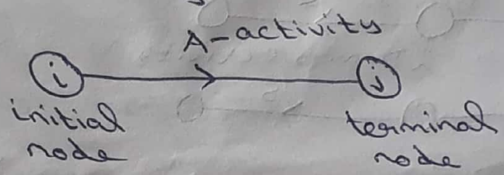
Unit - V

Networks

Basic Terminologies:

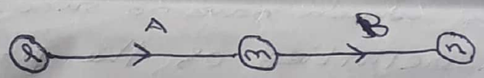
(i) **Activity** is a task or an item of work to be done in a project. An activity is represented by an arrow with a node at the beginning and a node at the end. Nodes are denoted by circles. The direction indicates the progress of the activity.

Eg:

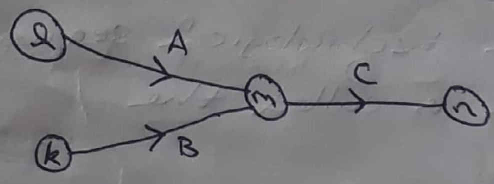


The diagram in which arrow represents an activity is called **arrow diagram**. The initial and terminal nodes of activities are also called tail and head events.

(ii)



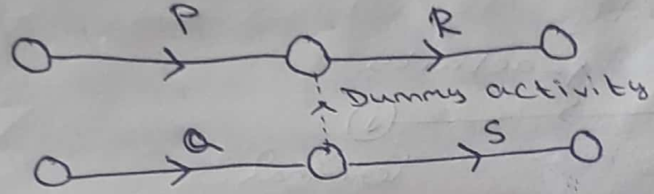
A is called the **immediate predecessor** of B and B is called the **immediate successor** of A. If C can start only after completing activities A and B then it is diagrammatically represented as follows:



(iii) If the project contains two or more activities which have some of their immediate predecessors in common then there is a need for introducing what is called dummy activity.

Dummy activity is an imaginary activity which does not consume any resource and which serves the purpose of indicating the predecessor or successor relationship clearly in any activity on arrow diagram.

Eg: Let P, Q be the predecessors of R and Q be the only predecessors of S



(iv) Activities which have no predecessors are called start activities of the project. All the start activities can be made to have the same initial node.

Activities which have no successors are called terminal activities of the project. These can be made to have the same terminal node of the project.

(v) A project consists of a number of activities to be performed in some technological sequence. The diagram denoting all the activities of a project by arrows taking into account the technological sequence of the activities is called the project network.

Rules For Constructing A Project Network:

1) There must be no loops
2) Only one activity should connect any two nodes.
3) No dangling should appear in a project network (ie) no nodes of any activity except the terminal node of the project should be left without any activity emanating from it. Such a node can be joined to the terminal node of the project to avoid.

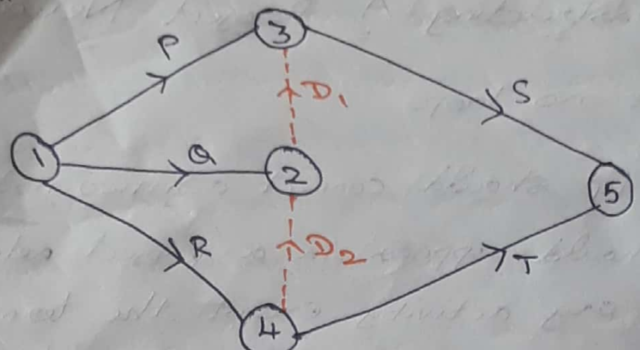
Ford And Fulkerson's Rule.

1) Number the start node which has no predecessor activity, as 1
2) Delete all the activities emanating from this node 1.
3) Number all the resulting start nodes without any predecessor as 2, 3, ...
4) Delete all the activities originating from the start nodes 2, 3, ... in step 3.
5) Number all the resulting new start nodes without any predecessor next to the last number used in step 3.
6) Repeat the process until the terminal node without any successor activity is reached and number this terminal node suitably.

Problems:

1) If there are five activities P, Q, R, S and T such that P, Q, R have no immediate predecessors but S and T have immediate predecessors P, Q and Q, R respectively. Represent this situation by a network.

Sol:



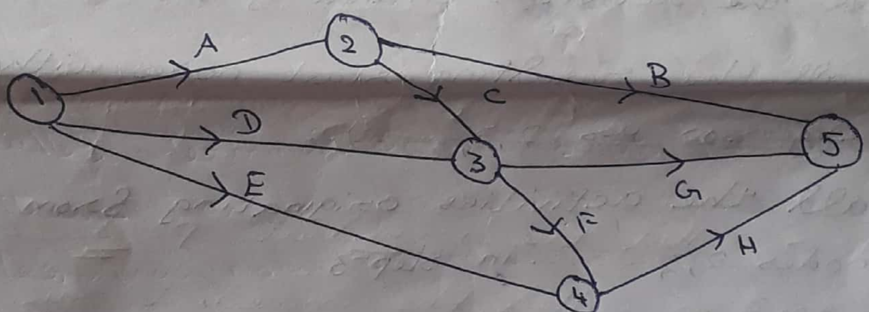
D_1 and D_2 are dummy activities.

2) Construct the network for the project whose activities and their relationships are as given below:

Activities : A, D, E can start simultaneously.

Activities : B, C > A ; G, F > D, C ; H > E, F

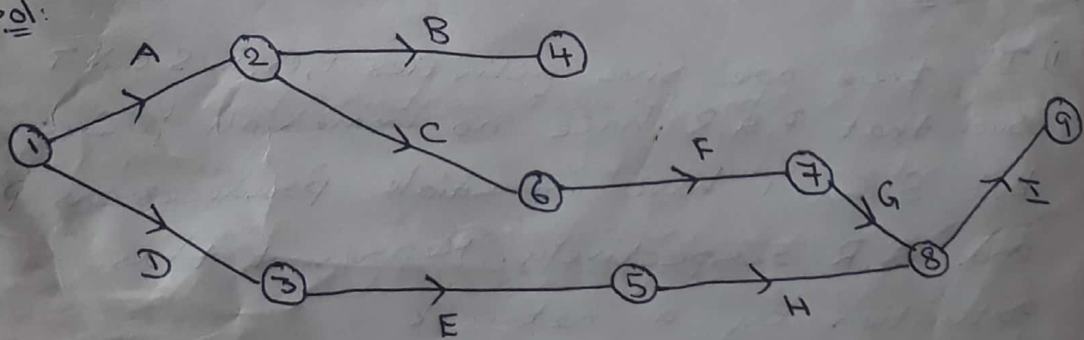
Sol:



3) Draw the network for the project whose activities and their precedence relationships are as given below:

Activities:	A	B	C	D	E	F	G	H	I
Immediate predecessor:	—	A	A	—	D	B,C,E	F	E	G,H

Sol:



Network Computations:

Critical path:

Path, connecting the first initial node to the very last terminal node, of longest duration in any project network is called the critical path.

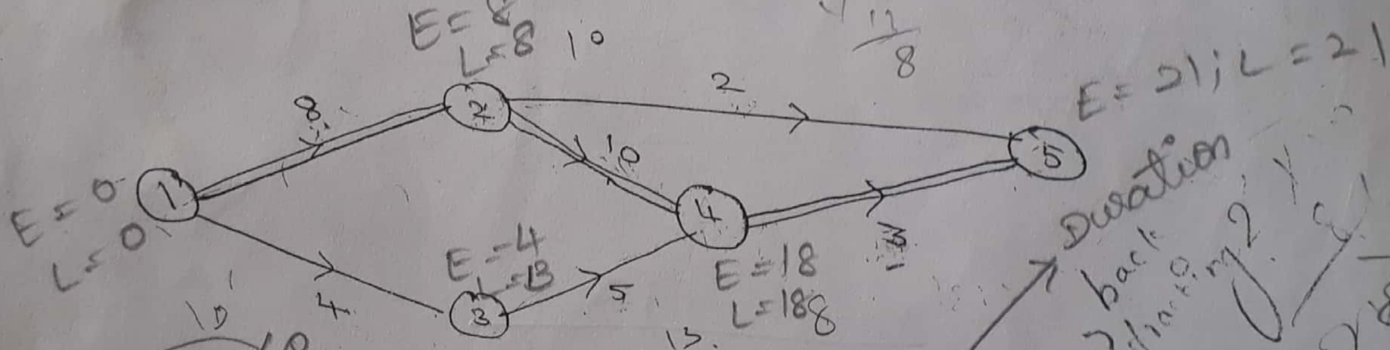
All the activities in any critical path are called Critical activities.

Compute the earliest start, earliest finish, latest start and latest finish of each activity of the project given below.

Activity :	1-2	1-3	2-4	2-5	3-4	4-5
Duration (in days):	8	4	10	2	5	3

Maximum

Sol.



Activity	Duration (days) t_{ij}	Earliest Start ES	Earliest Finish (EF) $EF=ES+t_{ij}$	Latest Start (LS) $LF-t_{ij}$ I Find	Latest Finish (LF) LF I Find
1-2	8	0	8	0	8
1-3	4	0	4	9	13
2-4	10	8	18	8	18
2-5	2	8	10	19	21
3-4	5	4	9	13	18
4-5	3	18	21	18	21

Critical path: 1 – 2 – 4 – 5

L for

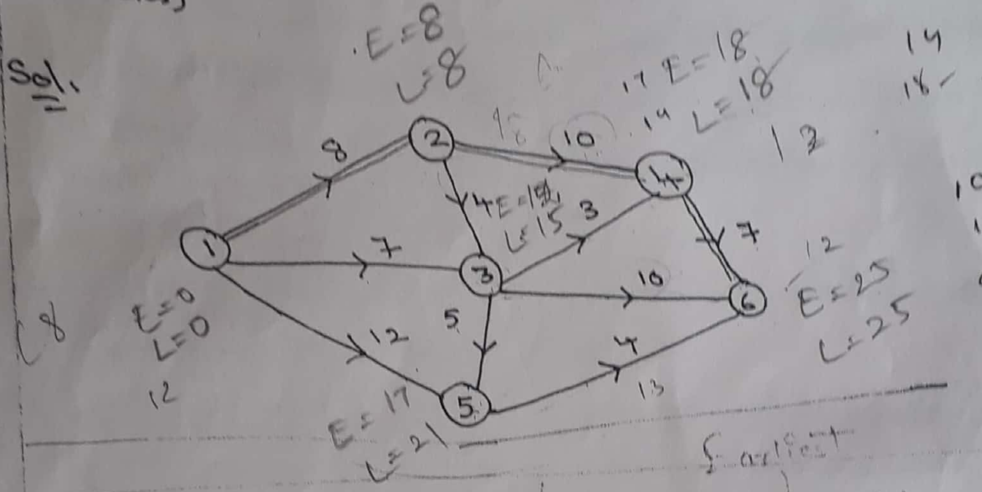
$EF = ES + t_{ij}$ LS =

2) Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the critical path

Activity :	1-2	1-3	1-5	2-3	2-4	3-4
Duration (in weeks) :	8	7	12	4	10	3

Activity :	3-5	3-6	4-6	5-6
Duration (in weeks) :	5	10	7	4

Sol.



Activity	Duration	Earliest Start	Earliest Finish	Latest Start (LF - ti)	Latest Finish
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	10	15	18
3-5	5	12	12	16	21
3-6	10	12	17	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

critical path = 1 - 2 - 4 - 6 = 25

Total float:

Total float of an activity is defined as the difference between the latest finish and the earliest finish of the activity or the difference between the latest start and the earliest start of the activity.

Total float of an activity i-j $= (LF)_{ij} - (EF)_{ij}$ (OR)
$(LS)_{ij} - (ES)_{ij}$

Leatest start . Earlest Start

Free float:

Free float of an activity is that portion of the total float which can be used for rescheduling that activity without affecting the succeeding activity.

Free float of an activity i-j = Total float of i-j − (L-E) of the event j

where L = Latest occurrence
E = Earliest occurrence

Note:

Free float ≤ Total float

Independent float:

I.F of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity.

I.F of an activity i-j = Free float of i-j − (L-E) of event i (Activity)

Note:

Independent float ≤ free float

$\Rightarrow$ I.F ≤ F.F ≤ T.F

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Interfering Float [or] Interference float:

Interfering float of an activity i-j is nothing but the slack of the head event j

Interfering Float of i-j = Total float of i-j − Free float of i-j

Calculate the total float, free float & independent float for the project whose activities are given below:

Activity:	1-2	1-3	1-5	2-3	2-4
Duration (in weeks):	8	7	12	4	10

Activity:	3-4	3-5	3-6	4-6	5-6
Duration:	3	5	10	7	4

Sol:

Activity	Duration (in weeks)	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Floats TF	FF	IF
1-2	8	0	8	0	8	0	0	0
1-3	7	0	7	8	15	8	5	5
1-5	12	0	12	9	21	9	5	5
2-3	4	8	12	11	15	3	0	0
2-4	10	8	18	8	18	0	0	0
3-4	3	12	15	15	18	3	3	0
3-5	5	12	17	16	21	4	0	-3
3-6	10	12	22	15	25	3	3	0
4-6	7	18	25	18	25	0	0	0
5-6	4	17	21	21	25	4	4	+0

TF = Total float, FF = Free float, IF = Independent float

T.F = E.S − L.S

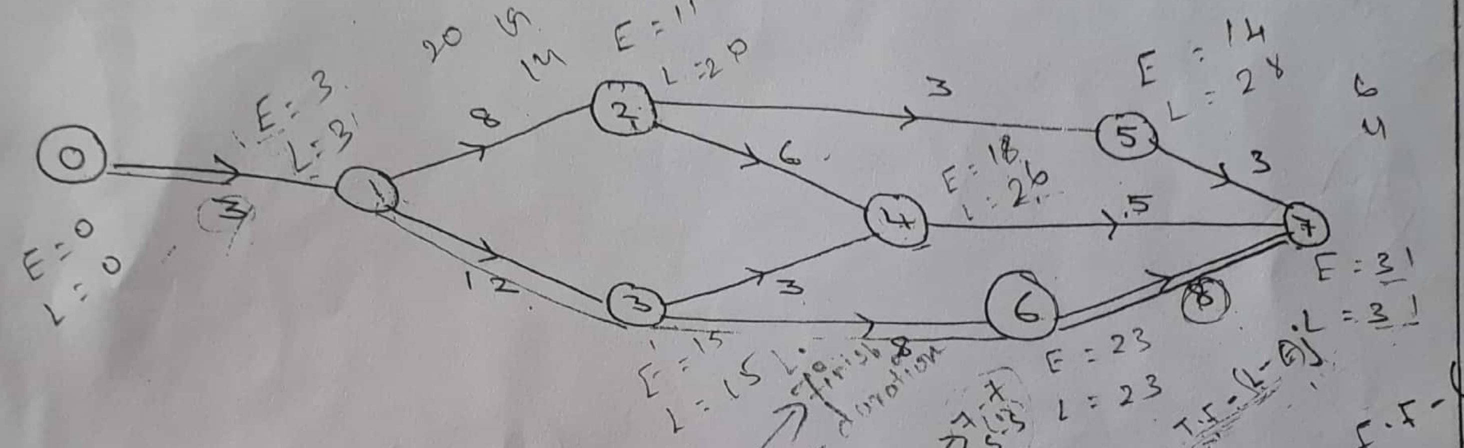
F.F = Total float − (L−E of j)

I.F = Free float − (L−E of i)

Construct the network for the project whose activities are given below and compute the total, free and independent float of each activity and hence determine the critical path and the project duration.

Activity :	0 – 1	1 – 2	1 – 3	2 – 4	2 – 5
Duration (in weeks) :	3	8	12	6	3
Activity :	3 – 4	3 – 6	4 – 7	5 – 7	6 – 7
Duration :	3	8	5	3	8

Sol:



Activity i j	Duration	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Floats TF	Floats FF	Floats IF
0 – 1	3	0	3	0	3	0	0	0
1 – 2	8	3	11	12	20	9	0	0
1 – 3	12	3	15	3	15	0	0	0
2 – 4	6	11	17	20	26	9	1	-8
2 – 5	3	11	14	25	28	14	0	-9
3 – 4	3	15	18	23	26	8	0	0
3 – 6	8	15	23	15	23	0	0	0
4 – 7	5	18	23	26	31	8	8	0
5 – 7	3	14	17	28	31	14	14	0
6 – 7	8	23	31	23	31	0	0	0

Programme Evaluation Review Technique (PERT):

PERT calculations depend upon the following three time estimates.

Optimistic (least) time estimate: (t_o or a)

It is the duration of any activity when everything goes on very well during the project.

(ie) labourers are available and come in time, machines are working properly, money is available wherever needed, there is no scarcity of raw material needed etc.

Pessimistic (greatest) time estimate: (t_p or b)

It is the duration of any activity when almost every thing goes against our will and a lot of difficulties is faced while doing a project.

Most likely time estimate: (t_m or m)

It is the duration of any activity when sometimes things going on very well, sometimes things go on very bad while doing the project.

Assumptions made in PERT Calculations:

(i) The activity durations are independent. (ie) the time required to complete any activity will have no bearing on the completion times of any other activity of the project.

(ii) The activity follows β-distribution

$$\text{Mean } t_e = \frac{1}{3}\left[2t_m + \frac{1}{2}(t_o - t_p)\right] \text{ and}$$

$$\text{Standard deviation } \sigma_t = \frac{t_p - t_o}{6}$$

Basic Differences Between PERT And CPM:

PERT:

1) PERT was developed in a brand new R and D project it had to consider and deal with the uncertainties associated with such projects. Thus the project duration is regarded as a random variable and therefore probabilities are calculated so as to characterise it.

2) Emphasis is given to important stages of completion of task rather than the activities required to be performed to reach a particular event or task in the analysis of network. (ie) PERT network is essentially an event-oriented network.

3) PERT is usually used for projects in which time estimates are uncertain.
Eg: R&D activities which are usually non-repetitive

4) PERT helps in identifying critical areas in a project so that suitable necessary adjustments may be made to meet the scheduled completion date of the project.

CPM

1) CPM was developed for conventional projects like construction project which consists of well known routine tasks whose resource requirement and duration were known with certainity.

2) CPM is suited to establish a trade off for optimum balancing b/w schedule time & cost of the project.

3) CPM is used for projects involving well known activities of repetitive in nature.

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II[nd] Topic

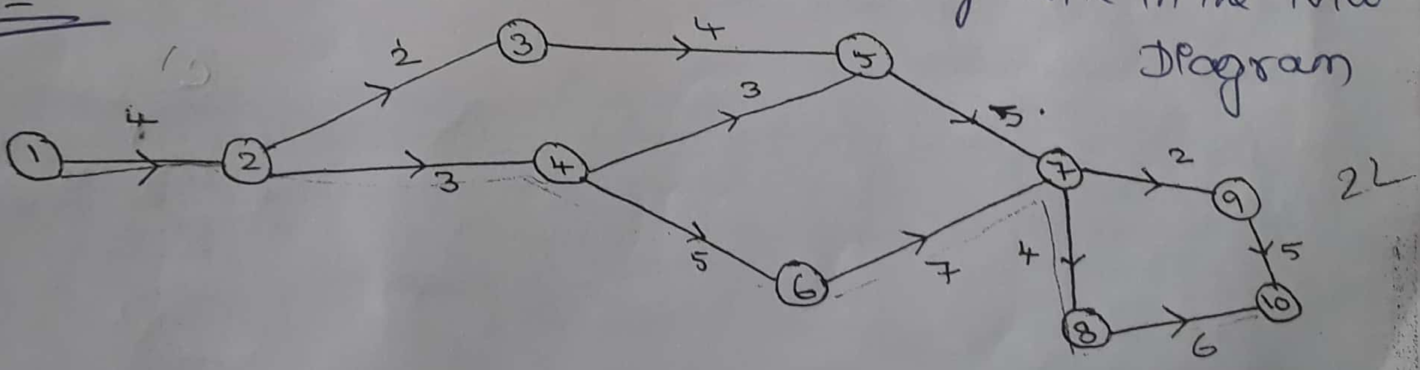
1) Construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute

a) Expected duration of each activity
b) Expected variance of each activity
c) Expected variance of the project length

Activity	t_o (a)	t_m (or) M	t_p (b)
1 – 2	3	4	5
2 – 3	1	2	3
2 – 4	2	3	4
3 – 5	3	4	5
4 – 5	1	3	5
4 – 6	3	5	7
5 – 7	4	5	6
6 – 7	6	7	8
7 – 8	2	4	6
7 – 9	1	2	3
8 – 10	4	6	8
9 – 10	3	5	7

↓ only write in the N/w Diagram

Sol:



Maximum path value is the project length.

Activity	t_o	t_m	t_p	Expected duration $t_e = \frac{t_o + 4t_m + t_p}{6}$	Expected Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	3	4	5	4	1/9 ✓
2-3	1	2	3	2	1/9
2-4	2	3	4	3	1/9 ✓
3-5	3	4	5	4	1/9
4-5	1	3	5	3	4/9
4-6	3	5	7	5	4/9 ✓
5-7	4	5	6	5	1/9
6-7	6	7	8	7	1/9 ✓
7-8	2	4	6	4	4/9 ✓
7-9	1	2	3	2	1/9
8-10	4	6	8	6	4/9 ✓
9-10	3	5	7	5	4/9

Critical path: 1-2-4-6-7-8-10 = 29 → project length.

Expected Variance of the project length = Sum of the expected variances of all the critical activities

$= \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9}$

$= \frac{15}{9} = \frac{5}{3}$

$= 1.67$

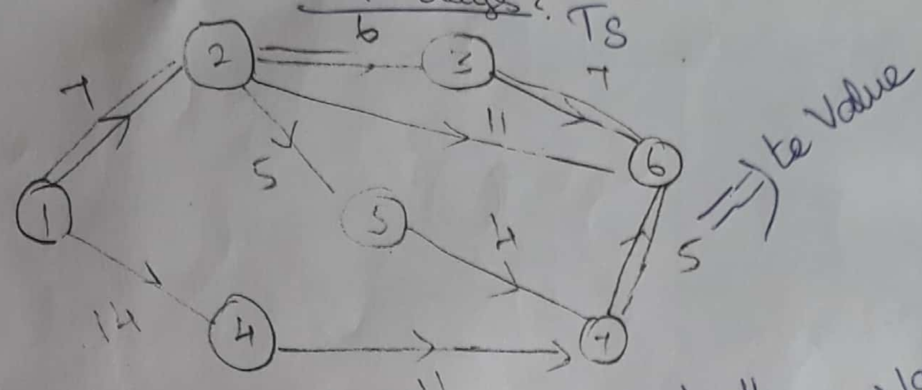
A project consists of the following activities and time estimates:

Activity	Least time (days) t_o	Greatest time (days) t_p	Most likely time (days) t_m
1-2	3	15	6
2-3	2	14	5
1-4	6	30	12
2-5	2	8	5
2-6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5

a) Draw the network
b) What is the probability that the project will be completed in 27 days? T_S

Sol:

a)



b) 1 - 2 - 3 - 6 - 7 = 25 → project length. Variance

Activity	t_o	t_p	t_m	Expected duration $t_e = (t_o + 4t_m + t_p)/6$	Expected variance $\sigma^2 = ((t_p - t_o)/6)^2$
1 – 2	3	15	6	7	4
2 – 3	2	14	5	6	4
1 – 4	6	30	12	14	16
2 – 5	2	8	5	5	1
2 – 6	5	17	11	11	4
3 – 6	3	15	6	7	4
4 – 7	3	27	9	11	16
5 – 7	1	7	4	4	1
6 – 7	2	8	5	5	1

critical activity

Critical path: ~~1 – 2 – 3 – 6 – 7~~ 1 – 4 – 7

Expected Project duration = 25 days

Expected variance of the project length = Sum of the expected variances of all the critical activities

$\sigma_c^2 = 16 + 16 = 32$

$= \sqrt{\text{Variance}}$

σ_c Standard deviation $= \sqrt{32} = 4\sqrt{2} = 5.656$ $\quad \sigma = \sqrt{32}$

T_S ⇒ compilation date

$$Z = \frac{T_S - T_E}{\sigma_c} = \frac{27 - 25}{5.656} = 0.35$$

Probability $= P(T_S \le 27) = P(Z \le 0.35) = 0.6368$

$= 63.68\%$

Expe ⇒ project length

$$\frac{T_S - T_E}{\sigma_c}$$

Three time estimates (in months) of all activities of a project are as given below:

Activity	a	m	b
1-2	0.8	1.0	1.2
2-3	3.7	5.6	9.9
2-4	6.2	6.6	15.4
3-4	2.1	2.7	5.1
4-5	0.8	3.4	3.6
5-6	0.9	1.0	1.1

a) Find the expected duration and standard deviation of each activity

b) Construct the project network

c) Determine the critical path, expected project length and expected variance of the project length.

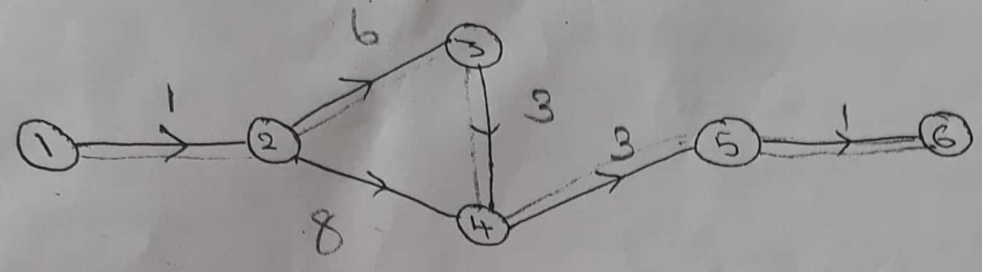
d) What is the probability that the project will be completed (i) two months later than expected. (ii) not more than 3 months earlier than expected (iii) What due date has about 90% chance of being met?

Sol:

Activity	t_o	t_m	t_p	t_e	σ^2	σ
1-2	0.8	1.0	1.2	1	0.004	0.063
2-3	3.7	5.6	9.9	6	1.06	1.032
2-4	6.2	6.6	15.4	8	2.35	1.533
3-4	2.1	2.7	5.1	3	0.25	0.5
4-5	0.8	3.4	3.6	3	0.217	0.465
5-6	0.9	1.0	1.1	1	0.001	0.031

$$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2 \qquad t_e = \frac{t_o + 4t_m + t_p}{6}$$

b)



c) Critical path : 1 – 2 – 3 – 4 – 5 – 6

Expected project length = 14 months

Expected variance = 0.004 + 1.06 + 0.25 + 0.217 + 0.004

$\sigma_c^2 = \sqrt{1.5374}$ = 1.5374

d) (i) $T_s = 16$, $T_E = 14$, $\sigma_c = 1.2399$

$Z = \frac{T_s - T_E}{\sigma_c} = \frac{16-14}{1.2399} = \frac{2}{1.2399} = 1.61$

$P(T_s \leq 16) = P(Z \leq 1.61) = 0.9463 = 94.63\%$

(ii) $T_s = 11$ $T_E = 14$, $\sigma_c = 1.2399$

$Z = \frac{11-14}{1.2399} = -2.42$

$P(T_s \leq 11) = 0.5 - 0.4922 = 0.0078 = 0.78\%$

(iii) Z value for 90% area in the table = 1.28 nearly

$1.28 = \frac{T_s - 14}{1.2399}$

$T_s = 14 + (1.28 \times 1.2399)$

= 15.59 months nearly.