BIG-M (or) PENALTY (or) CHARNE'S METHOD :-

1. Use Big-M method to solve the following LPP.

$$\text{Max } z = 6x_1 + 4x_2$$

subject to,
$$2x_1 + 3x_2 \le 30$$
$$3x_1 + 2x_2 \le 24$$
$$x_1 + x_2 \ge 3$$

and $x_1, x_2 \ge 0$

solution :-

The general form of the given L.P.P

$$\text{Max } z = 6x_1 + 4x_2 + 0.s_1 + 0.s_2 + 0.s_3 - MA_1$$

subject to,
$$2x_1 + 3x_2 + s_1 = 30$$
$$3x_1 + 2x_2 + s_2 = 24$$
$$x_1 + x_2 - s_3 + A_1 = 3$$

and $x_1, x_2, s_1, s_2, s_3, A_1 \ge 0$

The matrix form of the given LPP

$$Ax = b$$

$$A = \begin{bmatrix} 2 & 3 & 1 & 0 & 0 & 0 \\ 3 & 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & 1 \end{bmatrix} ; \; x = \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \\ A_1 \end{bmatrix} ; \; b = \begin{bmatrix} 30 \\ 24 \\ 3 \end{bmatrix}$$

To find IBFS :

$$x_B = B^{-1} b$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}.$$

$$x_B = [s_1, s_2, A_1] = [30, 24, 3]$$

C_B	Basis	x_1	x_2	s_1	s_2	s_3	A_1	x_B	Ratio	Remarks
	c_j	6	4	0	0	0	$-M$			
0	s_1	2	3	1	0	0	0	30	$\frac{30}{2}=15$	x_1-Enter
0	s_2	3	2	0	1	0	0	24	$24/3=8$	A_1-leave
$-M$	A_1	1	1	0	0	-1	1	3	$3/1=3^*$	
	z_j	$-M$	$-M$	0	0	M	$-M$			
	z_j-c_j	$-M-6$	$-M-4$	0	0	M	0			
0	s_1	0	1	1	0	2	-	24	$24/2=12$	s_3-Enter
0	s_2	0	-1	0	1	3	-	15	$15/3=5$	s_2-leave
6	x_1	1	1	0	0	-1	-	3	-	
	z_j	6	6	0	0	-6	-			
	z_j-c_j	0	2	0	0	-6	-			
0	s_1	0	$5/3$	1	$-2/3$	0	-	14		
	s_3	0	$-1/3$	0	$1/3$	1	-	5		
	x_1	1	$2/3$	0	$1/3$	0	-	8		
	z_j	6	4	0	2	0	-	$Z=48$		
	z_j-c_j	0	0	0	2	0	-			

Here all $z_j-c_j \geq 0$ and no artificial variable present at the basis.

Hence the optimum solution

$x_1 = 8$ (basic)

$x_2 = 0$ (non-basic)

and Max $z = 48$.

2. Max $z = 3x_1 + 2x_2$

subject to, $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

and $x_1, x_2 \geq 0$.

solution :-

The general form of the given LPP

$$\text{Max } z = 3x_1 + 2x_2 + 0.S_1 + 0.S_2 - MA_1$$

Subject to,

$$2x_1 + x_2 + S_1 = 2$$

$$3x_1 + 4x_2 - S_2 + A_1 = 12$$

and $x_1, x_2, S_1, S_2, A_1 \geq 0$

The matrix form of the given LPP

$$Ax = b$$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 \\ 3 & 4 & 0 & -1 & 1 \end{bmatrix}; \quad x = \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \\ A_1 \end{bmatrix}; \quad b = \begin{bmatrix} 2 \\ 12 \end{bmatrix}$$

To find IBFS,

$$x_B = B^{-1} b$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B^{-1}$$

$$x_B = [S_1, A_1] = [2, 12]$$

C_B	Basis	x_1	x_2	S_1	S_2	A_1	X_B	Ratio	Remark
0	S_1	2	1	1	0	0	2	$2/1 = 2$	x_2 - Enter
$-M$	A_1	3	4	0	-1	1	12	$12/4 = 3$	S_1 - leave
	z_j	$-3M$	$-4M$	0	M	$-M$			
	$z_j - c_j$	$-3M-3$	$-4M-2$	0	M	0			
2	x_2	2	1	1	0	0	2		
$-M$	A_1	-5	0	-4	-1	1	4		
	z_j	$4+5M$	2	$2+4M$	M	$-M$	$Z = 4-4M$		
	$z_j - c_j$	$1+5M$	0	$2+4M$	M	0			

Here all $z_j - c_j \geq 0$. But the artificial variable is present of the basis.

So the given LPP does not posses any optimum basic feasible solution.

3. Max $z = 2x_1 + 3x_2$

subject to, $x_1 + 2x_2 \leq 4$

$$x_1 + x_2 = 3$$

and $x_1, x_2 \geq 0$

solution :-

The general form of the given LPP

$$\text{Max } z = 2x_1 + 3x_2 + 0.s_1 - MA_1$$

subject to, $x_1 + 2x_2 + s_1 = 4$

$$x_1 + x_2 + A_1 = 3$$

and $x_1, x_2, s_1, A_1 \geq 0$

The Matrix form of the given LPP.

$$AX = b$$

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} ; \; X = \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ A_1 \end{bmatrix} ; \; b = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

To find IBFS,

$$X_B = B^{-1} b$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B^{-1}$$

$X_B = [S_1, A_1] = [4, 3]$

C_B	C_j Basis	x_1	x_2	S_1	A_1	X_B	Ratio	Re
0	S_1	1	2	1	0	4	$4/2 = 2$	x_2 - Enter
$-M$	A_1	1	1	0	1	3	$3/1 = 3$	S_1 - leav
	Z_j	$-M$	$-M$	0	$-M$			
	$Z_j - C_j$	$-M-2$	$-M-3$	0	0			
3	x_2	$1/2$	1	$1/2$	0	2	$2 \times \frac{2}{1} = 4$	x_1 -
$-M$	A_1	$1/2$	0	$-1/2$	1	1	$1 \times \frac{1}{2} = 2$	A_1 - le
	Z_j	$3/2 - M/2$	3	$3/2 + M/2$	$-M$			
	$Z_j - C_j$	$\frac{-M-1}{2}$	0	$\frac{3}{2} + \frac{M}{2}$	0			
3	x_2	0	1	1	-	1		
2	x_1	1	0	-1	-	2		
	Z_j	2	3	1	-	$Z = 7$		
	$Z_j - C_j$	0	0	1	-			

Here all $Z_j - C_j \geq 0$ and no artificial variab present at the basis. Hence the optimum solution.

$x_1 = 2$

$x_2 = 1$

and Max $Z = 7$.