c is a writing. .1. d(b) < d(a) 2 d(c) < d(a)

The product of a finite of prime demonds.

Let Q=P1P2...Pr=9,92...93 votrere P282 9;8 aue prime demente de R.

:- P1/2122---93.

Pila; for some i. assume the Pila,

Since P, 2 a, are both prime demants BR, P, 2 Q1.

: QI = U, P, where U, is a Unit in R.

:. PIP2 ... Pr = UI 9243 ... 95

Stres, ruines the left side becomes 1 and the eight side contains a impossible. 225.

SZY and hence r= 8.

Pi is an associate of some gi. Hence them