Simplex Method :-

Use simplex method to solve the following.

$$\text{Max } z = 4x_1 + 10x_2$$

subject to,

$$2x_1 + x_2 \le 50$$

$$2x_1 + 5x_2 \le 100$$

$$2x_1 + 3x_2 \le 90$$

and $x_1, x_2 \ge 0$.

Solution :-

The general form of the given LPP

$$\text{Max } z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$$

subject to,

$$2x_1 + x_2 + s_1 = 50$$

$$2x_1 + 5x_2 + s_2 = 100$$

$$2x_1 + 3x_2 + s_3 = 90$$

and $x_1, x_2, s_1, s_2, s_3 \ge 0$.

Matrix form :

$$AX = b$$

$$A = \begin{array}{c} \begin{matrix} x_1 & x_2 & s_1 & s_2 & s_3 \end{matrix} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 \\ 2 & 5 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 1 \end{bmatrix} \end{array}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}, \quad b = \begin{bmatrix} 50 \\ 100 \\ 90 \end{bmatrix}$$

To find IBFS :-

Here $n = 5$, $m = 3$.

no. of non-basic solution $= n - m = 5 - 3 = 2$

$\therefore$ No. of basic solution $= 3$

$$\text{Let } B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore x_B = B^{-1} b$

$[s_1, s_2, s_3] = [50, 100, 90]$

	C_j	4	10	0	0	0			
C_B	Basis	x_1	x_2	s_1	s_2	s_3	x_B	Ratio	Remark
0	s_1	2	10	1	0	0	50	$50/1 = 50$	$x_2 \to$ Enter the basis
0	$s_2 \rightarrow$	2	5^*	0	1	0	100	$100/5 = 20^*$	$s_2 \to$ leave the basis
0	s_3	2	3	0	0	1	90	$90/3 = 30$	
	z_j	0	0	0	0	0	$z = 0$		
	$z_j - c_j$	-4	-10 $\uparrow$	0	0	0			
0	s_1	8/5	0	1	-1/5	0	30		
10	x_2	2/5	1	0	1/5	0	20		
0	s_3	4/5	0	0	-3/5	1	30		
	z_j	4	10	0	2	0	$z = 200$		
	$z_j - c_j$	0	0	0	2	0			

Here all $z_j - c_j \geq 0$.

So the given LPP reached the optimum solution. Hence the optimum basic feasible solution is

$x_1 = 0, \; x_2 = 20$, and

Max $z = 200$.

2. Find the maximum value of Max $z = 107x_1 + x_2 + 2x_3$

Subject to, $14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + x_2 - 6x_3 \leq 5$

$3x_1 - x_2 - x_3 \leq 0$ and $x_1, x_2, x_3, x_4 \geq 0$.

using the simplex method.

The general form of the given L.P.P.

$$\text{Max } Z = 107x_1 + x_2 + 2x_3 + 0.x_4 + 0.S_1 + 0.S_2.$$

subject to, $16x_1 + x_2 - 6x_3 + S_1 = 5$

$$3x_1 - x_2 - x_3 + S_2 = 0$$

$$\frac{14}{3}x_1 + \frac{1}{3}x_2 - \frac{6}{3}x_3 + x_4 = \frac{7}{3}.$$

and $x_1, x_2, x_3, x_4, S_1, S_2 \geq 0.$

Matrix form:-

$$AX = b.$$

$$A = \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & S_1 & S_2 \end{array} \\ \begin{bmatrix} 14/3 & 1/3 & -6/3 & 1 & 0 & 0 \\ 16 & 1 & -6 & 0 & 1 & 0 \\ 3 & -1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ S_1 \\ S_2 \end{bmatrix}, b = \begin{bmatrix} 7/3 \\ 5 \\ 0 \end{bmatrix}$$

B

To find IBFS:-

Here $n = 6, m = 3.$

No. of non-basic solution $= n-m = 6-3 = 3.$

$\therefore$ No. of basic solution $= 3.$

$$\text{Let } B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore X_B = B^{-1}.b$$

Here

$$[x_4, S_1, S_2] = [7/3, 5, 0]$$

C_B	Basis (c_j)	x_1 (107)	x_2 (1)	x_3 (2)	x_4 (0)	S_1 (0)	S_2 (0)	x_B	Ratio	Remark
0	x_4	$14/3$	$1/3$	-2	1	0	0	$7/3$	$\frac{7}{3}\times\frac{3}{14}=\frac{1}{2}$	$x_1 \to$ Enter the basis,
0	S_1	16	1	-6	0	1	0	5	$\frac{5}{16}$	$s_2 \to$ leave
0	S_2	3^*	-1	-1	0	0	1	0	$\frac{0}{3}=0^*$	the basis
	z_j	0	0	0	0	0	0	$Z=0$		
	z_j-c_j	-107	-1	-2	0	0	0			
0	x_4	0	$17/9$	$-4/9$	1	0	$-14/9$	$7/3$		$x_3 \to$ Enter
0	S_1	0	$19/3$	$-2/3$	0	1	$-16/3$	5		
107	x_1	1	$-1/3$	$-1/3$	0	0	$1/3$	0		
	z_j	107	$-107/3$	$-107/3$	0	0	$\frac{107}{3}$	$Z=0$		
	z_j-c_j	0	$-110/3$	$-113/3$	0	0	$\frac{107}{3}$			

In the final iteration, x_3 Enter the basis, because here does not satisfy the condition $z_j - c_j \geq 0$.

Now the values are corresponding pivot column all are negative. so the given LPP has an unbounded solution.

1. Use simplex method to solve the following

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to, $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

and $x_1, x_2 \geq 0$.

Solution:-

The general form of the given LPP

$$\text{Max } Z = 3x_1 + 2x_2 + 0.S_1 + 0.S_2$$

subject to,

$$x_1 + x_2 + S_1 = 4$$

$$x_1 - x_2 + S_2 = 2$$

and $x_1, x_2, S_1, S_2 \geq 0$

The matrix form of the given LPP

$$AX = b$$

Here $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix}$; $b =$

To find IBFS ;

$$X_B = B^{-1} b$$

Here $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B^{-1}$

$$X_B = [S_1, S_2]$$

$$X_B = [4, 2]$$

	c_j	3	2	0	0			
C_B	Basis	x_1	x_2	s_1	s_2	x_B	Ratio	Remarks
0	s_1	1	1	1	0	4	$4/1 = 4$	x_1 - Enter
0	$s_2 \rightarrow$	1^*	-1	0	1	2	$2/1 = 2^*$	s_2 - leave
	z_j	0	0	0	0			
	$z_j - c_j$	$-3 \uparrow$	-2	0	0			
0	$s_1 \rightarrow$	0	2^*	1	-1	2	$2/2 = 1$	x_2 - Enter
3	x_1	1	-1	0	1	2	-	s_1 - leave
	z_j	3	-3	0	3			
	$z_j - c_j$	0	$-5 \uparrow$	0	3			
2	x_2	0	1	$1/2$	$-1/2$	1		
3	x_1	1	0	$1/2$	$1/2$	3		
	z_j	3	2	$5/2$	$1/2$	$Z = 11$		
	$z_j - c_j$	0	0	$5/2$	$1/2$			

Here all $z_j - c_j \geq 0$

so the given LPP reached the optimum solution.

$\therefore$ The solutions are

$x_1 = 3$

$x_2 = 1$

and Max $z = 11$.

2. Max $z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

and $x_1, x_2 \geq 0$.

solution :-

The general form of the given LPP

$$\text{Max } z = 5x_1 + 3x_2 + 0.S_1 + 0.S_2 + 0.S_3$$

subject to,

$$x_1 + x_2 + S_1 = 2$$

$$5x_1 + 2x_2 + S_2 = 10$$

$$3x_1 + 8x_2 + S_3 = 12$$

and $x_1, x_2, S_1, S_2, S_3 \geq 0.$

The Matrix form of the given L.P.P

$$AX = b$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 5 & 2 & 0 & 1 & 0 \\ 3 & 8 & 0 & 0 & 1 \end{bmatrix} ; X = \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} ; B = \begin{bmatrix} 2 \\ 10 \\ 12 \end{bmatrix}$$

To find IBFS,

$$X_B = B^{-1}b.$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}.$$

$$X_B = [S_1, S_2, S_3]$$

$$X_B = [2, 10, 12]$$

C_B	Basis	x_1	x_2	s_1	s_2	s_3	x_B	Ratio	Remark
	c_j	5	3	0	0	0			
0	s_1	1	1	1	0	0	2	$\frac{2}{1} = 2^*$	x_1 - Enter
0	s_2	5	2	0	1	0	10	$10/5 = 2$	s_1 - leave
0	s_3	3	8	0	0	1	12	$12/3 = 4$	
	z_j	0	0	0	0	0			
	$z_j - c_j$	-5	-3	0	0	0			
5	x_1	1	1	1	0	0	2		
	s_2	0	-3	-5	1	0	0		
	s_3	0	5	-3	0	1	6		
	z_j	5	5	5	0	0	$z=10$		
	$z_j - c_j$	0	2	5	0	0			

Here all $z_j - c_j \geq 0$

so the given L.PP reached the optimum solution

$\therefore$ The solution are

$x_1 = 2$

$x_2 = 0$

and Max $z = 10$.

3. Max $z = 4x_1 + x_2 + 3x_3 + 5x_4$

subject to, $4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20$

$3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$

and $x_1, x_2, x_3, x_4 \geq 0$

solution :-

The general form of the given LPP

$$\text{Max } z = 4x_1 + x_2 + 3x_3 + 5x_4 + 0.s_1 + 0.s_2 + 0.s_3$$

subject to,

$$-4x_1 + 6x_2 + 5x_3 + 4x_4 + s_1 = 20$$

$$3x_1 - 2x_2 + 4x_3 + x_4 + s_2 = 10$$

$$8x_1 - 3x_2 + 3x_3 + 2x_4 + s_3 = 20$$

and $x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0$

The Matrix form of the given LPP

$$Ax = b$$

$$A = \begin{bmatrix} -4 & 6 & 5 & 4 & 1 & 0 & 0 \\ 3 & -2 & 4 & 1 & 0 & 1 & 0 \\ 8 & -8 & 3 & 2 & 0 & 0 & 1 \end{bmatrix} ; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} ; b = \begin{bmatrix} 20 \\ 10 \\ 20 \end{bmatrix}$$

To find IBFS :-

$$x_B = B^{-1} b$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}$$

$$x_B = [s_1, s_2, s_3] = [20, 10, 20].$$

C_B	Basis	x_1	x_2	x_3	x_4	s_1	s_2	s_3	X_B	Ratio	Remark
0	$s_1 \rightarrow$	-4	6	5	4	1	0	0	20	20/4=5	x_4 - Enter
0	s_2	3	-2	4	1	0	1	0	10	10/1=10	s_1 - leave
0	s_3	8	-3	3	2	0	0	1	20	20/2=10	
	z_j	0	0	0	0	0	0	0			
	$z_j - c_j$	-4	-1	-3	$-5\uparrow$	0	0	0			
5	x_4	-1	3/2	5/4	1	1/4	0	0	5		x_1 - Enter
0	s_2	4	-7/2	11/4	0	-1/4	1	0			s_3 - leave
0	$s_3 \rightarrow$	10	-6	1/2	0	-1/2	0	1			
	z_j	-5	15/2	25/4	5	5/4	0	0			
	$z_j - c_j$	$-9\uparrow$	13/2	13/4	0	5/4	0	0	1		
5	x_4	0	9/10	13/10	1	1/5	0	1/10	6		
0	s_2	0	-1/10	59/20	0	-1/20	1	-2/5	1		
4	s_1	1	-3/5	1/20	0	-1/20	0	1/10	1		
	z_j	4	21/10	67/10	5	4/5	0	9/10	$z=34$		

Here all $z_j - c_j \geq 0$

so the given LPP reached the optimum solution.

$\therefore$ The solutions are

$x_1 = 1$

$x_2 = 0$

$x_3 = 0$

$x_4 = 6$

and Max $z = 34$.