1. Use the Binomial theorem find the 7th power of 11.

Solu:

$(11)^7 = (10+1)^7$.

$x = 10 \quad a = 1$.

$(x+a)^n = x^n + nc_1 x^{n-1} a + nc_2 x^{n-2} a^2 + \dots + a^n$

$= 10^7 + 7C_1 10^6 + 7C_2 10^5 + 7C_3 10^4$

$+ 7C_4 10^3 + 7C_5 10^2 \; 7C_6 10 + 7C_7$

$= 10000000 + 7000000 + 2100000$

$+ 350000 + 35000 + 2100$

$+ 70 + 1$.

$\boxed{= 19487171.}$

2. Write down the Middle term of

$$\left(\frac{y\sqrt{x}}{5} - \frac{5}{x\sqrt{y}}\right)^{12}.$$

Solu:

$$u_{r+1} = nc_r\, x^{n-r}\, a^r$$

$r=6.$

$$U_7 = 12C_6 \left(\frac{y\sqrt{x}}{5}\right)^{12-6}$$

$$= 12C_6 \frac{\cancel{y^6}^{3}\, \cancel{x^3}}{\cancel{5^6}} \frac{\cancel{5^6}}{3\, \cancel{x^6}\, \cancel{y^6}}$$

$$\boxed{U_7 = 12C_6 \left(\frac{y^3}{x^3}\right).}$$

3. Write down the Middle term $\left(2x - \frac{3}{x}\right)^{15}$

Soln:

Since there are 16 terms in the expansion the Middle is the 8^{th} 9^{th} Middle terms.

put $r=7.$

$$U_8 = 15C_7\,(2x)^{15-7}\left(\frac{-3}{x}\right)^7.$$

put $r=8$

$$u_9 = 15C_8\,(2x)^{15-8}\left(\frac{-3}{x}\right)^8$$

$$U_8 = 15C_7\,(2x)^8\left(-\frac{3}{x}\right)^8$$

$$= -15C_7\,2^8\,\cancel{x^8}\,\frac{3^7}{\cancel{x^7}}$$

$$\boxed{U_8 = -15C_7\,2^8 . 3^7 . x}$$

$$U_9 = 15C_8\,2^7 .\,\cancel{x^7}\,\frac{3^8}{\cancel{x^8}}$$

$$\boxed{U_9 = 15C_8\,2^7 . 3^8 . \frac{1}{x}.}$$

4. Find the coefficient of x^{32} in the expanssion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$

Soln :-

let x^{32} occur in the $(r+1)^{th}$ term in the expanssion

$$U_{r+1} = nC_r\,x^{n-r}a^r$$

$$U_{r+1} = 15C_r \cdot (x^4)^{15-r}\left(\frac{-1}{x^3}\right)^r.$$

$$= 15C_r \, x^{60-4r} \left(\frac{-1}{3r}\right)^r$$

$$= 15C_r \, (-1)^r \, . \, x^{60r - 4r - 3r}$$

$$U_{r+1} = 15C_r \, (-1)^r \, . \, x^{60-7r}.$$

$$60 - 7r = 32$$

$$-7r = 32 - 60$$

$$-7r = -28$$

$$\boxed{r = 4}$$

$$U_{4+1} = 15C_4 \, (-1)^4 \, . \, x^{60-7(4)}$$

$$U_5 = 15C_4 \, . \, x^{32}.$$

Coefficient of $\boxed{x^{32} = 15C_4.}$

5. Find the coefficient of x^7 in $(1-x-x^2+x^3)^6$

Soln:-

Given $(1-x-x^2+x^3)^6$

We can write it $(x^3-x^2-x+1)^6$.

$$x^3 - x^2 - x + 1$$

1	1	-1	-1	1
	0	1	0	-1
	1	0	-1	0

$$x^2 + 0x - 1 = 0$$
$$x^2 = 0 + 1$$
$$x^2 = 1.$$

$$x = 1 \Rightarrow (1-x)$$
$$\Rightarrow (1-x)^2$$

$$(1 - x - x^2 + x^3) = (1-x)(1-x^2)$$
$$(1 - x - x^2 + x^3)^6 = (1-x)^6 (1-x^2)^6.$$

Using Binomial theorem.

$$(1 - x - x^2 + x^3)^6 = (1 - 6C_1 x + 6C_2 x^2 - 6C_3 x^3 + 6C_4 x^4 - 6C_5 x^5 + 6x(1 - 6C_1 x^2 + 6C_2 x^4 - 6C_3 x^6 + 6C_4 x^8 - 6C_5 x^{10} + x^{12}).$$

The required coefficient of x^7

$$= \text{coeff. of } [(-6C_4 x) \times (-6C_3 x^6) + (-6C_3 x^3) \times (6C_2 x^4) + (-6C_5 x^5) \times (-6C_1 x^2)]$$

$$= 6 \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3} - \frac{6 \times 5 \times 4}{1 \times 2 \times 3} - \frac{6 \times 5}{1 \times 2} + \frac{6 \times 5 + 4 + 3 + 2}{1 \times 2 \times 3 \times 4 \times 5} \times 6$$

$$= 120 - 300 + 36$$

$$\boxed{= -144.}$$

6. If a_r be the coefficient of x^r in the expanssion of $(1+x)^n$. PT

$$\frac{a_1}{a_0} + 2\cdot\frac{a_2}{a_1} + 3\cdot\frac{a_3}{a_2} + \dots\dots + n\frac{a_n}{a_{n-1}} = \frac{n(n+1)}{2}.$$

Soln:

$$U_{r+1} = nC_r \cdot 1^{n-r} x^r$$

$$= nC_r \cdot x^r$$

given $a_r \to$ coeff. of x^r

$$\therefore a_r = nC_r$$

$$\therefore \frac{a_r}{a_{r-1}} = \frac{nC_r}{nC_{r-1}}$$

$$nC_r = \frac{n!}{r!\,(n-r)!}$$

$$\frac{1}{nC(r-1)} = \frac{(r-1)!\,(n-r+1)!}{n!}$$

$$\frac{a_r}{a_{r-1}} = \frac{n!}{r!\,(n-r)!} \quad \frac{(r-1)!\,(n-r+1)}{n!}$$

$$= \frac{n-r+1}{r}.$$

$$\frac{r a_r}{a_{r-1}} = n-r+1.$$

Put. $r = 1, 2, 3 \ldots\ldots$ we get.

$$1.\ \frac{a_1}{a_0} = n \quad .\ 2.\ \frac{a_2}{a_1} = n-1$$

$$3.\ \frac{a_3}{a_2} = n-2$$

$$n = \frac{a_n}{a_{n-1}} = 1.$$

Adding $\Rightarrow 1.\frac{a_1}{a_0} + 2\frac{a_2}{a_1} + 3\frac{a_3}{a_2} + \cdots\cdots$

$$\frac{n\,a_n}{a_{n-1}}$$

$$= n + (n-1) + (n-2) + \cdots + 1.$$

$$= \frac{n(n+1)}{2}$$

$\Rightarrow$

$$[1 + 2 + \cdots\cdots n].$$

$$\boxed{= \frac{n(n+1)}{2}.}$$

7. S.T $2^{2n} - 3n - 1$ is divisible by 9 for all positive integral values of n.

<u>soln</u>:

$$2^{2n} - 3n - 1 = (2^2)^n - 3n - 1.$$

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$= 4^n 3n - 1$

$= (3+1)^n - 3n - 1$

$= 3^n + nC_1 3^{n-1}$

$= 3^n + nC_1 3^{n-1} + nC_2 3^{n-2} + \dots$
$+ nC_{n-1} 3 + 1 \cdot 3n - 1$

$= 3^n + nC_1 3^{n-1} + nC_2 3^{n-2} + \dots\dots +$
$nC_{n-2} 3^2.$

Each term contions 3^2 is 9

$\therefore 2^{2n} - 3n - 1$ is exacty divisible by 9.

8. Using Binomial theorem find
$(1 \cdot 01)^5$

Soln:-

$= 1 + 5C_1 (0 \cdot 01) + 5C_2 (0 \cdot 01)^2 + 5C_3 (0 \cdot 01)^3 +$
$+ 5C_4 (0 \cdot 01)^4 + (0 \cdot 01)^5$

$= 1 + 5 (0 \cdot 01) + \frac{5 \times 4}{1 \times 2} (0 \cdot 0001) + \frac{5 \times 4 \times 3}{1 \times 2 \times 3} (0 \cdot 000001)$

$+ \frac{5 \times 4 \times 3 \cancel{\times 2}}{1 \times 2 \times 3 \times 4} (0 \cdot 00000001) + 0 \cdot 0000000001$

$= 1 + 0 \cdot 05 + 10 (0 \cdot 0001)$

$= 1 + 0 \cdot 05 + 0 \cdot 001$

$= 1 \cdot 051.$

9. $(9.999)^4$ to four place of decimals.

Soln:

$$(9.999)^4 = (9+0.999)^4$$

$$= 9^4 + 4C_1 (9)^3 (0.999) + 4C_2 (9)^2 (0.999)^2 + C_3 (9)(0.999)^3 + (0.999)^4.$$

$$= 6561 + (4 \times 9^3 \times 0.999) + \frac{4\times 3}{1\times 2}(9)^2 (0.999)^2 + \frac{4\times 3\times 2}{1\times 2\times 3}(9)(0.999)^3 + (0.999)^4$$

$$= 6561 + 2913.084 + 485.028486 + 35.892107964 + 0.996005996.$$

$$= 9996.0005996$$

$$= 9996.0006$$

10. Find the greatest term in $(x+a)^n$.....

$(3x+2)^{35}$.

$$(3x+2)^{35} = 2^{35}\left(1+\frac{x}{a}\right)^n$$

$$= 2^{35}\left(1+\frac{3x}{2}\right)^{35}$$

It will be sufficient to consider the expansion of $\left(1+\frac{3x}{2}\right)^{35}$

$$\frac{U_{r+1}}{U_r} = \frac{n-r+1}{r} \cdot \frac{3x}{2} \quad \text{Where } n=35,\ x=2$$

$$= \frac{35-r+1}{r} \cdot \frac{3\times 2}{2}$$

$$= \frac{108-3r}{r}$$

$\frac{U_{r+1}}{U_r} \geq 1$ according as $\frac{108-3r}{r} \geq 1$

" as $108-3r \geq r$

" as $4r \leq 108$

" as $r \leq 27$

$U_{r+1} > U_r$. but for $r = 27$ we got.

$U_{r+1} = U_r$

$U_{r+1} = U_r$

Thus the 27^{th}, 28^{th} terms are numerically equal and greater then any other term.

$$2^{36} \cdot 3^{8} \cdot 35C27$$

To find the 27^{th} or 28^{th} term

$$\boxed{\begin{matrix} n=35 & r=27 \\ x=3x & a=2 \end{matrix}}$$

$$U_{27+1} = 35C27\ (3x)^{35-r} \cdot 2^{27}$$

$$= 35C27\ 3^8 \cdot x^8 \cdot 2^{27}$$

$$= 35C27 \cdot 3^8 \cdot 2^8 \cdot 2^{27} = 2^{35} \cdot 3^8 \cdot 35C27$$

11. Find the greatest term in the expansion of $(2x-3)^9$ when $x=3$

$$(2x-3)^9 = (-3)^9 \left(1-\frac{2x}{3}\right)^9$$

$$\frac{U_{r+1}}{U_r} = \frac{n-r+1}{r} \cdot x = \frac{9-r+1}{r} \cdot \frac{2x}{3}$$

$$= \frac{9-r+1}{r} \cdot \frac{2\times\cancel{3}}{\cancel{3}} \text{ where } x=3$$

$$= \frac{10-r}{r} \cdot \times 2$$

$$\frac{U_{r+1}}{U_r} = \frac{20-2r}{r}$$

$\frac{U_{r+1}}{U_r} \geq 1$ according as $\frac{20-2r}{r} \geq 1$

is as $20-2r \geq r$

as $r \leq 6\,2/3$

The greatest term is therefore the 7th term and it value is (7th term $\Rightarrow r=6$)

$$U_{6+1} = 9C_6 (2x)^{9-6} (-3)^6$$

$$= 9C_6\, 2^3 . x^3 . (-3)^6$$

$$= 9C_6 . 2^3 . 3^3 . 3^6$$

$$\boxed{U_7 = 9C_6 . 3^9 . 2^3}$$

12. Find the coefficient of x^{11} in the expansion of $(x+\frac{2}{x^2})^{17}$.

Soln.

let x^{11} occur in the $(r+1)^{th}$ term in the expansion

$$U_{r+1} = 17C_r \, (x)^{17-r} \left(\frac{2}{x^2}\right)^r.$$

$$= 17C_r \, x^{17-r} \cdot 2^r \cdot x^{-2r}$$

$$U_{r+1} = 17C_r \cdot 2^r \cdot x^{17-3r}$$

$$17-3r = 11$$

$$3r = 17-11$$

$$3r = 6$$

$$\boxed{r = 2}$$

Hence U_{2+1} (i.e.) 3^{rd} term contains x^{11}

coefficient of x^{11} in the expansion

$$= 17C_2 \, (2)^2$$

$$= \frac{17 \times 16}{1 \times 2} \times 4$$

$$\boxed{= 544}$$

13. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$.

P.T $C_0 + 2C_1 + 3C_2 + \dots + (n+1) C_n = 2^n + n \cdot 2^{n-1}$.

Soln:

$C_0 + 2C_1 + 3C_2 + \dots + (n+1) C_n$

$= (C_0 + C_1 + C_2 + \dots + C_n) + (C_1 + 2C_2 + \dots + nC_n)$

Now, $C_0 + C_1 + C_2 + \dots + C_n = 2^n \rightarrow (1)$

and $C_1 + 2C_2 + 3C_3 + nC_n$

$$= n + 2\frac{n(n-1)}{2} + 3 \cdot \frac{n(n-1)(n-2)}{3!} + \dots + n \dots 1$$

$$= n\left[n + \frac{n-1}{1!} + \frac{(n-1)(n-2)}{2!} + \dots + 1\right]$$

$$= n[1+1]^{n-1}$$

$$= n \cdot 2^{n-1} \rightarrow (2)$$

Adding (1) & (2)

$(C_0 + C_1 + C_2 + \dots + C_n) + (C_1 + 2C_2 + 3C_3 + \dots nC_n)$

$$= 2^n + n \cdot 2^{n-1}$$

$$= 2^{n-1}(2+n).$$

$$\boxed{C_0 + 2C_1 + 3C_2 + \dots + (n+1) C_n = 2^{n-1}(n+2)}$$

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14. Expand $(1+3x)^{5/2}$ given $|x| < 1/3$.

Soln :-

$$(1+3x)^{5/2} = 1 + 5/2\,(3x) + \frac{(5/2)(5/2-1)}{2!}(3x)^2 + \frac{(5/2)(5/2-1)(5/2-2)}{3!}(3x)^3 + \frac{(5/2)(5/2-1)(5/2-2)(5/2-3)}{4!}(3x)^4 + \dots$$

$$= 1 + 5\left(\frac{3x}{2}\right) + \frac{(5/2)(3/2)}{2!}(3x)^2 + \frac{(5/2)(3/2)(1/2)}{3!}(3x)^3 + \frac{(5/2)(3/2)(1/2)(-1/2)}{4!}(3x)^4 + \dots$$

$$= 1 + 5\left(\frac{3x}{2}\right) + \frac{5\cdot 3}{2!}\left(\frac{3x}{2}\right)^2 + \frac{5\cdot 3 \cdot 1}{3!}\left(\frac{3x}{2}\right)^3 + \frac{5\cdot 3\cdot 1(-1)}{4!}\left(\frac{3x}{2}\right)^4 + \dots$$

The terms which follow are alternately positive and negative and the general term is

$$\frac{5\cdot 3\cdot 1(-1)(-3)(-5)\cdots(-2r+7)}{r!}\left(\frac{3x}{2}\right)^r$$

$$(-1)^{r-3}\,\frac{5\cdot 3\cdot 1\cdots 1\cdot 3\cdot 5\cdots(2r-7)}{r!}\left(\frac{3x}{2}\right)^r$$

$r > 3$

15. Sum the Series the infinity

$$\frac{1.4}{5.10} - \frac{1.4.7}{5.10.15} + \frac{1.4.7.10}{5.10.15.20} - \ldots\ldots$$

Soln:-

$$S = \frac{\frac{1}{3}.\frac{4}{3}}{1.2}\left(-\frac{3}{5}\right)^2 + \frac{\frac{1}{3}.\frac{4}{3}.\frac{7}{3}}{1.2.3}\left(\frac{-3}{5}\right)^3 + \frac{\frac{1}{3}.\frac{4}{3}.\frac{7}{3}.\frac{10}{3}}{1.2.3.4}\left(\frac{-3}{5}\right)^4 + \ldots$$

Take $n = \frac{1}{3}$ and $x = -\frac{3}{5}$

$$S = \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \frac{n(n+1)(n+2)(n+3)}{4!}x^4 \ldots$$

$$= 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \frac{n(n+1)(n+2)(n+3)}{4!}x^4 + \ldots \ldots - 1 - nx$$

$$= (1-x)^{n} - 1 - nx$$

$$= \left(1+\frac{3}{5}\right)^{-1/3} - 1 - (1/3)(-3/5)$$

$$= (1+3/5)^{-1/3} + 1/5 - 1$$

$$= (8/5)^{-1/3} - 4/5$$

$$= 1/(8/5)^{1/3} - 4/5$$

$$= 1/8^{1/2} - 4/5 \Rightarrow \frac{1}{8^{1/2}/5^{1/2}} - 4/5$$

$$\boxed{S = \frac{5^{1/3}}{2} - \frac{4}{5}}$$

16. Sum the series to infinity

$$\frac{15}{16}+\frac{15\cdot 21}{16\cdot 24}+\frac{15\cdot 21\cdot 27}{16\cdot 24\cdot 32}+\cdots$$

Soln:-

$$s=\frac{\frac{15}{6}}{2}\left(\frac{6}{8}\right)+\frac{\left(\frac{15}{6}\right)\left(\frac{21}{6}\right)}{2\cdot 3}\left(\frac{6}{8}\right)^2+\frac{\left(\frac{15}{6}\right)\left(\frac{21}{6}\right)\left(\frac{27}{6}\right)}{2\cdot 3\cdot 4}\left(\frac{6}{8}\right)^4$$

We introduce an additional factor..

$\frac{9}{6}$ in the numerator because the factors of

With 1.

$$\therefore \frac{9}{6}s=\frac{\frac{9}{6}\cdot\frac{15}{6}}{1\cdot 2}\left(\frac{6}{8}\right)+\frac{\frac{9}{6}\cdot\frac{15}{6}\cdot\frac{21}{6}}{1\cdot 2\cdot 3}\left(\frac{6}{8}\right)^2+\cdots$$

$$\frac{\frac{9}{6}\cdot\frac{15}{6}\cdot\frac{21}{6}\cdot\frac{27}{6}}{1\cdot 2\cdot 3\cdot 4}\left(\frac{6}{8}\right)^3+\cdots$$

Multiply by $\frac{6}{8}$ on both sides. We get.

$$s\cdot\frac{9}{6}\cdot\frac{6}{8}=\frac{\frac{9}{6}\cdot\frac{15}{6}}{1\cdot 2}\left(\frac{6}{8}\right)^2+\frac{\frac{9}{6}\cdot\frac{15}{6}\cdot\frac{21}{6}}{1\cdot 2\cdot 3}\left(\frac{6}{8}\right)^3+\cdots$$

$$n=\frac{9}{6} \text{ and } x=\frac{6}{8}$$

$$\frac{9}{8}s=\frac{n(n+1)}{2!}x^2+\frac{n(n+1)(n+2)}{3!}x^3+\cdots$$

$$=1+\frac{n}{1!}x+\frac{n(n+1)}{2}x^2+\frac{n(n+1)(n+2)}{3!}x^3+\cdots$$

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17. S.T the coeff. of x^n in the infinite series

$$1+\frac{b+ax}{1!}+\frac{(b+ax)^2}{2!}+\dots+\frac{(b+ax)^n}{n!}+\dots \text{ is } \frac{e^b a^n}{n!}$$

Soln:

$$\begin{aligned}\text{The given series} &= e^{b+ax}\\ &= e^b.e^{ax}\\ &= e^b.\left[1+\frac{ax}{1!}+\frac{(ax)^2}{2!}+\dots+\frac{(ax)^n}{n!}\right]\end{aligned}$$

$$\boxed{x^n = \frac{e^b.a^n}{n!}}$$

18. $\dfrac{1+2x-3x^2}{e^x}$.

Soln:

$$\frac{1+2x-3x^2}{e^x} = (1+2x-3x^2)\,e^{-x}$$

$$= (1+2x-3x^2)\left[1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots+(-1)^n\frac{x^n}{n!}+\dots\right]$$

$$= 1.\frac{(-1)^n}{n!}+2.\frac{(-1)^{n-1}}{(n-1)!}-3\frac{(-1)^{n-2}}{(n-2)!}$$

$$= \frac{(-1)^n}{n!}\left[1-2n-3n(n-1)\right]$$

$$= \frac{(-1)^n}{n!}(1-2n-3n^2+3n)$$

$$= \frac{(-1)^n}{n!}(1+n-3n^2)$$

$= (1-x)^{-n} - (1+nx)$

$= \left(1-\frac{6}{8}\right)^{-9/6} - \left(1+9/6 \cdot 6/8\right)$

$= \left(\frac{2}{8}\right)^{-9/6} - \left(1+ 9/8\right)$

$= \left(1/4\right)^{-3/2} - \left(17/8\right)$

$= \frac{1}{(1/4)^{3/2}} - 17/8$

$= \frac{1}{\frac{(1)^{3/2}}{(4)^{3/2}}} - 17/8$

$= \frac{1}{\frac{1}{[(4)^{1/2}]^3}} - \frac{17}{8}$

$= \frac{1}{\frac{1}{(2)^3}} - \frac{17}{8}$

$= \frac{1}{1/8} - \frac{17}{8}$

$= 8 - \frac{17}{8}$

$\frac{9}{8} 8 = \frac{47}{8}$

$$\boxed{8 = \frac{47}{9}}$$

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19. S.T. $\log\sqrt{12} = 1 + \left(\frac{1}{2}+\frac{1}{3}\right)\cdot\frac{1}{4} + \left(\frac{1}{4}\cdot\frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6}+\frac{1}{7}\right)\frac{1}{4^3} + \dots$

<u>Soln:</u>

R.H.S. can be written as,

$$\left(\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{4^2}+\frac{1}{6}\cdot\frac{1}{4^3}+\dots\right) + \left(1+\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{1}{4^2}+\frac{1}{7}\cdot\frac{1}{4^3}+\dots\right)$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^4 + \frac{1}{6}\cdot\left(\frac{1}{2}\right)^6 + 1 + \frac{1}{3}\left(\frac{1}{2}\right)^2 + \frac{1}{5}\left(\frac{1}{2}\right)^4 + \frac{1}{7}\left(\frac{1}{2}\right)^6 \dots$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}\right)^4 + \frac{1}{3}\left(\frac{1}{2}\right)^6 + \dots\right] + 2\left[\frac{1}{2} + \frac{1}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{5}\left(\frac{1}{2}\right)^5 + \frac{1}{7}\left(\frac{1}{2}\right)^7 + \dots\right]$$

$$= -\frac{1}{2}\log\left(1-\left(\frac{1}{2}\right)^2\right) + \log\frac{1+\frac{1}{2}}{1-\frac{1}{2}}$$

$$= -\frac{1}{2}\log\left(1-\frac{1}{4}\right) + \log\frac{3/2}{1/2}$$

$$= -\frac{1}{2}\log(3/4) + \log 3$$

$$= \log 3 - \frac{1}{2}\log 3/4$$

$$= \log (3^2)^{1/2} - \frac{1}{2}\log 3/4$$

$$= \frac{1}{2}\log 9 - \frac{1}{2}\log 3/4.$$

$= \frac{1}{2} [\text{Log } 9 - \text{Log } 3/4]$

$= 1/2 \log \frac{9}{3/4}$

$= 1/2 \log \frac{9 \times 4}{\cancel{3}}$

$= 1/2 \text{ Log } 12$

$= \text{Log } (12)^{1/2}$

$\boxed{= \text{Log } \sqrt{12}}$

20. Sum the series $\log_3 e - \text{Log}_9 e + \text{Log}_{27} e - \text{Log}_{81} e + \cdots -\infty$

<u>Sol:</u>

$\text{Log}_x a \cdot \log_a x = 1.$

$\log_x a = \frac{1}{\text{Log}_a x}$

$\text{Log}_3 e = \frac{1}{\log_e 3}$

$\text{Log}_9 e = \frac{1}{\log_e 9} = \frac{1}{\log_e 3^2} = \frac{1}{2 \log_e 3}$

$\text{Log}_{27} e = \frac{1}{\log 27} = \frac{1}{\log_e 3^3} = \frac{1}{3 \log_e 3}.$

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and so on

$$\log_3 e - \log_9 e + \log_{27} e - \log_{81} e \dots \infty$$

$$= \frac{1}{\log_e 3} - \frac{1}{2\log_e 3} + \frac{1}{3\log_e 3} \dots \infty$$

$$= \frac{1}{\log_e 3}\left[1 - \frac{1}{2} + \frac{1}{3} + \dots\right]$$

$$= \frac{1}{\log_e 3}\left[\log(1+1)\right]$$

$$= \frac{\log_e 2}{\log_e 3}.$$

21. Series of n terms:

$$\frac{8}{1\cdot2\cdot3}\left(\frac{5}{7}\right) + \frac{9}{2\cdot3\cdot4}\left(\frac{5}{7}\right)^2 + \frac{10}{3\cdot4\cdot5}\left(\frac{5}{7}\right)^3 + \dots$$

<u>Soln:</u>

$$U_n = \frac{n+7}{n(n+1)(n+2)}\left(\frac{5}{7}\right)^n$$

$$\frac{n+7}{n(n+1)(n+2)} \Rightarrow \frac{A}{n(n+1)} + \frac{B}{(n+1)(n+2)}$$

$$A(n+2) + Bn = n+7$$

put $n=0 \Rightarrow A(0+2)+B(0)=0+7$

$$2A=7$$

$$A=7/2$$

put $n=-2 \Rightarrow A(-2+2)+B(-2)=-2+7$

$$\boxed{B=-\frac{5}{2}}$$

put $n=-2 \Rightarrow A(-2+2)+B(-2)=-2+7$

$$A(0)-2B=5$$

$$\boxed{B=-\frac{5}{2}}$$

$$U_n=\left[\frac{7/2}{n(n+1)}-\frac{5/2}{(n+1)(n+2)}\right](5/7)^n$$

$$=\frac{1}{2}\left[\frac{7}{n(n+1)}-\frac{5}{(n+1)(n+2)}\right]\left(\frac{5}{7}\right)^n$$

$$=\frac{7}{2}\left[\frac{1}{n(n+1)}-\frac{5/7}{(n+1)(n+2)}\right]\left(\frac{5}{7}\right)^n$$

$$U_n=\frac{7}{2}\left[\frac{(5/7)^n}{n(n+1)}-\frac{(5/7)^{n+1}}{(n+1)(n+2)}\right].$$

$$\Rightarrow \frac{2}{7}U_n=\frac{(5/7)^n}{n(n+1)}-\frac{(5/7)^{n+1}}{(n+1)(n+2)}$$

$$\frac{2}{7}U_{n-1}=\frac{(5/7)^{n-1}}{(n-1)n}-\frac{(5/7)^n}{n(n+1)}$$

$$\frac{2}{7}U_{n-2}=\frac{(5/7)^{n-2}}{(n-2)(n-1)}-\frac{(5/7)^{n-1}}{(n-1)(n)}$$

$$\frac{2}{7}U_2 = \frac{(5/7)^2}{(2)(3)} - \frac{\left(\frac{5}{7}\right)^3}{(3)(4)}$$

$$\frac{2}{7}U_1 = \frac{(5/7)}{1\cdot 2} - \frac{(5/7)^2}{2\cdot 3}.$$

Adding we get $\frac{2}{7}S_n = \frac{5/7}{1\cdot 2} - \frac{(5/7)^{n+1}}{(n+1)(n+2)}$

$$\therefore S_n = \frac{7}{2}\left[\frac{5/7}{1\cdot 2} - \frac{(5/7)^{n+1}}{(n+1)(n+2)}\right]$$

$$= \frac{7\times\frac{5}{7}}{1\times 2\times 2} - \frac{7(5/7)^{n+1}}{2(n+1)(n+2)}$$

$$S_n = \frac{5}{4} - \frac{7}{2(n+1)(n+2)}\left(\frac{5}{7}\right)^{n+1}$$

$$\therefore \boxed{S_\infty = \frac{5}{4}}$$

22. $\frac{7}{1\cdot 3}\left(\frac{1}{5}\right) + \frac{11}{3\cdot 5}\,\frac{1}{5^2} + \frac{15}{5\cdot 7}\,\frac{1}{5^3} + \cdots \infty$

Soln:

$$U_n = \frac{4n+3}{(2n-1)(2n+1)}\left(\frac{1}{5}\right)^n$$

$$\frac{A}{2(n-1)} + \frac{B}{2n+1} = 4n+3$$

$$A(2n+1) + B(2n-1) = 4n+3.$$

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$$n=\frac{1}{2} \Rightarrow A\left(2\times\frac{1}{2}+1\right)+B\left(2\times\frac{1}{2}-1\right)=4\left(\frac{1}{2}\right)+3$$

$$2A+0B=2+3$$

$$\boxed{A=\frac{5}{2}}$$

$$\text{put } n=-\frac{1}{2} \Rightarrow A\left(2\times-\frac{1}{2}+1\right)+B\left(2\times-\frac{1}{2}\right)=4\left(-\frac{1}{2}\right)+3$$

$$(0)A-2B=-2+3$$

$$\boxed{B=-\frac{1}{2}}$$

$$U_n=\left[\frac{5/2}{(2n-1)}+\frac{-1/2}{(2n+1)}\right]\left(\frac{1}{5}\right)^n$$

$$=\frac{1}{2}\left[\frac{5}{2n-1}-\frac{1}{2n+1}\right]\left(\frac{1}{5}\right)^n$$

$$=\frac{5}{2}\left[\frac{5}{2n-1}-\frac{1}{2n+1}\right]\left(\frac{1}{5}\right)^n$$

$$=\frac{5}{2}\left[\frac{(1/5)^n}{2n-1}-\frac{(1/5)^{n+1}}{2n+1}\right]$$

$$\frac{2}{5}U_n=\frac{(1/5)^n}{2n-1}-\frac{(1/5)^{n+1}}{2n+1}$$

$$\frac{2}{5}U_{n-1}=\frac{\left(\frac{1}{5}\right)^{n-1}}{2(n-1)-1}-\frac{(1/5)^n}{2(n-1)+1}$$

$$=\frac{(1/5)^{n-1}}{2n-3}-\frac{(1/5)^n}{2n-1}$$

$$\frac{2}{5} U_{n-2} = \frac{(1/5)^{n-2}}{2n-5} - \frac{(1/5)^{n-1}}{2n-3}$$

$$\frac{2}{5} U_2 = \frac{(1/5)^2}{3} - \frac{(1/5)^3}{5}$$

$$\frac{2}{5} U_1 = \frac{\frac{1}{5}}{1} - \frac{(1/5)^2}{3}$$

adding $\frac{2}{5} S = \left[\frac{1}{5} - \frac{(1/5)^n}{5(2n+1)}\right]$

$$S = \frac{5}{2}\left[\frac{1}{5} - \frac{(1/5)^n}{5^n\,2(2n+1)}\right]$$

$$S = \frac{5}{2\times 5}\left[1 - \frac{1}{5^n(2n+1)}\right]$$

$$\boxed{S = \frac{1}{2}\left[1 - \frac{1}{5^n(2n+1)}\right]}$$

23. $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots \text{ to } \infty$

Soln:

$$U_n = \frac{1+3+3^2+\dots+3^{n-1}}{n!}$$

$$= \frac{3^n - 1}{3-1} \cdot \frac{1}{n!}$$

$$= \frac{1}{2}\left(\frac{3^n}{n!} - \frac{1}{n!}\right)$$

$$u_1 = \frac{1}{2}\left(\frac{3}{1!} - \frac{1}{1!}\right)$$

$$u_2 = \frac{1}{2}\left(\frac{3^2}{2!} - \frac{1}{2!}\right)$$

$$u_3 = \frac{1}{2}\left(\frac{3^3}{3!} - \frac{1}{3!}\right)$$

$$u_n = \frac{1}{2}\left(\frac{3^n}{n!} - \frac{1}{n!}\right)$$

$$S = \frac{1}{2}\left(\frac{3^1}{1!} + \frac{3^2}{2!} + \cdots + \frac{3^n}{n!} + \cdots\right)$$

$$- \left(\frac{1}{2}\left(\frac{1}{1!} + \frac{1}{2!} + \cdots \frac{1}{n!} + \cdots\right)\right)$$

$$= \frac{1}{2}(e^3 - 1) - \frac{1}{2}(e-1)$$

$$\boxed{= \frac{1}{2} e(e^2-1)}$$