IDHAYA COLLEGE FOR WOMEN

KUMBAKONAM – 612 001

DEPA*RTMENT OF PHYSICS*

SEMESTER	**:**	***II***
CLASS	**:**	**I B.SC., *PHYSICS***
SUBJECT- INCHARGE	**:**	**M*S. A. BHARATHI***
SUBJECT NAME	**:**	***MECHANICS***
SUBJECT CODE	**:**	***16SCCPH2***
TOPIC	***:***	***UNIT – (1-5)***
		(Q&A)

Unit-1

1. Define projectile?

Any object or particle which is projected into space with a given velocity in a particular direction. It is called projectile. And its motion is called projectile motion.

Example: A ball is hit by a bat

2.Define Angle of projection?

When a particle is projected in any direction from the point on the earth the angle which a direction of projection makes the horizontal plane through the point of projection. This is called Angle of projection.

3.What is meant by time of flight?

The interval of time from the instant of projection to the instant the particle reaches the horizontal plane through the point of projection is called Time of flight.

4.Define impulsive force?

An impulsive is an infinitely great force acting for a very short interval of time such that their product is finite.

Example: The sudden force is experienced when the ball is hit by the bat.

5.What is meant by trajectory?

When a particle is projected in any direction from the point on the earth the angle which a projection makes the horizontal plane through the point of projection. This is called angle of projection. The path is described by the particle is called trajectory.

6.Define Range of projectile?

The distance between point of projection and the point where the projector meets the plane through the point of projection is called range of projectile.

7.State Principle of conservation of momentum?

The total momentum of two bodies after impact along the common normal must be equal to the total momentum of two bodies before impact along the same direction.

8.Define direct impact?

If the velocities of two colloiding bodies act along the line of impact, that impact is called direct impact.

9.Define Oblique Impact?

If the velocities of two colloiding bodies act along the lines other than the line of impact. That impact is called oblique impact.

10.Show that the path of projectile is a parabola?

The particle is projected from the point p with an initial velocity "u". α be the angle of projection. Draw the line AM perpendicular to PP'. Q is the position of the particle after t seconds in the trajectory. From the point Q draw QN perpendicular to AM and QL perpendicular to PP'.

We know that, the maximum height of the projectile,

$$AM = \frac{u^2 sin^2 \alpha}{2g} \longrightarrow 1$$

The range of angle of projectile PP,

$$PP' = \frac{2\, u^2 \sin\alpha \cos\alpha}{g} \longrightarrow 2$$

PM is the half of the distance of PP',

$$PM = \overline{u^2 \sin\alpha \cos\alpha /g} \longrightarrow 3$$

The distance of QL is,

$$QL = u\sin\alpha t - ½\, g t^2 \longrightarrow 4$$

$$PL = u\cos\alpha t \longrightarrow 5$$

From the diagram,

AN= AM – MN

= AM – QL

From equation [1] and [4]

$$AN = \frac{u^2 sin^2 \alpha}{2g} - (u \sin\alpha t - ½\, g t^2) \longrightarrow 6$$

$$= \frac{u^2}{2g} sin^2 \alpha - u\sin\alpha t + ½\, g\, t^2$$

,

$$= ½\, g\, (u^2 sin^2 \alpha/g - 2u\sin\alpha t/g + t^2)$$

$$AN= ½\, g\, \{ usin\alpha/g - t \}^2 \longrightarrow 7$$

QN= PM – PL

$= [u^2 \sin\alpha \cos\alpha /g - u\cos\alpha\, t]$

$QN = u \cos\alpha \{u \sin\alpha/g - t\} \longrightarrow 8$

from equation (8),

$QN^2 = u^2 sin^2\alpha\ [u \sin\alpha/g - t]^2$

From equation (7),

$AN = \frac{1}{2}\, g \left[\frac{usin\alpha}{g} - t\right]^2$

$\left[\frac{usin\alpha}{g} - t\right]^2 = 2AN / g$

$QN^2 = u^2cos^2\alpha\ (2AN / g) \longrightarrow 9$

In the diagram, S is the point on AM, so that AS= $u^2cos^2\alpha/2g$

$AS = u^2cos^2\alpha/2g$

$u^2cos^2\alpha = As\ .\ 2g$

$QN = AS\ .\ 2g\ [\ 2AN/\ g\ ]$

$QN = 4AS\ .\ AN \longrightarrow (10)$

EQN (10) represents Q parabola having QS is its focus with its t vertical with a vertex at A and having a latus rectum which is 4AS.

$4AS = 4 * u^2cos^2\alpha$

$AS = 2\,u^2cos^2\alpha\ / g$

11.Explain the laws of impact?

1st law: The total momentum of the two bodies after impact along the common normal must be equal to the total momentum of two bodies before impact along the same direction.

2nd law: The relative velocity of the spheres after impact along the common normal bears a constant ratio to the relative velocity before impact along the same direction and is of the opposite side. The constant ratio is known as Co-efficient of restitution. It is denoted by the letter " e".

3rd law: There is no tangential action between the two spheres at the point of contact, from this it follows that due to the impact there is no change in velocity of each sphere in the direction perpendicular to the common normal at their point of contact.

12.Explain direct impact between two smooth spheres?

Let a smooth sphere of mass m_1 moving with a velocity u_1 impinge directly on another

Smooth sphere of mass m_2 moving with the velocity u_2 in the same direction. Let 'e' be the Co-efficient restitution between them. Since is the impact is direct there is no force along the common normal between the two spheres at the point of contact. Hence the velocity of two spheres after the impact are v_1 and v_2.

By the principle of conversation of Momentum, the total momentum after the impact Along the common normal at the point of contact is equal to the total momentum before impact in the same directions

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \longrightarrow 1$$

By the Newton's experimental law;

$$v_1 - v_2 = -e\,(u_1u_2) \longrightarrow 2$$

Multiplying equation 2 by m_2 and adding it to equation 1;

$$v_1m_2 - v_2m_2 = -e\,(m_2u_1 - m_2u_2)$$

$$v_1m_2 - v_2m_2 = -em_2u_1 + em_2u_2$$

$$v_1(m_1 + m_2) = u_1(m_1 - em_2) + u_2m_2(1 + e) \longrightarrow 3$$

$$v_1 = \frac{[u_1(m_1 - em_2) + u_2m_2(1+e)]}{m_1 + m_2} \longrightarrow 4$$

Multiplying equation 2 by m_1 and subtracting it by equation 1;

$$v_1m_1 - v_2m_2 = -eu_1m_1 + eu_2m_1$$

$$v_2(m_1 + m_2) = m_1u_1(1 + e) + u_2(m_2 - em_2) \longrightarrow 5$$

$$v_2 = \frac{[m_1u_1(1+e) + u_2(m_2 - em_2)]}{m_1 + m_2} \longrightarrow 6$$

Equation 4 and 6 gives the velocities of two spheres after impact along the common normal.

Special Cases:

Corollary 1:

If the two spheres are in equal mass and perfectly elastic.

$m_1 = m_2$, e = 0

$$v_1 = \frac{[m_1u_2(2) + u_2(m_2 - m_2)]}{2m_1}$$

Therefore, $v_1 = u_2$

Similarly $v_2 = u_1$

The two spheres interchange their velocities after impact.

Corollary 2:

The impulse of the blow on the sphere of mass m1 is equal to the change of momentum produced in it.

$$I = m_1(v_1 - u_1)$$

$$I = m_1v_1 - m_1u_1$$

Substitute the value of $v_1$and we get,

$$I = \frac{[m_1m_2(u_2-u_1)(1+e)]}{m_1+m_2}$$

Corollary 3:

The two spheres are inelastic. E = 0, therefore $v_1 = v_2$

13. Explain Loss of kinetic energy due to direct impact between two smooth spheres?

The impulse of the blow 'I' on the sphere of mass m_1 in the direction o_1o_2, while the impulse of the mass m_2 is also I but in the direction o_2o_1.

The change in kinetic energy of $m_1 =$ ½ $m_1(v_1^2 - u_1^2)$

= ½ $m_1(v_1 + u_1)(v_1 - u_1)$

I = m $(v_1 - u_1)$

= ½ I $(v_1 + u_1)$

The change in kinetic energy of m_2 = ½ I $(v_2 + u_2)$

But the direction of I in m_2 is opposite to that of m_1, therefore

the change in kinetic energy of m_2 = - ½ I $(v_2 + u_2)$

the total change in kinetic energy = ½ I $(u_1 + v_1)$ – ½ I $(u_2 + v_2)$

= ½ I [$(v_1 - v_2)$ + $(u_1 - u_2)$]

Substitute the valve of I, then we get,

$$= ½ \ [m_1m_2(u_2 - u_1)(1 + e)/(m_1 + m_2)][(v_1 - v_2) + (u_1 - u_2)]$$

Therefore the total change in kinetic energy = ½ $\frac{m_1m_2}{m_1+m_2}[u_1 - u_2]^2(1 + e^2)$

14.Explain the oblique impact between two smooth spheres?

Let a smooth of mass m_1 is moving with a velocity u_1 impinge obliquely on a smooth sphere of mass m_2 is moving with a velocity u_2. let the direction of motion of sphere before angle makes an angles α and β with common normal at the point of contact. The velocities of spheres be $v_1 and v_2$ is making an angles θ and Φ after impact.

By the principle of conservation of momentum,

The total momentum of two bodies after impact along the common normal must be equal to the total momentum of two bodies before impact along the same direction.

$$m_1 v_1 cos\theta + m_2 v_2 cos\Phi = m_1 u_1 cos\alpha + m_2 u_2 cos\beta \longrightarrow 1$$

By Newton's experimental law,

$$v_1 cos\theta - v_2 cos\Phi = -e\,(u_1 cos\alpha - u_2 cos\beta) \longrightarrow 2$$

Multiplying equation 2 by m_2 and adding it to equation 1 ,

$$v_1 cos\theta m_2 - v_2 cos\Phi m_2 = -e\, v_2 cos\alpha u_1 + e m_2 u_2 cos\beta$$

$$v_1 cos\theta(m_1 + m_2) = u_1 cos\alpha(m_1 - e m_2) + m_2 u_2 \cos\beta\,(1+e)$$

$$v_1 cos\theta = u_1 cos\alpha(m_1 - e m_2) + m_2 u_2 \cos\beta(1+e) / (m_1 + m_2) \longrightarrow 3$$

Multiplying equation 2 by m_1 and subtracting it by equation 1,

$$m_1 v_1 cos\theta - m_1 v_1 cos\Phi = -e m_1 u_1 cos\alpha + e m_1 u_2 cos\beta$$

$$v_2 cos\Phi = \frac{[m_1 u_1 cos\alpha(1+e) - u_2 cos\beta(m_2 - e m_1)]}{m_1 + m_2} \longrightarrow 4$$

Since there is no tangential force, there is no change in velocity of either sphere perpendicular to the common normal.

$$v_1 \sin\theta = u_1 sin\alpha \longrightarrow 5$$

$$V_2 \sin\Phi = u_2 \sin\beta \longrightarrow 6$$

v_1 can be obtained by squaring equation 3 and 5. Then adding it,

$$V_1^2 = [u_1^2 \sin^2\alpha + \frac{u_1^2\cos^2\alpha(m_1 - em_2)^2 + m_2^2 u_2^2 cos^2\beta(1+e)^2]}{(m_1+m_2)^2}$$

v_2 can be obtained by squaring equation 4 and 6,

$$V_2^2 = \frac{[u_2^2\sin^2\alpha + m_1^2 u_1^2 \cos^2\alpha(1+e)^2 - u_2^2\cos^2\beta(m_1 - m_2)^2]}{(m_1+m_2)^2}$$

Θ can be obtained by dividing 3 and 5,

$$\frac{V_1\sin\theta}{v_1\cos\theta} = \frac{u_1\sin\alpha\,(m_1+m_2)}{[u_1\cos\alpha(m_1 - em_2) + m_2 u_2 \cos\beta\,(1+e)]}$$

$$\tan\theta = \frac{u_1\sin\alpha(m_1+m_2)}{[u_1\cos\alpha(m_1 - em_2) + m_2 u_2\cos\beta(1+e)]}$$

similarly we can get the value of Φ,

$\tan\Phi = \frac{u_2\sin\beta(m_1+m_2)}{m_1u_1\cos\alpha(1+e)- u_2\cos\beta(m_2-em_1)}$

Special cases:

Corollary 1:

If $e = 1$ and $m_1 = m_2$

$V_1\cos\theta = [m_1u_2\cos\beta(2) + u_1\cos\alpha(m_1 - m_1)]/(m_1 + m_1)$

$V_1\cos\theta = u_2\cos\beta$

Similarly , $v_2\cos\Phi = u_2\cos\alpha$

Corollary 2:

The impulse of the blow on m_1 ,

$I = m_1(v_1\cos\theta - u_1\cos\alpha)$

$I = m_1\left[\frac{m_2u_2\cos\beta(1+e)+u_1\cos\alpha(m_1- em_2)}{m_1+m_2}\right] - m_1u_1\cos\alpha$

$= \{(m_1m_2)(1 + e)[v_1\cos\theta - u_1\cos\alpha]\}/(m_1 + m_2)$

The impulse of blow on $m_2$is equal and opposite to the impulse of the blow on m_1.

Corollary 3:

If the two spheres are inelastic e = 0.

Therefore, $v_1c\ os\theta = v_2\cos\Phi$

15.Explain the impact of a smooth sphere on a smooth fixed horizontal plane?

Let a smooth sphere of mass m and co-efficient of restitution 'e' is impact obliquely on the smooth fixed horizontal plane PQ. Let A be the point of contact and AO is the common normal at the point of contact.

Let 'u' be the velocity of smooth sphere before impact in the direction is making an angle α 'v' be the velocity of sphere after impact inclined at an angle θ with the common normal.

By Newton's experimental law;

$v\cos\theta = e\,u\cos\alpha \longrightarrow 1$

Since both the sphere and the plane are smooth. The is no change in velocity in the direction perpendicular to the common normal.

Therefore,

$v\sin\theta = u\sin\alpha \longrightarrow 2$

squaring (1) and (2) and adding it,

$$v^2\cos^2\theta = e^2u^2\cos^2\alpha$$

$$V^2\sin^2\theta = u^2\sin^2\alpha$$

$$V^2 = u^2(e^2\cos^2\alpha + \sin^2\alpha) \quad v = u\sqrt{(e^2cos^2\alpha + sin^2)}$$

Dividing equation 2 by 1,

For finding the inclined angle θ,

$$\frac{vsin\theta}{vcos\theta} = \frac{usin\alpha}{eucosa}$$

$$\Theta = \tan^{-1}\left[\frac{\tan\alpha}{e}\right]$$

Special cases:

Corollary 1:

V=u , θ = α

Thus, If a perfectly elastic sphere impinge obliquely on a fixed smooth plane, the velocity is unchanged in magnitude but the direction of motion before and after impact make equal angles with the common normal.

2nd Corollary:

If α = 0, θ = 0, v= eu, then the smooth fixed plane it rebounced along the common normal with its velocity reduced to e times its velocity before impact.

3rd Corollary:

If e = 0 and the sphere is inelastic θ =0 and v=usinα and inelastic smooth sphere after oblique impact with a smooth fixed plane, slides along the plane with velocity u sinα.

4th Corollary:

The impulse of the pressure on the sphere is measured by the change of momentum produced in the sphere.

I= m[v cosθ – (-v cosα)]

= m [v cosθ + u cosα]

=m[e u cosα + u cosα]

I= m u cosα (1+ e)

The impulse of the force on the plane is equal and opposite to the impulse of a pressure on the sphere.

5th Corollary:

The change in kinetic energy of the sphere due to impact on the plane is given by,

Kinetic energy = ½ mv^2

Here, K.E = ½ $m(v^2 - u^2)$

$= \frac{1}{2}$ m (v + u)(v – u)

We know that, I=m (v - u)

Kinetic energy = $\frac{1}{2}$ I (v + u)

UNIT- 2

1.Define centripetal force?

This force directed towards the centre . A particle of mass ‘m’ is moving in a circular path of radius ‘r’ with a uniform velocity ‘v’, a force is required to impact the normal acceleration. The magnitude of these force is $\frac{mv^2}{r}$.

Example: Incase of the planet moving round to the sun is an approximately circular orbit the centripetal force is provided by the gravitational force exerted by the sun on the planet.

2.Define centrifugal force?

There must be acting on the particle is described uniform circular motion and equal and opposite force. This force is known as centrifugal force and it always directed away from the centre.

Example :

A stone is tied to one end of the string is rotated in a circular with uniform speed. The stone in turn exerted and equal and opposite force on a hand. It is account for centrifugal reaction.

3.Define hodograph?

A curve the radius vector of which represents in magnitude and dire3ction the velocity of a moving object.

4. Derive an expression for the normal acceleration by the hodograph?

Let the particle ‘p’ move along a circular path of centre O and radius ‘r’ with uniform velocity ‘v’. Let $p_1$andp_2 represents the position of the particle ‘p’ before and after a short interval of time dt. Let $O_1Q_1$and O_2Q_2 be drawn from O_1 parallel and proportional to the velocities of the particle at $p_1$and$p_2$respectively. Then, Q_1Q_2 is the hodograph of the particle ‘p’ at the time dt. Q_1Q_2 is an arc of circle of radius.

The velocity of Q in the hodograph is, v= $\frac{Q_1Q_2}{dt}$

If the ‘$\angle p_1Op_2$= dθ

$\angle Q_1O_1Q_2$= dθ

Now the arc , $p_1Op_2 = r\,d\theta$

The arc $Q_1O_1Q_2 = v\,d\theta$

$$\frac{RQ_1Q_2}{Rp_1p_2} = \frac{v\,d\theta}{r\,d\theta}$$

$$\frac{RQ_1Q_2}{Rp_1p_2} = \frac{v}{r} \longrightarrow \quad 1$$

$$\frac{\frac{RQ_1Q_2}{dt}}{\frac{Rp_1p_2}{dt}} = \frac{v}{r} \longrightarrow \quad 2$$

When dt tends to zero, equation 2 reduces to

Velocity of the hodograph = $\frac{v^2}{r}$

But the velocity of Q in the hodograph is equal to the acceleration of p in its circular path.

Therefore normal acceleration of p is $\frac{v^2}{r}$.

5.Explain the motion of a cyclist along a curved path?

If the cyclist is to negotiate in a circular path. He invariably leaves from a vertical 'l' towards the centre of the circular path. And the pressures the ground in an inclined position. The horizontal component of a reaction of ground supplies the centripetal force necessary for circular motion. AB represent a section of cycle with the cyclist. D is the centre of the circular path. Mg is the total weight of the cycle and cyclist. R is the reaction of the ground and θ is the inclination of the cycle to the vertical. The vertical component R cosθ of the reaction balances mg. and the horizontal component of the reaction R sinθ supplies the centripetal force needed for circular motion.

$$R\cos\theta = mg \longrightarrow 1$$

$$R\sin\theta = \frac{mv^2}{r} \longrightarrow 2$$

V is the velocity of the cyclist and 'r' is the radius of circular path.

For getting θ, dividing 2 by 1,

$$\frac{R\sin\theta}{R\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{rg} \longrightarrow 3$$

from equation 3 we find that, v increases and r decreases, θ increases and the cyclist runs a risk of falling to ground. If he takes a sharp turn while moving with a great speed.

6. Derive an expression for the variation of 'g' with altitude?

Consider a unit mass on the surface of the earth of radius R, the mass of the earth is M. let 'g' be the acceleration due to gravity on the surface of the earth. Therefore the gravitational force on the unit mass due to the mass 'M' acting at the centre

$$g = \frac{GM}{R^2} \longrightarrow 1$$

consider the same unit mass at an altitude 'h' per acceleration due to gravity g,

$$g' = \frac{GM}{(R+h)^2} \longrightarrow 2$$

Now dividing 2 by 1,

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} \longrightarrow 3$$

$$\frac{g'}{g} = \frac{1}{\left(1+\frac{n}{R}\right)^2}$$

$$\frac{G'}{g} = \left(1- \frac{2h}{R}\right)$$

g decreases as altitude increases.

7. Derive an expression for the variation of 'g' with depth?

Let g and g$'$be the accelerations due to gravity at P and Q respectively. At P the whole mass of the earth attracts the body and at Q it is attracted by the mass of the earth of radius (R – h).

$$mg = \frac{GMm}{R^2} \longrightarrow 1$$

$$mg' = \frac{GM'm}{(R-h)^2} \longrightarrow 2$$

$$m = 4/3\ \pi R^3 \rho$$

$$m' = \frac{4}{3}\pi\rho(R-h)^3$$

Dividing 2 by 1

$$\frac{mg'}{mg} = \frac{\frac{GM'm}{(R-h)^2}}{\frac{GMm}{R^2}}$$

$$= \frac{\frac{m'}{(R-h)^2}}{\frac{m}{R^2}}$$

$$= \frac{R-h}{R}$$

$$\frac{g'}{g} = \frac{1-h}{R}$$

$$g' = g[\frac{1-h}{R}]$$

8.Write about the Effect of the earth's rotation on the value of the acceleration due to gravity?

Let OA and OB represent the equatorial and polar radii of the earth respectively. Let P be a particle on the surface whose latitude ∟POA = λ. Consider a particle of mass 'm' situated

at latitude λ. The gravitational pull acts along PO. This force be represented by PD. As the earth rotates about its polar axis with angular velocity ω. The particle which shares the earth rotation about it axis describes a circle with C as centre and PC as radius. If the radius PC = R cosλ.

To enable the particle at p to execute circular motion with angular velocity ω. The centripetal force is supplied by the earth's pull mg on the particle. Complete the parallelogram PEDF. PF represents the effective pull of the earth. Let this cause an acceleration G' along PF.

Resolve mg along PO into two component,

i. mg cosλ (along PC)
ii. mg sinλ (perpendicular to PC)

out of mg cosλ a part of it namely $m\omega^2 R\cos\lambda$ to produce centripetal force and the rest force along PC is $mg\cos\lambda - m\omega^2 R\cos\lambda$

the component mg sinλ is not affected by rotation. Therefore the effective weight of the body mg' along PF is,

$$mg' = \sqrt{(mg\cos\lambda - m\omega^2 R\cos\lambda)^2 + (mg\sin\lambda)^2}$$

$$mg' = m[g^2\cos^2\lambda + \omega^4 R^2\cos^2\lambda - 2\omega^2 gR\cos^2\lambda + g^2\sin^2\lambda]^{\frac{1}{2}}$$

$$g' = [g^2 + \omega^4 R^2\cos^2\lambda - 2\omega^2 gR\cos^2\lambda]^{\frac{1}{2}}$$

Neglecting higher powers, we get

$$g' = [g^2\left(1 - \frac{2\omega^2 R\cos^2\lambda}{g}\right)]^{\frac{1}{2}}$$

$$g' = g\,[\,1 - \frac{\omega^2 R\cos^2\lambda}{g}]$$

From the above equation it is easily seen that $R\omega^2 = g$

$$g' = g\,[1 - \frac{g\cos^2\lambda}{g}]$$

$$g' = g\sin^2\lambda$$

9.Explain the Motion of a carriage on a banked up curve?

If the rails are ride along a curve at the same horizontal level. The centripetal force required for the circular portion is supplied by the pressure exerted by rails on the flanges of the wheels. By Newton's law, the flanges of the wheel exert equal and opposite pressure on the rails. This would result in the wearing out of rails due to the large amount of friction that it is called into play. To avoid this wearing out of the rails the plane of the track is tilted suitably so as to completely eliminate the flange pressure on the rails. This is done by tilting the sleepers up so that the outer rail is raised above the inner one, so that the floor of the carriage is inclined to the horizontal. The normal reactions in this case will be inclined to vertical so that the vertical components balance the weight of carriage while the horizontal components supply the necessary force for circular the motion.

Let ABCD be a vertical section of the carriage through the line joining the centre of gravity 'G' and the centre O of the circular track. Let the outer rail be raised over the inner so that the floor of the carriage AB is inclined at an angle θ to the horizontal, and there is no lateral pressure exerted by the flanges of the wheels on the rails. If R_1 and R_2 be the normal reactions at the inner and outer rails.

Resolving vertically we have,

$$(R_1 + R_2)\cos\theta = mg \longrightarrow 1$$

Resolving horizontally we have,

$$(R_1 + R_2)\sin\theta = \frac{mv^2}{r} \longrightarrow 2$$

Where,

V= velocity of the carriage, r = radius of the circular path,

Dividing 2 by 1,

$$\tan\theta = \frac{v^2}{rg} \longrightarrow 3$$

equation 3 gives the angle through which the sleepers are to be tilted from the horizontal, so that there is no lateral flange pressure on the rails.

The carriage moving with different velocity has to pass around the curve. It is not possible to eliminate completely the lateral pressure exerted by the flanges on the rails. Assuming that the height of the rail over the inner is adjusted so that there is no flange pressure for a critical speed B, let F be the additional lateral flange pressure acting from B to A for a carriage moving along the curve with a velocity v.

Then resolving vertically and horizontally we have,

$$(R_1 + R_2)\cos\theta - F\sin\theta = mg \longrightarrow 4$$

$$(R_1 + R_2)\sin\theta + F\cos\theta = \frac{mv^2}{r} \longrightarrow 5$$

Multiplying 4 by sinθ and 5 by cosθ and subtracting ,

$$(R_1 + R_2)\sin\theta\cos\theta - F\sin^2\theta - [(R_1 + R_2)\sin\theta\cos\theta - F\cos^2\theta = mg\sin\theta - \frac{mv^2}{r}\cos\theta$$

$$-F(\sin^2\theta + \cos^2\theta) = mg\sin\theta - \frac{mv^2}{r}\cos\theta$$

$$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

$$F = \frac{m\cos\theta}{r}[v^2 - \frac{g\sin\theta r}{\cos\theta}]$$

$$F = \frac{m\cos\theta}{r}[v^2 - g\tan\theta]$$

$$F = \frac{m\cos\theta}{r}[V^2 - v^2]$$

Notes:

1. If V > v F is positive value and the additional lateral pressure acts along BA. The pressure is exerted at the outer rails.
2. If V < v, F is negative and therefore acts along AB and the flange pressure in this case is exerted at the inner rails.

Unit: 3

1.State Newton's law of gravitation?

Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of distance between them.

$$F = \frac{Gm_1m_2}{d^2}$$

2.Define inertial mass?

The mass of a body may be determined by measuring the acceleration 'a' produced on it by a known force F. thus $m = \frac{F}{a}$. the mass m is called inertial mass.

3.Define gravitational mass?

The mass of the body may also be determined by measuring the gravitational force exerted on it by earth.

$$m = \frac{FR^2}{GM}$$

Thus the mass m is called gravitational mass.

4.State Kepler's 1st law.

Every planet moves in an elliptical orbit around the sun, the sun is being one of the foci.

5. State Kepler's 2nd law.

The radius vector drawn from the sun to a planet sweeps out equal areas in equal interval of times. The areal velocity of the radius vector is constant.

$$\frac{dA}{dt} = \text{constant}$$

6. State Kepler's 3rd law.

The square of the period of revolution of the planet around the sun is proportional to the cube of the semi major axis of the ellipse.

$$T^2 = a^3$$

7. **What is meant by gravitational field?**

The space around a body with in which its gravitational force of attraction is perceptible is called its gravitational field. The gravitational field is an example for vector field.

8. Define gravitational potential?

The work done in moving a unit mass from infinity to the point in a gravitational field is called gravitational potential at the point.

9. Define orbital velocity?

The velocity which an object must acquire to a circle, the earth in the circular path of radius ‘r’ is called orbital velocity.

Example : the motion of planets around the sun.

10.Define escape velocity?

The velocity with which a body should be projected to unable it to escape from the gravitational pulled of the earth is called escape velocity.

Example : A space craft leaves the surface of the earth.

11.Deduce Newton’s law of gravitation from Kepler’s law?

Consider two planets of masses $m_1$and m_2. let $r_1$and $r_2$be the radii of circular orbit. $T_1$and T_2 be the period of revolution around the sun.

The centrifugal force acting on the 1st planet is $F_1 = m_1 r_1 \omega^2 \longrightarrow 1$

Where, $\omega = \frac{2\pi}{T_1}$

Substitute the value of ω in equation 1,

$$F_1 = m_1 r_1 (\frac{2\pi}{T_1})^2 \longrightarrow 2$$

The centrifugal force acting on the 2nd planet is $F_2 = m_2 r_2 (\frac{2\pi}{T_2})^2 \longrightarrow 3$

Dividing , $\frac{F_1}{F_2} = \frac{m_1 r_1 (\frac{2\pi}{T_1})^2}{m_2 r_2 (\frac{2\pi}{T_2})^2}$

$$\frac{F_1}{F_2} = \frac{m_1 r_1 {T_2}^2}{m_2 r_2 {T_1}^2}$$

From kepler’s 3rd law, $T^2 \alpha\, r^3$,

$$\frac{F_1}{F_2} = \frac{m_1 {r_2}^2}{m_2 {r_1}^2}$$

The force on the planet is directly proportional to $\frac{m}{r^2}$

$$F = \frac{m}{r^2}$$

The force is proportional to the mass of the planet since the attraction is mutual, the force is also directly proportional to the mass of the sun M,

$$F \ \alpha \ \frac{Mm}{r^2}$$

$$F = \frac{GMm}{r^2}$$

12. Derive an expression for the escape velocity?

Suppose a body of mass m is situated at a height x from the centre of the earth. M is the mass of the earth.

The force of attraction on the body $F = \frac{GMm}{x^2}$

If the body moved upwards through a distance dx, $F = \frac{GMm}{x^2} \square\square$

Total work done in moving the body from the surface of earth to infinity,

$$W = \int_{\square}^{\infty} \frac{\square\square\square}{\square^2} \square\square$$

$$= -\frac{GMm}{R}$$

Here R is radius of the earth. If the body is projected upwards with velocity v. the initial kinetic energy of the body = ½ mv^2

The kinetic energy = work done of the body can escape from the gravitational pull of the earth. ½ $mv^2 = \frac{GMm}{R}$

$$v = \sqrt{\frac{2\square\square}{\square}}$$

$g = \frac{GM}{R^2}$ where g = acceleration due to gravity.

Therefore , $v = \sqrt{2\square\square}$

13. Derive an expression for orbital velocity?

The body of mass m should be projected from the earth surface so as to revolve around (ie) to become a satellite.

Let it be v_0. Let the distance of the satellite from the centre of the earth be ‘r’.

Centrifugal force on the body tending to take it away from the surface of the earth = $\frac{mv^2}{r}$

The gravitational force acting on the body must be counter balance the centrifugal force.

$$\frac{GMm}{r^2} = \frac{mv_0^2}{r}$$

$$v_0^2 = \frac{GM}{r}$$

$$v_0 = \sqrt{\frac{\square\square}{\square}}$$

But $mg = \frac{GMm}{r^2}$

$$g\,r = \frac{GM}{r}$$

$$v_0 = \sqrt{gr}$$

The velocity of projection of a body to become a satellite of the earth, $v_0 = \sqrt{gr}$.

14.Write about 'G' by boy's method?

Boy's method for the determination of gravitational constant . two sets of balls AC and BD are kept at different levels. So that attraction between a bigger ball and a distance of small ball is negligible. The apparatus consists of a torsion head 'T', quartz suspension wire 'x', and a mirror strip RS. Two small balls A and B are made up of gold and suspended from the ends of the mirror strip RS. The length of RS is 2.3 cm. the suspension wire has a length of 43.2 cm and diameter of 0.5 cm. The difference in level of A and B is 15 cm. C and D are lead balls of diameter of 11cm. the mass of the each ball is 7.5 kg. the angle of deflection of the mirror strip is measured with the help of lamp and scale arrangement, kept at a distance of 7m from the mirror. As the apparatus is enclosed in the small chamber, convection currents are avoided.

Initially two large balls C and D adjusted so that they are on opposite sides of two small balls and not in line with the mirror strip RS.

This is done by rotating the lid of the outer chamber about the vertical axis. Now bring the ball C near to the ball A. and the ball D near to the ball B. the ball C attracts the ball A and the ball D attracts ball B. There is a twist in the suspension wire and the mirror strip is deflected through a certain angle 'α'. In the equilibrium position, the deflecting couple is equal to the restoring couple. Force of attraction between A and C,

$$F \alpha \frac{Mm}{r^2}$$

$$F = \frac{GMm}{r^2}$$

Force of attraction between B and D,

$$F \alpha \frac{Mm}{r^2}$$

$$F = \frac{GMm}{r^2}$$

Deflecting couple = force * perpendicular distance between them.

$$= F * 2d$$

$$= \frac{GMm}{r^2} * 2d$$

In the triangle AOC, applying cosine's law,

$$r^2 = a^2 + l^2 - 2al\cos\alpha$$

$$r = (a^2 + l^2 - 2al\cos\alpha)^{\frac{1}{2}}$$

also, $\frac{\sin\alpha}{r} = \frac{\sin\beta}{l}$

$$r = \frac{l\sin\alpha}{\sin\beta}$$

In triangle FOC,

$$\sin\beta = \frac{d}{a}$$

$$r = \frac{al\sin\alpha}{d}$$

$$d = \frac{al\sin\alpha}{r}$$

$$d = \frac{al\sin\alpha}{(a^2+l^2-2al\cos\alpha)^{\frac{1}{2}}}$$

substituting the value of 'r' and 'd' in deflecting couple equation,

$$D.C = \frac{GMm}{r^2} * 2d$$

$$= \frac{2al\sin\alpha\, GMm}{(a^2+l^2-2al\cos\alpha)^{3/2}}$$

Let couple per unit twist of the wire be 'c'. and deflection in the mirror is 'θ'.

Restoring couple = $c\theta$

For equilibrium D.C = R.C

$$c\theta = \frac{2al\sin\alpha\, GMm}{(a^2+l^2-2al\cos\alpha)^{3/2}}$$

$$G = \frac{c\theta(a^2+l^2-2al\cos\alpha)^{3/2}}{2al\sin\alpha\, Mm}$$

To find the value of 'c' the mirror strip is allowed to oscillate and the time period T is noted.

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$T^2 = 4\pi^2\frac{I}{C}$$

$$C = \frac{4\pi^2 I}{T^2}.$$

I is the moment of inertia of the oscillating system. G= $6.676*10^{-11}\ Nm^{-2}kg^2$

15. Explain the Gravitational potential field at a point due to spherical shell?

Consider a uniform section of spherical shell with mass ‘M’ and radius ‘a. ‘ρ’ be the density per unit of the shell. The planes AC and BD cut the shell vertically and the element between the two planes is ‘ring’ of radius AB. From the diagram the radius AB = a . dθ . therefore surface area of the element is,

$$\text{Surface area} = 2\pi rh$$

$$= 2\pi\ a.d\theta\ a.\sin\theta$$

$$= 2\pi\ a^2\sin\theta\ d\theta.$$

The relation between mass and density,

$$\rho = \frac{m}{v}$$

$$M = \rho v$$

$$M = \rho 2\pi a^2 \sin\theta\ d\theta$$

from the diagram AP= x

$$\text{potential at a point p } dv = \frac{-GM}{x}$$

$$dv = \frac{-G\ \rho 2\pi a^2 \sin\theta\ d\theta}{x}$$

by cosine’s formula,

In triangle AOP,

$$x^2 = r^2 + a^2 - 2ar\cos\theta$$

Differentiating,

$$2x\ dx = 2ar\ \sin\theta\ d\theta$$

$$x = \frac{ar\ \sin\theta\ d\theta}{dx}$$

substitute the value of ‘x’ in equation 1,

$$dv = \frac{-\ G\ \rho 2\pi a^2 \sin\theta\ d\theta\ dx}{ar\ \sin\theta\ d\theta}$$

$$= \frac{-G\rho 2\pi a\ dx}{r}$$

$$dv = \frac{-2\pi aG\rho\ dx}{r}$$

the potential at a point due to the whole shell

$$\int dv = \int_{r-a}^{r+a} \frac{-2\pi aG\rho\, dx}{r}$$

$$V = \frac{-2\pi a\rho G}{r} \int_{r-a}^{r+a} dx$$

$$= \frac{-2\pi a\rho G}{r} [r + a - r + a]$$

$$= \frac{-2\pi a\rho G[2a]}{r}$$

$$= \frac{-4\pi a^2\rho G}{r}$$

Mass of the shell M = $4\pi a^2\rho$

Then $V = \frac{-GM}{r}$

Thus for a point outside the shell, the shell behaves as if the whole of its concentrated at the centre of the shell.

Point on the surface of the shell:

Let us consider a point which lies on the surface of the shell itself. The limits for the value x will be 0 and 2a.

$$\int dv = \int_{0}^{2a} \frac{-2\pi aG\rho\, dx}{r}$$

$$V = \frac{-G4\pi a^2\rho}{r}$$

$$V = \frac{-GM}{r}$$

Where r=a, $V = \frac{-GM}{a}$

Point inside the shell:

Let the point p is situated at inside the shell. The limits are (a – r) to (a + r).

$$\int dv = \int_{a-r}^{a+r} \frac{-2\pi aG\rho\, dx}{r}$$

$$V = \frac{-G4\pi a\rho\, a}{a}$$

$$V = \frac{-GM}{a}$$

The potential at all points inside the spherical shell is same and is equal to the gravitational potential on the surface.

Gravitational field due to a spherical shell:

The intensity of the gravitational field F is given by

$$F = \frac{-dx}{dr}, \quad V = \frac{-GM}{r}$$

$$F = \frac{-d}{dr}\left[\frac{-GM}{r}\right]$$

$$F = \frac{-GM}{r^2}$$

The negative sign indicates that the force is towards the centre O.

At the point on the outer surface of the shell:

Putting r = a , we get the intensity of the gravitational field at a point on the outer surface of the shell.

$$F = \frac{-GM}{a^2}$$

At the point inside the shell:

Potential, $V = \frac{-GM}{a} =$ constant

$$F = \frac{-dv}{dr} = 0$$

There is no gravitational field inside the spherical shell.

Unit- 4

1.**What is moment of inertia?**

Moment of inertia of a body is its inability to change by itself its state of rest (or) uniform rotatory about an axis.

2.What is meant by radius of gyration?

Suppose the whole mass of the body is concentrated at a point distant k from the axis. Such that mk^2= I. Then k is called radius of gyration of the body about the given axis.

$$K = \sqrt{\frac{I}{m}}.$$

3.Define centre of percussion?

When a body is capable of rotation about a fixed axis given a blow at a suitable point such that there is no impulsive force exerted on a fixed axis that point is known as centre of percussion.

4.Define centre of suspension?

The intersection of the axis of rotation of a pendulum with a plane perpendicular to the axis that passes through the centre of mass is called centre of suspension.

5.Define centre of oscillation?

The point on the line through the point of suspension and the centre of mass which moves as if all the ,masses of the pendulum hence concentrated there.

6.State perpendicular axis theorem?

If $I_x$and I_y are the moments of inertia of the lamina about two rectangular axis ox and oy in its plane, its moment of inertia about an axis oz, perpendicular to its plane is $I_z = I_x + I_y$.

7.State parallel axis theorem?

If I is the moment of inertia of a body about an axis through its centre of mass and I' its moment of inertia about a parallel axis, then $I' = I + mh^2$.

8. Explain the Perpendicular axis theorem:

If $I_x$and I_y are the moments of inertia of lamina about two rectangular axis ox and oy in its plane, its moment of inertia about an axis oz, perpendicular to its plane is $I_z = I_x + I_y$.

Proof:

Let ox and oy be the perpendicular axis in the plane of lamina and oz axis perpendicular to the lamina.

Consider a particle p, mass m in the plane of lamina x and y. Here r is the distance from ox, oy, oz respectively. So, the moment of the particle about oz = Σmr^2

Similarly, moment of inertia about ox = Σmy^2

oy = Σmx^2

but , $r^2 = x^2 + y^2$

$\Sigma mr^2 = \Sigma mx^2 + \Sigma my^2$

$I_z = I_y + I_x$

9. Explain the Parallel axis theorem:

If I is the moment of inertia of the body about an axis through its centre of mass and I' its moment of inertia about a parallel axis, then $I' = I + mh^2$.

Proof:

AB is an axis passing through the centre of mass of the body G. CD is parallel axis at a perpendicular distance h from AB. Consider a particle p from mass m at a distance AB.

Moment of inertia of a particle p about AB = mx^2

Moment of inertia of a particle p about CD = $m[x + h]^2$

$$= m(x^2 + h^2 + 2xh)$$

$$= mx^2 + mh^2 + m2xh$$

Moment of inertia of whole body CD = $I' = \Sigma mx^2 + \Sigma mh^2 + \Sigma 2mxh$

$\Sigma \square x^2 = I$

$\Sigma \square h^2 = mh^2$

$I' = I + mh^2 + \Sigma 2mxh$

Σmx is the algebraic sum of moments of all the particles about g, since the body is balanced about the centre of mass.

$\Sigma mx = 0$

$I' = I + mh^2$

10. Explain about a Compound pendulum:

A compound pendulum consist of a rigid body capable of rotation about a fixed horizontal axis under gravity. Let the axis of rotation pass through the point o in a vertical section of the body taken through the centre of gravity g of the body.

In the equilibrium position OG will be in vertical, OG = h.

If θ is small angular displacement of the body from the equilibrium position in time t. and 'M' mass of the body.

The couple tending to restore the body to its equilibrium position = $Mgh \sin\theta$

The couple will produce angular acceleration = $\frac{d^2\theta}{dt^2}$

If I be the moment of inertia of the body about the axis of rotation.

The product of moment of inertia and the angular acceleration = the couple acting

$$I * \frac{d^2\theta}{dt^2} = -mgh \sin\theta$$

The significance of negative sign is that the angular acceleration and the angular displacement are oppositely directed.

Then θ = small,

$\sin\theta = \theta$

$$I * \frac{d^2\theta}{dt^2} = -mgh\,\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{-mgh\,\theta}{I}$$

'k' be the radius of gyration.

$$I = mk^2$$

$$\frac{d^2\theta}{dt^2} = \frac{-mgh\,\theta}{mk^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{-gh\,\theta}{k^2}$$

It represents a simple harmonic oscillation of period $T = \frac{2\pi}{\sqrt{\frac{gh}{k^2}}}$

$$T = 2\pi\sqrt{\frac{k^2}{gh}}$$

'k' be the radius of gyration about an axis through G parallel to axis of rotation.

Parallel axis theorem is given by

$$I' = I + mh^2$$

$$mK^2 = mk^2 + mh^2$$

$$K^2 = k^2 + h^2$$

Then $T = 2\pi\sqrt{\frac{k^2+h^2}{gh}}$.

Centre of suspension :

The point O where the axis of rotation meets the vertical plane through the centre of gravity G of the rigid body is called centre of suspension.

A simple pendulum which has the same period as the given component pendulum is called the equivalent simple pendulum.

The equivalent simple pendulum,

$$L = \frac{k^2}{h}$$

$$L = \frac{k^2+h^2}{h}$$

If OG produced to point c such that OC=L, the length of the equivalent simple pendulum, the point c is called centre of oscillation.

The centre of oscillation is a point at which the mass of the body may be considered to be concentrated without any change in periodic time.

If the body is suspended about a parallel axis through c we have CG= L-h.

The length of the simple pendulum will be,

$$L_1 = \frac{k^2+(L-h)^2}{L-h}$$

But $L = \frac{k^2+h^2}{h}$

$$L_1 = \frac{Lh-h^2+(L-h)^2}{L-h}$$

$L_1 = L$

The centre of oscillation and the centre of suspension will be interchangeable.

11. Explain about the Oscillation of the small sphere on the large concave smooth surface:

Let m be the mass of the small sphere of radius r oscillating on a large smooth concave radius R. let A be the position of the centre of ball in its equilibrium position. A now be vertically blow the centre of the concave surface. Let B be the position of the centre of the sphere at an instant of time after it has passed the equilibrium position.

Let ∟AOB = θ (small) and AB which is small θ.

The potential energy of the sphere at B = mg *AD

AD = OA – OD

$= (R - r)(1 - \cos\theta)$

$= 2(R - r)\sin^2 \frac{1}{2}\theta$

$= 2(R - r)\frac{1}{4}\theta^2$

When θ is small, potential energy at the instant of the sphere is at B = mg*2(R – r) $\frac{1}{4}\theta^2$

$= \frac{1}{2} mg (R - r)\theta^2$

$\Theta = \frac{x}{R-r}$

$= \frac{1}{2} mg (R - r)\left(\frac{x}{R-r}\right)^2$

$= \frac{1}{2} mg \frac{x^2}{(R-r)}$

If v and ω represent the linear and angular velocities of the sphere at the instant it is at B.

Kinetic energy of rotation of this sphere at B,

$= \frac{1}{2} I \omega^2$

$= \frac{1}{2} I \frac{v^2}{r^2}$

$$= \frac{1}{2}\,\frac{2}{5}mr^2 - \frac{v^2}{r^2}$$

$$= \frac{1}{5}\,mv^2$$

Kinetic energy of translation at B = ½ mv^2

The total kinetic energy at B = ½ m v^2 + $\frac{1}{5}$ □v^2

$$= \frac{5mv^2+2mv^2}{10}$$

$$= \frac{7mv^2}{10}$$

$$V= \frac{dx}{dt}$$

$$= \frac{7m\,(\frac{dx}{dt})^2}{10}$$

$$= \frac{7}{10}m\,(\frac{dx}{dt})^2$$

By the principle of conservation of energy,

Kinetic energy + potential energy at B = constant

$$\frac{7}{10}m\,(\frac{dx}{dt})^2 + \frac{1}{2}\,mg\,\frac{x^2}{(R-r)} = 0$$

Differentiate with respect to t :

$$\frac{7}{5}m\,2\,\frac{d^2x}{dt^2}\,\frac{dx}{dt} = -\,\frac{mg}{2\,(R-r)}x\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} = -\,\frac{5gx}{7(R-r)}$$

Since $\frac{5g}{(R-r)}$ is a constant.

The acceleration of the ball is directly proportional to its displacement. The oscillation of the ball on the concave surface are simple harmonic for small oscillation.

The period of oscillation is given by $T = \frac{2\pi}{\sqrt{\frac{5g}{7(R-r)}}}$

$$= 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

If we measured the period of oscillation of the sphere on the concave surface and if we know r and R, we can calculate the value of g at the place. If we assume the value of g and measure r we can calculate the various of curvature of the concave surface. When the above arrangement is used for the determination of R, it is known as dynamical spherometer.

12. Derive an expression for minimum time period of a Compound pendulum..

A compound pendulum consist of a rigid body capable of rotation about a fixed horizontal axis under gravity. Let the axis of rotation pass through the point o in a vertical section of the body taken through the centre of gravity g of the body.

In the equilibrium position OG will be in vertical, OG = h.

If θ is small angular displacement of the body from the equilibrium position in time t. and 'M' mass of the body.

The couple tending to restore the body to its equilibrium position = Mgh sinθ

The couple will produce angular acceleration = $\frac{d^2\theta}{dt^2}$

If I be the moment of inertia of the body about the axis of rotation.

The product of moment of inertia and the angular acceleration = the couple acting

$$I * \frac{d^2\theta}{dt^2} = -mgh\sin\theta$$

The significance of negative sign is that the angular acceleration and the angular displacement are oppositely directed.

Then θ = small,

Sinθ = θ

$$I * \frac{d^2\theta}{dt^2} = -mgh\,\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{-mgh\,\theta}{I}$$

'k' be the radius of gyration.

$$I = mk^2$$

$$\frac{d^2\theta}{dt^2} = \frac{-mgh\,\theta}{mk^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{-gh\,\theta}{k^2}$$

It represents a simple harmonic oscillation of period T = $\frac{2\pi}{\sqrt{\frac{gh}{k^2}}}$

$$T = 2\pi\sqrt{\frac{k^2}{gh}}$$

'k' be the radius of gyration about an axis through G parallel to axis of rotation.

Parallel axis theorem is given by

$$I' = I + mh^2$$

$$mK^2 = mk^2 + mh^2$$

$$K^2 = k^2 + h^2$$

Then $T = 2\pi\sqrt{\frac{k^2+h^2}{gh}}$.

Centre of suspension:

The point O where the axis of rotation meets the vertical plane through the centre of gravity G of the rigid body is called centre of suspension.

A simple pendulum which has the same period as the given component pendulum is called the equivalent simple pendulum.

The equivalent simple pendulum,

$$L = \frac{k^2}{h}$$

$$L = \frac{k^2+h^2}{h}$$

If OG produced to point c such that OC=L, the length of the equivalent simple pendulum, the point c is called centre of oscillation.

The centre of oscillation is a point at which the mass of the body may be considered to be concentrated without any change in periodic time.

If the body is suspended about a parallel axis through c we have CG= L-h.

The length of the simple pendulum will be,

$$L_1 = \frac{k^2+(L-h)^2}{L-h}$$

But $L = \frac{k^2+h^2}{h}$

$$L_1 = \frac{Lh-h^2+(L-h)^2}{L-h}$$

$$L_1 = L$$

The centre of oscillation and the centre of suspension will be interchangeable.

centre of percussion:

When a body is capable of rotation about a fixed axis given a blow at a suitable point such that there is no impulsive force exerted on a fixed axis that point is known as centre of percussion.

Centre of percussion of the body with respect to axis:

If a pendulum supported on the axis through O is given by a blow at the centre of oscillation c. it will rotate about O without any jar on the axis of rotation. The centre of oscillation C on account of the reason is also called centre of percussion.

Minimum period of a compound pendulum:

$$T = 2\pi\sqrt{\frac{k^2+h^2}{gh}}$$

To find the value of T is depends on the length of the simple pendulum.

$$L = \frac{k^2+h^2}{h}$$

If T is minimum, $\frac{dt}{dh} = 0$

$$\frac{d}{dh}\left\{\frac{k^2}{h} + h\right\} = 0$$

$$-\frac{k^2}{h^2} + 1 = 0$$

$$1 - \frac{k^2}{h^2} = 0$$

$$h^2 - k^2 = 0$$

$$h^2 = k^2$$

$$K = \pm h$$

A compound pendulum will have its period a minimum when the depth of the centre of gravity of the pendulum below the centre of suspension is equal to magnitude to the radius of gyration about an axis through the centre of gravity parallel to the axis of rotation.

Unit : 5

1.Define centre of gravity of body?

The centre of gravity of a body may therefore be defined as a point through which the line of action of weight of the body always passes in whatsoever manner the body is placed.

2.Define centre of pressure?

The centre of pressure of a plane surface in contact with a fluid is a point on the surface through which the line of action of the resultant of the trusts on the various elements of the area passes.

3.What is the uses of barometer?

The barometer is used to measure the atmospheric pressure.

4.What is centre of buoyancy?

When a body floats freely in a liquid the resultant thrust acts through the centre of gravity of the liquid displaced. This point is called centre of buoyancy.

5.What is meant by metacentre?

If a floating body be slightly displaced by it remains the same, the point in which the vertical line through the new centre of buoyancy meets the line joining the centre of gravity of the body to the original centre of buoyancy is called metecentre.

6.What is plane of floatation?

When a body floats in a liquid the section in which the surface on the liquid intersects the floating body is called plane of floatation.

7. Derive an expression for Centre of gravity of solid hemisphere?

Let ACB represent a section of a solid hemisphere of radius ‘r’ and centre ‘o’. consider a thin slice of the hemisphere of thickness ‘dx’ at a distance ‘x’ from ‘o’. ‘w’ be the weight per unit volume of the material of the hemisphere. ‘y’ be the radius of the slice. Then , y = $\sqrt{r^2 - x^2}$

Volume of the slice = $\pi r^2 h$

$$= \pi (r^2 - x^2)\ dx$$

Weight of the slice = $\pi (r^2 - x^2)\ dx * w$

Distance of the centre of gravity of slice from o = x

Moment of the weight of slice = w * x

$$M = \pi (r^2 - x^2)\ dx\ .\ w * x$$

$$= \pi\ w\ (r^2 - x^2) x\ dx$$

Algebraic sum of the moments of all slices M = $\int_0^r \pi\ w\ (r^2 - x^2) x\ dx$

Weight of the hemisphere = volume * w

$$= \frac{2}{3}\pi r^3 * w$$

Le t the distance of centre of gravity of hemisphere from o = $\overline{\square}$

Moment of the weight of hemisphere about o = force * distance

$$= \frac{2}{3}\pi r^3 w * \overline{\square}$$

$$\int_0^r \pi\ w\ (r^2 - x^2) x\ dx = \frac{2}{3}\pi r^3 w * \overline{\square}$$

$$\overline{\square} = \frac{\int_0^r \pi\ w\ (r^2 - x^2) x\ dx}{\frac{2}{3}\pi r^3 w}$$

$$= \frac{\int_0^r (r^2 x - r^3) dx}{\frac{2}{3} r^3}$$

$$= \frac{\frac{r^4}{2} - \frac{r^4}{4}}{\frac{2}{3} r^3}$$

$$= \frac{\frac{r^4}{4}}{\frac{2}{3}r^3}$$

$$\bar{x} = \frac{3}{8}r$$

8. Derive an expression for Centre of gravity of solid cone?

Let ABC be the section of a solid cone of height h. the radius of the base is r. join AD then AD=h. consider a slice B_1C_1 of the cone parallel to the base of thickness dx. The slice is at a depth x below the vertex A. y be the radius of slice. Then

$$\tan\alpha = \frac{y}{x}$$

$$y = x \tan\alpha$$

volume of the slice = $\pi r^2 h$

$$= \pi\, x^2 \tan^2\alpha\, dx$$

W be the weight per unit volume of the cone.

Then the weight of the slice is $= \pi\, x^2 \tan^2\alpha\, dx\, .\, w$

Moment of slice about A $= \pi\, x^2 \tan^2\alpha\, dx.\, w\, .\, x$

Sum of the moments of all such slices of the cone = $\int_0^h \pi x^3 \tan^2\alpha\, dx\, w$

$$= \pi\, \square\, \tan^2\alpha \int_0^h x^3 dx$$

$$= \pi\, w \tan^2\alpha \frac{h^2}{4}$$

Volume of cone = $1/_3 \pi r^2 h$ cu.units

$$= 1/_3\, \pi \tan^2\alpha h^3$$

Weight of the cone $= 1/_3\, \pi \tan^2\alpha h^3\, .\, w$

Let $\bar{x}$ be the depth of the centre of gravity of cone below A.

Moment of weight of the cone about A = $1/_3\, \pi \tan^2\alpha h^3 w * \bar{x}$

$$1/_3\, \pi \tan^2\alpha h^3 w * \bar{x} = \pi\, w \tan^2\alpha \frac{h^2}{4}$$

$$\bar{x} = 3/_4\, h$$

The centre of gravity of a solid cone is a distance $3/_4$ h below the vertex.

9. Derive an expression for Centre of pressure of a rectangular lamina?

A rectangular lamina ABCD is immersed in a liquid of density 'ρ'. Let AB = a and AD = b. if the rectangle is divided into number of small strips parallel to AB of width dh. One such strip is considered at a depth 'h' below the surface of the liquid, we have the area of the strip is a*dh.

And the thrust acting on it is hρg a dh.

The moment of the strip about AB is hρg a dh. h = h^2ρg a dh

The sum of moments of the thrust on all the strips = $\int_0^b h^2 \rho g\, a\, dh$

Then the resultant of thrust on the rectangular lamina = $\int_0^b h \rho g\, a\, dh$

The moment of the resultant of strips about AB = H $\int_0^b h \rho g\, a\, dh$

Where H is the depth of the centre of pressure below AB.

$$\int_0^b h^2 \rho g\, a\, dh = H \int_0^b h \rho g\, a\, dh$$

$$H = \frac{\int_0^b h^2 dh}{\int_0^b h^3 dh}$$

$$H = \frac{\frac{b^3}{3}}{\frac{b^2}{2}}$$

$$H = \frac{2}{3}b$$

The centre of pressure will lie on the line EF, E and F are the midpoints of AB and DC.

10. . Derive an expression for Variation of atmospheric pressure with altitude?

The atmosphere pressure decreases with increase in altitude of a place.

Let A and B be the two points at height x and x + dx : p and p + dp be the pressure at places.

'a' is the cross section of a cylindrical column of air. P be the density of air.

Downward thrust = (p + dp)a

Upward thrust = pa

Weight of the air = aρg dx where g is the acceleration due to gravity.

At equilibrium, pa = (p + dp)a + aρg dx

$$0 = dp\, a + a\rho g\, dx$$

$$-dp\, a = a\rho g\, dx$$

$$dp = -\rho g\, dx$$

temperature of the air column is constant at a height 'x'. from Boyle's law ,

p = kρ

$\rho = \frac{p}{k}$

$dp = -\frac{p}{k} g\ dx$

divided by p, we get $\frac{dp}{p} = -\frac{g}{k} g\ dx$

$$\int \frac{dp}{p} = \int -\frac{g}{k} dx$$

$$\square\square\square_{\square}\square = -\frac{\square}{\square}\square + c$$

'c' is the integration constant. p_0= pressure at sea level.

$c = \log_e p_0$

$$\square\square\square_{\square}\square = -\frac{\square}{\square}\square + \log_e p_0$$

$$\log_e\left(\frac{p}{p_0}\right) = -\frac{gx}{k}$$

$$\frac{p}{p_0} = e^{-\frac{gx}{k}}$$

$$P = p_0 e^{-\frac{gx}{k}}$$

p_1, p_2, p_3.. be the pressures of altitudes. Here x = 1,2,3,…

$$p_1 = p_0 e^{-\frac{g}{k}}$$

$$p_2 = p_0 e^{-\frac{2g}{k}}$$

$$p_3 = p_0 e^{-\frac{3g}{k}}$$

$$\frac{p_1}{p_2} = \frac{p_0 e^{-\frac{g}{k}}}{p_0 e^{-\frac{2g}{k}}}$$

$$= e^{\frac{g}{k}}$$

$$\frac{p_2}{p_3} = e^{\frac{g}{k}}$$

$$\frac{p_1}{p_2} = \frac{p_2}{p_3} = \frac{p_3}{p_4} = \cdots . = \text{constant}$$

p_1, p_2, p_3 are in the geometric progression. If points are taken in earth atmosphere whose vertical height about earth surface are in arithmetic progression. A corresponding pressure are in geometrical progression.

11. Explain the Experimental determination of metacentric height of the ship?

Metacentre :

The floating body be slightly displaced such that the volume of the liquid displaced by it remains the same. Then the point in which the vertical line through the new centre of buoyancy meet is called metacentre.

Stability of equilibrium of floating body:

A freely suspended body is in equilibrium if the centre of gravity of the body is vertically below or above the point of suspension. But the equilibrium is stable only if the centre of gravity of the body is below the point of suspension.

A floating body will be in equilibrium;

1) The weight of the displaced liquid is equal to the weight of the floating body.
2) The centre of gravity of the body and that of the displaced liquid lie in the same vertical line.

This two conditions has to be satisfied. Then the equilibrium is stable.

Centre of buoyancy:

When the body floats freely in a liquid the resultant thrust acts through the centre of gravity of the liquid displaced. This point is called centre of buoyancy.

Meta centric height:

The distance between centre of gravity and the meta centre is called the meta centric height.

Determination of meta centric height of the ship:

The weight of the ship w is determined by the displacement method. The ship has two boats on the deck at a distance ‘l’ apart. If A and B represents the boats at a distance ‘l’ apart on the deck. Filling A and B alternatively with water is equivalent to moving a weight mg from A to B across the deck. The filling of water at B with same mass of water at A turns the ship through a small angle θ. The angle θ is determined by pump line suspended in the ship. Since the ship slightly inclined the centre of buoyancy is changed. ‘H’ and ‘H'’ be the original and changed position of centre of buoyancy. G is the centre of gravity of the ship, M is the meta centre and GM is the meta centric height of ship.

The deflecting couple due to filling the boats alternatively with water = mgl cosθ = w l cosθ

The restoring couple due to the weight of the ship acting at G and the force of buoyancy acting through H' = W * GM $\sin\theta$

$$\sin\theta = \frac{HH'}{GM}$$

$$HH' = GM \sin\theta$$

Restoring couple = deflecting couple

$$w * GM \sin\theta = w\, l\, \cos\theta$$

$$GM = \frac{W\, l \cos\theta}{w \sin\theta}$$

$$GM = \frac{w\, l}{W \tan\theta}$$

When θ is small, $GM = \frac{w\, l}{W\theta}$.