IDHAYA COLLEGE FOR WOMEN, KUMBAKONAM

DEPARTMENT OF MATHEMATICS

CLASS	:	**I B.SC., MATHS**
SUBJECT NAME	:	**ANALYTICAL GEOMETRY 3D**
SUBJECT CODE	:	**16SCCMM4**
SEM	:	**II**
UNIT	:	**V (CONE)**
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UNIT - V

PART- A

1. **Define central quadrics**.

If P (x_1, y_1, z_1) lie on the surface $Ax^2+By^2+Cz^2= 1$ and Q ($-x_1$, $-y_1$, $-z_1$) also lie on the surface and O the origin is the midpoint of PQ. Hence all the chords of $Ax^2+By^2+Cz^2= 1$ which passes through O are bisected at O. For the reason $Ax^2+ By^2+ Cz^2= 1$ is called central quadric. O is the centre and a chord through O is called diameter.

2. **Define ellipsoid**.

Let A, B, C all positive put $A=1/a^2$; $B=1/b^2$; $c= 1/c^2$ and the equation $Ax^2+ By^2+ Cz^2 =1$ becomes

$$\mathbf{x^2/a^2+ y^2/b^2+ z^2/c^2 =1}$$

The surface meets the coordinate axis in the point (±a,0,0), (0, ±b,0), (0,0, ±c).

It can be seen the x, y, z satisfying the equation $\mathbf{x^2/a^2+ y^2/b^2+ z^2/c^2 =1}$ cannot have numerical values which is exceeds a, b, c is respectively. Hence the surface is closed the surface is called ellipsoid.

3. **Write down the condition for the plane to touch the quadric cone.**

The condition for the plane to touch the quadric cone,

$$\begin{vmatrix} a & h & g & l \\ h & b & f & m \\ g & f & c & n \\ l & m & n & o \end{vmatrix} = 0$$

4. **Write down the condition for the plane lx+ my+ nz= p to touch the conicoid ax^2+ by^2+ cz^2= 1.**

The condition for the plane lx+ my+ nz= p to touch the conicoid $ax^2+ by^2+ cz^2= 1$ is

$$P^2 = l^2/a + m^2/b + n^2/c$$

5. **Which surface is called hyperboloid of one sheet ?**

Let A and B be positive and C be negative put $A=1/a^2$; $B=1/b^2$; $c = -1/c^2$ and then the equation $Ax^2+ By^2+ Cz^2 =1$ becomes $x^2/a^2+ y^2/b^2- z^2/c^2 =1$ **.**The section z = k is given by, $x^2/a^2+ y^2/b^2 = 1+ k^2/c^2$, z = k and for all values of k the equation becomes ellipse.This surface is called hyperboloid of one sheet.

PART- B

1. **Derive the condition for the plane to touch the conicoid.**

Let the plane to touch the conicoid at (x_1, y_1, z_1). The equation of tangent plane at (x_1, y_1, z_1) is

$$\mathbf{ax x_1+ by y_1+ cz z_1 = 1} \quad \rightarrow (1)$$

This plane is also represented by the equation **lx+ my+ nz = p**

$ax_1/l = by_1/m = cz_1/n = 1/p$

$ax_1/l = 1/p$; $by_1/m = 1/p$; $cz_1/n = 1/p$

$x_1 = l/ap$; $y_1 = m/bp$; $z_1 = n/cp$

Since (x_1, y_1, z_1) is lie on conicoid $ax_1^2+ by_1^2+ cz_1^2 = 1$

$al^2/a^2p^2 + bm^2/b^2p^2 + cn^2/c^2p^2 = 1$

$l^2/ap^2 + m^2/bp^2 + n^2/cp^2 = 1$

$p^2 = l^2/a + m^2/b + n^2/c$

COROLLARY: 1

The equation of any tangent plane to the conicoid $ax_1^2+ by_1^2+ cz_1^2=1$ is of the form

$$lx + my + nz = \pm (l^2/a + m^2/b + n^2/c)^{1/2}$$

COROLLARY: 2

There are two tangent planes to a conicoid parallel to the plane lx+my+nz=0 and their equations are

$$lx + my + nz = \pm (l^2/a + m^2/b + n^2/c)^{1/2}$$

2. Derive the equation of tangent and tangent planes.

Any line through P (x_1, y_1, z_1) is of the form,

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} \quad \rightarrow (1)$$

and if this line meets the conicoid $ax^2+by^2+cz^2=1$ at the point (x_1+lr, y_1+mr, z_1+nr).

The parameter **r** is given by the equation

$$r^2(al^2+bm^2+cn^2) + 2r(alx_1+bmy_1+cnz_1) + ax_1^2+ by_1^2+ cz_1^2 - 1 = 0 \rightarrow (2)$$

If x_1, y_1, z_1 lie on the conicoid $ax_1^2+by_1^2+cz_1^2 = 1$ →(A)

Put the equation (A) in (2)

$$r^2(al^2+bm^2+cn^2) +2r(alx_1+bmy_1+cnz_1) = 0 \rightarrow (3)$$

If the line (1) is tangent to the conicoid line (1) will meet the conicoid in two coincident points. Hence the equation (3) has two zero roots.

$$alx_1+bmy_1+cnz_1 = 0 \quad \rightarrow (4)$$

Equation (4) is the condition that the line (1) is perpendicular to the line whose direction cosines are proportional to $ax_1:by_1:cz_1$.

Hence all tangent line at P of P (x_1, y_1, z_1) to the conicoid is perpendicular to the line whose direction ratios are $ax_1+by_1+cz_1$.

Hence all the tangent line at P lies in a plane through P perpendicular to this direction. This plane is known as tangent plane at P and its equation is

$$ax_1(x-x_1) + by_1(y-y_1) + cz_1(z-z_1) = 0$$

$$axx_1+byy_1+czz_1 = ax_1^2+ by_1^2+ cz_1^2$$

$$axx_1+byy_1+czz_1 = 1$$

3. Find the point of intersection of line $-\frac{1}{3}(x+5) = (y-4) = \frac{1}{7}(z-11)$ and cut the conicoid $12x^2-17y^2+7z^2 = 7$.

Solution:

Given:

$$-\tfrac{1}{3}(x+5) = (y-4) = \tfrac{1}{7}(z-11) \quad \rightarrow (1)$$

$$12x^2-17y^2+7z^2 = 7 \quad \rightarrow (2)$$

From (1)

$-(x+5)/3 = (y-4) = (z-11)/7 = r$

$-(x+5)/3 = r$; $y-4 = r$; $(z-11)/7 = r$

$-(x+5) = 3r$; $y = r+4$; $z-11 = 7r$

$x+5 = -3r$

$x = -3r-5$

the point (-3r-5, r+4, 7r+11) lies on equation (2),

$12(-3r-5)^2 - 17(r+4)^2 + 7(7r+11)^2 = 7$

$12(9r^2+25+30r)-17(r^2+16+8r) +7(49r^2+121+154r) = 7$

$434r^2+1302r+868 = 0$

$r^2+3r+2 = 0$

$(r+1)(r+2) = 0$

$r = -1, -2$

The point is (3-5, -1+4, -7+11) for r = -1, (-2, 3, 4)

The point is (6-5, -2+4, -14+11) for r= -2, (1, 2, -3).

PART -C

1. **Find the equation of the tangent planes to $x^2+y^2+4z^2=1$ which intersect in the line whose equations are 12x-3y-5=0, z=1.**

solution:

Any plane which passes through the line is given by

$$12x-3y-5+\lambda(z-1) = 0$$

$$\mathbf{12x-3y+\lambda z-(\lambda+5) = 0} \quad \rightarrow(1)$$

Let the plane touch the conicoid at x_1, y_1, z_1.

The equation of the tangent plane at x_1, y_1, z_1 is $\mathbf{xx_1+yy_1+4zz_1=1} \rightarrow (2)$

Equation (1), (2) represents the same plane.

$x_1/ 12 = y_1/ -3 = 4z_1/ \lambda = 1/ \lambda+5$

$x_1/12 = 1/ \lambda+5 \Rightarrow x_1 = 12 /\lambda+5$

$y_1/-3 = 1 \lambda+5 \Rightarrow y_1 = -3/ \lambda+5$

$4z_1 /\lambda = 1/\lambda+5n \Rightarrow z_1 = \lambda/4(\lambda+5)$

Since x_1, y_1, z_1 lies on the conicoid $x_1^2+ y_1^2+ 4z_1^2 = 1$

$(12/ \lambda+5)^2 + (-3/ \lambda+5)^2 + 4(\lambda /4 (\lambda+5))^2 = 1$

$144 /(\lambda+5)^2 + 9/(\lambda+5)^2 + 4\lambda2/16(\lambda+5)^2 = 1$

$144\times16 + 9\times16 + 4 \lambda^2 /16(\lambda+5)^2 = 1$

$2304 + 144 + 4 \lambda^2 /16(\lambda+5)^2 = 1$

$4 \lambda^2 + 2448 = 16 (\lambda^2 + 25 +10 \lambda)$

$4 (\lambda^2+ 612) = 16 (\lambda^2 + 25 + 10 \lambda)$

$\lambda^2 + 612 = 4\lambda^2 + 100 + 40\lambda$

$612 - 100 + \lambda^2 = 4\lambda^2 + 40\lambda$

$512 - 3\lambda^2 - 40\lambda = 0$

$3\lambda^2 + 40\lambda - 512 = 0$

$\lambda = \frac{-40 \pm \sqrt{40^2 - 4(3)(-512)}}{2(3)}$

$\lambda = \frac{-40 \pm \sqrt{1600+6144}}{6}$

$\lambda = \frac{-40 \pm 88}{6}$

$= \frac{-40+88}{6}, \frac{-40-88}{6}$

$= \frac{-128}{6}, \frac{48}{6}$

$= \frac{-64}{3}, 8$

Hence the equation of the tangent plane is put $\lambda = 8$ in (1) we get,

$12x-3y-(8)z-(8+5) = 0$

$12x-3y+8z-13 = 0$

(or)

Put $\lambda = \frac{-64}{3}$ in equation (2) we get,

$12x-3y-5-\frac{64}{3}(z-1) = 0$

$36x-9y-15-64z+64 = 0$

$36x-9y-64z+49 = 0$

2. **If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of the set of three mutually perpendicular generator of the cone 5yz-8zx-3xy = 0. Find the equation of the other two.**

Solution:

The given line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ → **(1)** and the cone is 5yz-8zx-3xy = 0 →**(2)**

Let the line perpendicular to the equation (1) be $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ → (**3**)

$\therefore l + 2m + 3n = 0 \quad \rightarrow (4)$

If equation (3) is generator of (1) then x = l, y = m, z = n

$\therefore 5mn - 8nl - 3lm = 0 \rightarrow (5)$

Eliminating n between (4)and (5) equation we get,

(4) ⇒ l + 2m + 3n = 0

3n = -l – 2m

n = $\frac{-(l+2m)}{3}$

substitute n value in equation (5)

(5) ⇒ n (5m – 8l) – 3lm = 0

$\frac{-(l+2m)}{3}$ (5m-8l) – 3lm = 0

$(-l - 2m)\,(5m - 8l) - 9lm = 0$

$-5lm + 8l^2 - 10m^2 + 16\,ml - 9\,lm = 0$

$8l^2 - 10m^2 + 2ml = 0$

$8l^2 + 2ml - 10m^2 = 0$

$4l^2 + lm - 5m^2 = 0$

$4\,(l/m)^2 + l\,(m/m^2) - 5 = 0$

$4\,(l/m)^2 + (l/m) - 5 = 0$

$(l/m - 1)\,(4l/m + 5) = 0$

l/m = 1 ; l/m = -5/4

l = m ; 4l = -5m

l-m=0 ; 4l + 5m = 0

when 4l+ 5m = 0

l = $\frac{-5}{4}$ m

(4) ⇒ n = $\frac{-(l+2m)}{3}$

= $\frac{-(-\frac{5}{4}m+2m)}{3}$

= $\frac{(-5m+8m)/4}{3}$

= $\frac{\left(-\frac{3m}{4}\right)}{3}$

n = $\frac{-m}{4}$

∴ $\frac{l}{5} = \frac{-m}{4} = \frac{n}{1}$ → (6)

When l – m = 0

l = m

(4) ⇒ n = $\frac{-(l+2l)}{3}$

n = -l ,

∴ $\frac{l}{1} = \frac{m}{1} = \frac{n}{-1}$ → (7)

From the equation (6)and (7) the other two mutually perpendicular generators are

$$\frac{x}{5} = \frac{-y}{4} = \frac{z}{1} \quad ; \quad \frac{x}{1} = \frac{y}{1} = \frac{z}{-1}$$