I B.Sc [MATHEMATICS]

DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

16SCCMM3

UNIT- II

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Particular Integral of second order Differential Equations with Constant co-efficients

TYPE

1) $(a_0D^2+a_1D+a_2)y=0$
2) $(a_0D^2+a_1D+a_2)y= e^{ax}$
3) $(a_0D^2+a_1D+a_2)y = \sin ax$ (or) $\cos ax$
4) $(a_0D^2+a_1D+a_2)y= x^n$
5) $(a_0D^2+a_1D+a_2)y = e^{ax}.f(x)$
6) $(a_0D^2+a_1D+a_2)y= x^n \sin ax$ (or) $x^n.\cos ax$

S.NO	Roots	Complementary function
1)	$m_1=m_2$	$(Ax+B)e^{mx}$
2)	$m_1\neq m_2$	$Ae^{m_1x}+Be^{m_2x}$
3)	$m_1=\alpha\pm i\beta$	$e^{\alpha x}[A\cos\beta x+B\sin\beta x]$

Note

PI - Particular Integral

C.F - Complementary function

A.E - Auxilliary Equation.

1) Solve $(D^2+2D+2)y=0$

Sol

A.E is, $D=m$,

$m^2+2m+2=0$

$a=1,\ b=2,\ c=2$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{4-4(1)(2)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{4-8}}{2}$$

$$=\frac{-2\pm\sqrt{-4}}{2}$$

$$=\frac{-2\pm\sqrt{2^2 i^2}}{2}\quad (\because i^2=-1)$$

$$=\frac{-2\pm 2i}{2}=\frac{\cancel{2}[-1\pm i]}{\cancel{2}}$$

$\boxed{m=-1\pm i}$, $\Rightarrow \alpha=-1,\ \beta=1$

$$\therefore \boxed{C.F = e^{-x}[A\cos x + B\sin x]}$$

— x —

Solve: $(D^2+6D+8)y = e$

Sol

A.E is,

Put $D = m$,

$m^2+6m+8=0$

4 6 2 8

$(m+4)(m+2)=0$

$m+4=0$ | $m+2=0$

$m=-4$ | $m=-2$

$\therefore \boxed{m_1 \neq m_2}$

$\therefore \boxed{C.F = A e^{-4x} + B.e^{-2x}}$

To find P.I

$$P.I = \frac{1}{D^2+6D+8} . e^{-2x}$$

Put $D=-2$,

$$= \frac{e^{-2x}}{(-2)^2+6(-2)+8}$$

$$= \frac{e^{-2x}}{4-12+8}$$

$$= \frac{e^{-2x}}{0}$$

$$\Rightarrow \frac{x}{2D+6} . e^{-2x}$$

(Put $D=-2$)

$$= \frac{x.e^{-2x}}{2(-2)+6}$$

$$= x \cdot \frac{e^{-2x}}{-4+6}$$

$$\boxed{P.I = x \cdot \frac{e^{-2x}}{2}}$$

$\therefore$ The General Solution,

$$y = C.F + P.I$$

$$\boxed{y = Ae^{-4x} + Be^{-2x} + \frac{x}{2} \cdot e^{-2x}}$$

— + —

Solve

3 $(D^2+3D+2)y = \sin 3x$

Sol

G.T, $(D^2+3D+2)y = \sin 3x$

A.E is,

$m^2+3m+2=0$

$(m+1)(m+2)=0$

$m+1=0$ | $m+2=0$

$m=-1$ | $m=-2$

$\therefore \boxed{m_1 \neq m_2}$

$$\therefore \boxed{C.F = Ae^{-x} + B.e^{-2x}}$$

To find P.I

$$P.I = \frac{1}{D^2+3D+2} \cdot \sin 3x$$

$(\text{put } D^2 = -3^2)$

$$= \frac{1}{-9+3D+2} \cdot \sin 3x$$

$$= \frac{1}{3D-7} \sin 3x$$

$$= \frac{1}{3D-7} \times \frac{3D+7}{3D+7} \sin 3x$$

$$\boxed{\because (a+b)(a-b) = a^2-b^2}$$

$$= \frac{3D+7}{(3D)^2 - 7^2} \cdot \sin 3x$$

$$= \frac{3D+7}{9D^2-49} \sin 3x$$

$$(\text{Put } D^2 = -3^2)$$

$$= \frac{3D+7}{9(-9)-49} \cdot \sin 3x$$

$$= \frac{3D+7}{-81-49} \sin 3x$$

$$= \frac{3D+7}{-130} \sin 3x$$

$$= \frac{-1}{130}\left[3D[\sin 3x] + 7 \sin 3x\right]$$

$$= \frac{-1}{130}\left[3 \times 3 \times \cos 3x + 7 \sin 3x\right]$$

$$\boxed{P.I = \frac{-1}{130}\left[9 \cos 3x + 7 \sin 3x\right]}$$

$$\therefore y = C.F + P.I$$

$$\Rightarrow \boxed{y = Ae^{-x} + Be^{-2x} - \frac{[9 \cos 3x + 7 \sin 3x]}{130}}$$

$(D^2+5D+4)y = x^2+7x+9$

sol

A.E becomes,

$m^2+5m+4=0$

$(m+4)(m+1)=0$

$m+4=0 \quad | \quad m+1=0$

$m=-4 \quad | \quad m=-1$

$\therefore \boxed{m_1 \neq m_2}$

$$\boxed{C.F = Ae^{-4x} + Be^{-x}}$$

To find P.I

$$P.I = \frac{1}{D^2+5D+4}(x^2+7x+9)$$

$$= \frac{1}{4\left[1+\frac{D^2+5D}{4}\right]}(x^2+7x+9)$$

$$= \frac{1}{4} \times \left[1+\left(\frac{D^2+5D}{4}\right)\right]^{-1}(x^2+7x+9)$$

$\boxed{\because (1+x)^{-1}}$

$(1+x)^{-1} = 1-x+x^2 - \cdots$

$f = x^2+7x+9$

$D(f) = f' = 2x+7$

$D^2(f) = f'' = 2$

$D^3(f) = f''' = 0$

$$\frac{1}{4}\left\{1-\left(\frac{D^2+5D}{4}\right)+\left(\frac{D^2+5D}{4}\right)^2-\cdots\right\}(x^2+7x+9)$$

$$=\frac{1}{4}\left\{1-\left(\frac{D^2+5D}{4}\right)+\left[\frac{\cancel{D^4}+25D^2+\cancel{10D^3}}{16}\right]\right\}(x^2+7x+9)$$

$\boxed{\text{Omit } D^3, D^4}$

$$=\frac{1}{4}\left\{1-\left(\frac{D^2+5D}{4}\right)+\left(\frac{25D^2}{16}\right)\right\}(x^2+7x+9)$$

$$=\frac{1}{4}\left\{(x^2+7x+9)-\frac{1}{4}\left[D^2(x^2+7x+9)+5D(x^2+7x+9)\right]+\frac{25}{16}D^2(x^2+7x+9)\right\}$$

$$=\frac{1}{4}\left\{x^2+7x+9-\frac{1}{4}\left[(2)+5(2x+7)\right]+\frac{25}{\cancel{16}_{8}}(\cancel{2})\right\}$$

$$=\frac{1}{4}\left\{x^2+7x+9-\frac{2}{4}-\frac{5(2x+7)}{4}+\frac{25}{8}\right\}$$

$$=\frac{1}{4}\left\{x^2+7x+9-\frac{\cancel{2}}{\cancel{4}_{2}}-\frac{\cancel{10}^{5}x}{\cancel{4}_{2}}-\frac{35}{4}+\frac{25}{8}\right\}$$

$$=\frac{1}{4}\left\{x^2+\left(7-\frac{5}{2}\right)x+9-\frac{1}{2}-\frac{35}{4}+\frac{25}{8}\right\}$$

$$=\frac{1}{4}\left\{x^2+\left(\frac{14-5}{2}\right)x+\frac{72-4-70+25}{8}\right\}$$

$$\boxed{P.I=\frac{1}{4}\left\{x^2+\frac{9}{2}x+\frac{23}{8}\right\}}$$

$$y=C.F+P.I$$

$$\boxed{y=A\cdot e^{-x}+B\cdot e^{-4x}+\frac{1}{4}\left\{x^2+\frac{9}{2}x+\frac{23}{8}\right\}}$$