IDHAYA COLLEGE FOR WOMEN, KUMBAKONAM

DEPARTMENT OF MATHEMATICS

CLASS	**:**	**I MBA**
SUBJECT NAME	**:**	**OPERATIONS RESEARCH**
SUBJECT CODE	**:**	**P16MBA7**
SEMESTER	**:**	**II**
UNIT	**:**	**V (QUEUING THEORY)**
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UNIT V
QUEUING THEORY

1. What is Queuing Theory?

Queuing theory is the mathematical study of the congestion and delays of waiting in line. Queuing theory examines every component of waiting in line to be served, including the arrival process, service process, number of servers, number of system places, and the number of customers (which might be people, data packets, cars, etc.)

2. Elements of Queuing system

A queuing system can be completely described by:

i) The input (Arrival Pattern)

ii) The service mechanism

iii) The queue discipline

iv) Customer's behavior

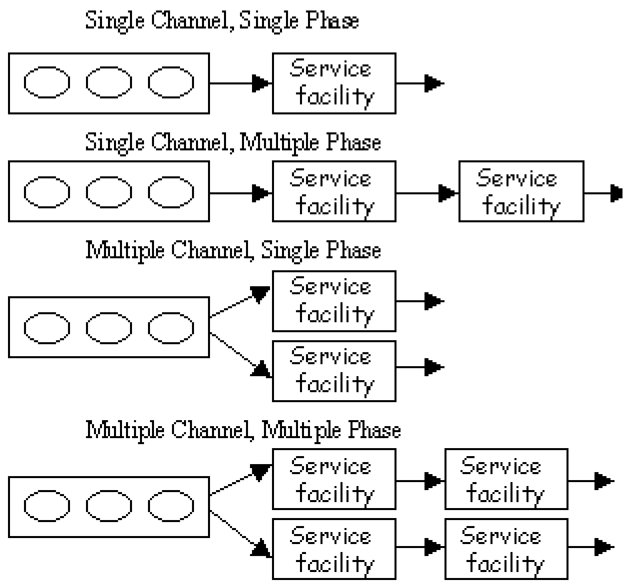
3. Define Arrival Pattern

Input describes the way in which the customers arrive and join the system. Generally, customers arrive in a more or less random fashion, which is not possible to predict. we deals with those queuing system in which the customers arrive in Poisson fashion.

4. Service Mechanism:

This means, the arrangement of service facility to serve customer. If there is an infinite number of servers, then all the customers are served instantaneously on arrival, and there will be no queue.

A specification of the service mechanism includes a description of time to complete a service and the number of customers who are satisfied at each service event. The service mechanism also prescribes the number and configuration of servers. If there is more than one service facility, the calling unit may receive service from a sequence of these. At a given facility, the unit enters one of the parallel service channels and is completely serviced by that server. Most elementary models assume one service facility with either one or a finite number of servers. The following figure shows the physical layout of service facilities.



5. **Queue discipline**

It is a rule according to which the customer are selected for service when a queue has been formed. The most common disciplines are

- First come First served

-First In First Out

-Last in first out

-Selection for service in random order

6. **Customer's Behavior**

The customers generally behavior in the following 4 ways

i) **Priority** is where customers are served based on their priority level; these levels could be based on status, task urgency, or some other criteria.

ii) **Balking:** when a customer decides not to wait for service because the wait time threatens to be too long.

iii) **Reneging** is similar, but when a customer who has waited already decides to leave because they've wasted too much time.

iv) **Jockeying** is when a customer switches between queues in a tandem queue system, trying to orchestrate the shortest wait possible. (or) When there are a number of queues, a customer may move from one queue to another in hope of receiving the service quickly.

7. Kendall’s Notation for Representing Queuing Model

Generally, queuing model may be completely specified in the following symbol form (a/b/c): (d/e) where

a: the arrival process probability distribution.

b: the service process probability distribution.

c: the number of servers.

d: the maximum number of customers allowed in the system at any given time, waiting or being served

e:queue discipline.

8. Classification of Queuing models

Model I: (M/M/1): (∞/FCFS)

This denotes Poisson arrival, Possion depature, Single sever, Infinite capacity and First come first served service discipline.

Model II: (M/M/1): (N/FCFS)

In this model the capacity system is limited (finite), say N, Obviously, the number of arrivals ill not exceed the number N.

Model III : (M/M/S): (∞/FCFS)

This model takes the number of service channels as S

Model IV: (M/M/S): (N/FCFS)

This model is essentially the same as model II, except the maximum number of customers in the system is limited to N, N>S

Model I Formula

λ: the mean arrival rate.

μ: the mean service rate.

n: the number of people in the system.

1. Expected (Average) **number** of customers waiting in the **system** Ls= $\frac{\lambda}{\mu-\lambda}$
2. Expected (Average) **number** of customers waiting in the **queue** Lq= $\frac{\lambda^2}{\mu(\mu-\lambda)}$
3. Expected (Average) **waiting time** of customers waiting in the **system** Ws= $\frac{1}{\mu-\lambda}$
4. Expected (Average) **waiting time** of customers waiting in the **queue** Wq= $\frac{\lambda}{\mu(\mu-\lambda)}$
5. Traffic intensity $\rho=\frac{\lambda}{\mu}$
6. No customer in the system (System being idle) $P_0=1-\rho$

Hints:

*P[server is idel]= P[no customer in the system] =p[arrival need not wait]=P0 in model I

*P[server is busy]=P[arriving customer has to wait]=P(n≥1) in single server

=P(n≥S) in multiple server

*Waiting to get service or waiting for service – Queue

Problem 1. A TV mechanic finds that the spent on his jobs has an exponential distribution with mean 30 min, if he repairs sets in the order in which they come in. if the arrival of sets is approximately poisson with an average rate of 10 per eight- hour day, what is the mechanic's expected idle time each day? How many jobs are ahead of the average set just brought in?

Solution: Step 1. Model identification

Server = 1 TV repair man

System capacity= No information (∞)

This is **Model I:** (M/M/1): (∞/FCFS)

Step 2: Parameters

Arrival rate λ =10 per 8 hour (unit time) day

Service rate μ =30 min

1/ μ =1/30 min=60/30 hours=2x8 day

1/ μ =16

$\mu = 16$

Step 3: understand the question, what is asked?

i) The probability for the repairman to be idle P0=1- ρ

$$\rho = \frac{\lambda}{\mu} = \frac{10}{16} = 0.625$$

P0=1-0.625=0.375

ii) Expected idle time per day= 8x 0.375=3 hours

iii) Expected number of TV sets in the system Ls= $\frac{\lambda}{\mu - \lambda}$

=10/(16-10)=1.6=2 sets

Model II: **(M/M/1): (N/FCFS) formula**

1. Probability that the system is idle P0= $\frac{1-\rho}{1-\rho^{N+1}}$, $\rho = \frac{\lambda}{\mu}$
2. $P_N = \frac{1-\rho}{1-\rho^{N+1}} \rho^n$, n=0,1,2……N
3. Average number of customers in the system $Ls = P_0 \sum_{n=0}^{N} n\rho^n$
4. Average number of customers in the queue $Lq = Ls - \frac{\lambda}{\mu}$
5. The effective Arrival rate $\lambda' = \mu(1 - P_0)$

6. Average waiting time of a customers in the system $Ws = \frac{Ls}{\lambda}$

7. Average waiting time of a customers in the queue $Wq = \frac{Lq}{\lambda}$

Problem2: A car park contains 5 cars. The arrival of cars is poisson at a mean rate of 10 per hour. The length of time each car spends in the car park is exponentially distributed with mean of 0.5 hour. How many cars are there in the car park on an average?

Solution: Step 1: Model Identification

Server = A car park

System capacity= 5 cars

This is **Model II:** (M/M/1): (5/FCFS)

Step 2: Parameters

Arrival rate λ =10 per hour

=10/60 per min=1/6 per min

Service rate μ =0.5 per hour=1/2x60 per min

Capacity of the system N=5

Step 3: understand the question, what is asked?

$$\rho = \frac{\lambda}{\mu} = 20$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$$

$$P_0 = \frac{1-20}{1-20^6} = \frac{-19}{-6399} = 0.00297$$

i) Average cars in the car park is given by $Ls = P_0 \sum_{n=0}^{N} n\rho^n$

$$= P_0[\rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + 5\rho^5]$$

$$= 0.00297[20+2(20)^2+3(20)^3+4(20)^4+5(20)^5] = 4 \text{ (approximately)}$$