LINEAR ALGEBRA (P16MA22)

INCHARGE: Ms. A.HELEN SHOBANA

CLASS : I M.Sc., MATHEMATICS

UNIT – I

ELEMENTARY ROW OPERATION:

Elementary row operations on $m \times n$ matrix A over the field F are

1. Multiplication of one row of A by a non-zero scalar c
2. Replacement of the r^{th} row of A by row r plus c times row s, c any scalar and $r \neq s$.
3. Interchange of two rows of A

ROW EQUIVALENT:

If A and B are $m \times n$ matrices over the field F , then B is row equivalent to A if B can be obtained from A by a finite sequence of elementary row operations.

ROW REDUCED:

An $m \times n$ matrix R is called row- reduced if

1. The first non -zero entry in each non-zero row of R is equal to 1
2. Each column of F which contains the leading non-zero entry of some row is all its other entries 0

ROW REDUCED ECHELON MATRIX:

An $m \times n$ matrix R is called a row- reduced echelon matrix if

a) R is row-reduced
b) Every row R which has all its entries 0 occurs below every row which has a non-zero entry
c) If rows 1,......,r are the non zero rows of R and if the leading non-zero entry of row i occurs in column k_i , I = 1, 2, ... r, $k_1 < k_2 < ..... < k_r$

INVERTIBLE:

Let A be an $n \times n$ matrix over the field F. An $n \times n$ matrix B such that BA = I is called a left inverse of A; an $n \times n$ matrix B such that BA = I is called a right inverse of A. If AB = BA = I, then B is called a two sided inverse of A and A is said to be invertible.

VECTOR SPACE:

A vector space consists of the following

- a field F of scalars
- a set V of objects called vectors
- A rule called vector addition which associates with each pair of vectors $\alpha \text{ and } \beta$ in V a vector $\alpha+\beta$ in V called the sum of $\alpha \text{ and } \beta$ in such a way that
  1. Addition is commutative, $\alpha+\beta=\beta+\alpha$
  2. Addition is associative, $\alpha+(\beta+\gamma)=(\alpha+\beta)+\gamma$
  3. There is a unique vector 0 in V, called the zero vector, such that $\alpha+0=\alpha$ for all α in V
  4. For each vector α in V, there is a unique vector $-\alpha$ in V such that $\alpha+(-\alpha)=0$
- A rule called scalar multiplication, which associates with each scalar c in F and a vector α in V , a vector cα in V, called the product of c and α in such a way that
  1. $1.\alpha=\alpha \text{ for every } \alpha \text{ in } V$
  2. $(c_1c_2)\alpha=c_1(c_2\alpha)$
  3. $c(\alpha+\beta)=c\alpha+c\beta$
  4. $(c_1+c_2)\alpha=c_1\alpha+c_2\alpha$

SUBSPACE:

Let V be a vector space over the field F. A subspace of V is a subset W of V which is itself a vector space over F with the operations of vector addition and scalar multiplication on V.

LINEAR COMBINATION:

A vector β in V is said to be a linear combination of the vectors $\alpha_1,...,\alpha_n$ in V provided there exists scalars $c_1,...,c_n$ in F such that $\beta=c_1\alpha_1+...+c_n\alpha_n$

LINEARLY DEPENDENT AND INDEPENDENT:

Let V be a vector space over F. A subset S of V is said to be linearly dependent if there exists distinct vectors $\alpha_1,...,\alpha_n$ in S and scalars $c_1,...,c_n$ in F, not all of which are 0, such that $c_1\alpha_1+...+c_n\alpha_n=0$. A set which is not linearly dependent is called linearly independent

FINITE DIMENSIONAL:

Let V be a vector space. A basis for V is a linearly independent set of vectors in V which spans the space V. The space V is finite dimensional if it has a finite basis.

ORDERED BASIS:

If V is a finite dimensional vector space, an ordered basis for V is a finite sequence of vectors which is linearly independent and spans V.

UNIT – II

LINEAR TRANSFORMATION:

Let V and W be vector space over the field F. A linear transformation from V into W is a function T from V into W such that

$$T(c\alpha+\beta)=c(T\alpha)+T\beta \ \ for \ all \ \alpha,\beta \in V \, and \ c \in F$$

NULL SPACE:

Let V and W be vector space over the field F and let T be a linear transformation from V into W. The null space of T is the set of all vectors α in V such that Tα = 0.

RANK AND NULLITY:

If V is finite dimensional, the rank of T is the dimension of the range of T and the nullity of T is the dimension of the null space of T.

LINEAR OPERATOR:

If V is a vector space over the field F, a linear operator on V is a linear transformation from V into V.

ISOMORPHISM:

If V and W are vector spaces over the field F, any one to one linear transformation T of V onto W is called an isomorphism of v onto W. If there exists an isomorphism of v onto W, then V is isomorphic to W.

LINEAR FUNCTIONAL:

If V is a vector space over the field F, a linear transformation f from V into the scalar field F is called a linear functional on V.

DUAL SPACE:

If V is a vector space, the collection of all functionals on V forms a vector space. It is defined by V^* = L(V, F)

HYPERSPACE:

If V is a vector space, a hyperspace in V is a maximal proper subspace of V.

ANNIHILATOR:

If V is a vector space over the field F and S is a subset of V, then the annihilator of S is the set S^0 of linear functionals f on V such that $f(\alpha)=0, \ for \ every \ \alpha \, in \, S$

UNIT – III

LINEAR ALGEBRA:

Let F be a field. A linear algebra over the field F is a vector space $\mathcal{a}$ over F with an additional operation called multiplication of vectors which associates with each pair of vectors $\alpha \text{ and } \beta$ in $\mathcal{a}$ a vector $\alpha\beta$ in $\mathcal{a}$ in such a way that

1. $\alpha(\beta\gamma) = (\alpha\beta)\gamma$
2. $\alpha(\beta+\lambda) = \alpha\beta + \alpha\gamma \, and \, (\alpha+\beta)\gamma = \alpha\gamma + \beta\gamma$
3. $c(\alpha\beta) = (c\alpha)\beta = \alpha(c\beta)$

POLYNOMIAL:

Let F[x] be the subspace of F^{∞} spanned by the vectors1, x, x^2, …. An element of F[x] is called a polynomial over F.

LAGRANGE'S INTERPOLATION FORMULA:

Let V be a vector space. For each f in V, $f = \sum_{i=0}^{n} f(t_i)P_i$ is called the Lagrange's Interpolation Formula.

MULTIPLICITY:

If c is a root of the polynomial f, the multiplicity of c as a root of f is the largest positive integer r such that $(x-c)^r$ divides f.

IDEAL:

Let F be a field. An ideal in F[x] is a subspace M of F[x] such that fg belongs to M whenever f is in F[x] and g is in M.

GREATEST COMMON DIVISOR AND RELATIVELY PRIME:

If $p_1,...p_n$ are polynomials over a field F, not all of which are 0, the monic generator d of the ideal $p_1 F[x] + ... + p_n F[x]$ is called the greatest common divisor of $p_1,...p_n$.

The polynomials $p_1,...p_n$ are relatively prime if their greatest common divisor is 1.

REDUCIBLE AND IRREDUCIBLE:

Let F be afield. A polynomial f in F[x] is said to be reducible over F if there exists polynomials g, h in F[x] of degree > 1 such that f = gh. If not, f is said to be irreducible over F.

PRIME POLYNOMIAL:

A non-scalar irreducible polynomial over F is called a prime polynomial.

ALGEBRAICALLY CLOSED:

The field F is called algebraically closed if every prime polynomial over F has degree 1.

RING:

A ring is a set K together with two operations $(x, y) \rightarrow x + y$ *and* $(x, y) \rightarrow xy$ satisfying

1. K is a commutative group under addition
2. Multiplication is associative
3. Two distributive laws hold

n - LINEAR:

Let K be a commutative ring with identity, n a positive integer and D be a function which assigns to each $n \times n$ matrix A over K a scalar D(A) in K. D is n-Linear if for each i, $1 \leq i \leq n$, D is a linear function of the i^{th} row when the other (n-1) rows are held fixed

ALTERNATING:

Let D be an n-lineaα r functional. D is alternating if

- D(A) = 0 whenever two rows of A are equal
- If A' is a matrix obtained from A by interchanging two rows of A, then
  D(A') = -D(A)

DETERMINANT FUNCTION:

Let K be a commutative ring with identity and n be a positive integer. Suppose D is a function from v matrices over K into K. Then D is a determinant function if D is n-linear, alternating and D(I) = 1.

UNIT – IV

CHARACTERISTIC VALUE:

Let V be a vector space over the field F and T be a linear operator on V. A characteristic value of T is a scalar c in F such that there is a non-zero vector α in V with
Tα = cα .

CHARACTERISTIC VECTOR AND SPACE:

Let V be a vector space over the field F and T be a linear operator on V. If c is a characteristic value of T, then any α such that Tα = cα is called a characteristic vector of T and the collection of all α such that Tα = cα is called the characteristic space associated with c.

DIAGONALIZABLE:

Let T be a linear operator on the finite-dimensional space V. T is diagonalizable if there is a basis for V each vector of which is a characteristic vector of T.

MINIMAL POLYNOMIAL:

Let T be a linear operator on a finite-dimensional vector space V over the field F. The minimal polynomial for T is the unique monic generator of the ideal of polynomials over F which annihilates T.

CAYLEY-HAMILTON THEOREM:

Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, then f(T) = 0

UNIT – V

INVARIANT:

Let V be a vector space and T a linear operator on V. If W is a subspace of V, the W is invariant under T if for each vector α in W the vector Tα is in W, that is T(W0 is contained in W.

T-CONDUCTOR:

Let W be an invariant subspace for T and α be a vector in V. The T-conductor of α into W is the set $S_T(\alpha;W)$, which consists of all polynomials g such that g(T)α is in W.

INDEPENDENT:

Let $W_1,...W_k$ be subspaces of the vector space V. Then $W_1,...W_k$ are independent if $\alpha_1+...+\alpha_K=0,\alpha_i\in W_i$ implies that each α_i is 0.

DIRECT SUM:

The direct sum of subspaces $W_1,...W_k$ of the vector space V is $W=W_1\oplus...\oplus W_k$

PROJECTION:

If V is a vector space, then a projection of V is a linear operator E on V such that $E^2 = E$

NILPOTENT:

Let N be a linear operator on the vector space V. Then N is nilpotent if there is some positive integer r such that $N^r = 0$