## Bisection Method

i) $f(x) = x^4 - 7$

soln:

Given equation $f(x) = x^4 - 7$

$f(-1) = (-1)^4 - 7 \Rightarrow 1 - 7 = -6$ (-ve)

$f(0) = (0)^4 - 7 \Rightarrow 0 - 7 = -7$ (-ve)

$f(1) = (1)^4 - 7 \Rightarrow 1 - 7 = -6$ (-ve)

$f(2) = (2)^4 - 7 \Rightarrow 16 - 7 = 9$ (+ve)

| n | a | b | $x_n = \frac{a+b}{2}$ | $f(x_n)$ |
|---|---|---|---|---|
| 1 | 1 | 2 | 1.5 | -1.9375 |
| 2 | 1.5 | 2 | 1.75 | 2.3789 |
| 3 | 1.5 | 1.75 | 1.625 | -0.0270 |
| 4 | 1.75 | 1.625 | 1.6875 | 1.091 |
| 5 | 1.625 | 1.6875 | 1.6562 | 0.5242 |
| 6 | 1.6875 | 1.6562 | 1.6718 | 0.8115 |
| 7 | 1.6562 | 1.6718 | 1.664 | 0.6667 |
| 8 | 1.6718 | 1.664 | 1.6679 | 0.7389 |
| 9 | 1.664 | 1.6679 | 1.6659 | 0.7018 |
| 10 | 1.6679 | 1.6659 | 1.6669 | 0.7203 |
| 11 | 1.6659 | 1.6669 | 1.6664 | 0.7111 |
| 12 | 1.6667 | 1.6664 | 1.6666 | 0.7148 |
| 13 | 1.6664 | 1.6266 | 1.6665 | 0.7129 |
| 14 | 1.6665 | 1.6265 | 1.6665 | 0.7129 |

∴ 1625 is the root of equation.

2) $f(x) = 3x^3 + 5x^2 + 14x - 16$

soln:

Given equation $f(x) = 3x^2 + 5x^2 + 14x - 16$

$f(1) = 3(1)^3 + 5(1)^2 + 14(1) - 16 \Rightarrow 0$

$f(2) = 3(2)^3 + 5(2)^2 + 14(2) - 16 \Rightarrow 8$ (+ve)

$f(3) = 3(3)^3 + 5(3)^3 + 14(3) - 16 \Rightarrow -10$ (-ve)

| n | a | b. | $x_n = \frac{a+b}{2}$ | $f(x_n)$ |
|---|---|---|---|---|
| 1 | 2 | 3 | 2.5 | 3.375 |
| 2 | 3 | 2.5 | 2.75 | -2.0781 |
| 3 | 2.5 | 2.75 | 2.625 | 0.9394 |
| 4 | 2.75 | 2.625 | 2.6875 | -0.4943 |
| 5 | 2.625 | 2.6875 | 2.6562 | 0.2421 |
| 6 | 2.6875 | 2.6562 | 2.6718 | -0.1202 |
| 7 | 2.6718 | 2.6718 | 2.664 | 0.620 |
| 8 | 2.6718 | 2.664 | 2.6679 | -0.0288 |
| 9 | 2.664 | 2.6679 | 2.6659 | 0.178 |
| 10 | 2.6679 | 2.6659 | 2.6669 | -0.005 |
| 11 | 2.6659 | 2.6669 | 2.6664 | -0.006 |
| 12 | 2.6669 | 2.6664 | 2.6666 | -0.001 |

$\therefore$ 2.666 is the real root of the equation

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3) $f(x) = 3x - 2\sin x - 1$

Soln:

Given equation $= 3x - 2\sin x - 1$

$f(-1) = 3(-1) - 2\sin(-1) - 1$

$= -3.96$ (-ve)

$f(0) = 3(0) - 2\sin(0) - 1$

$= -1$ (-ve)

$f(1) = 3(1) - 2\sin(1) - 1$

$= 1.96$ (+ve)

| $n$ | $a$ | $b$ | $x_n = \frac{a+b}{2}$ | $f(x_n)$ |
|---|---|---|---|---|
| 1 | 0 | 1 | 0.5 | 0.4825 |
| 2 | 1 | 0.5 | 0.75 | 1.2238 |
| 3 | 0.5 | 0.75 | 0.625 | 0.8531 |
| 4 | 0.75 | 0.625 | 0.6875 | 1.0385 |
| 5 | 5.625 | 0.6875 | 0.6562 | 0.9456 |
| 6 | 0.6875 | 0.6562 | 0.6718 | 0.9919 |
| 7 | 0.6562 | 0.6718 | 0.664 | 0.9688 |
| 8 | 0.6718 | 0.664 | 0.6679 | 0.9803 |
| 9 | 0.664 | 0.6679 | 0.6659 | 0.9744 |
| 10 | 0.6679 | 0.6659 | 0.6669 | 0.9774 |
| 11 | 0.6659 | 0.6669 | 0.6664 | 0.9759 |
| 12 | 0.6669 | 0.6664 | 0.6666 | 0.9765 |

$\therefore$ 0.666 is the real root of the equation

## Iteration Method:

1) $f(x) = x^3 + x^2 - 1$

Soln:

$f(x) = x^3 + x^2 - 1$

$f(0) = (0)^3 + (0)^2 - 1 = -1$ (-ve)

$f(1) = (1)^3 + (1)^2 - 1 = 1$ (+ve)

$\frac{0+1}{2} = \frac{1}{2} = 0.5$

$x^3 + x^2 - 1 = 0$

$x^2(x+1) - 1 = 0$

$x^2 = \frac{1}{x+1} \Rightarrow x = \frac{1}{\sqrt{x+1}}$

$x_1 = \phi(x_0) = \frac{1}{\sqrt{0.5+1}} = 0.81649$

$x_2 = \phi(x_1) = \frac{1}{\sqrt{0.81649+1}} = 0.74196$

$x_3 = \phi(x_2) = \frac{1}{\sqrt{0.74196+1}} = 0.75767$

$x_4 = \phi(x_3) = \frac{1}{\sqrt{0.75767+1}} = 0.75427$

$x_5 = \phi(x_4) = \frac{1}{\sqrt{0.75427+1}} = 0.75500$

$x_6 = \phi(x_5) = \frac{1}{\sqrt{0.75500+1}} = 0.75485$

$x_7 = \phi(x_6) = \frac{1}{\sqrt{0.75485+1}} = 0.75488$

$\therefore$ 0.75488 is the real root of the equation.

2) $f(x) = \cos x - 3x + 1$

Soln:

Given $f(x) = \cos x - 3x + 1$

$f(0) = \cos(0) - 3(0) + 1 = 2 \ (+ve)$

$f(1) = \cos(1) - 3(1) + 1 = -1.45 \ (-ve)$

$\Rightarrow \frac{0+1}{2} = \frac{1}{2} = 0.5$

$\phi(x) = \frac{1}{3}(1 + \cos x)$

$x_1 = \phi(x_0) = \frac{1}{3}\left[1 + \cos(0.5)\right] = 0.625$

$x_2 = \phi(x_1) = \frac{1}{3}\left[1 + \cos(0.625)\right] = 0.6036$

$x_3 = \phi(x_2) = \frac{1}{3}\left[1 + \cos(0.6036)\right] = 0.6077$

$x_4 = \phi(x_3) = \frac{1}{3}\left[1 + \cos(0.6077)\right] = 0.6069$

$x_5 = \phi(x_4) = \frac{1}{3}\left[1 + \cos(0.6069)\right] = 0.6071$

$x_6 = \phi(x_5) = \frac{1}{3}\left[1 + \cos(0.6071)\right] = 0.6071$

$\therefore$ 0.6071 is the real root of the equation.

# Newton Raphson Method



1) $f(x) = \cos x - x^3$

soln:

$f(x) = \cos x - x^3$

$f(0) = 1 = +ve$

$f(1) = -0.1 = -ve$

$\therefore x_0 = \frac{0+1}{2} = 0.5$

Formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 - \frac{0.8749}{(-0.7581)}$$

$$= 0.5 + 1.653 = 1.1653$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.1653 - \frac{f(1.1653)}{f'(1.1653)} = 1.1653 - \frac{(-0.5825)}{(-4.0941)}$$

$$= 1.1653 - 0.1422 = 1.0231$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.0231 - \frac{f(1.0231)}{f'(1.0231)} = 1.0231 - \frac{(-0.0710)}{(3.1580)}$$

$$= 1.0231 - 0.0224 = 1.0007$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.0007 - \frac{f(1.0007)}{f'(1.0007)} = 1.0007 - \frac{(-0.0022)}{(-3.0216)}$$

$$= 1.0007 - 0.00072 = 0.9999$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.9999 - \frac{f(0.9999)}{f'(0.9999)} = 0.9999 - \frac{-0.00009}{(-3.0168)}$$

$$= 0.9999 - 0.00002$$

$$= 0.9999$$

$\therefore$ 0.9999 is the real root of the equation

2) $f(x) = x^3 - 3x - 5$ 

Soln:

$f(x) = x^3 - 3x - 5$ ; $f'(x) = 3x^2 - 3$

$f(0) = -5 = -ve$

$f(1) = -7 = -ve$

$f(2) = -2 = -ve$

$f(3) = 2 = +ve$

$\therefore x_0 = \frac{3+2}{2} = 5/2 = 2.5$

Formula $\Rightarrow$ $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.5 - \frac{f(2.5)}{f'(2.5)} = 2.5 - \frac{3.125}{5.75}$

$= 2.5 - 0.1984 = 2.3016$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_0)} = 2.3016 - \frac{f(2.3016)}{f'(2.3016)} = 2.3016 - \frac{0.2876}{12.8920}$

$= 2.3016 - 0.0223 = 2.2793$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.2793 - \frac{f(2.2793)}{f'(2.2793)} = 2.2793 - \frac{0.0035}{6.5856}$

$= 2.2993 - 0.0005 = 2.2788$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.2788 - \frac{f(2.2788)}{f'(2.2788)} = 2.2788 + 0.00021$

$= 2.2790$

$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 2.2790 - \frac{f(2.2790)}{f'(2.2790)} = 2.2790 - \frac{(-0.00023)}{12.5815}$

$= 2.2790 + 0.00008$

$= 2.2790$

$\therefore$ 2.2790 is the real root of the equation

3) $f(x) = 2x - 6 - \log_{10} x$ 

Soln:

$f(x) = 2x - 6 \log_{10} x$ ; $f'(x) = 2 - \frac{1}{x} = \frac{2x-1}{x}$

$f(1) = -4 = -ve$

$f(2) = -2.3 = -ve$

$f(3) = -0.4771 = -ve$

$f(4) = 1.3979 = +ve$

$\therefore x_0 = \frac{3+4}{2} = 7/2 = 3.5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3.5 - \frac{f(3.5)}{f'(3.5)} = 3.5 - \frac{6.4559}{1.7142}$

$= 3.5 - 0.2659 = 3.2341$

$x_2 = x_1 = \frac{f(x_1)}{f(x_1)} = 3.2341 - \frac{f(3.2341)}{f'(3.2341)} = 3.2341 - \frac{(-0.0415)}{1.6907}$

$= 3.2341 + 0.0245 = 3.2586$

$x_3 = x_2 = \frac{f(x_2)}{f'(x_2)} = 3.2586 - \frac{f(3.2586)}{f'(3.2586)} = 3.2586 - \frac{0.0041}{1.6931}$

$= 3.2586 - 0.00242 = 3.2561$

$x_4 = x_3 = \frac{f(x_3)}{f'(x_3)} = 3.2561 - \frac{f(3.2561)}{f'(3.2561)} = 3.2561 - \frac{(-0.00049)}{1.6928}$

$= 3.2561 + 0.00028 = 3.2563$

$x_5 = 3.2563 - \frac{f(3.2563)}{f'(3.2563)} = 3.2563 - \frac{(-0.00012)}{1.6929}$

$= 3.2563 + 0.00007 = 3.2563$

$\therefore$ 3.2563 is the real root of the equation

## Newton's Forword Interpolation Method

1)

| year (x) | 1931 | 1941 | 1951 | 1961 | 1971 | 1981 |
|---|---|---|---|---|---|---|
| sale in (y) thousands | 12 | 15 | 20 | 27 | 39 | 59 |

soln:

| x | y | $\Delta$ | $\Delta^2$ | $\Delta^3$ | $\Delta^4$ | $\Delta^5$ |
|---|---|---|---|---|---|---|
| 1931 | 12 | | | | | |
| | | 3 | | | | |
| 1941 | 15 | | 2 | | | |
| | | 5 | | 0 | | |
| 1951 | 20 | | 2 | | 3 | |
| | | 7 | | 3 | | -10 |
| 1961 | 27 | | 5 | | -7 | |
| | | 12 | | -4 | | |
| 1971 | 39 | | 1 | | | |
| | | 13 | | | | |
| 1981 | 59 | | | | | |

To Find : 1966 $\therefore h = 10$

$x = x_0 + ph$

$1966 = 1931 + p(10)$

$1966 - 1931 = 10p$

$10p = 35$

$p = 3.5$

$$y(1966) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \dots$$

$$= 12 + (3.5)(3) + \frac{(3.5)(3.5-1)}{2}(2) + \frac{(3.5)(3.5-1)(3.5-2)}{6}(0) + \frac{(3.5)(3.5-1)(3.5-2)(3.5-3)}{24}(3) + \frac{(3.5)(3.5-1)(3.5-2)(3.5-3)(3.5-4)}{120}(-10)$$

$$= 12 + 10.5 + 8.75 + 0 + \frac{6.5625}{8} + \frac{(3.28125)}{12}$$

$$= 31.25 + \frac{6.5625}{8} + \frac{3.2815}{12}$$

4 | 8, 12
2 | 2, 3
3 | 1, 3
1, 1
= 24

$$= \frac{750 + 19.6875 + 6.5625}{24} \Rightarrow \frac{750 + 13.125}{24} = \frac{763.125}{24}$$

$$= \frac{776.25}{24} = 32.84375$$

The year of Sale 1966 is 32.34375

## Newtons BackWord Interpolation Method (11)

| $x$ | 0 | 10 | 20 | 30 | 40 |
|---|---|---|---|---|---|
| $y$ | 7 | 18 | 32 | 48 | 85 |

To Find = 35

Given

| $x$ | $y$ | $\Delta$ | $\Delta^2$ | $\Delta^3$ | $\Delta^4$ |
|---|---|---|---|---|---|
| 0 | 7 | | | | |
| | | 11 | | | |
| 10 | 18 | | 3 | | |
| | | 14 | | -1 | |
| 20 | 32 | | 2 | | 20 |
| | | 16 | | 19 | |
| 30 | 48 | | 21 | | |
| 40 | 85 | 37 | | | |

$$x = x_0 + ph \Rightarrow 35 = 0 + 10P \Rightarrow p = \frac{35}{10} = 3.5$$

$$f(x) = y_0 - p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 - \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$\Rightarrow 7 - 3.5(11) + 3.5\frac{(3.5-1)}{2}(3) - 3.5\frac{(3.5-1)(3.5-2)}{6}(-1) + \frac{3.5(3.5-1)(3.5-2)(3.5-3)}{24}(20)$$

$$= 38.5 + \frac{26.25}{2} + \frac{13.125}{6} + \frac{32.8125}{6}$$

$$= 3.15 + \frac{78.75 + 13.125 + 32.8125}{6}$$

$$= \frac{18.9 + 78.75 + 13.125 + 32.8125}{6}$$

$$= \frac{105.7875}{6}$$

$$= 17.63125$$

2 | 6, 6, 2
3 | 3, 3, 1
1, 1, 1

$2 \times 3 = 6$

The Value of $x = 35$ is 17.63125

# Lagrange Interpolation

1) Find the value of $y$ at $x=8$, given $y(0)=18$, $y(1)=42$, $y(7)=57$ and $y(9)=90$

Given

| $x$ | 0 | 1 | 7 | 9 |
|---|---|---|---|---|
| $y$ | 18 | 42 | 57 | 90 |

To Find

$y$ at $x=8 \Rightarrow y(8)=?$

$$y=f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1+$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$\Rightarrow \frac{(x-1)(x-7)(x-9)}{(0-1)(0-7)(0-9)}(18)+\frac{(x-0)(x-7)(x-9)}{(1-0)(1-7)(1-9)}(42)+$$

$$\frac{(x-0)(x-1)(x-9)}{(7-0)(7-1)(7-9)}(57)+\frac{(x-0)(x-1)(x-7)}{(9-0)(9-1)(9-7)}(90)$$

$$\Rightarrow \frac{(x-1)(x-7)(x-9)}{-63}\overset{2}{\cancel{(18)}}+\frac{x(x-7)(x-9)}{\cancel{48}\ 8}\overset{7}{\cancel{(42)}}+$$

$$\frac{x(x-1)(x-9)}{-\cancel{84}\ 28}\overset{19}{\cancel{(57)}}+\frac{x(x-1)(x-7)}{\cancel{144}\ \cancel{24}\ 8}\overset{5}{\cancel{(90)}}$$

put $x=8$

$$=-\frac{2(8-1)(8-7)(8-9)}{7}+7\frac{8(8-7)(8-9)}{8}-19\frac{8(8-1)(8-9)}{28}+5\frac{8(8-1)(8-7)}{8}$$

$$=\frac{-2\times-7}{7}+\frac{7\times-8}{8}-\frac{19\times-56}{28}+\frac{5\times 56}{8}$$

$$=2-7+38+35$$

$$=-5+38+35$$

$y(8)=68$