IDHAYA COLLEGE FOR WOMEN, KUMBAKONAM

DEPARTMENT OF MATHEMATICS

CLASS	:	*I –INFORMATION TECHNOLOGY*
SUBJECT NAME	:	*OPERATION RESEARCH*
SUBJECT CODE	:	*16SACMA3*
SEM	:	*II*
UNIT	:	*III (TRANSPORTATION AND ASSIGNMENT PROBLEM)*
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UNIT III
PART-A

1. **Define Transportation problem.**
   The transportation problem is a special type of LLP where the objective is to minimize the cost of distributing a product from a number of sources or origins to a number of destinations.

2. **Define:**
   (i) Feasible solution
   (ii)Basic feasible solution
   Feasible solution
   Any set of non-negative allocations which satisfies the row and column sum is called a feasible solution.
   Basic feasible solution
   A feasible solution is called as a basic feasible solution if the number of non-negative allocation is equal to m+n-1.

3. **Write about non-degenerate solution and degenerate basic feasible solution.**
   Non-degenerate solution.
   Any feasible solution is said to be non-degenerate solution if the number of allocation is equal to m+n-1.
   Degenerate basic feasible solution.
   If a basic feasible solution contains less than m+n-1 non-negative allocations, it is said to be degenerate.

4. **Define Assignment problem.**
   The problem is to find an assignment in which job should be assigned to which person, on a one-to-one basis, so that the total cost of performing all the jobs is minimum.

5. **Define Unbalanced Assignment problem**
   Any Assignment problem is said to be unbalanced if the cost matrix is not a square matrix. Number of rows is not equal to number of columns to make it balanced add dummy row or dummy column with all the entries as zero.

6. **Write down the mathematical formulation of an Assignment problem**.

Mathematical Formulation of an Assignment problem.

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}, i = 1,2....n; \ j = 1,2,....n.$$

$$x_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

Where x_{ij} denotes that the j^th^ job is to be assigned to the i^th^ person.

PART – B

1. **Determine an initial basic feasible solution to the following transportation problem using least cost method.**

	D_1	D_2	D_3	D_4	Supply
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	35

Solution:

Step 1

To find a basic feasible solution. Using the steps in least cost method, to make the first allocation to the cell (1, 3) with magnitude X13 = Min (14, 15) = 14 as it is the cell having the least cost. This allocation exhausts the first row supply. Hence, the first row is deleted.

Step 2

From the reduced table, the next allocation is made in the next least cost cell (2, 3) which is chosen arbitrarily with magnitude X23 = Min (1, 16) = 1.This exhausts the third column destination.

	D_1	D_2	D_3	D_4	Supply
O_1	6	4	1 (14)	5	14
O_2	8 (6)	9 (9)	2 (1)	7	16
O_3	4	3 (1)	6	2 (4)	5
Demand	6	10	15	4	

Step 3

From the reduced table, the next least cost cell is (3, 4) for which allocation is made with magnitude Min (4, 5) = 4. This exhausts the destination D4 requirement.

Step 4

Delete this fourth column from the table. The next allocation is made in the cell (3, 2) with magnitude X32 = Min (1, 10) = 1 which exhausts the third origin capacity.

Hence, the third row is exhausted. From the reduced table, the next allocation is given to the cell (2,1) with magnitude X21 = Min (6, 15) = 6. This exhausts the first column requirement. Hence, it is deleted from the table.

Step 5

Finally, the allocation is made to the cell (2, 2) with magnitude X22 = Min (9, 9) = 9 which satisfies the rim requirement.

The initial basic feasible solution to the transportation problem using least cost method is given by $x_{13} = 14$; $x_{21} = 6$; $x_{22} = 9$; $x_{23} = 1$; $x_{32} = 1$; $x_{34} = 4$.

(14x1) + (6x8) + (9x9) + (1x2) + (1x3) + (4x2) = 156

2. **Determine an initial basic feasible solution to the following transportation problem using least cost method.**

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	24

Solution:

Step 1

To find a feasible solution to the Transportation Problem, using the steps in the least cost method, the first allocation is made in the cell (3, 1) the magnitude being X31 = 4. This satisfies the demand at the destination D1 and delete this column from the table as it is exhausted.

	D_1	D_2	D_3	D_4	Supply
O_1	1	2 (6)	3	4	~~6~~ 0
O_2	4	3	2 (2)	0 (6)	~~8~~ ~~2~~
O_3	0 (4)	2	2 (6)	1	~~10~~ ~~6~~
Demand	~~4~~ 0	~~6~~ 0	~~8~~ ~~2~~ 0	~~6~~ 0	24

Step 2

The second allocation is made in the cell (2, 4) with magnitude
X24 = Min (6,8) = 6. Since it satisfies the demand at the destination D4, it is deleted from the table.

Step 3

From the reduced table, the third allocation is made in the cell (3, 3) with magnitude X33 = Min (8, 6) = 6. The next allocation is made in the cell (2, 3) with magnitude X23 of Min (2, 2) = 2.

Step 4

Finally, the allocation is made in the cell (1, 2) with magnitude
X12= Min (6, 6) = 6. Now, all the requirements have been satisfied and hence, the initial feasible solution is obtained.
The initial basic feasible solution to the transportation problem using least cost method is given by $x_{12} = 6$; $x_{23} = 2$; $x_{24} = 6$; $x_{31} = 4$; $x_{33} = 6$.

(6x2) + (2x2) + (6x0) + (4x0) + (6x2) = 28

3. **Determine an initial basic feasible solution to the following transportation problem using North west method.**

	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	34

Solution:

Step 1

To find a feasible solution to the transportation problem. The first allocation is made in the cell (1, 1), the magnitude beingX11 = Min (5, 7) = 5.

Step 2

The second allocation is made in the cell (2, 1) and the magnitude of the allocation is given by X21 = Min (8, 7 – 5) = 2.

	D_1	D_2	D_3	Supply
O_1	2 (5)	7	4	~~5~~ 0
O_2	3 (2)	3 (6)	1	~~8~~ ~~6~~ 0
O_3	5	4 (3)	7 (4)	~~7~~ ~~4~~ 0
O_4	1	6	2 (14)	~~14~~ 0
Demand	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~18~~ 14 0	34

Step 3

The third allocation is made in the cell (2, 2) the magnitude
X22 = Min (8 – 2,9) = 6.

Step 4

The magnitude of the fourth allocation is made in the cell (3, 2) given by
X32 = Min (7, 9 – 6) = 3.

Step 5

The fifth allocation is made in the cell (3, 3) with magnitude
X33 = Min (7 – 3, 14)= 4.

Step 6

The final allocation is made in the cell (4, 3) with magnitude
X43 = Min(14, 18 – 4) = 14.
The initial basic feasible solution to the transportation problem using North west method is given by $x_{11} = 5$; $x_{21} = 2$; $x_{22} = 6$; $x_{32} = 3$; $x_{33} = 4$; $x_{43} = 14$

(5x2) + (2x3) + (6x3) + (3x4) + (4x7) + (14x2) = 102

4. Using the following cost matrix, determine the optimal job assignment and minimum cost of assignment.

Jobs		I	II	III	IV	V
Mechanic	**A**	10	3	3	2	8
	B	9	7	8	2	7
	C	7	5	6	2	4
	D	3	5	8	2	4
	E	9	10	9	6	10

Solution :

Step 1
Select the smallest element in each row and subtract this smallest element from all the elements in its row.

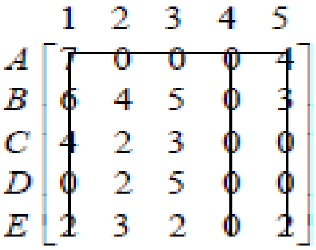
	1	2	3	4	5
A	8	1	1	0	6
B	7	5	6	0	5
C	5	3	4	0	2
D	1	3	6	0	2
E	3	4	3	0	4

Step 2
Select the minimum element from each column and subtract from all other elements in its column. With this we get the first modified matrix.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c}\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}\\ \begin{bmatrix} 7 & 0 & 0 & 0 & 4 \\ 6 & 4 & 5 & 0 & 3 \\ 4 & 2 & 3 & 0 & 0 \\ 0 & 2 & 5 & 0 & 0 \\ 2 & 3 & 2 & 0 & 2 \end{bmatrix}\end{array}$$

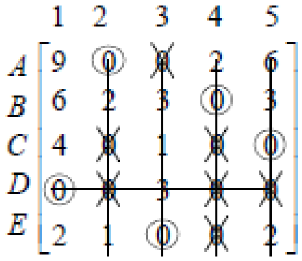
Step 3

In this modified matrix we draw the minimum number of lines to cover all zeros horizontal or vertical.



Step 4

Number of lines drawn to cover all zeros is 4 = N.
The order of matrix is n = 5
hence, N < n. Now we get the second modified matrix by subtracting the smallest uncovered element from the remaining uncovered elements and adding it to the element at the point of intersection of lines.



Step 5

Number of lines drawn to cover all zeros = N = 5
The order of matrix is n = 5.
Hence N = n. Now we determine the optimum assignment.
Optimal assignment and optimum cost of assignment

Job	Mechanic	Cost
1	D	3
2	A	3
3	E	9
4	B	2
5	C	4

Optimal job assignment 1→D, 2→A, 3→E, 4→B, 5→C

The minimum cost of assignment = 21

5. **A company has 4 machines to do 3 jobs. The cost of each job on each machine is given below. Determine the job assignments and that will minimize the total cost.**

Machine		**W**	**X**	**Y**	**Z**
Job	**A**	18	24	28	32
	B	8	13	17	18
	C	10	15	19	22

Solution :

Step 1
Since the cost matrix is not a square matrix, we add a dummy row D with all the elements 0.

Step 2
Subtract the minimum element in each row from all the elements in its row.

Step 3
Since each column has a minimum element 0, we draw minimum number of lines to cover all zeros.

Step 4
The number of lines drawn to cover all zeros = 2 < the order of matrix, we form a second modified matrix.

Step 5
Here, N = 3 < n = 4.
Again we subtract the smallest uncovered element from all the uncovered elements and add to the element at the point of intersection and here, N = 4 = n.

Optimal assignment and optimum cost of assignment

Job	**Machine**	**Cost**
A	W	18
B	X	13
C	Y	19
D	Z	0

Since D is a dummy job, machine Z is assigned no job.

Optimal job assignment A→W, B→X, C→Y, D→Z
The minimum cost of assignment = 50

6. Determine optimal job assignment and the minimum cost of assignment.

Machines		**A**	**B**	**C**	**D**	**E**
Jobs	**1**	4	3	6	2	7
	2	10	12	11	14	16
	3	4	3	2	1	5
	4	8	7	6	9	6

Solution :

Step 1
Since the cost matrix is not a square matrix, the problem is unbalanced.
We add a dummy job 5 with corresponding entries zero.

Step 2
We subtract the smallest element from all the elements in the respective rows.

Step 3
Since each column has minimum element as zero, we draw minimum number of lines to cover all zeros.

Step 4
The number of lines to cover all zeros = 4 < the order of matrix. We form the 2nd modified matrix by subtracting the smallest uncovered element from the remaining uncovered elements and adding to the element at the point of intersection of lines.

Step 5
Here the number of lines drawn to cover all zeros = 5 = Order of matrix.
Therefore, we can make the assignment
Optimal assignment and optimum cost of assignment

Job	**Machines**	**Cost**
1	B	3
2	A	10
3	D	1
4	C	6
5	E	0

Since 5 is a dummy job, Machine E is assigned no job.

Optimal job assignment 1→B, 2→A, 3→D, 4→C, 5→E

The minimum cost of assignment = 20

7. **A marketing manager has 5 salesmen and there are 5 sales districts. Considering the capabilities of the salesmen and the nature of districts, the estimates made by the marketing manager for the sales per month in 1,000 rupees for each salesman in each district would be as follows.**

District		**A**	**B**	**C**	**D**	**E**
Salesmen	**1**	32	38	40	28	40
	2	40	24	28	21	36
	3	41	27	33	30	37
	4	22	38	41	36	36
	5	29	33	40	35	39

Find the assignment of salesmen to the districts that will result in the maximum sales.

Solution :

Step 1
To maximize the profit, first convert it into a loss matrix, which can be minimized. To convert it into loss matrix, subtract all the elements from the highest element 41. Subtract the smallest element from all the elements in the respective rows and columns, to get the first modified matrix.

Step 2
We now draw minimum number of lines to cover all zeros.
N = 4 < n = 5

Step 3
We subtract the smallest uncovered element from the remaining uncovered elements and add to the elements at the point of intersection of lines, to get the second modified matrix.

Step 4
Again, N = 4 n = 5. Repeat the above step to get the assignment.

Optimal assignment and optimum cost of assignment

Salesmen	District	Maximum sales
1	B	38
2	A	40
3	E	37
4	C	41
5	D	35

Optimal job assignment 1→B, 2→A, 3→E, 4→C, 5→D

The maximum profit of assignment =191

8. Write down the difference between Transportation and Assignment problem.

TRANSPORTATION PROBLEM	ASSIGNMENT PROBLEM
(i) This is about reducing cost or improving profit involving in transportation merchandize.	(i) This is about assigning finite sources to finite destinations in a way where only one destination is allotted for one source with minimum cost.
(ii) Number of sources and number of demand need not be equal	(ii) Number of sources and number of destination must be equal
(iii) Matrix need not to be a square matrix	(iii) Matrix must be square matrix
(iv) If total demand and total supply are not equal them problem is said to be unbalanced.	(iv) Problems are said to be unbalanced if number of rows are not equal to the number of columns
(v) It requires 2 stages to solve :IBFS by North west corner rule, vogel's approximation method, least cost method and optimal solution by MODI method.	(v)First stage is sufficient for obtaining optimal solution by Hungarian method

PART – C

1. **Determine an initial basic feasible solution to the following transportation problem using VAM method.**

	D_1	**D_2**	**D_3**	**D_4**	**Supply**
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	950

Solution

Step 1

To find a feasible solution to the problem.
First, we find the row and column penalty PI as the difference between the least and the next least cost. The maximum penalty is 5. Choose the first column arbitrarily. In this column, choose the cell having the least cost name (1, 1).

Step 2

Allocate to this cell with minimum magnitude (that is, Min (250, 200) = 200.) This exhauststhe first column. Delete this column. Since a column is deleted, there is a change in row penalty PII while column penalty PII remains the same. Continuing in this manner to get the remaining allocations.

I Allocation

	D_1	D_2	D_3	D_4	Supply	P_1
O_1	11 (200)	13	17	14	50 ~~250~~	2
O_2	16	18	14	10	300	4
O_3	21	24	13	10	400	3
Demand	~~200~~ 0	225	275	250		
P_1	5↑	5	(1)	0		

II Allocation

	D_2	D_3	D_4	*Supply*	P_{II}
O_1	13 (50)	17	14	~~50~~	(1)
O_2	18	14	10	300	4
O_3	24	13	10	400	3
Demand	~~225~~ 175	275	250		
P_{II}	5↑	(1)	0		

III Allocation

	D_2	D_3	D_4	*Supply*	P_{III}
O_2	18 (175)	14	10	~~300~~ 125	4
O_3	24	13	10	400	3
Demand	~~175~~ 0	275	250		
P_{III}	6↑	1	0		

IV Allocation

	D_3	D_4	*Supply*	P_{IV}
O_2	14	10 (125)	~~125~~ 0	4 ←
O_3	13	10	400	3
Demand	275	~~250~~ 125		
P_{IV}	1	0		

V Allocation

	D_3	D_4	*Supply*	P_V
O_3	13 (275)	10	~~400~~ 125	3
Demand	~~275~~ 0	125		
P_V	13↑	10		

VI Allocation

	D_4	*Supply*	P_{VI}
O_3	10 (125)	~~125~~ 0	10 ←
Demand	~~125~~ 0		
P_{VI}	10		

	D_1	D_2	D_3	D_4	Supply
O_1	11 (200)	13 (50)	17	14	250
O_2	16	18 (175)	14	10 (125)	300
O_3	21	24	13 (275)	10 (125)	400
Demand	200	225	275	250	

Step 3

The solution is a non-degenerate basic feasible solution because 6 positive independent allocation are equal to m+n-1=3+4-1.

$x_{11} = 200$; $x_{12} = 50$; $x_{22} = 175$; $x_{24} = 125$; $x_{33} = 275$; $x_{34} = 125$

(200x11) + (50x13) + (175x18) + (125x10) + (275x13) + (125x10) = 12,075.

2. **The owner of a small machine shop has four mechanics available to assign jobs for the day. Five jobs are offered with expected profit for each mechanic on each jobs, find the assignment of mechanics to the job that will result in maximum profit. Which job should be declined?**

Job		**A**	**B**	**C**	**D**	**E**
Mechanic	**1**	62	78	50	111	82
	2	71	84	61	73	59
	3	87	92	111	71	81
	4	48	64	87	77	80

Solution :

Step 1
The given profit matrix is not a square matrix as the number of jobs is not equal to the number of mechanics. Hence, we introduce a dummy mechanic 5 with all the elements 0.

Step 2
Now we convert this profit matrix into loss matrix by subtracting all the elements from the highest element 111.
We subtract the smallest element from all the elements in the respective rows.

Step 3
Since each column has minimum element as zero, we draw minimum number of lines to cover all zeros.

Step 4
Here the number of lines drawn to cover all zeros = N = 4, is less than the order of matrix.

Step 5
We form the 2nd modified matrix by subtracting the smallest uncovered element
from the remaining uncovered elements and adding to the element that is at the point of intersection of lines.
Hence, N = 5 = n be the order of matrix.

Optimal assignment and optimum cost of assignment

Job	**Mechanic**	**Maximum profit**
A	5	84
B	2	111
C	3	111
D	1	80
E	4	0

Since the 5^{th} mechanic is a dummy, job A is assigned to the 5^{th} mechanic , this job is declined. Optimal job assignment A→5, B→2, C→3, D→1, E→4
The maximum profit = 386.

3. Determine optimal job assignment and the minimum cost of assignment.

Operators		**I**	**II**	**III**	**IV**	**V**
Machines	**A**	30	25	33	35	36
	B	23	29	38	23	26
	C	30	27	22	22	22
	D	25	31	29	27	32
	E	27	29	30	24	32

Solution :

Step 1
We form the first modified matrix by subtracting the least element from all the elements in the respective rows and then in the respective columns.

Step 2
Since each column has the minimum element 0, the first modified matrix is obtained. We draw the minimum number of lines to cover all zeros.
The number of lines drawn to cover all zeros = 4 < the order of matrix = 5. Hence, we form the second modified matrix by subtracting the smallest uncovered element from the remaining uncovered elements and adding to the element that is at the point of intersection of lines.

Step 3
N = 5, i.e., the number of lines drawn to cover all zeros = order of matrix. Hence,
we can make an assignment.

Optimal assignment and optimum cost of assignment

Operator	**Machines**	**Cost**
I	D	25
II	A	25
III	C	22
IV	E	24
V	B	26

Since 5 is a dummy job, Machine E is assigned no job.
Optimal job assignment I→D, II→A, III→C, IV→E, V→B
The minimum cost of assignment = 122.

4. A company has 5 machines to do 5 jobs. The cost of each job on each machine is given below. Determine the job assignments and that will minimize the total cost.

Machines		**A**	**B**	**C**	**D**	**E**
Job	**1**	13	8	16	18	19
	2	9	15	24	9	12
	3	12	9	4	4	4
	4	6	12	10	8	13
	5	15	17	18	12	20

Solution :

Step 1

We form the first modified matrix by subtracting the minimum element from all the elements in the respective row, and the same with respective columns.

Step 2

Since each column has the minimum element 0, we have the first modified matrix.

Now we draw the minimum number of lines to cover all zeros.

Step 3

Number of lines drawn to cover zero is N = 4 < the order of matrix n = 5.

We find the second modified matrix by subtracting the smallest uncovered element (3) from all the uncovered elements and adding to the element that is the point of intersection of lines.

Step 5

Number of lines drawn to cover all zeros = 5, which is the order of matrix. Hence, we can form an assignment.

Optimal assignment and optimum cost of assignment

Job	**Machine**	**Cost**
1	B	8
2	E	12
3	C	4
4	A	6
5	D	12

Since 5 is a dummy job, Machine E is assigned no job.
Optimal job assignment 1→B, 2→E, 3→C, 4→A, 5→D
The minimum cost of assignment = 42.