Q. Solve the following LPP by two phase method.

$$\text{Max } z = 5x_1 - 4x_2 + 3x_3.$$

Subject to,
$$2x_1 + x_2 - 6x_3 = 20$$
$$6x_1 + 5x_2 + 10x_3 \le 76$$
$$8x_1 - 3x_2 + 6x_3 \le 50$$

and $x_1, x_2, x_3 \ge 0$.

Solution :-

General form of L.PP

$$\text{Max } Z = 5x_1 - 4x_2 + 3x_3 + 0.S_1 + 0.S_2 - A_1$$

subject to,
$$2x_1 + x_2 - 6x_3 + A_1 = 20$$
$$6x_1 + 5x_2 + 10x_3 + S_1 = 76$$
$$8x_1 - 3x_2 + 6x_3 + S_2 = 50$$

$$A = \begin{bmatrix} 2 & 1 & -6 & 1 & 0 & 0 \\ 6 & 5 & 10 & 0 & 1 & 0 \\ 8 & -3 & 6 & 0 & 0 & 1 \end{bmatrix};\ X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ S_1 \\ S_2 \\ A_1 \end{bmatrix};\ b = \begin{bmatrix} 20 \\ 76 \\ 50 \end{bmatrix}$$

IBFS :-

$$X_B = B^{-1}b.$$

$$\text{Here, } B = B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore X_B = \begin{bmatrix} A_1 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 76 \\ 50 \end{bmatrix}$$

Phase I :-

C_B	Basis	x_1	x_2	x_3	S_1	S_2	A_1	x_B	Ratio	Remarks
	c_j	0	0	0	-1	0	0			
-1	A_1	2	1	-6	1	0	0	20	$20/2=10$	x_1-Enter
0	S_1	6	5	10	0	1	0	76	$76/6=12.6$	S_2-leave
0	S_2	8*	-3	6	0	0	1	50	$50/8=6.25$	
	z_j	-2	-1	6	-1	0	0	$z^*=-20$		
	z_j-c_j	-2	-1	6	0	0	0			
-1	A_1	0	7/4*	-15/2	1	0	-1/4	15/2	$\frac{15}{2}\times\frac{4}{7}=\frac{30}{7}$	x_2-Enter
0	S_1	0	29/4	11/2	0	1	-3/4	77/2	$\frac{77}{2}\times\frac{4}{29}=\frac{154}{29}$	A_1-leave
0	x_1	1	-3/8	3/4	0	0	1/8	25/4	-	
	z_j	0	-7/4	15/2	-1	0	1/4			
	z_j-c_j	0	-7/4	15/2	0	0	1/4			
0	x_2	0	1	-30/7	4/7	0	-1/7	30/7		
0	S_1	0	0	256/7	-29/4	1	1/3	75/2		
0	x_1	1	0	-6/7	3/14	0	1/14	55/7		
	z_j	0	0	0	0	0	0	$z^*=0$		
	z_j-c_j	0	0	0	1	0	0			

Here all $z_j-c_j \geq 0$ and max $z^*=0$, there is no artificial variable present in the basis. Hence the given L.P.P can reached the optimum solution. so we can proceed phase 2.

PHASE -2

C_B	c_j Basis	5 x_1	-4 x_2	3 x_3	0 S_1	0 S_2	x_B	Ratio
-4	x_2	0	1	$-30/7$	0	$-1/7$	$\frac{30}{7}$	
0	s_1	0	0	$256/7$	1	$1/3$	$75/2$	
5	x_1	1	0	$-6/7$	0	$1/14$	$55/7$	
	z_j	5	-4	$90/7$	0	$13/4$	$z = \frac{155}{7}$	
	$z_j - c_j$	0	0	$69/7$	0	$13/4$		

Here all $z_j - c_j \geq 0$. so the given LPP reached the optimum solution.

$\therefore$ The solution is $x_1 = 55/7$; $x_2 = 30/7$; $x_3 = 0$

$\therefore$ Max $z = \frac{155}{7}$

and Max $z = 34$.

3. Use two-phase method to solve the following LPP.

$$\text{Min } z = -x_1 - 2x_2 - x_3$$

subject to,

$$x_1 + 2x_2 \leq 2$$

$$x_1 + 2x_2 + x_3 \geq 6$$

$$x_1 + x_3 \leq 4$$

and $x_1, x_2, x_3 \geq 0$

Solution:-

The general form of the given LPP is

$$\text{Max } z^* = (-z) = x_1 + 2x_2 + x_3 + 0.S_1 + 0.S_2 + 0.S_3 - A_1$$

subject to,

$$x_1 + 2x_2 + S_1 = 2$$

$$x_1 + 2x_2 + x_3 - S_3 + A_1 = 6$$

$$x_1 + x_3 + S_2 = 4$$

and $x_1, x_2, x_3, S_1, S_2, S_3, A_1 \geq 0$

The matrix form of the given LPP is

$$Ax = b$$

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}; \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ S_1 \\ S_2 \\ S_3 \\ A_1 \end{bmatrix}; \; b = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

To find IBFS:

$$x_B = B^{-1} b.$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}$$

$$x_B = [s_1, A_1, s_2] = [2, 6, 4]$$

Phase I:-

$$\text{Max } z^* = -A_1$$

subject to,

$$x_1 + 2x_2 + s_1 = 2$$

$$x_1 + 2x_2 + x_3 - s_3 + A_1 = 6$$

$$x_1 + x_3 + s_2 = 4$$

and $x_1, x_2, x_3, s_1, s_2, s_3, A_1 \geq 0.$

C_B	C_j / Basis	x_1 (0)	x_2 (0)	x_3 (0)	s_1 (0)	s_2 (0)	s_3 (0)	A_1 (-1)	x_B	Ratio	Remarks
0	s_1	1	2	0	1	0	0	0	2	$2/2 = 1^*$	x_2 - Enter
-1	A_1	1	2	1	0	0	-1	1	6	$6/2 = 3$	s_1 - leave
0	s_2	1	0	1	0	1	0	0	4	–	
	z_j	-1	-2	-1	0	0	1	-1			
	$z_j - c_j$	-1	-2	-1	0	0	1	0			
0	x_2	1/2	1	0	1/2	0	0	0	1	–	x_3 - Enter
-1	A_1	0	0	1	-1	0	-1	1	4	4^*	A_1 - leave
0	s_2	1	0	1	0	1	0	0	4	4	
	z_j	0	0	-1	1	0	1	-1			
	$z_j - c_j$	0	0	-1	1	0	1	0			
0	x_2	1/2	1	0	1/2	0	0	0	1		
0	x_3	0	0	1	-1	0	-1	1	4		
0	s_2	1	0	0	1	1	1	-1	0		
	z_j	0	0	0	0	0	0	0	$z^* = 0$		
	$z_j - c_j$	0	0	0	0	0	0	+1			

Here all $z_j - c_j \geq 0$ and Max $z^* = 0$

There is no artificial variable present in the basis.

Hence the given LPP can reached the optimum solution.

So we can proceed Phase-II

Phase-II

C_B	c_j / Basis	x_1 (1)	x_2 (2)	x_3 (1)	s_1 (0)	s_2 (0)	s_3 (0)	X_B	Ratio	Remar
2	x_2	1/2	1	0	1/2	0	0	1	$1/0 = -$	s_3- En
1	x_3	0	0	1	-1	0	-1	4	-	s_2- Le
0	s_2	1	0	0	1	1	1	0	0	
	z_j	1	2	1	0	0	-1			
	$z_j - c_j$	0	0	0	0	0	-1			
2	x_2	1/2	1	0	1/2	0	0	1		
1	x_3	1	0	1	0	1	0	4		
0	s_3	1	0	0	1	1	1	0		
	z_j	2	2	1	1	1	0	$z^* = 6$		
	$z_j - c_j$	1	0	0	1	1	0			

Here all $z_j - c_j \geq 0$

Min $z = -$Max z^*

So the given L.P.P reached the optimum solution.

$\therefore$ The solutions are

$x_1 = 0$,

$x_2 = 1$,

$x_3 = 4$

and Min $z = -6$.