IDHAYA COLLEGE FOR WOMEN, KUMBAKONAM

DEPARTMENT OF MATHEMATICS

CLASS : II BBA

SUBJECT NAME : OPERATIONS RESEARCH

SUBJECT CODE : 16CCBB8

SEM : IV

UNIT : V (REPLACEMENT MODELS)

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UNIT - V

PART- A

1. What are the replacement situations?

The replacement situations may be divided in to four categories.

i. Replacement of capital equipment that suffers heavy depreciation in the course of time,eg.,machines tools,planes,etc.

ii. Group replacement of items that fails completely.eg.,light bulbs

iii. Problems of mortality and staffing

iv. Miscellaneous problems.

2. **Write down the formula for Average annual total cost**.

Average annual total cost,A(N) = $\frac{TC}{N} = \frac{C-S}{N} + \frac{1}{N} \int_0^n f(t)dt$

where, TC = capital cost - scrap value + maintenance cost

C = capital cost

S = scarp value

N = number of years

F(t) = maintenance cost

3. what is money value?

Since money has a value over time,we often say money is worth 10 percent per year.If we borrow Rs.100 at a 10 percent rate of interest per year and spend this amount today,then we have to pay Rs.110 after one year.

4. Define present worth factor.

If r is the rate of interest,then $(1+r)^{-n}$ is called the **present worth factor (PWF)**.The expression $(1+r)^{-n}$ is known as the payment **compound amount factor** (caf) of one rupee spent in n years duration.

5. Define Discount rate.

The present worth factor of unit amount to be spent after one year is given by $V= (1+r)^{-1}$,when r is called the rate of interest and V is called **discount rate**.

6. Write about the two types of replacement policies.

We shall consider the following two types of replacement policies.

(i)**Individual replacement policy:** Under this policy,an item is replaced immediately after it fails.

(ii)**Group replacement policy:** Under this policy,we take decisions as to when all the items must be replaced,irrespective of the fact that items have failed or not,with a provision that if any item fails before the optimal time,it may be individually replaced.

PART –B

1)The cost of a machine is Rs.61,000 and its scrap value is Rs.1,000.The maintenance costs found from the past experiences are as follows:

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Maintenance cost in rupees	**1000**	**2500**	**4000**	**6000**	**9000**	**12000**	**16000**	**20000**

When should the machine be replaced?

Solution:

Given , the cost of a machine = Rs.61,000 and its scrap value = Rs.1,000.We calculate an Average total cost using the following table.

Year **1**	**Running cost** **2**	**Cumulative running cost** **3**	**Depreciation cost C-S** **4**	**Total cost TC** **5=3+4**	**Average cost A(n)** **6=5/1**
1	1000	1000	60000	61000	61000
2	2500	3500	60000	63500	31750
3	4000	7500	60000	67500	22500
4	6000	13500	60000	73500	18375
5	9000	22500	60000	82500	16500
6	12000	34500	60000	94500	15750
7	16000	50500	60000	110500	15785.71
8	20000	70500	60000	130500	16312.50

From the table, it is noted that the Average cost A(n) is minimum in the 6^{th} year (Rs.15750).Hence the machine should be replaced after every 6 years.

2)A machine costs Rs.10,000.Its operating cost and resale values are given below:

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Operating cost	**1000**	**1200**	**1400**	**1700**	**2000**	**2500**	**3000**	**3500**
Resale value	**6000**	**4000**	**3200**	**2600**	**2500**	**2400**	**2000**	**1600**

Determine at what time it should be replaced.

Solution:

Given , the cost of a machine = Rs.10,000. We calculate an Average total cost using the following table.

Year **1**	**f(n)** **2**	$\sum$**f(n)** **3**	**S** **4**	**C-S** **5**	**TC** **6=3+5**	**A(n) = TC/n** **7 = 6/1**
1	1000	1000	6000	4000	5000	5000
2	1200	2200	4000	6000	8200	4100
3	1400	3600	3200	6800	10400	3466.70
4	1700	5300	2600	7400	12700	3175
5	2000	7300	2500	7500	14800	2960
6	2500	9800	2400	7600	17400	2900
7	3000	12800	2000	8000	20800	2971.4
8	3500	16300	1600	8400	24700	3087.5

From the table, it is noted that the Average cost A(n) is minimum at the end of 6^{th} year (Rs.2900).Hence the machine should be replaced at the end of 6^{th} year.

3) The cost pattern for two machines A and B,when money value is not considered is given in the table below.

Year	**Cost at the beginning of year**	
	Machine A	**Machine B**
1	**900**	**1400**
2	**600**	**100**
3	**700**	**700**

Find the cost pattern for each machine when money is worth 10 percent per year and hence find which machine is less costly.

Solution:

The total outlay for the three years for machine A = 900 + 600 +700=2200.

Also for machine B = 1400+ 100 + 700 = 2200.

The total outlay for both the machines is the same for 3 years.Both the machines appear to be equally good in this case.

Present worth factor = $(1+r)^{-n} = (1+\frac{10}{100})^{-1} = \left(\frac{110}{100}\right)^{-1} = \frac{100}{110}$

Year	Discount cost (10% rate)	
	Machine A	Machine B
1	900	1400
2	$600\times\frac{100}{110} = 545.45$	$100\times\frac{100}{110} = 90.91$
3	$700\times\left(\frac{100}{110}\right)^{-1} = 578.51$	$700\times\left(\frac{100}{110}\right)^{2} = 578.51$
Total outlay	Rs.2,023.96	Rs.2,069.42

The outlay of machine A is less that of machine B.Hence,machine A will be preferred.

PART-C

1) **A machine costs Rs.15000.The running cost for the different years are given below.**

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Running							

cost	2500	3000	4000	5000	6500	8000	10000

Find the optimum replacement period if the capital is worth 10 percent per annum and has no salvage value.

Solution:

Present worth factor = V= $(1 + r)^{-1} = (1+\frac{10}{100})^{-1} = 0.9091$.

Weighted average cost = W(n) = $\frac{C+\sum R_n V^{n-1} - S_n V^n}{\sum_V n-1}$

Year (n)	Running cost R_{n-1}	V^{n-1}	$\sum V^{n-1}$	$R_{n-1}V^{n-1}$	$\sum R_n V^{n-1}$	c+ $\sum R_n V^{n-1}$	W(n)
1	2500	1	1	2500	2500	17500	17500
2	3000	0.9091	1.9091	2727.3	5227.30	20227.30	10595.2
3	4000	0.8265	2.7356	3306	8533.30	23533.30	8602.60
4	5000	0.75134	3.486	3756.68	12289.98	27289.98	7826.34
5	6500	0.6830	4.169	4439.76	16729.74	31729.74	7610.87
6	8000	0.6209	4.7899	4967.61	21697.35	36697.35	7661.40
7	10000	0.5645	5.3544	5645	27342.42	42342.43	7907.96

The weighted average cost is minimum at the end of 5^{th} year.

$$R_4 < W(5) < R_6$$

$$6500 < 7610.87 < 8000$$

Hence, the optimum replacement period is every 5^{th} year.