COMPLEX ANALYSIS (16SCCMM13)

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CLASS : III B.Sc., MATHEMATICS 'A'

QUESTION BANK - TWO MARKS

APRIL - 2019

1. If $\lim_{z \to z_0} f(z) = l$, then prove that $\lim_{z \to z_0} \overline{f(z)} = \bar{l}$

Sol: Let $\varepsilon > 0$ be given. Then there exists $\delta > 0$ such that $0 < |z - z_0| < \delta \Rightarrow |f(z) - l| < \varepsilon$

Now, $|\overline{f(z)} - \bar{l}| = |\overline{f(z) - l}|$

$= |f(z) - l| < \varepsilon \quad [\because |\bar{z}| = |z|]$

Hence, $0 < |z - z_0| < \delta \Rightarrow |\overline{f(z)} - \bar{l}| < \varepsilon$

$$\boxed{\therefore \lim_{z \to z_0} \overline{f(z)} = \bar{l}}$$

Hence proved.

2. prove that the function $u = 2x(1-y)$ is harmonic.

Sol: Let $f(z) = u + iv$

Here, $u = 2x(1-y)$

$\Rightarrow u = 2x - 2xy$

$u = 2x - 2yx$

Diff. with respect to x

$u_x = 2 - 2y$

Diff with respect to y

$u_y = -2x$

Again differentiate with respect to x & y

$u_{xx} = 0$ & $u_{yy} = 0$

$\therefore u_{xx} + u_{yy} = 0 + 0$

$\boxed{u_{xx} + u_{yy} = 0}$

$\therefore$ It is harmonic function

Hence proved.

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3. Define Bilinear transformation

A transformation of the form $w = T(z) = \dfrac{az+b}{cz+d}$ where a, b, c, d are complex constants and $ad - bc \neq 0$, is called a bilinear transformation (or) Mobius transformation.

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4). $w = \dfrac{z}{2-z}$, find the fixed point

Sol. Let $w = f(z)$

$$\therefore f(z) = \frac{z}{2-z}$$

Now, $f(z) = z \Rightarrow z - \frac{z}{2-z}$

$\Rightarrow z(2-z) = z$

$\Rightarrow 2z - z^2 = z$

$\Rightarrow z^2 - 2z + z = 0$

$\Rightarrow z^2 - z = 0$

$\Rightarrow z(z-1) = 0$

$\Rightarrow z = 0$ & $z = 1$

$\therefore$ 0 & 1 are the fixed points for $w = \frac{z}{2-z}$

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5. Evaluate using Cauchy's integral formula, $\int_C \frac{dz}{z-3}$ where $C : |z| = 1$

Sol.

Here, $f(z) = 1$

The point $z - 3 = 0$

$z = 3$

The point 3 lies outside on $|z| = 1$

The Cauchy's integral formula is $\frac{1}{2\pi i}\int_C \frac{dz}{z-z_0} f(z) = f(z_0)$

$\Rightarrow \frac{1}{2\pi i}\int_C \frac{dz}{z-3} = 0$ $\because$ [The point 3 lies outside C]

$\Rightarrow \int_C \frac{dz}{z-3} = 2\pi i(0)$

$$\boxed{\therefore \int_C \frac{dz}{z-3} = 0}$$ on C at $|z| = 1$

6. What is the Maclaurin's series expansion for e^{-x}?

Sol.

Maclaurin's series expansion for e^{-x} is

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \dots\dots\dots + (-1)^n \frac{x^n}{n!}$$

7. Define Singular point

A point a is called a singular point or singularity of a function $f(z)$ if $f(z)$ is analytic at a and f is analytic at some point of every disc $|z-a|<r$.

8. Find the poles of $f(z) = \frac{z+1}{z^2-2z}$

Sol.

Let $f(z) = \frac{z+1}{z^2-2z}$

$= \frac{z+1}{z(z-2)}$

$\therefore$ $z=0$ & $z=2$ are simple poles for $f(z)$.

9. Find the residue of $f(z) = \frac{e^z}{z^2}$

Sol.

Given, $f(z) = \frac{e^z}{z^2}$

$$\frac{e^z}{z^2} = \frac{1}{z^2}\left[1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots\right]$$

$$\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \frac{z^2}{4!} + \cdots$$

$\therefore f(z)$ has a double poles at $z=0$

$\therefore \text{Res}\{f(z);0\} =$ coefficient of $1/z = 1$

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10. Evaluate $\displaystyle\int_{|z|=2} \frac{dz}{2z+3}$

sol.

The pole is $2z+3=0$

$2z=-3$

$z=-3/2$ is a simple pole.

Here, $z=-3/2$ lies inside at $|z|=2$

we know that,

$$\boxed{\therefore \text{Res}\{f(z);a\} = \frac{h(a)}{k'(a)}}$$

$$\text{Res}\{f(z);-3/2\} = \frac{h(-3/2)}{k'(-3/2)}$$

where $h(-3/2)=1$ & $k'(-3/2)=2$

$\therefore \text{Res}\{f(z);-3/2\} = 1/2$

By residue theorem $\Rightarrow \displaystyle\int_C f(z)dz = 2\pi i\left(\frac{1}{2}\right)$

$$\int_{|z|=2} \frac{dz}{2z+3} = \pi i$$

$$\boxed{\therefore \int_{|z|=2} \frac{dz}{2z+3} = \pi i}$$

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APRIL - 2018

1. write down the complex form of the C.R equations.

Sol.

Statement:

Let $f(z) = u(x,y) + iv(x,y)$ be differentiable. Then C.R equation in the complex form as $f_x = -if_y$

Proof:

Let $f(z) = u(x,y) + iv(x,y)$

$f_x = u_x + iv_x$

$f_y = u_y + iv_y$

$\therefore f_x = -if_y$

$\Leftrightarrow u_x + iv_x = -i(u_y + iv_y)$

$\Leftrightarrow u_x + iv_x = v_y - iu_y$

$\Leftrightarrow \boxed{u_x = v_y \ \& \ v_x = -u_y}$

Thus the two C.R eqns are equivalent to equation $f_x = -if_y$.

2. prove that $u = 2x - x^3 + 3xy^2$ is harmonic function

Sol.

Given, $u = 2x - x^3 + 3xy^2$

$\therefore$ Diff with respect to x

$u_x = 2 - 3x^2 + 3y^2$

Diff with respect to y

$u_y = 6xy$

Again differentiate with respept to x & y

$u_{xx} = -6x$, $u_{yy} = 6x$

$\therefore \quad u_{xx} + u_{yy} = -6x + 6x$

$$\boxed{u_{xx} + u_{yy} = 0}$$

$\therefore$ It is harmonic function

3\. If $w = T(z) = \dfrac{az+b}{cz+d}$ is a bilinear transformation

find $T^{-1}(w)$.

Sol.

Given, $w = T(z) = \dfrac{az+b}{cz+d} \Rightarrow w = \dfrac{az+b}{cz+d} \Rightarrow T(w) = z$

$$z = T^{-1}(w) \Rightarrow \quad w(cz+d) - az - b = 0$$

$$wcz + dw - az - b = 0$$

$$z(wc - a) = -wd + b$$

$$\therefore \quad z = \frac{-dw+b}{cw-a}$$

$$\boxed{z = T^{-1}(w) = \frac{-dw+b}{cw-a}}$$

is also a bilinear transformation

4\. Write down the cross ratio of (z_1, z_2, z_3, ∞)

Sol.

The cross ratio of (z_1, z_2, z_3, ∞) is $\dfrac{z_1 - z_3}{z_2 - z_3}$

$$\boxed{\therefore (z_1, z_2, z_3, \infty) = \frac{z_1 - z_3}{z_2 - z_3}}$$

5. Evaluate $\int_C \frac{dz}{z-3}$ where C is the circle $|z-2| = 5$

Sol.

Let $f(z) = 1$

The point $z=3$ lies inside C

Hence by Cauchy's integral formula,

$$f(z_0) = \frac{1}{2\pi i}\int \frac{f(z)}{z-z_0}\,dz$$

$$\int_C \frac{dz}{z-3} = 2\pi i f(3)$$

$$\therefore \boxed{\int_C \frac{dz}{z-3} = 2\pi i}$$

6. State the Morera's theorem

If $f(z)$ is continuous in a simply connected domain D and if $\int_C f(z)dz = 0$ for every simple closed curve C lying in D then $f(z)$ is analytic in D.

7. Find the Laurent's series expansion of $f(z) = z^2 e^{1/z}$ about $z=0$

Sol. Given, $f(z) = z^2 e^{1/z}$

$f(z)$ is analytic at all points $z \neq 0$

$$f(z) = z^2\left[1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} + \cdots\right]$$

$$f(z) = z^2 + z + \frac{1}{2} + \frac{1}{3!z} + \frac{1}{4!z^2} + \cdots$$

This is the required Laurent series expansion for $f(z)$, at $z=0$

8. Find the zeros of $f(z) = e^z (z-2i)^2 (z+3)^3$

sol.

$2i$ is a zero of order 2 is

-3 is a zero of order 3 for $f(z)$

9. Find the residue of $f(z) = \tan z$ at $\pi/2$

sol.

Given, $f(z) = \tan z$

Here, $z = \pi/2$ is simple pole for $\tan z$

$$\text{Let } f(z) = \frac{\sin z}{\cos z} = \frac{h(z)}{k'(z)}$$

$$\therefore \text{Res}\{f(z), \pi/2\} = \frac{h(\pi/2)}{k'(\pi/2)}$$

$$= \frac{\sin \pi/2}{-\sin \pi/2}$$

Here, $h(\pi/2) = \sin \pi/2$ ie) $h(z) = \sin z$

$k(z) = \cos z$

$k'(z) = -\sin z$

$\therefore \text{Res}\{f(z), \pi/2\} = 1/-1$

$$\boxed{\text{Res}\{f(z), \pi/2\} = -1}$$

10. Evaluate $\int_C \frac{dz}{2z+3}$ where C is the circle $|z| = 2$

sol.

The poles are $2z + 3 = 0$

$z = -3/2$ are simple poles

$z = -3/2$ lies inside at $|z| = 2$

$\therefore \text{Res}\{f(z); 0\} = \dfrac{h(a)}{k'(a)}$

$\text{Res}\{f(z); -3/2\} = \dfrac{h(-3/2)}{k'(-3/2)}$

where $h(-3/2) = 1$ & $k'(-3/2) = 2$

$\therefore \text{Res}\{f(z); -3/2\} = 1/2$

By residue theorem

$$\int_C f(z)dz = 2\pi i(1/2)$$

$$\boxed{\therefore \int_C \frac{dz}{2z+3} = \pi i}$$

APRIL - 2017

1) If $\lim_{z \to z_0} f(z) = l$, then prove that $\lim_{z \to z_0} \overline{f(z)} = \bar{l}$

Sol.

Let $\varepsilon > 0$ be given. Then there exist $\delta > 0$ such that

$0 < |z - z_0| < \delta \Rightarrow |f(z) - l| < \varepsilon$

Now, $|\overline{f(z)} - \bar{l}| = |\overline{f(z) - l}|$

$= |f(z) - l| < \varepsilon \quad [\because |\bar{z}| = |z|]$

Hence, $0 < |z - z_0| < \delta \Rightarrow |\overline{f(z)} - \bar{l}| < \varepsilon$

$$\therefore \lim_{z \to z_0} \overline{f(z)} = \bar{l}$$

Hence proved

2. Define an Analytic function

A function f defined in a region D of the complex plane is said to be analytic at a point $a \in D$ if f is differentiable at every point of some neighbourhood of a.

3. prove that the transformation $w = \bar{z}$ is not a bilinear transformation

Sol.

Any bilinear transformation, other than the identity transformation has two fixed points. However the transformation $w = \bar{z}$ has infinitely many fixed points, namely all real numbers. Hence it is not a bilinear transformation.

4. Define cross ratio

Let z_1, z_2, z_3, z_4 be four distinct points in the extended complex plane. The cross ratio of these four points denoted by (z_1, z_2, z_3, z_4) is defined by

$$(z_1, z_2, z_3, z_4) = \begin{cases} \dfrac{(z_1-z_3)(z_2-z_4)}{(z_1-z_4)(z_2-z_3)} & \text{if none of } z_1, z_2, z_3, z_4 \text{ is } \infty \\[2ex] \dfrac{z_1-z_3}{z_1-z_4} & \text{if } z_2 \text{ is } \infty \\[2ex] \dfrac{z_2-z_4}{z_1-z_4} & \text{if } z_3 \text{ is } \infty \\[2ex] \dfrac{z_1-z_3}{z_2-z_3} & \text{if } z_4 \text{ is } \infty \\[2ex] \dfrac{z_2-z_4}{z_2-z_3} & \text{if } z_1 \text{ is } \infty \end{cases}$$

5. state cauchy's integral theorem

Let $f(z)$ be a function which is analytic inside and on a simple closed curve C. Let z_0 be any point in interior of C.

Then, $\boxed{f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_0}\,dz}$

6. state the fundamental theorem of Algebra

Every polynomial of degree ≥ 1 has atleast one zero(root) in C.

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7. Define Meromorphic function

A function $f(z)$ is said to be meromorphic function if its analytic except at a finite set of points are poles.

8. State Taylor's theorem

Let $f(z)$ be analytic in a region D containing z_0.

Then $f(z)$ can be represented as a power series in $z-z_0$ given by

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z-z_0) + \frac{f''(z_0)}{2!}(z-z_0)^2 + \cdots$$
$$\cdots + \frac{f^{(n)}(z_0)}{n!}(z-z_0)^n + \cdots$$

9. Find the residue of $f(z) = \frac{z}{z^2+1}$ at $\boxed{z=i}$

Sol.

$z=i$ is a simple pole

Let $f(z) = \frac{h(z)}{k(z)}$

where $h(z) = z$ and $k(z) = z^2+1$

Now, $\text{Res}\{f(z); i\} = \frac{h(i)}{k'(i)}$

$= \frac{i}{2(i)}$ $[k'(z) = 2z]$

$$\boxed{\therefore \text{Res}\{f(z); i\} = 1/2}$$

10. State Rouche's theorem

If $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C and if $|g(z)|<|f(z)|$ on C then $f(z)+g(z)$ and $f(z)$ have the same number of zeros inside C.

NOVEMBER - 2015

1. If $\lim_{z\to z_0} f(z)=l$, then prove that $\lim_{z\to z_0} Im f(z) = Im\, l$

Sol.

Let $\lim_{z\to z_0} f(z) = l$

Since, $Im f(z) = \frac{1}{2i}[f(z) - f(\bar{z})]$

Now, $\lim_{z\to z_0} Im f(z) = \frac{1}{2i}\left[\lim_{z\to z_0} f(z) - \lim_{z\to z_0} f(\bar{z})\right]$

$= \frac{1}{2i}[l - \bar{l}]$ Given $\lim_{z\to z_0} f(z) = l$

$\lim_{z\to z_0} Im f(z) = \frac{l-\bar{l}}{2i}$

$\boxed{\therefore \lim_{z\to z_0} Im f(z) = Im\, l}$

Hence proved

2. prove that the function $f(z)=z^4$ is differentiable at every point.

Sol.

Now, $\lim_{h\to 0} \frac{f(z+h)-f(h)}{h} = \lim_{h\to 0} \frac{(z+h)^4 - z^4}{h}$

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$$\lim_{h\to 0} \frac{f(z+h)-f(z)}{h} = \lim_{h\to 0} \frac{z^4+4z^3h+6z^2h^2+4zh^3+h^4-z^4}{h}$$

$$= \lim_{h\to 0} \frac{h[4z^3+6z^2h+4zh^2+h^3]}{h}$$

$$\lim_{h\to 0} \frac{f(z+h)-f(z)}{h} = \lim_{h\to 0} [4z^3+6z^2h+4zh^2+h^3]$$

$$= 4z^3$$

$\therefore$ The function $f(z)=z^4$ is differentiable at every point.

3. When the four points z_1, z_2, z_3, z_4 are said to be collinear?

Four distinct points z_1, z_2, z_3, z_4 are collinear if (z_1, z_2, z_3, z_4) is real.

4. Define parabolic transformation

A bilinear transformation with only one fixed point is called parabolic

5. State Maximum Modulus theorem

Let $f(z)$ be continuous in a closed and bounded region D and analytic and non constant in the interior of D. Then $|f(z)|$ attains its maximum value on the boundary D and never in the interior of D.

6. State Cauchy's inequality

Let $f(z)$ be analytic inside and on the circle c with centre z_0 and radius r. Let M denote the maximum of $|f(z)|$ on C. Then $|f^n(z_0)| \leq \frac{n!\,M}{r^n}$

7. Write the Maclaurin's series for $\cosh z$ and $\log(1+z)$

Maclaurin's series for $\cosh z$ is

$$\cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{2!} + \cdots + \frac{z^{2n}}{(2n)!} + \cdots \quad (|z| < \infty)$$

Maclaurin's series for $\log(1+z)$ is

$$\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots (-1)^{n-1}\frac{z^n}{n} + \cdots \quad (|z| < 1)$$

8. Define poles

Let a be isolated singularity of $f(z)$. The point a is called a pole if the principal part of $f(z)$ at $z=a$ has a infinite number of terms. If the principal part of $f(z)$ at $z=a$ is given by

$$\boxed{\frac{b_1}{z-a} + \frac{b_2}{(z-a)^2} + \cdots + \frac{b_r}{(z-a)^r}}$$

where $b_r \neq 0$, we say that a is a pole of order r for $f(z)$.

9. State Rouche's theorem

If $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C and if $|g(z)| < |f(z)|$ on c then $f(z)+g(z)$ and $f(z)$ have same number of zeros inside c.

10. Define Residue of a function

Let a be an isolated sigularity for $f(z)$. Then the residue of $f(z)$ at a is defined to be coefficient of $\frac{1}{z-a}$ in the Laurent's series expansion about a of $f(z)$ and is denoted by $Res\{f(z);a\}$

$Res\{f(z);a\} = \frac{1}{2\pi i}\int_C f(z)dz$, where C is circle $|z-a|=r$, f is analytic in $0<|z-a|<r$

APRIL - 2009

1) Evaluate $\lim\limits_{z\to 0} \dfrac{x^2y^2}{(x+y^2)^3}$

sol.

Along the parabola $y^2 = mx$,

$$f(z) = \frac{mx^3}{(x+mx)^3}$$

$$\boxed{f(z) = \frac{m}{(1+m)^3}}$$

Hence if $z\to 0$ along the parabola $y^2 = mx$

$f(z)$ tends to $\dfrac{m}{(1+m)^3}$ which depends on m.

Hence $f(z)$ does not have a limit as $z \to 0$

2. Examine the differentiability of $f(z) = \bar{z}$

sol.

Now, $\lim\limits_{h\to 0} \dfrac{f(z+h)-f(z)}{h} = \lim\limits_{h\to 0} \dfrac{(\overline{z+h}) - \bar{z}}{h}$

$$= \lim_{h\to 0} \frac{\cancel{\bar{z}} + \bar{h} - \cancel{\bar{z}}}{h}$$

$$\lim_{h\to 0} \frac{f(z+h)-f(z)}{h} = \lim_{h\to 0} \frac{\bar{h}}{h}$$

$$\boxed{\therefore \lim_{h\to 0} \frac{\bar{h}}{h} \text{ does not exist}}$$

$\therefore f(z) = \bar{z}$ is nowhere differentiable

3. what are the fixed points of transformation $w = az$,

$(|a| = 1)$?

sol. $w = az$, $|a| = 1$

0 and ∞ are the fixed points of this transformation.

4. state the geometrical meaning of the transformation

$w = \frac{1}{z}$.

sol. The inversion is $w = 1/z$

This transformation can be expressed as a product

of two transformations

$T_1(z) = (1/r)e^{i\theta}$ & $T_2(z) = re^{-i\theta} = \bar{z}$

Now, $(T_1 o T_2)(z) = T_1(T_2(z))$

$= T_1(re^{-i\theta})$

$= \left(\frac{1}{r}\right) e^{-i\theta}$

$$\boxed{(T_1 o T_2)(z) = \frac{1}{z}}$$

The transformation $T_1(z) = (1/r)ei\theta$ represents the inversion

with respect to unit circle $|z| = 1$ and $T_2(z) = \bar{z}$ represents the

reflection about real axis.

Hence the transformation $w = 1/z$ is the inversion

w.r.t the unit circle followed by the reflection about the

real axis.

5. Evaluate : $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}\,dz,\ C: |z|=4$

Sol.

Here, $f(z)=z^2+5$ is analytic inside and on $|z|=4$ and $z=3$ lies inside it.

$\therefore$ By cauchy's integral formula, $\frac{1}{2\pi i}\int_C \frac{dz}{z-z_0} f(z) = f(z_0)$

$$\Rightarrow \frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}\,dz = f(3)$$
$$= 3^2+5$$
$$= 14$$

6. Evaluate : $\int_C \frac{e^z}{z^n}\,dz,\ C: |z|=1$

Sol.

Let $f(z)=e^z$

clearly $f(z)$ is analytic and $f^n(z)=e^z$ for all n

The formula for higher derivatives is,

$$\boxed{f^n(z) = \frac{n!}{2\pi i}\int \frac{f(z)}{(z-a)^{n+1}}\,dz}$$

$$\therefore \int_C \frac{e^z}{z^n}\,dz = \int_C \frac{e^z}{(z-0)^n}\,dz$$
$$= \frac{2\pi i}{(n-1)!}\,e^0$$

$$\boxed{\therefore \int_C \frac{e^z}{z^n}\,dz = \frac{2\pi i}{(n-1)!}}$$

7. Expand $f(z) = 1/z$ about $z=1$ as a Taylor series

Sol. $\frac{1}{z} = f(z)$

By Taylor serie's expansion,

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z-z_0) + \frac{f''(z_0)}{2!}(z-z_0)^2 + \cdots + \frac{f^{(n)}(z_0)}{n!}(z-z_0)^n + \cdots$$

$$f(z) = \frac{1}{z} = f(1) + \frac{f'(1)}{1!}(z-1) + \frac{f''(1)}{2!}(z-1)^2 + \frac{f'''(1)}{3!}(z-1)^3 + \cdots$$

Now, $f(z) = \frac{1}{z} \Rightarrow f(1) = 1$

$f'(z) = \frac{-1}{z^2} \Rightarrow f'(1) = -1$

$f''(z) = \frac{2}{z^3} \Rightarrow f''(1) = 2$

$f'''(z) = \frac{-6}{z^4} \Rightarrow f'''(1) = -6$

....

Hence the Taylor's series expression for $1/z$ about 1 is

$$f(z) = \frac{1}{z} = 1 - (z-1) + (z-1)^2 - (z-1)^3 + \cdots$$

8. State the zero and its order of $f(z) = z^2 \sin z$

Sol.

$$f(z) = z^2\left(z - \frac{z^3}{3!} + \frac{z^5}{5!} \cdots\right)$$

$$= z^3\left(1 - \frac{z^2}{3!} + \frac{z^4}{5!} \cdots\right)$$

$$f(z) = z^3\phi(z)$$

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where $\phi(z) = 1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \ldots$

obviously $\phi(z)$ is analytic and $\phi(0) \neq 0$

$\therefore z=0$ is a zero of order 3 for $f(z) = z^2 \sin z$

9. Find the residue of $\cot z$ at $z=0$

sol.

$z=0$ is a simple pole for $\cot z$.

Let $f(z) = \frac{\cos z}{\sin z} = \frac{h(z)}{k(z)}$

where $h(z) = \cos z$ & $k(z) = \sin z$

$$\therefore \operatorname{Res}\{f(z); 0\} = \frac{h(0)}{k'(0)}$$

$$= \frac{\cos 0}{\cos 0} \qquad (k'(z) = \cos z)$$

$$\boxed{\operatorname{Res}\{f(z); 0\} = 1}$$

10. Define: Residue at infinity

The residue at infinity is

$$\boxed{\operatorname{Res}\{f(z); \infty\} = -\frac{1}{2\pi i} \int_C f(z)\, dz}$$

where C is a simple closed path in $R < |z| < \infty$. If we expand $f(z)$ as $f(z) = \sum a_n z^n$ in $R < |z| < \infty$, $\operatorname{Res}\{f(z); \infty\} = -a_{-1}$