TRANSPORTATION

INTRODUCTION:

Transportation deals with the transportation of a commodity [single product from 'm' Sources [orgin or Supply or capacity centers) to 'n' destinations [sinks or demand or required centers]

Mathematical of a Transportation problem:

$$\text{minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i , \quad i = 1, 2, 3 \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j , \quad j = 1, 2, 3 \ldots n$$

and $x_{ij} \geq 0$ for all i and j.

Definition :

OR (operation Research) :

OR is a scientific method for problem solved for executive management.

Transportation

NWCR - North west corner Rule

LCM - Least Cost Method

VAM - vogels Approximation Method

Problems :

Method-1 (NWCR)

1). Find the IBFS (Intial Basic feasible soln)

Destination

					Supply
Orgin	2	5	3	1	20
	4	6	2	3	10
	1	4	1	2	10
Demand	5	10	15	10	

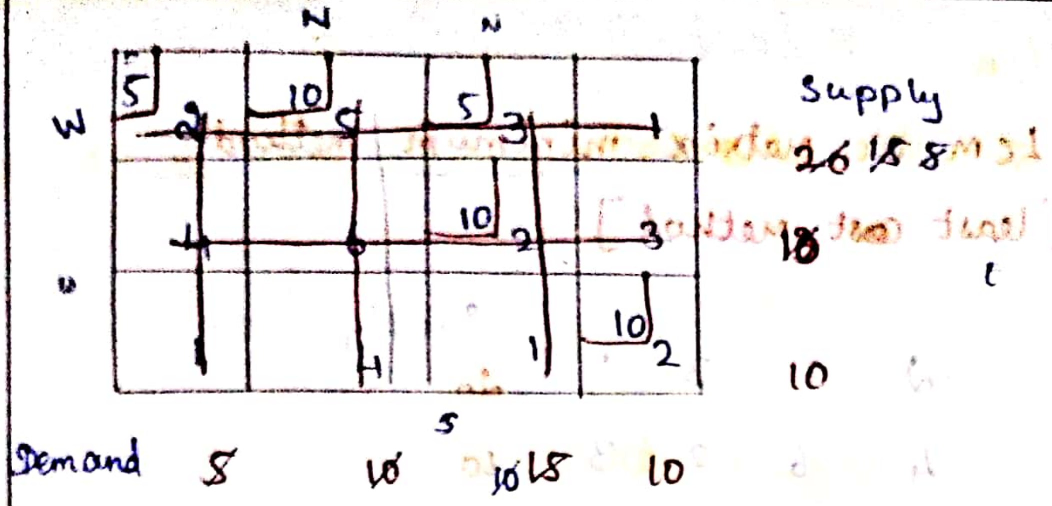
Soln :

∴ The given problem is balanced.

(ie) Demand value = Supply value

40 = 40

by using NWCR (North west corner Rule)



[5] 2	[10] 5	[5] 3	1
4	6	[10] 2	3
1	4	1	[10] 2

∴ total Transportation cost is =

$2 \times 5 + 10 \times 5 + 5 \times 3 + 10 \times 2 + 2 \times 10 = 115$

$m + n - 1$ = number of allocation

$3 + 4 - 1 = 5$

$6 \neq 5$

∴ The solution is degenerasy

Formula:

degeneracy:

$m + n - 1 \neq$ number of allocation

non-degeneracy:

$m + n - 1 =$ number of allocation

Method - III (VAM)

Find VAM (Volgel's Approximation method).

2	5	3	1	20
4	6	2	3	10
1	4	1	2	10
5	10	15	10	

Soln:

2	5	3	[10] 1	~~20~~ 10	(1)
4	6	2	3	10	(1)
1	4	1	2	10	(0)
5	10	15	~~10~~		
(1)	(1)	(1)	(1)		

2	5	3	10	(1)
~~4~~	~~6~~	[10] ~~2~~	~~10~~	(2)
[5] 1	4	1	~~10~~ 5	(0)
~~5~~	10	~~15~~ 5		
(1)	(1)	(1)		

2	5	3	10	(1)
1	4	[5] 1	~~10~~ 5	(0)
5	10	~~5~~		
(1)	(1)	(2)		

2	5	10	(3)
[5] 1	4	~~5~~	(3)
~~5~~	10		
(1)	(1)		

[10] 5	10
10	

∴ Total transportation cost :

2	[10] 5	3	[10] 1
4	6	[10] 2	3
[5] 1	4	[5] 1	2

Total transporatation cost $= 10\times5 + 1\times10 + 2\times10 + 1\times5 + 1\times5$

$= 90$

$m+n-1$ = number of allocation

$3+4-1 = 5$

$6 \neq 5$

∴ This Solution is degeneracy

Scanned with CamScanner

Solve Assignment problem:

10	5	13	15
3	9	18	3
10	7	3	2
5	11	9	7

Soln:

Step 1: The given matrix is square

Step 2: row reduction

$$\begin{pmatrix} 5 & 0 & 8 & 10 \\ 0 & 5 & 15 & 0 \\ 8 & 5 & 1 & 0 \\ 0 & 6 & 4 & 2 \end{pmatrix}$$

Step 3: Column reduction:

$$\begin{pmatrix} 5 & 0 & 7 & 10 \\ 0 & 5 & 14 & 0 \\ 8 & 5 & 0 & 0 \\ 0 & 6 & 3 & 2 \end{pmatrix}$$

Step 4: assignment

$$\begin{pmatrix} 5 & \boxed{0} & 7 & 10 \\ \times & 5 & 14 & \boxed{0} \\ 8 & 5 & \boxed{0} & \times \\ \boxed{0} & 6 & 3 & 2 \end{pmatrix}$$

1 → 2
2 → 4
3 → 3
4 → 1

Assignment Cost

5 + 3 + 3 + 5

= 16//.

Find the assignment of salesman various districts which will yield maximum profit.

16	10	14	11
14	11	15	15
15	15	13	12
13	12	14	15

Soln:

The maximum element is 16. subtract all the element from 16, the problem reduces to minimazing of 'loss' given by table.

0	6	2	5
2	5	1	1
1	1	3	4
3	4	2	1

Step 1:

The given matrix is square.

Step 2:

Row Reduction

0	6	2	5
1	4	0	0
0	0	2	3
2	3	1	0

Step 3:

[0]	6	2	5
1	4	[0]	~~0~~
~~0~~	[0]	2	3
2	3	1	[0]

The optimal assignment is

1→1, 2→3, 3→2, 4→4

The cost assignment is 16+ 15 +15 +15 = 61.

Max $z = 5x_1 + 3x_2$ Simplex method.

$x_1 + x_2 \le 2$

$5x_1 + 2x_2 \le 10$

$3x_1 + 8x_2 \le 10$

$x_1, x_2 \ge 0$

Sn:

Max $z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$

$x_1 + x_2 + s_1 = 2$

$5x_1 + 2x_2 + s_2 = 10$

$3x_1 + 8x_2 + s_3 = 10$

$x_1, x_2, s_1, s_2, s_3 \ge 0$

		c_j	5	3	0	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	θ
0	S_1	2	[1]	1	1	0	0	$2/1 = 2$
0	S_2	10	5	2	0	1	0	$10/5 = 2$
0	S_3	10	3	8	0	0	1	$10/3 = 10/3$
		z_j	0	0	0	0	0	
		$z_j - c_j$	-5 ↑	-3	0	0	0	
5	x_1	2	1	1	1	0	0	$R_1 \div 1$
0	S_2	0	0	-3	-5	1	0	$R_2 - 5R_1$
0	S_3	4	0	5	-3	0	1	$R_3 - 3R_1$
		z_j	5	5	5	0	0	
		$z_j - c_j$	0	2	5	0	0	

Hence All $z_j - c_j \ge 0$

Hence the optimal soln is reached

max $z = 5x_1 + 3x_2$ [illegible]

$x_1 = 2$

$x_2 = 0$

max $z = 10 + 0$ ∴ $x_1 = 2,\ x_2 = 0$

max $z = 10$

) max $z = x_1 + 2x_2 + 3x_3$

$x_1 + 2x_2 + 3x_3 \le 10$

$x_1 + x_2 \le 5$

$x_1, x_2, x_3 \ge 0$

Soln: max $z = x_1 + 2x_2 + 3x_3 + 0S_1 + 0S_2$

$x_1 + 2x_2 + 3x_3 + S_1 = 10$

$x_1 + x_2 + S_2 = 5$

$x_1, x_2, x_3, S_1, S_2 \ge 0$

C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	θ
		c_j	1	2	3	0	0	
0	S_1	10	1	2	[3]	1	0	$10/3 = 10/3 = 3.33$
0	S_2	5	1	1	[0]	0	1	$5/0 = \infty$
		z_j	0	0	0	0	0	
		$z_j - c_j$	-1	-2	-3 ↑	0	0	
3	x_3	-8 10/3	-5 1/3	-4 2/3	0	-2	-6	$R_1 - 3R$ [illegible]
0	S_2	6 5	2 1	2 1	1 0	1 0	2	$R + 1$ [illegible]
		z_j	6	6	3	3	6 0	
		$z_j - c_j$	5	4	0	3	6	

Hence all $z_j - c_j \ge 0$

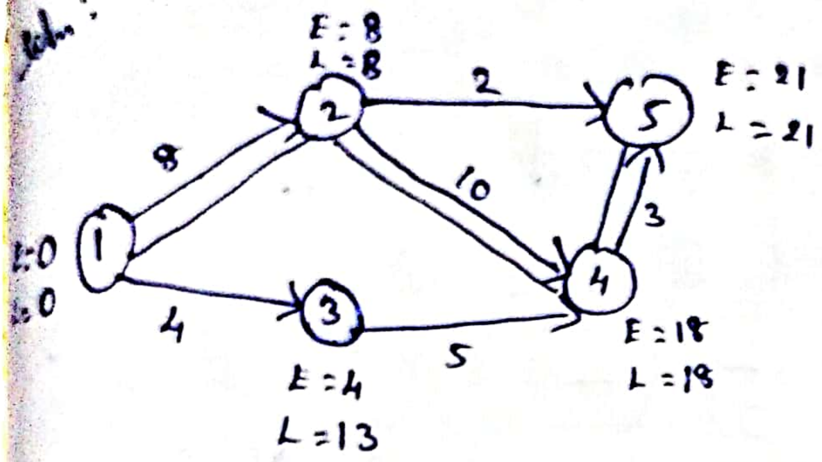
Hence the optional soln is reached.

CPM Method

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Time	8	4	10	2	5	3

Find critical path.

Sol:



Activity	Time	Earliest		Latest		TF	FF
		Start	Finish	Start	Finish		
1-2	8	0	8	0	8	0	0
1-3	4	0	4	9	13	9	0
2-4	10	8	18	8	18	0	0
2-5	2	8	10	19	21	11	0
3-4	5	4	9	13	18	9	0
4-5	3	18	21.	18	21	0	0

Critical path:

1-2-4-5

Time duration:

8+10+3 = **21**