BBA- OPERATIONS RESEARCH

UNIT-1

Linear Programming Formulation

1.1 Introduction

Linear programming problem *deals with the optimization (Maximization or Minimization) of a function of decision variables (The variables whose values determine the solution of a problem are called* ***decision variables*** *of the problem) known as* ***objective function*** *subject to a set of simultaneous linear equations (or inequalities) known as* ***constraints****.*

(The term linear means that all the variables occurring in the objective function and the constraints are of the first degree in the problems under consideration and the term programming means the process of determining a particular course of action.

1.2 Requirements for employing LPP Technique :

1. There must be a well-defined objective function.
2. There must be alternative courses of action to choose.
3. At least some of the resources must be in limited supply, which give rise to constraints.
4. Both the objective function and constraints must be linear equations or inequalities.

1.3 Mathematical Formulation of L.P.P

If x_j (j= 1,2, ... , n) are the n decision variables of the problem and if the system is subject to m constraints, the general Mathematical model can be written in the form:

$$\text{Optimize } Z = f(x_1, x_2, \ldots x_n)$$

$$\text{subject to } g(x_1, x_2, \ldots x_n) \leq, =, \geq b_i, \ (i = 1,2, \ldots , m)$$

(called structural constraints)

and $x_1, x_2, ...x_n \geq 0$,

(called the non-negativity restrictions or constraints)

Procedure for forming a LPP Model :

Step 1: Identify the unknown decision variables to be determined and assign symbols to them.

Step 2: Identify all the restrictions or constraints (or influencing factors) in the problem and express them as linear equations or inequalities of decision variables.

Step 3: Identify the objective or aim and represent it also as a linear function of decision variables.

Step 4: Express the complete formulation of LPP as a general mathematical model.

We consider only those situations where this will help the reader to put proper inequalities in the formulation.

1. Usage of manpower, time, raw materials etc are always less than or equal to the availability of manpower, time, raw materials etc.

2. Production is always greater than or equal to the requirement so as to meet the demand.

Example 1: *A firm manufactures two types of products A and B and sells them at a profit of Rs.2 on type A and Rs.3 on type B. Each product is processed on two machines M_1 and M_2. Type A requires 1 minute of processing time on M_1 and 2 minutęs on M_2. Type B requires 1.minute on M_1 and 1 minute on M_2 Machine M_1 is available for not more than 6 hours 40 minutes while machine M_2 is available for10 hours during any working day. Formulate the problem as a LPP so as to maximize the profit.*

Solutlon :

Let the firm decide to produce x_1 units of product A and x_2 units of product B to maximize its profit. To produce these units of type A and type B products, it requires

$x_1 + x_2$ processing minutes on M_1

$2x_1 + x_2$ processing minutes on M_2

Since machine M_1, is available for not more than 6 hours and 40minutes and machine M_2 is available for 10 hours doing any working day, the constraints are

$$x_1 + x_2 \leq 400$$
$$2x_1 + x_2 \leq 600$$
$$and\ x_1, x_2 \geq 0.$$

Since the profit from type A is Rs. 2 and profit from type B is Rs. 3, the total profit is $2x_1 + 3x_2$. As the objective is to maximize the profit, the objective function is

$$\text{maximize } Z = 2x_1 + 3x_2$$

∴The complete formulation of the LPP is

$$\text{maximize } Z = 2x_1 + 3x_2$$

subject to the constraints

$$x_1 + x_2 \leq 400$$
$$2x_1 + x_2 \leq 600$$
$$and\ x_1, x_2 \geq 0.$$

Example 2: (Production Allocation Problem)

A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below:

Time per unit (minutes)	*Time per unit (minutes)*			*Machine capacity (Minutes/day)*
	Product 1	*Product 2*	*Product 3*	
M_1	*2*	*3*	*2*	*440*
M_2	*4*	*-*	*3*	*470*
M_3	*2*	*5*	*-*	*430*

It is required to determine the number of units manufactured for each product daily. The profit per unit for product 1, 2 and 3 is Rs.4, Rs.3 and Rs.6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical model for the problem.

Solution :

Let x_1, x_2 and x_3 be the number units of products 1, 2 and 3 produced respectively.

To produce these amount of products 1, 2 and 3, it requires:

$$2x_1 + 3x_2 + 2x_3 \text{ minutes on } M_1$$

$$4x_1 + 3x_3 \text{ minutes on } M_2$$

$$2x_1 + 5x_2 \text{ minutes on } M_3.$$

But the capacity of the machines $M_1, M_2 and\ M_3$ are 440,470 and 430 (minutes/day).

∴The constraints are

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 \leq 430$$

$$x_1, x_2, x_3 \geq 0.$$

Since the profit per unit for product 1, 2, and 3 is Rs.4, Rs. 3 and Rs.6 respectively, the total profit is $4x_1 + 3x_2 + 6x_3$. As the objective is to maximize the profit, the objective function is maximize $Z = 4x_1 + 3x_2 + 6x_3$

∴The complete formulation of the LPP is

$$\text{Maximize } Z = 4x_1 + 3x_2 + 6x_3$$
$$2x_1 + 3x_2 + 2x_3 \leq 440$$
$$4x_1 + 3x_3 \leq 470$$
$$2x_1 + 5x_2 \leq 430$$
$$x_1, x_2, x_3 \geq 0.$$

Example 3: (Blending Problem)

A firm produces an alloy having the following specifications:

(i) Specific gravity ≤ 0.98

(ii) Chromium $\geq 8\%$

(iii) Melting point $\geq 450°C$

Raw materials A, B and C having the properties shown in the table can be used to make the alloy.

Property	*Raw material*		
	A	*B*	*C*
Specific gravity	*0.92*	*0.97*	*1.04*
Chromium	*7%*	*13%*	*16%*
Melting point	*440°C*	*490°C*	*480°C*

Cost of the various raw materials per unit ton Rs. 90 for A, Rs. 280 for B and Rs. 40 for C. Find the proportions in which A, B and C be used to obtain an alloy of desired properties while the cost of raw materials is minimum.

Solution :

Let x_1, x_2 and x_3 be the tons of raw materials A, B and C to be used for making the alloy.

From these raw materials, the firm requires:

$$0.92x_1 + 0.97x_2 + 1.04x_3 \text{ specific gravity}$$

$$7_1 + 13x_2 + 16x_3 \quad \text{chromium}$$

$$440x_1 + 490x_2 + 480x_3 \text{ melting point.}$$

∴By the given specifications, the constraints are

$$0.92x_1 + 0.97x_2 + 1.04x_3 \leq 0.98,$$

$$7_1 + 13x_2 + 16x_3 \geq 8,$$

$$440x_1 + 490x_2 + 480x_3 \geq 450$$

$$\text{and } x_1, x_2, x_3 \geq 0.$$

Since the cost of the various raw materials per unit ton are Rs. 90 for A, Rs. 280 for B and Rs. 40 for C, the total cost is $90x_1 + 280x_2 + 40x_3$. As the objective is to minimize the total cost, the objective function is

Minimize $Z = 90x_1 + 280x_2 + 40x_3$.

∴The complete formulation of the LPP is

$$\text{Minimize } Z = 90x_1 + 280x_2 + 40x_3$$

subject to

$$0.92x_1 + 0.97x_2 + 1.04x_3 \leq 0.98,$$

$$7_1 + 13x_2 + 16x_3 \geq 8,$$

$$440x_1 + 490x_2 + 480x_3 \geq 450$$

$$\text{and } x_1, x_2, x_3 \geq 0.$$

Example 4: (Diet Problem)

A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table:

Food type	*Yield/unit*			*cost/unit(Rs)*
	Proteins	*Fats*	*carbohydrates*	
1	*3*	*2*	*6*	*45*
2	*4*	*2*	*4*	*40*
3	*8*	*7*	*7*	*85*
4	*6*	*5*	*4*	*65*
Minimum requirement	*800*	*200*	*700*	

Formulate the L.P model for the problem.

Solution :

Let x_1, x_2, x_3 and $x_4$be the units of food of type 1,2,3 and 4 used respectively.

From these units of food of type 1, 2, 3 and 4 he requires

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \text{ Proteins/day}$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \text{ Fats / day}$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \text{ Carbohydrates/day}$$

Since the minimum requirement of these proteins, fats and carbohydrates are 800, 200 and 700 respectively, the constraints are

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$

$$and\ x_1, x_2, x_3, x_4 \geq 0.$$

Since, the costs of these food of type 1. 2. 3 and 4 are Rs. 45, Rs.40, Rs.85 and Rs. 65 per unit, the total cost is Rs. $45x_1 + 40x_2 + 85x_3 + 65x_4$. As the objective is to minimize the total cost, the objective function is

$$\text{Minimize } Z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

∴The complete formulation of the L.P.P is

$$\text{Minimize } Z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

subject to

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$

$$and\ x_1, x_2, x_3, x_4 \geq 0.$$