operation Research

UNIT - I

operation research - Concept - Models - scope - phases - Limitations - operations research and Decision Making - Linear programming problem formulation of L.P.P - Graphical Method.

UNIT - II

Transportation problem : North west Corner rules - Least Cost method - vogel's approximation Method.

UNIT - III

Inventory Control : Categories of Inventory - reasons for Carrying Inventory - Costs and terms associated with Inventory - Determindistic and probabilistic Inventory problem.

UNIT - IV

Assignment problem : solving assignment problem - Travelling salesman model - maxima, and method - Hungarian method.

UNIT - V

Replacement decisions : Replacement of equipement the deteriorates gradually - replacement of equipement that fails suddenly.

Marks : Theory 20% & problem 80%.

Text Book Recommended

=> operation Research : kanti swarup, P.K Gupta and Man Mohan, sultan chand.

=> operation research : S. kalavathy, vikas, publishing House private limited.

Book for Reference :

1) Quantitative Techniques - C.R kothari, vikas publishing house.

2) Quantitative Techniques - for decision Making Anand sharma, Himalaya publishing house.

UNIT- II

Transportation problem :

The Transportation problem deals with the transportation of a single product from different origins to serval demand.

Let there be m sources $S_1, S_2 \ldots S_m$ with a_i $(i = 1, 2, \ldots n)$ available supplies or capacity at each source, to be allocated among n destinations $D_1, D_2, \ldots D_n$ with b_j $(j = 1, 2, \ldots n)$ specified requirements at each destinations j. let c_{ij} be the cost of shipping one unit from source i to destinations j for each route. Then if x_{ij} be the units shipped per route from source i to destinations j, the problem is to determine the transportation schedule so as to minize the total transportation Cost satisfying the supply and demand Conditions.

Mathematically, the problem may be states as follows :

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to the constraints

$$x_{i1} + x_{i2} + \dots + x_{in} = a_i \; ; \; i = 1, 2, \dots n$$

$$x_{1j} + x_{2j} + \dots x_{mj} = b_j \; ; \; j = 1, 2, \dots n$$

and $x_{ij} \geq 0$ for all i and j for a ~~feasible~~ feasible solutions to exits.

its is necessary that total supply equals total requirement (ie) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$]

Initial basic feasible solutions :

An Initial basic feasible solutions can be constructed by selecting the $(m+n-1)$ basic variables (allocations or occupied cells) x_{ij} one at a time After each selection, we assign a value to that varible so as to satisfy a linear constraint There, are severable methods available to obtain an intial basic feasible solutions.

North - West Corner Method :

step : 1

starting with the cell at the upper left (North - west) Corner of the Transportation matrix, we allocate as much as possible so that either the capacity of the 1st row is exahausted or the destinations requirement of the 1st column is statified (i.e) $x_{11} = \min(a_1, b_1)$.

step : 2 :

If $b_1 > a_1$ we move down vertically to the 2nd row and make 2nd allocation of the magnitude $x_{12} = \min(a_1 - a; b_1) = 0$ in the cell (1,2) (or) $x_{21} = \min(a_2 \cdot b_1 - b_1) = 0$ in the cell (2,1).

Least - Cost Method :

step : 1

Determine the smallest cost in the Cost matrix of the transportation table. Let be c_{ij} allacate $x_{ij} = \min(a_i; b_j)$ in the cell (i,j)

step : 2

If $x_{ij} = a_1$ Cross of the ith columns

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of the transportation Table and decrease a_i by b_j. go to step 3.

If $x_{ij} = a_i = b_j$ Cross of either the i^{th} row or the j^{th} columns but not both.

Step 3:

Repeat step 1 & 2 for the resulting seduced transportation table until all the sim requirements are satisfied Conever the minimum is not unique make an orbitary choice among tho minima.

Vogel's approximation method:

step : 1

calculate penalties by taking differences b/w the minimum and next to minimum unique, transportation Cost in each row. and each column.

Step : 2

circle the largest row difference are or column difference In the event of the choice either.

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step : 3

allocate as much as possible in the lowest cost cell of the row or column having a circled row or coloumn difference.

step : 4

In case the allocation is made fully to the row (column) ignore that row column for the consideration by crossing it.

step : 5

Revcies the difference again & cross out the earlier figures. Go to step 2

step : 6

Continue the produce until all rows & columns have been crossed out i.e) distribution is complete.

Transportation Algoritham (Modi Method)

various steps involved in solveing any transportation problem may be summarized in the following.

procedure :

step : 1

Find the initial basic feasible solution by using any of the three methods discussed about.

Step: 2

check the number of occupied cells. If there are less than $m+n-1$, there exist decenercy and be introduce a very small +ve assoginment of ϵ (≈ 0), In suitable independent position so that the ~~values~~ Numbers of occupied cells is exactly equal to $m+n-1$.

Step: 3 For each occupied cells in the current solutions solve the system of equation $u_i + v_j = c_{ij}$ starting Initinally with $u_1 = 0$ & $v_i = 0$ and entering successfully the values of u_i and v_j in the transporations table Margins.

Step 4: Compute the net evolutations $z_{ij} - c_{ij} = u_i + v_j - c_{ij}$ for all an occupied basic cells and entre them in the upper wright corner of the corresponding cells.

Step: 5

Examine the sign of each $z_{ij} - c_{ij}$. If for all $z_{ij} - c_{ij} \le 0$, then the current basic feasible solution is an optain one. If atleast one $z_{ij} - c_{ij} > 0$, select the unoccupied cells having the largest +ve not evaluation to entire the basic.

step: 6 Let the unoccupied cells (R,S) entire the basic, allocate unkown quantity, say, to the cells (R,S) identityfy loop that starts and ends at the cells (R,S) and connects the some of the basic cells, Add & subbract interchangebly, θ_2 to and from the transition of the loop in search a way that the rim requirements remains staisfied

step: 7 Assign a max value to in such a way that the values of to in such a way that the values of one basic variable becomes 0. and the others basic variable remain non-ve. the basic cell whose allocation has been reduced to 0, leaves the basic.

step: 8 Return to step 3 & repeat the process until an optimum basic feasible soln has

North West Corner Method

problems :

1. To Find the initial basic Feasible solution to the Transportation problem using North West Corner method.

	D	E	F	G	
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
requirement	200	225	275	250	

$\Sigma ai = 950$

$\Sigma bj = 950.$

soln :

since $\Sigma ai = 950.$

$\Sigma bj = 950$

$\Sigma ai = \Sigma bj = 950.$

The Given broblom is balanced.

200	11	13	17	14	250
	16	18	14	10	300
	21	24	13	10	400
	200	225	275	250	

50	13	17	14	50
	18	14	10	300
	24	13	10	400
	225	275	250	

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175	18	14	10	300
	24	13	10	400
	175	275	250	

125	14	10	1250
	13	10	400
	275	250	

150	13	10	400
	150	250	

250	10	250
	250	

$m + n - 1 =$ occupied cells.

$3 + 4 - 1 = 6$ m = Row

$7 - 1 = 6$ n = colum

$6 = 6$ The non degenerate solu

Total Initial Basic feasible solu (I BFS) $= (200 \times 11) + (50 \times 13) + (175 \times 18) + (125 \times 14) + (150 \times 13) + (250 \times 10)$

$=$

$= 12200$

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problems :

2) Find the IBFS using NWCM

	D_1	D_2	D_3	Supply
F_1	5	4	3	20
F_2	2	8	6	10
F_3	4	7	9	15
F_4	3	1	4	5
Demand	25	18	7	

Soln :

since $\Sigma a_i = 50$

$\Sigma b_j = 50$

$\Sigma a_i = \Sigma b_j = 50$.

The Given problem is balanced.

20	5	~~4~~	~~3~~	20
	2	8	6	10
	4	7	9	15
	3	1	4	5
	25	18	7	

5	2	8	6	10
	4	7	9	15
	3	1	4	5
	5	18	7	

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5	8	6	15
	7	9	15
	1	4	5
	18	7	

13	7	9	15
	1	4	5
	13	7	

2	9	2
	4	5
	7	

5	4	5
	5	

$m+n-1 =$ occupied cells.

$4+3-1 = 6$

$6 = 6$

The non degenerate soln

Total IBFS $= (20\times5)+(5\times2)+(5\times8)+(13\times7)+(2\times9)+(5\times4)$

$=$

$= 279$

3. Find IBFS using NWCM:

	D_1	D_2	D_3	D_4	
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Requir	6	10	15	4	

Since $\Sigma a_i = 35$

$\Sigma b_j = 35$.

$\Sigma a_i = \Sigma b_j = 55$

The Given problem is balanced.

6	6	4	1	5	14
	8	9	2	7	16
	4	3	6	2	5
	6	10	15	4.	

				8
8	4	~~1~~ 5		8
	9	2	7	16
	3	6	2	5
	10	15	4.	
	2			

				14
2	9	2	7	16
	3	6	2	5
	2	15	4	

14	2	~~7~~	14
	6	2	5.
	15	4	
	1		

			4
1	6	~~2~~	5.
	~~15~~ 1	4	

2	4
4	

$m + n - 1 =$ occupied. cells

$3 + 4 - 1 = 6$

$6 = 6$.

The non degenerate soln

Total IBFS $= (6 \times 6) + (8 \times 4) + (2 \times 9) + (14 \times 2) + (1 \times 6) + (4 \times 2)$

$= 36 + 32 + 18 + 28 + 6 + 8$

$= 128$.

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4 Find the IBFS Using NWCM.

	W_1	W_2	W_3	W_4	Supply
F_1	10	2	20	11	10
F_2	1	17	9	20	5
F_3	5	14	16	18	15
Demand	5	10	8	7	

Since $\sum a_i = 30$

$\sum b_j = 30$

$\sum a_i = \sum b_j = 30$

The Given problem is balanced.

5	10	2	20	11	~~10~~ 5
	1	17	9	20	5
	5	14	16	18	15
	5	10	8	7	

5	2	20	11	5
	17	9	20	5
	14	16	18	15
	~~10~~ 5	8	7	

5	17	9	20	5
	14	16	18	15
	5	8	7	

0 ✗	9	20	0
	16	18	15
	8	7	

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8	16	18	15.
	8	7	

18	7
7	

$m+n-1 = \text{occupied cells}$

$3+4-1 = 6$

$6 = 6.$

Non degenerate solu

Total IBFS $= (5\times10)+(5\times2)+(15\times17)+(0\times9)+(8\times16)+(18\times7)$

$= 50+10+85+0+128+126$

$= 399.$

Least cost Method [LCM] (or) Matrix Minima Method [MMM]

1) Find the IBFS using LCM supply.

	I	II	III	supply
O_1	5	7	2	10
O_2	3	9	12	18
O_3	4	15	10	17
O_4	8	3	6	22
Demand	15	20	32	

Since $\Sigma a_i = 67$

$\Sigma b_j = 67$

$\Sigma a_i = \Sigma b_j = 67$ The given problem is balanced.

5	7	20 $\boxed{10}$	10
3	9	12	18
4	15	10	17
8	3	6	32
15	20	~~32~~ 22	

15	3	9	12	18 3
	4	15	10	17
	8	3	6	22
	~~15~~	20	22	

	9	12	3
	15	10	17
20	3	6	~~22~~ 2
	~~20~~	22	

	12	3
	10	17
2	6	~~2~~
	~~22~~ 20	

	12	3
17	10	17
	~~20~~ 3	

3	12	~~3~~
	~~3~~	

$m+n-1 =$ occupied cells.

$4+3-1 = 6$

$7-1 = 6$

$6 = 6$ Non degenerate soln.

Total IBFS = (2×10) + (3×15) + (3×20) + (2×6) + (10×17) + (3×12).

= 343.

② Find IBFS using MMM.

	D_1	D_2	D_3	D_4	Supply
A	6	4	1	5	14
B	8	9	2	7	16
C	4	3	6	2	5
Demand	6	10	15	4	

Since $\Sigma a_i = 35$

$\Sigma b_j = 35$

$\Sigma a_i = \Sigma b_j = 35$ The given problem is balanced.

6	4	1 (14)	5	14
8	9	2	7	16
4	3	6	2	5
6	10	15	4.	

8	9	2 (1)	7	16 15
4	3	6	2 (4)	5
6	10	1	4.	

8	9	7	15
4	3	2 (4)	~~5~~, 1
6	10	~~4~~	

8	9	15
4	3 (1)	~~1~~
6	10 9	

6	8	9	15
	6	9	

9	9
9.	

m+n-1 = occupied cells.

3+4 -1 = 6

7-1 = 6

6 = 6

Non degenerate soln.

Total IBFS = (14×1) + (1×2) + (2×4) + (3×1) + (6×8) + (9×9).

= 14 + 2 + 8 + 3 + 48 + 81.

= 156.

3. Find the IBFS using MMM or LCM

	D	E	F	G	
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requ	200	225	275	250	

Since Σai = 95

Σbj = 950.

$\Sigma ai = \Sigma bj$ = 950

The given problem balanced.

200	11	13	17	~~250~~ 50
	16	18	14	300
	21	24	13	150
	200	225	275	

50	13	17	~~50~~
	18	14	300
	24	13	150
	~~225~~ 175	275	

18	14	300
24	13 (150)	~~150~~
175	~~275~~ 125	

18	14 (125)	~~300~~ 175
~~175~~	~~125~~	

175	$\boxed{18}$	~~175~~
	~~175~~	

m + n - 1 = occupied cells

3 + 4 - 1 = 6

6 = 6   Non degenerate soln.

Total IBFS = (250×10) + (11×200) + (50×13) + (150×13) + (14×125) + (18×175)

= 2500 + 2200 + 650 + 1950 + 1750 + 3150

= 12200.

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Vogel's Approximation Method (VAM)
(or) unit cost penalty method (UCPM)

1) Determine IBFS using Vogel's Approximation
Ex Method :

	D_1	D_2	D_3	D_4	supply
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
demand	6	10	15	4	

Since $\sum a_i = 35$
$\sum b_j = 35$

$\sum a_i = \sum b_j = 35$

The given problem in balanced.

6	4	1	5	14	(3)
8	9	15 2	7	16	(5)
4	3	6	2	5	(1)
6	10	~~15~~	4		
(2)	(1)	(1)	(3)		

6	4	5	14	(1)
8	9	7	1	(1)
4	3	4 2	5 1	(1)
6	10	4		
(2)	(1)	(3)		

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6	4	14	(2)
8	9	1	(1)
[1] 4	3	1	(1)
5 6	10		
(2)	(1)		

6	[10] 4	14	(2)
8	9	1	(1)
5	10		
(2)	(5)		

6	4	(6)
[1] 8	1	(8)
5		
(2)		

4 | 6 | 4

4

$m+n-1$ = occupied cells

$3+4-1 = 6$

$6 = 6.$

Non degenerate soln.

Total IBFS = $(2\times15)+(4\times2)+(4\times1)+(4\times10)+(1\times8)+(4\times6)$

$= 114.$

Vogel's Approximation Method (VAM)

1) Determine IBFS using UCPM

	D	E	F	G	Avaliable
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

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$\Sigma b_j = 950$

$\Sigma a_i = \Sigma b_j = 950$

The given problem is balanced.

$\boxed{200}$ 11	13	17	14	~~250~~ 50	(2)
16	18	14	10	300	(4)
21	24	13	10	400	(3)
200	225	275	250		
(5)	(5)	(1)	(0)		

$\boxed{50}$ 13	17	14	~~50~~	(1)
18	14	10	300	(4)
24	13	10	400	(3)
175 ~~225~~	275	250		
(5)	(1)	(0)		

$\boxed{175}$ 18	14	10	~~300~~ 125	(4)
24	13	10	400	(3)
175	275	250		
(6)	(1)	(0)		

14	$\boxed{125}$ 10	~~125~~	(4)
13	10	400	(3)
275	~~250~~ 125		
(1)	(0)		

275	13	10	~~400~~ 125 (3)
	~~275~~	125	
	(13)	(10)	

m+n-

3+L

NO

Tota

3. Find

Dem

Soln

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125 | 10 | 125

125

$m+n-1$ = occupied cells.

$3+4-1 = 6$

$6 = 6$

Non degenerate soln

Total IBFS $= (200 \times 11) + (50 \times 13) + (175 \times 18) + (10 \times 125) + (275 \times 13) + (125 \times 10)$

$= \cancel{2200}\ 12{,}075$

3). Find the IBFS using (VAM)

	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

soln: Since $\Sigma a_i = 34$

$\Sigma b_j = 34$

$\Sigma a_i = \Sigma b_j = 34$

The given problem is balanced

8	2	7	4	5	(2)
	3	3	1 [8]	~~8~~	(2)
	5	4	7	7	(1)
	1	6	2	14	(1)
	7	9	~~18~~ 10		
	(1)	(1)	(1)		

$\boxed{5}$ 2	7	4	~~5~~	(2)
5	4	7	7	(1)
1	6	2	14	(1)
~~7~~ 2	9	10		
(1)	(2)	(2)		

5	4	~~7~~	7	(1)
1	6	$\boxed{10}$ 2	14 4	(1)
2	9	~~10~~		
(4)	(2)	(5)		

5	4	7	(1)
$\boxed{2}$ 1	6	~~4~~ 2	(5)
2	9		
(4)	(2).		

4	7	(4)
$\boxed{2}$ 6	~~2~~	(6)
~~9~~ 7		
(2).		

7	4	~~7~~
	~~7~~	

$m+n-1$ = occupied cells

$4+3-1 = 6$

$7-1 = 6$

$6 = 6$

NON degenerate solution

Total IBFS $= (8\times1)+(5\times2)+(10\times2)+(2\times1)(2\times6)$
$+ (4\times7)$

$= 80$

Unbalanced Transportation problem

Determine the IBFS using Transportation problem:

				Sup
4	6	8	13	500
13	11	10	8	700
14	4	10	13	300
9	11	13	3	400
Demand 250	350	1050	200	

soln:

Since $\sum a_i = 1900$

$\sum b_j = 1850$.

$\sum a_i \neq \sum b_j$

$1900 \neq 1850$.

The given problem in unbalanced

4	6	8	13	0	500
13	11	10	8	0	700
14	4	10	13	0	300
9	11	13	3	0	400
250	350	1050	200	50	

$\sum a_i = 1900$

$\sum b_j = 1900$. balanced

1) NWCM:

250 4	6	8	13	0	250 ~~500~~
13	11	10	8	0	700
14	4	10	13	0	300
9	11	13	3	0	400

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250	6	8	13	0	250
	11	10	8	0	700
	4	10	13	0	300
	11	13	3	0	400
	350	1050	200	50	
	100				600.

100	11	10	8	0	700
	4	10	13	0	300
	11	13	3	0	400.
	100	1050	200	50.	

600	10	8	0	600
	10	13	0	300
	13	3	0	400.
	1050	200	50	
	450.			

300	10	13	0	300
	13	3	0	400.
	450	200	50	
	150			

				250
150	13	3	0	400
	150	200	50	

200	3	0	250.
	200	50	

50	0	50
	50	

$m + n - 1$ = occupied calls

$4 + 5 - 1 = 8$

$8 = 8$

Non degenerate solution

Total IBFS $= (250\times4)+(250\times6)+(100\times11)+(600\times10)+(300\times10)$
$+ (150\times13) + (200\times3) + (50\times0)$

$= 1000 + 1500 + 1100 + 6000 + 3000 + 1950 + 600$

$= 15150.$

Non Degenerate

ii) Lest Cost Method [MMM] or [LCM]

4	6	8	13	0	500
13	11	10	8	0	700
14	4	10	13	0	300
9	11	13	3	0	400
250	350	1050	200	50	

$\Sigma a_i = 1900$

$\Sigma b_j = 1900$

$\Sigma a_i = \Sigma b_j$

4	6	8	13	0	500
13	11	10	8	0	700
14	4	10	13	0	300
9	11	13	3	0	~~400~~ 350
250	350	1050	200	50	

4	6	8	13	500
13	11	10	8	700
14	4	10	13	300
9	11	13	3 (200)	~~350~~ 150
250	350	1050	200	

4	6	8	500
13	11	10	700
14	4 (300)	10	300
9	11	13	150
250	~~350~~ 50	1050	

4 (250)	6	8	~~500~~ 250
13	11	10	700
9	11	13	150
250	50	1050	

6 (50)	8	250
11	10	700
11	13	150
50	1050	

8	~~200~~
10	~~700~~
13	150
~~1050~~ 850	

$= (0\times50) + (3\times200) + (4\times300) + (4\times250) + (6\times50) + \ldots$

10	700
13	150

850

13	150

150

$m+n-1 =$ occupied

$4+5-1 = 8$

$8 = 8$

Non degenerate

$= (0\times 50) + (3\times 200) + (4\times 300) + (4\times 250) + (6\times 5$

$(8\times 200) + (10\times 700) + (13\times 150)$

$= 13650$

Vogels Approximation Method :

4	6	8	13	0	500	(2)
13	11	10	8	0	700	(2)
14	4	10	13	0	300	(3)
9	11	13	3	0	400	(6)
250	350	1050	200	50		
(5)	(2)	(5)	(5)	(0)		

4	6	8	13	500	(
13	11	10	8	700	(
14	4	10	13	300	(
9	11	13	3	350	(6
250	350	1050	200		
(5)	(2)	(5)	(5)		

4	6	8	500	(2)
13	11	10	700	(3)
14	4	10	300	(6)
9	11	13	150	(2)
250	350	1050		
(5)	(2)	(5)		

4	6	8	500	(4)
13	11	10	700	(1)
9	11	13	150	(4)
250	50	1050		
(5)	(5)	(2)		

6	8	250	(2)
11	10	700	(1)
11	13	150	(2)
50	1050		
(5)	(2)		

8	200	(8)
10	700	(10)
13	150	(13)
1050		
(2)		

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8 200

~~700~~ 10 ~~700~~

900 .

200 | 8 | 200

200

$M+n-1$ = occupied cell .

$8 = 8$

Non degenerate

Total IBFS = $(0\times50) + (3\times200) + (4\times300) + (4\times250)$
$+ (6\times50) + (13\times150) + (10\times700) + (200\times8)$

$= 600 + 1200 + 1000 + 300 + 1950 + 7000 + 1600$

$= 13650$.

UNBALANCED TRANPORTATION

Problem :

If the given Transportation problem is unbalanced one that is if $\Sigma a_i \neq b_j$ Then Convert into a balanced one by Introducing a dummy source or dummy destination with zero cosets vector as the case may be and then solved by usual method.

when the total supply is greater than (the total demand, A dummy destination is Included in the Matrix with zero Cosets vector) the excess supply dummy destination.

when the total demand is (greater than total suppy A dummy destination is include in the Matrix with zero Cosets vectors). the excess demand is ended as the rim requirement for the dummy destination

2. determine the IBFS using Transportation problem :

6	1	9	3	70
11	5	2	8	55
10	12	4	7	75
85	35	50	45.	

since $\Sigma a_i = 200$

$\Sigma b_j = 215$

The given problem in unbalanced add the dummy destination whice zero vectors in a Column.

6	1	9	3	70.
11	5	2	8	55
10	12	4	7	75
0	0	0	0	15
85	35	50	45.	

$\Sigma a_i = 215$

$\Sigma b_j = 215$

$\Sigma a_i = \Sigma b_j$

The given problem is balanced.

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6 (70)	1	9	3	~~70~~
11	5	2	8	55
10	12	4	7	75
0	0	0	0	15
15 ~~85~~	35	50	45	

11 (15)	5	2	8	~~55~~ 40
10	12	4	7	~~75~~ 75
0	0	0	0	15
15	35	50	45	

5 (35)	2	8	40 5
12	4	7	75
0	0	0	15
35	50	45	

2 (5)	8	5
4	7	75
0	0	15
~~50~~ 45	45	

4 (45)	7	~~75~~ 30
0	0	15
45	45	

7	~~30~~
0	15
~~45~~ 15	

0	15
15	

$m+n-1 =$ occupied cells

$4+4-1 = 7$

$8-1 = 7$

$7 = 7$

Total IBFS $= (70\times6) + (11\times15) + (35\times5) + (2\times5) + (4\times45) + (7\times30) + (0\times15)$

$= 1,160.$

i) Least Cost Method :

6	1	9	3	70
11	5	2	8	55
10	12	4	7	75
~~0~~ (15)	~~0~~	~~0~~	0	15
85 70	35	50	45	

6	1 (35)	9	3	~~70~~ 35
11	5	2	8	55
10	12	4	7	75
70	35	50	45	

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11	8	5
10	7	~~75~~ 65
70	10	

11	5
10	65
70 5	

11	5
5	

m+n-1 = occupied cells

4+4-1 = 7

7 = 7

Non degenerate

Total IBFS = $(15\times0)+(1\times35)+(2\times50)+(3\times35)+$
$(7\times10)+(10\times65)+(11\times5)$

$= 1015$

Vogel's approximation :

6	1	9	3	70	(2)
11	5	2	8	55	(3)
10	12	4	7	75	(3)
~~0~~	0	0	0	15	(0)
~~70~~ 85	35	50	45		
(6)	(1)	(2)	(3)		

6	1	9	3	~~70~~ 35	(2)
11	5	2	8	55	(3)
10	12	4	7	75	(3)
70	35	50	45		
(4)	(4)	(2)	(4)		

6	9	3	35	(3)
11	5	8	55	
10	12	4	75	
70	35	50		
(4)	(4)	(		

6	35 1	9	3	85 70	(2)
11	5	2	8	55	(3)
10	12	4	7	75	(3)
70	35	50	45		
(4)	(4)	(2)	(4)		

6	9	3	35
11	2	8	55
10	4	7	75
70	50	45	
(4)	(2)	(4)	

~~6~~	~~3~~	~~35~~ (3)
11	8	5 (3)
10	7	75 (3)
70	~~45~~ 10	
(4)	(4)	

11	8	5 (3)
10	7	75 (3) 65
70	10	
(1)	(1)	

11	5 (11) 65
10	65 (10)
70 65	

10 65
65

m+n-1 = occupied cells

4+4-1 = 7

7 = 7

Non degenerate

Total IBFS $= (15\times0)+(1\times35)+(2\times50)+(3\times35)+$
$(7\times10)+(11\times5)+(65\times10)$

$= 1015$

Modi Method (or) Modified distribution Method (or) (Test for optimal solution)

1. Find the optimal soln of the following TP problem :

	1	2	3	4	supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	

Soln :

Since $\Sigma a_i = 43$

$\Sigma b_j = 43$.

~~21~~	~~16~~	~~25~~	$\overset{11}{13}$	11	(3)
17	18	14	23	13	(3)
32	27	18	41	19	(9)
6	10	12	~~15~~ 4		
(4)	(2)	(4)	(10)		

17	18	14	$\overset{4}{23}$	13	(3)
32	27	18	41	19	(9)
6	10	12	4		
(15)	(9)	(4)	(18)		

$\overset{6}{17}$	18	14	9	(3)
~~32~~	27	18	19	(9)
6	10	12		

3	18	14	3 (4)
	27	18	19 (9)
	10 (9)	12 (4)	

7	27	18	19 (9)
	7	12	

(2) (18).

12	18	12
	12	

m+n-1 = occupied cells

3+4-1 = 6

6 = 6.

Non degenerate

Total ~~I~~ BFS = (11×13) + (4×23) + (6×17) + (3×
+ (7×27) + (12×18)

= 796

21	16	25	13 [11]	$u_1 = -10$
[6] 17	[3] 18	14	23 [4]	$u_2 = 0$
32	[7] 27	[12] 18	41	$u_3 = 9$.

$V_1 = 17$ $V_2 = 18$ $V_3 = 9$. $V_4 = 23$.

For the occupied cells.

(1,4). (2,1) (2,2). (2,4), (3,2), (3,3).

$u_2 = 0$.

(2,1)

$\boxed{C_{ij} = u_i + v_j}$

$C_{21} = u_2 + v_1$

$17 = 0 + v_1$

$v_1 = 17$

(2,2)

$C_{22} = u_2 + V_2$

$18 = 0 + V_2$

$\boxed{V_2 = 18}$

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(2,4)

$C_{24} = u_2 + V_4$

$23 = 0 + V_4$

$\boxed{V_4 = 23}$

(3,2)

$C_{32} = u_3 + V_2$

$27 = u_3 + 18$

$\boxed{u_3 = 9}$

(3,3)

$C_{33} = u_3 + V_3$

$18 = 9 + V_3$

$\boxed{V_3 = 9}$

(1,4)

$C_{14} = u_1 + V_4$

$13 = u_1 + 23$

$\boxed{u_1 = -10}$

For the unoccupied cells :

$$\boxed{d_{ij} = C_{ij} - (u_i + v_j)}$$

$d_{11} = C_{11} - (u_1 + v_1).$

(1,1) $d_{11} = \cancel{2}\ 21 - (-10+17)$

$= 21 - (-7)$

$\boxed{d_{11} = 14}$

(1,2) $d_{12} = C_{12} - (u_1 + V_2)$

$= 16 - (-10+18)$

$\boxed{d_{12} = 8}$

(1,3) $d_{13} = C_{13} - (u_1 + v_3)$

$= \cancel{26}\ 25 - (-10+9).$

$= 25 - (-1)$

$\boxed{d_{13} = 26}$

(2,3) $d_{23} = C_{23} - (u_2 + V_3)$

$= 14 - (0+9)$

$\boxed{d_{23} = 5}$

(3,1)
$d_{31} = C_{31} - (u_3 + v_1)$

$= 32 - (9 + 17)$

$= 32 - 26$

$\boxed{d_{31} = 6.}$

(3,4)
$d_{34} = C_{34} - (u_3 + v_4)$

$= 41 - (9 + 23)$

$= 41 - (32)$

$\boxed{d_{34} = 9}$

since all $d_{ij} > 0$ the soln is optimal and unique.

T P Cost $= (11 \times 13) + (4 \times 23) + (6 \times 17) + (3 \times 18) + (7 \times 27) + (12 \times 11)$

$= 796.$

2. find the optimal solution for the followi T.P. problem:

7	3	2	2
2	1	3	3
3	4	6	5
4	1	5	

since $\Sigma a_i = 10$
$\Sigma b_j = 10.$

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7	3	2	2	(1)
2	$\boxed{1}$ 1	3	3	(1)
3	4	6	5	(1)
4	1	~~5~~ 3		
(1)	(2)	(1)		

2	1	3	3	(1)
3	4	6	5	(1)
4	1	3		
(1)	~~(3)~~	(3)		

7	$\boxed{2}$ 2	2	(5)
2	3	2	(1)
3	6	5	(3)
4	~~5~~ 3		
(1)	(1)		

2	$\boxed{3}$ 3	2	(1)
3	6	5	(3)
4	3		
(1)	(3)		

2

3	6	5	(3)
4	1		
(3)	(6)		

3	4
4	

$m+n-1$ = occupied cell.

$3+3-1 = 5$

$5 = 5$. Non degenerate

Total IBFS $= (2\times 2) + (1\times 1) + (2\times 3) + (6\times 1) + (3\times 4)$

$= 29$

7	3	2 $\boxed{2}$	2
2	$\boxed{1}$ 1	3 $\boxed{2}$	
$\boxed{4}$ 3	4	6 $\boxed{1}$	

For the occupied cells:

$(1,3)$, $(2,2)$, $(2,3)$, $(3,1)$, $(3,3)$

$v_3 = 0$.

$c_{ij} = u_i + v_j$

$\cancel{c_{31}} = \cancel{u_3 + v_1}$

$\cancel{4} =$

$c_{13} = u_1 + v_3$

$c_{ij} = u_i + v_j$

$c_{23} = u_2 + v_3$

$3 = u_2 + 0$

$u_2 = 3$

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$C_{ij} = u_i + v_j$

$C_{22} = u_2 + v_2$

$1 = 3 + 0$

$C_{31} = u_3 + v_1$

$4 = u_3 + v$

For occupied cells:

(1, 3) (2, 2) (2, 3) (3, 1) (3; 3)

$V_3 = 0$.

$C_{ij} = u_i + v_j$

$C_{13} = u_1 + v_3$

$2 = u_1 + 0$

$u_1 = 2$

$C_{23} = u_2 + v_2$

$C_{23} = u_2 + V_3$

$3 = u_2 + 0$

$3 = u_2$

$C_{22} = u_2 + V_2$

$1 = 3 + V_2$

$V2 = -2$

$C_{31} = u_3 + V_1$

$3 = 6 + V_1$

$V_1 = -3$.

$C_{33} = u_3 + V_3$

$6 = u_3 + 0$

$u_3 = 6$

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For unoccupied cells:

$d_{ij} = c_{ij} - (u_i + v_j)$

$(1,1)$; $(1,2)$, $(2,1)$, $(3,2)$.

$(1,1)$ $d_{11} = c_{11} - (2 + -3)$.

$= 7 - (2 + -3)$.

$= 7 - (-1)$.

$d_{11} = 8$

$(1,2)$

$d(1,2)$ $d_{12} = c_{12} - (2 \cancel{+3} -2)$

$= 3 - (0)$.

$= 3$.

$(2,1)$ $d_{21} = c_{21} - (3 + (-3))$.

$= 2 - (0)$

$d_{21} = 2$.

$(3,2)$ $d_{32} = c_{32} - (6 + -2)$.

$= 4 - 4$

$d_{32} = 0$

Since all $d_{ij} > 0$ with $d_{32} = 0$, the current solution is optimal and the exists an alternetive optimal solution Hence the optimum transportation.

$cost = (2 \times 2) + (1 \times 1) + (3 \times 2) + (3 \times 4) + (6 \times 1)$

$= 29$

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Transportation Aegorithm(or) MODI Method (Modified distribution Method) Test for optimal solution.

Step : 1

Find the initial basic feasible solution of the given problem by north - west Corner rule (or) least Cost Method (or) VAM.

Step : 2 check the number of occupied cell. If these are less than $m+n-1$ there exist degeneracy and we introduce a very small +ve assignment of ε (≈ 0) in suitable independent position. So that the number of occupied cells is exactly equal to $m+n-1$.

Step : 3

Find the set of value u_i, v_j ($i = 1, 2, 3 \ldots m$) $j = (1, 2, 3 \ldots m)$ from the reaction $c_{ij} = u_i + v_j$ for each occupied cell (i, j) by starting initially with $u_i = 0$ (or) $v_j = 0$ preferably for which the Corresponding row(or) Column has maximum no of individual allocation.

Step : 4

Find $u_i + v_j$ for each unoccupied cell(i,j) and enter at the upper right Corner of the corresponding (i, j)

Step : 5

Find the cell evaluation $d_{ij} = c_{ij} - (u_i + v_j)$ (d_{ij} = upper left - upper right) for each unoccupied all(i, j) and enter at the lower right Corner of the Corresponding.

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step : 6 :

Examine the cell evaluation d_{ij} for all unoccupied cell (i, j) and unclude that.

i) If all $d_{ij} > 0$ then the solution under the test is optimal and unique.

ii) If all $d_{ij} > 0$ which atleast one $d_{ij} = 0$ then the solution under the test is optimal and an alternative optimal solution exists.

iii) If atleast one $d_{ij} < 0$ then the solution is not optimal, Go to the next step.

step : 7

From a new BFS by given maximum allocation to the cell for which d_{ij} is most negative by making an occupied cell empty for that draw a closed path consisting of horizontal and vertical lines begining and ending its other corners at some allocated cells. Along this closed loop indicates $+\theta$ and $-\theta$ alternatively at the corners. choose minimum of the allocations from the cells having $-\theta$. Add this maximum allocation to the cells with $-\theta$.

step : 8

Repeat step (2) to (6) to test the optimality of this new basic feasible solution.

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Step : 9

Continue the above procedure till on optimum solution is attained.

Note : 1

The two set of Constraints will be consident if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

(total supply) . (total demand)

~~which for a transportation~~

which is the necessary and sufficient Condition for a transportation problem to have a feasible solutions problem satisfying this Condition are called balanced transportation.

problem :

Note : 2

If $\sum a_i \neq \sum b_j$, then the transportation problem is said to be unbalanced.

Definition : 1

A set of non-negative value x_{ij}, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots n$ that satisfied the Constraints (rim Conditions and also the non negative restrictions) is called a feasible solution. to the transportation problem.

Note :

A balanced transportation probelem will always have a feasible solution.

Definition : 2

A feasible solution to a $(m \times n)$ transportation problem that contains no more than $m+n-1$ non-negative allocation is called a basic feasible solution to the transportation problem.

Definition : 3

A basic feasible solution to a $(m \times n)$ transportation problem is said to be a non-negative basic feasible solution if it contains exactly $m+n-1$ non-negative allocations in independent position.

Definition : 4

A basic feasible solution that contains less than $m+n-1$ non-negative allocations is said to be a degenerate basic feasible solution.

Definition : 5

A feasible solution (non necessary basic) is said to be an optimal solution. If it minimizes the total transportation cost.

Definition : operation reserch (OR).

operation reserch is the art of winning without the war without actually finding by Authon. clark

Definition : 2 Lpp

OR is the systemetic application of quantity Methods, techincis and tools to the Analysis of problems involving. the operation of system.

Linear programming problem (LPP).

Linear programming problem deals. with the optimazation (Maximization or Minimization) of a function of decision variables (The variables whole values. determine the solution of a problem are called decision variable. of the. problem known as objective function; subject to a set of simultaneous. linear equations (or. inequalities). known as Constraints.

Mathematical function of LPP :

If x_j ($j = 1, 2, \ldots n$) are n decision variables - of the problems and if the system is subject to the m Constraints the general Mathematical Mode can be written in the form.

optimize $z = f(x_i, x_2, \dots x_n)$

Subject to $g_i(x_1, x_2, \dots x_n) \leq = \geq b_i \; (i = 1, \dots n)$

and $x_1, x_2, \dots x_n \geq 0$

procedure for forming a LPP Model

step : 1

Identify the unknown decision variable to be determined and asign symbols to them.

step : 2 :

Identify the all restrictions or unstraints in the problem and express them as linear equation or inequalities of decision variables.

step : 3

Identify the objective or aim and represent is also a linear function of decision variable.

step : 4.

Express the Complete formulation of LPP as a general Mathematical Models.

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Definition ; Recision :

The variable is $x_1, x_2, \dots, x_n$ all called docision variables.

objective function :

It is of each Lpp is expressed in teems of decision variables to optimise the objective such as profit, cost, distance, ... etc. In the above modal equation is objective function

Ex : max (or) min $z = cx$

Non - Negative restriction :

The set of inequalities from equati ③ in the general Lpp is called non - negative restrictions.

Eq : $x \geq 0$.

$x_1, x_2 \geq 0$

Definition : solution of Lpp :

A set of numerical value for the variables which satisfies the Constraints is called the solution of the Lpp.

Feasible solution : $(2, 1/2)$

A solution which satisfies the non - nogative restrictions is called feasible solution.

Infeasible solution : $(-2, -1/2)$

Infeasible solution is a solution which does not satisfies non - negative

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optimum Solution:

A feasible solution which optimizes a objective function is called optimum solution.

Eq: max = positive;
min = negative

Basic Solution:

Assume that there are m simultaneous linear equation with n variable [n > m]. solving this m equation for m variable by setting the remaining n-m variables equal to 0, gives a solution. This is called basic solution.

Ex: $x_1 + 2x_2 + x_3 = 4$ $n = 3$

$x_1 + x_2 + 5x_3 = 5$ $m = 2$

$n - m = 3 - 2$

$= 1$

$x_1 = 0 \quad 2x_2 + x_3 = 4$

$x_1 = 0 \quad x_2 + 5x_3 = 5$

Non-basic variable and basic variable:

The variable set equal to 0 in the basic solution is called non-basic variable. otherwise it is basic variable.

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Basic feasible solution:

Any basic solution which satisfies the non-negative restriction is called basic feasible solution.

Basic Infeasible solution:

Any basic solution which does not satisfies the non-negative restriction is called basic Infeasible solution.

Degenerate solution: any one value is (or)

A basic feasible solution of an LPP is set to degenerate if the value of atleast ~~or~~ the one of the basic value is 0

Eg: $x_1 = 2$; $x_2 = 0$; $x_3 = 3$.

Remark:

For a system of m equations with n variable [n > m], in ~~table~~ total number of basic feasible solution is [≤] ^nC_m

Eg: $^4C_2 \quad \frac{4 \times 3}{1 \times 2}$

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Constraint:

There are some limitations or Conditions on the use of resources [Ex: money machine, raw materials, space, etc...] that limit the degree to which an objective can be achived.

Such constraints must be expressed as linear equalities (or) inequalities ($=$, $\geq$, $\leq$) in terms of decision variables. In the above mode equation ⑤ is called Constraint.

Eg: $Ax \ (\leq, =, \geq) \ B$,

$3x_1 + 4x_2 \leq 5$.

$4 - \frac{1}{3}\frac{1}{2}$

$= 4/2$

problem:

① Find the basic solution for the system of linear equation:

$$x_1 + 2x_2 + x_3 = 4$$
$$x_1 + x_2 + 5x_3 = 5$$

Given $x_1 + 2x_2 + x_3 = 4 \rightarrow ①$

$x_1 + x_2 + 5x_3 = 5 \rightarrow ②$

$n = 3 \Rightarrow$ (variable) $\cdot x_1, x_2, x_3, x_4, x_5$

$m = 2 \Rightarrow$ eqn).

$n - m = 3 - 2 = 1$

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Case (i) $x_1 = 0$

$$① \Rightarrow 2x_2 + x_3 = 4 \rightarrow ③.$$
$$x_2 + 5x_3 = 5 \rightarrow ④.$$

$$2x_2 + x_3 = 4$$
$$④ \times 2 \Rightarrow 2x_2 + 10x_3 = 10$$
$$(-) \quad (-) \quad (-)$$
$$-9x_3 = -6.$$
$$x_3 = -6/-9 = 2/3.$$

$$④ \Rightarrow \cancel{2x} \quad x_2 + 5x_3 = 5$$
$$x_2 + 5(2/3) = 5$$
$$x_2 + \frac{10}{3} = 5$$
$$x_2 = 5 - 10/3$$
$$x = +5/3.$$

$x_1 = 0 \quad x_2 = +5/3 \quad x_3 = 2/3.$

~~In~~ feasible solution

ii) $x_2 = 0$

$$① \Rightarrow x_1 + x_3 = 4 \rightarrow ⑤$$
$$x_1 + 5x_3 = 5 \rightarrow ⑥$$

$$x_1 + x_3 = 4$$
$$x_1 + 5x_3 = 5.$$
$$(-) \quad (-) \quad (-)$$
$$-4x_3 = -1$$

$x_3 = 1/4 \quad x_2 = 0$

feasible solution

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$x_3 + 1/4$

(5) . $x_1 + 1/4 = 4$

$x_1 = 4 - 1/4$

$x_1 = \frac{15}{4}$.

$x_2 = 0, x_1 = \frac{15}{4}$,

feasible solution

case iii) $x_3 = 0$

$x_1 + 2x_2 = 4 \rightarrow (7)$

$x_1 + x_2 = 5. \rightarrow (8)$

$$\begin{array}{l} \cancel{x_1} + 2x_2 = 4 \\ \cancel{x_1} + x_2 = 5 \\ (-)\quad (-) \quad\quad (-) \\ \hline x_2 = -1. \end{array}$$

(8) $x_2 = -1$

~~x+5~~ $x_1 - 1 = 5$.

$x_1 = 5 + 1$

$x_1 = 6$

$x_1 = 6, x_2 = -1. \; x_3 = 0$

It is a Infeasible solution.

(2) $2x_1+6x_2+2x_3+x_4=3$.

$6x_1+4x_2+4x_3+6x_4=2$.

$x_1=0,\ x_2=0$
$x_1=0\ x_3=0$
$x_1=0\ x_4=0$
$x_2=0,\ x_3=0$
$x_2=0\ x_4=0$
$x_3=0\ x_4=0$.

$n=4$

$m=2$

$n-m=2$

$x_1=0,\ x_2=0$

$2x_1+6x_2+2x_3+x_4=3$

$6x_1+4x_2+4x_3+6x_4=2$

$1\times\cancel{6}$ $2x_3+x_4=3 \rightarrow (1)$

$4x_3+6x_4=2 \rightarrow (2)$

$\times 6$ $12x_3+\cancel{6x_4}=18$

$4x_3+\cancel{6x_4}=2$

$(-)\quad(-)\quad(-)$

$8x_3= \ =16$

$x_3=2$.

$x_1=0,\ x_3=2$

(1) $2\times 2+x_4=3$

$4+x_4=3$

$x_4=3/4$

$x_1=0,\ x_3=0$

$6x_2+x_4=3$

$4x_2+6x_4=2$

$6\times\ 6x_2+x_4=3 \rightarrow (3)$

$4x_2+6x_4=2 \rightarrow (4)$

$36x_2+\cancel{6x_4}=18$

$(-)4x_2(-)+\cancel{6x_4}(-)=2$

$32x_2=16$

$x_2=16/32$ $\quad x_2=1/2$

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$x_2 = 1/2$

$6x_2 + x_4 = 3$

$6(1/2) + x_4 = 3$

$6/2 + x_4 = \cancel{3} / \cancel{6}/2$

$x_4 = 0$

$x_1 = 0, \; x_4 = 0$

$6x_2 + 2x_3 = 3 \to (1)$

$4x_2 + 4x_3 = 2 \to (2)$

$(-)$

$2 \times \; 6x_2 + 2x_3 = 3$ ($12 \quad 4 \quad 6$)

$4x_2 + 4x_3 = 2$

$(-) \quad (-) \quad (-)$

$8x_2 = 4$

$x_2 = 4/8$

$x_2 = 1/2$

$\cancel{6(1/2) + 2x_3 = 3}$

$\cancel{6/2 + 0}$

$4x_2 + 4x_3 = 2$

$4 \times 1/2 + 4x_3 = 2$

$8/2 + 4x_3 = 2$

$4x_3 = 2/8$

$4x_3 = 1/4$

$x_3 = 1/2 / 4$

$x_3 = 1/8$

Formulation of LPP graphical M

① solve the following LPP graphical Maximi

$Z = 100x_1 + 40x_2$ subject to

$5x_1 + 2x_2 \leq 1000$

$3x_1 + 2x_2 \leq 900$

$x_1 + 2x_2 \leq 500$

$x_1, x_2 \geq 0$

i) Max ≤ In -> Fesible s bounded

ii) max Z out - unfesible. un bounded.

iii) ≤, ≥ -> Inf out (Ff un) (bo &

Soln :

$5x_1 + 2x_2 = 1000 \rightarrow (1)$

$3x_1 + 2x_2 = 900 \rightarrow (2)$

$x_1 + 2x_2 = 500 \rightarrow (3)$

Con eqn (1) $x_1 = 0$.

(1) => $5x_1 + 2x_2 = 1000$

$5(0) + 2x_2 = 1000$

$x_2 = 1000/2$

$\boxed{x_2 = 500}$

A (0, 500)

Con eqn (1) $x_2 = 0$

$5x_1 + 2x_2 = 1000$

$5x_1 = 1000$

$x_1 = \frac{1000}{5}$ $\boxed{x_1 = 200}$ B(200, 0)

Con equ ② $x_1 = 0$.

$3x_1 + 2x_2 = 900$

$2x_2 = 900$

$x_2 = \frac{900}{2}$

$\boxed{x_2 = 450}$

C (0, 450).

Con equ ② $x_2 = 0$

$3x_1 + 2x_2 = 900$

$3x_1 = 900$

$x_1 = \frac{900}{3}$

$\boxed{x_1 = 300}$

D (300, 0).

Con eque (3) $x_1 = 0$

$x_1 + 2x_2 = 500$

$2x_2 = 500$

$x_2 = \frac{500}{2}$

$\boxed{x_2 = 250}$.

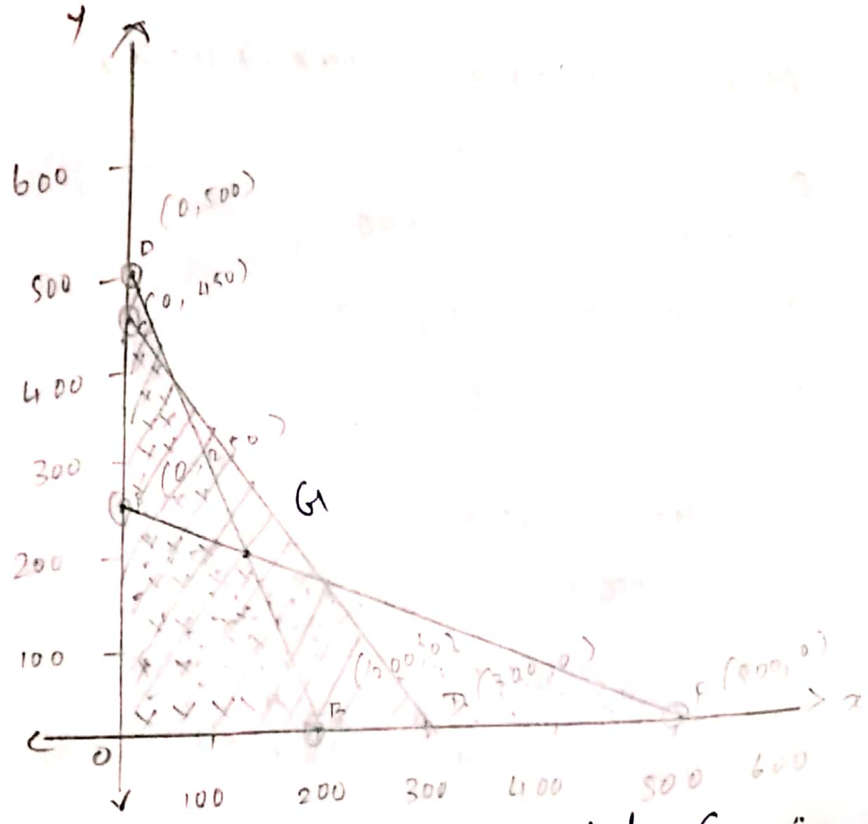
E (0, 250).

Con equ ③ $x_2 = 0$

$x_1 + 2x_2 = 500$

$\boxed{x_1 = 500}$

F (500, 0).



We want to Calculate G

G in the intersection ① & ③.

$$5x_1 + 2x_2 = 1000$$
$$x_1 + 2x_2 = 500$$
$$(-)\ (-)\quad (-)$$

$$4x_1 = 500$$

$$x_1 = \frac{500}{4}$$

$$\boxed{x_1 = 125}$$

(8) $\Rightarrow 125 + 2x_2 = 500$

$$2x_2 = 500 - 125$$

$$2x_2 = 375$$

$$x_2 = \frac{375}{2}$$

$$x_2 = 187.5$$

G $(125, 187.5)$

OBGE

points	Max $z = 100x_1 + 40x_2$
O $(0,0)$	0
B $(200, 0)$	20,000
G $(125, 187.5)$	20,000
E $(0, 250)$	10,000

$$100x_1 + 40x_2$$
$$100(200) + 40(0)$$

B = 20,000

$$100(125) + 40(187.5)$$

$$12500 + 7500$$

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$E = (0)(100) + 40(250)$

$I = 10,000$.

B. G

B $(200, 0) \rightarrow (20,000)$

G $(125, 187.5) \rightarrow 20,000$

Fesible soln & bounded.

(2) Max $Z = 2x_1 + 4x_2$

subject to $x_1 + 2x_2 \leq 5$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$.

solution:

$x_1 + 2x_2 = 5 \rightarrow (1)$

$x_1 + x_2 = 4 \rightarrow (2)$.

Con eqn (1) $x_1 = 0$.

$x_1 + 2x_2 = 5$

$2x_2 = 5$

$x_2 = \frac{5}{2}$

$x_2 = 2.5$

D $(0, 2.5)$

Con eqn (1) $x_2 = 0$

$x_1 + 2x_2 = 5$

$x_1 = 5$.

B $(5, 0)$

Con eqn (2) $(x_1 = 0)$

$x_1 + x_2 = 4$

$x_2 = 4$

C $(0, 4)$

Con eqn (2) $x_2 = 0$

$x_1 + x_2 = 4$

$x_1 = 4$

D $(4, 0)$

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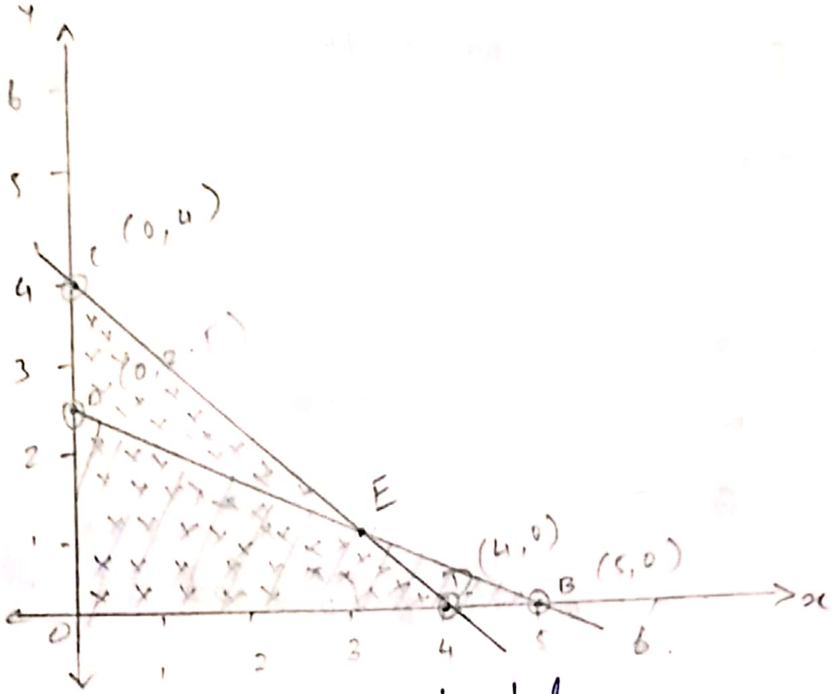
①



we want to . calculate E

E in the intersection ① & ②

$$x_1 + 2x_2 = 5$$
$$\underline{\overset{(-)}{x_1} + \overset{(-)}{x_2} = \overset{(-)}{4}}$$
$$x_2 = 1$$

$D = 2(4) + 4(0)$

$D = 8$

② $\Rightarrow x_1 + x_2 = 4$.

$x_1 = 4 - 1$

$x_1 = 3$

$E = 2(3) + 4(1)$
$= 6 + 4$
$E = 10$

$E = (3, 1)$

$A = 2(0) + 4(2.5)$

$A = 10$

Max $Z = 2x_1 + 4x_2$

O D E A

O	=	(0,0)	=	0
D	=	(4,0)	=	8 .
E	=	(3,1)	=	10
A	=	(0,2.5) .	=	10

②

E-A

E (3,1) → (10) fesible soln & bounded

A (0,2.5) → (10)

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(1) Maxi $z = 6x_1 + x_2$

subject to

$2x_1 + x_2 \geq 3$
$-x_1 + x_2 \geq 0$
$x_1, x_2 \geq 0$

sol:

$2x_1 + x_2 \geq 3 \rightarrow (1)$
$-x_1 + x_2 \geq 0 \rightarrow (2)$

$2x_1 + x_2 = 3$
$-x_1 + x_2 = 0.$

Con equ (1) $x_1 = 0$

$x_2 = 3$

A $(0, 3)$

Con equ (1) $x_2 = 0$

$2x_1 = 3$
$x_1 = 3/2 = 1.5$
$x_1 = 1.5$ B $(1,5,0)$.

Con equ (2) $x_1 = 0$

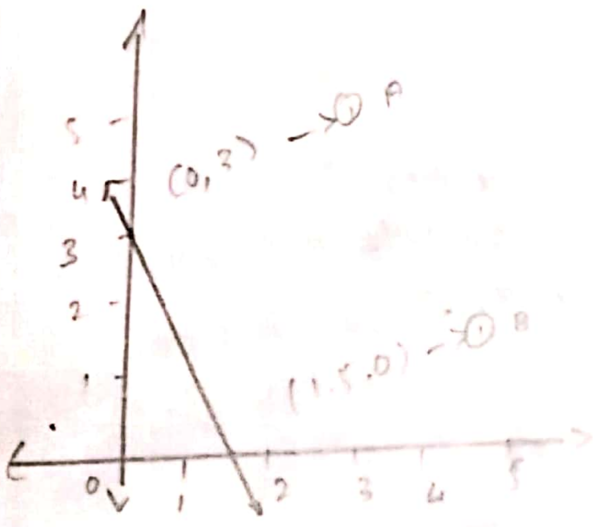
$x_2 = 0$

C $(0,0)$

Con equ (2) $x_2 = 0$

$x_1 = 0.$

D $(0,0)$.



The feasiable solu in bounded.
The problem is in feasible

(2) solu the followin Lpp.

Max $z = x_1 + x_2$

subject to $x_1 + x_2 \leq 1$

$-3x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

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$x_1 + x_2 = 1 \rightarrow (1)$

$-3x_1 + x_3 = 3 \rightarrow (2)$

Con eque (1) $x_1 = 0$ $x_2 = 1$

A $(0,1)$

Con equ (1) $x_2 = 0$ $x_1 = 1$

$B = (1,0)$

Con equ (2) $x_1 = 0$

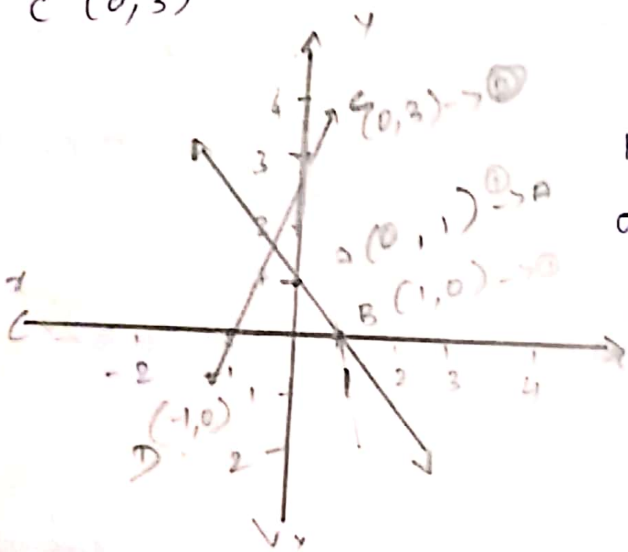
$x_2 = 3$

C $(0,3)$

Con equ (2) $x_2 = 0$

$x_1 = -1$

D $(-1,0)$



Hence not exist any solution, so the given problem in feasible solution.

(3) Max $Z = 3x_1 - 2x_2$

subject to

$x_1 + x_2 \le 1$

$2x_1 + 2x_2 \ge 4$

$x_1, x_2 \ge 0$

$x_1 + x_2 = 1 \rightarrow (1)$

$2x_1 + 2x_2 = 4 \rightarrow (2)$

Con equ (1) $x_1 = 0$

$x_2 = 1$

A $(0,1)$

Con equ (1) $x_2 = 0$

$x_1 = 1$

B $(1,0)$

Con equ ② $x_1 = 0$

$2(0) + 2x_2 = 4$

$x_2 = 4/2$

$x_2 = 2$

C (0, 2)

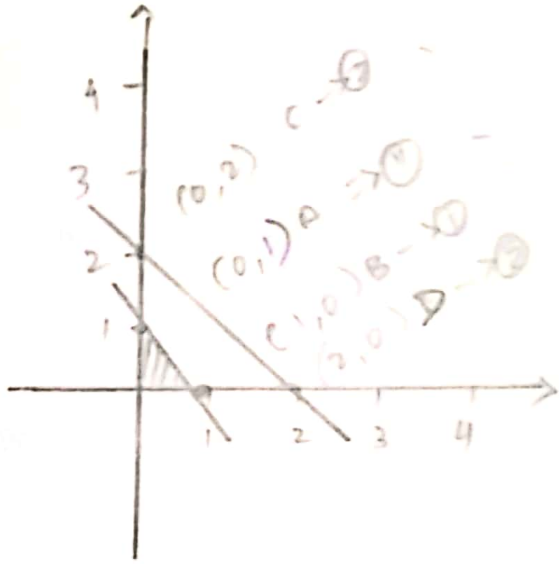
Con equ ② $x_2 = 0$

$2x_1 + 2(0) = 4$

$2x_1 = 4/2$

$x_1 = 2$

D (2,0)



Hence the feasible soln in unbounded the problem is infeasible.

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4. Max $z = 3x + 2y$ $-2x + 3y \le 9$

$x - 5y \ge 0$

$x, y \ge 0$

$-2x + 3y = 9 \rightarrow$ ①

$x - 5y = 0 \rightarrow$ ②

Con equ ① $x_1 = 0$

$-2(0) + 3y = 9$

$3y = 9$

$y = 9/3 = 3$

A (0,3)

Con equ ① $x_2 = 0$

$-2x + 3(0) = 9$

$x = 9/2 = 4.5$

B (−4.5, 0)

Con equ ② $x_1 = 0$

$x - 5y = 0$

$y = 0$

C (0,0)

Con equ ② $x_2 = 0$

$x - 0 = 0$

D (0,0).

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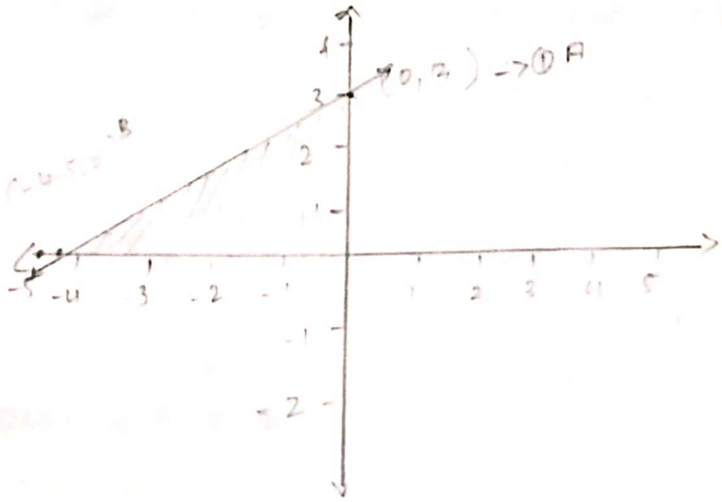


5) Max $z = 5x + 8y$

$3x + 2y \leq 36$

$x + 2y \leq 20$

$3x + 4y \leq 42$

$x, y \geq 0$

$3x + 2y = 36 \rightarrow (1)$

$x + 2y = 20 \rightarrow (2)$

$3x + 4y = 42 \rightarrow (3)$.

Con equ (1) $x_1 = 0$

$3x + 2y = 36$

$2y = 36$

$y = \frac{36}{2} = 18$

A $(0, 18)$

Con equ (1) $y = 0$

$3x + 2(0) = 36$

$3x = 36$

$x = 36/3 = 12$

B$(12, 0)$

Con equ (2) $x_1 = 0$

$x + 2y = 20$

$2y = 20$

$y = \frac{20}{2} = 10$

$y = 10$

C $(0, 10)$

Con equ (2) $y = 0$

$x + 2(0) = 20$

$x = 20$

D $(20, 0$

con eqn 3 $x = 0$

$3x + 4y = 42$

$4y = 42$

$y = 42/4 = 10.5$

E (0, 10.5)

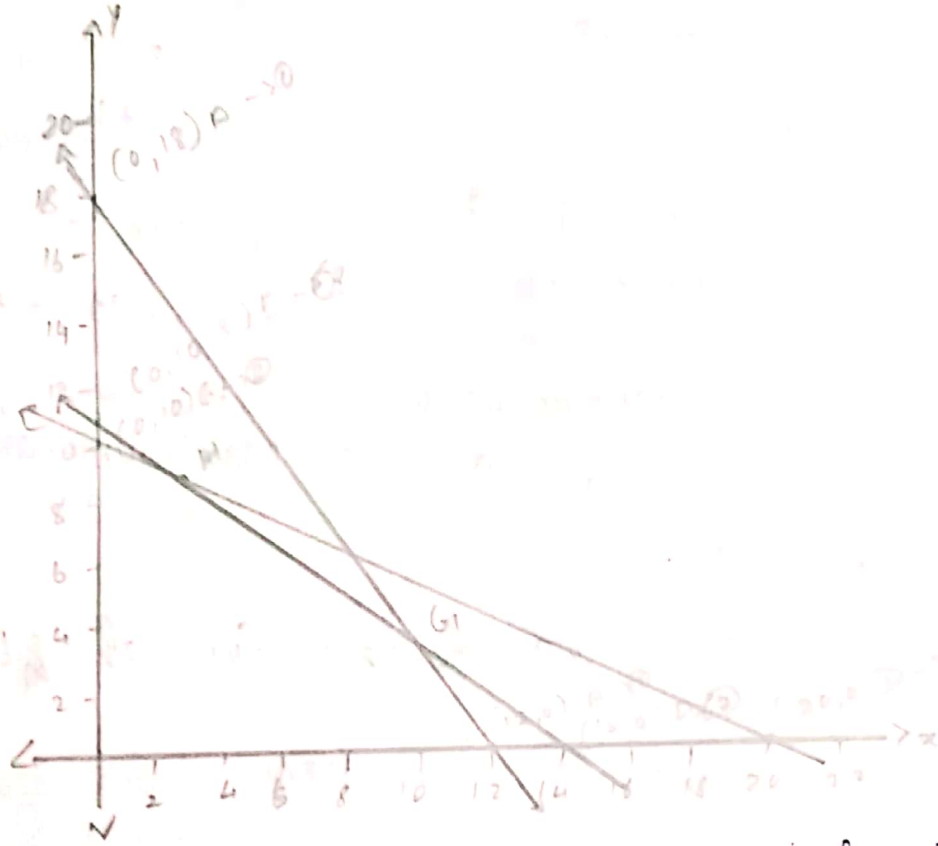
con eqn $y = 0$

$3x + (0) = 42$

$x = 42/3 = 14$

$x = 14$

F (14, 0)



we want calculate G. G in the intersection ① to ③

To Find G

① => $3x + 2y = 36$

③ => $3x + 4y = 42$

(-) (-) (-)

$-2y = -6$

$y = -6/2$

$y = +3$.

$3x + 2(3) = 36$

$3x + 6 = 36$

$3x = 36 - 6$

$3x = 30$

$x = 30/3$

$x = 10$

(10, 3) G

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To Find H

$3x+4y=42$

$x+2y=20$

3×2 $3x+4\cancel{y}=42$

$2x+\cancel{4y}=40$

(-) (-) (-)

$x=20$

$x=2$

$x=2$ Con equ ②

$2(2)+4y=40$

$4+4y=40$

$4y=40-4=36$

$y=36/4$ $y=9$

H $(2,9)$

O $(0,0)$

B $(12,0)$

G $(10,3)$

H $(2,9)$

$B=5(12)+8(0)$

$B=60$

$G=5(10)+8(3)$

$=50+24$

$G=74$

$H=5(2)+18(9)$

$=10+72$

$H=82$

H = Maximum value = 82

Canonical and standard form of LPP:

The Canonical form

$$\max z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Subject to

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \le b_i$$

$$x_1, x_2, \dots, x_n \ge 0$$

These form of LPP is called the canonical form of LPP

The characteristic of these form are subdivision → ① : The objective function of the maximization type.

The minimization of a function $f(x)$ is equivalent to the maximization of the negative expression of these function $-f(x)$.

$$\therefore \min f(x) = \max \{-f(x)\}.$$

Eg:

$$\min z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

$$\max n = -c_1x_1 - c_2x_2 \dots c_nx_n.$$

where $z = -n$.

subdivision -2 : the Constraints are of the less than ~~of~~ or equal to type. except for the non-negative restructions.

An inequality of gratter than or equal to [$\geq$] type can be changed to on inequality of the less than or equal to type by multiplying both the sides of the ~~negative~~ inequality by -1.

Foreq: The unier constrainsts.

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq b_i,$$

is eqivalent to.

$$-a_{i1}x_1 - a_{i2}x_2 - \dots - a_{in}x_n \geq b_i,$$

$$x_1 \geq 1$$

$$-x_1 \leq -1$$

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An equation may be replaced by two weak inequality in opposite directions.

eg: $a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i$

is equivalent to,

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq b_i$$
$$\leq b_i$$

Subdivision -> ③ : All the variable ~~att~~ are non-negative.

A variable which is unresitricted in sign. [positive, negative ~~&~~ or zero] is equivalent to the difference between two non-negative variables. Thus, if x_j is unrestricted in sign, it can be replaced by $(x_j' - x_j'')$

where x_j' and x_j'' are both non-negative that is,

$$x_j = x_j' - x_j''$$

where $x_j' \geq 0$ &

$x_j'' \geq 0$

The standard form of LPP:

The general form of LPP in the form.

$$\max \text{ (or) } \min z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

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subject to

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i$$
$$\forall \; i = 1 \text{ to } n.$$
$$x_1, x_2, \dots, x_n \geq 0$$

is known as the standard from of LPP.

The characterists of this form are subdivision → ① . All the Constraints are expressed in the form of equations. except for the non-negative instrictions

The right hand side of each constrain equation is non-negative.

The inequality constraints can be changed into equation by introducing a non-negative variables on the left hand side of such constraints, It is to be added slack variable. If the constraint is of ≤ type and subtrected [surplus variable] If the Constraint is ≥ type.

maximize or minimize $z = cx$

subject to the Constraints.

$$Ax = b$$
$$x \geq 0$$

where $x = (x_1, x_2, \dots x_n)$ $c = (c_1, c_2, \dots, c_n)$

$b^T = (b_1, b_2, \dots, b_m)$.

$A = (a_{ij})$, $i = 1, 2, \dots, m$; $j = (1, 2, \dots n)$

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Remark :

The coefficient of slack or surplus variables in the objective function are always assumed to be zero, so that the conversation of the constraints to a system of simultanuos, linear equation does not change the objectives function under consideration.

Remark 2 :

The linear ~~pe~~ programming for max $z = cx$ subject to the constraints :

$$Ax \le b$$
$$x \ge 0$$

is known as the canonical form of the LPP.

Canonical & standard form of LPP :

① Express the following LPP in standard form ⑤.

minimize $z = 5x_1 + 7x_2$ subject to

$$x_1 + x_2 \le 8$$
$$3x_1 + 4x_2 \ge 3$$
$$6x_1 + 7x_2 \ge 5 \quad \& \quad x_1, x_2 \ge 0$$

since mini $z = -\text{Max}(-z)$

$$\text{Mini } z = -\text{Max } z^*$$

The given LPP becomes

$$\text{Max } z^* = -5x_1 - 7x_2$$

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$x_1 + x_2 \leq 8$

$3x_1 + 4x_2 \geq 3$

$6x_1 + 7x_2 \geq 5$

The standard LPP form is

$Max\ z^* = -5x_1 - 7x_1 - 0S_1 + 0S_2 + 0S_3$.

subject to.

$\leq$ → Add → slack variables.

$\geq$ → sub → surplus variables.

$x_1 + x_2 + S_1 = 8$.

$3x_1 + 4x_2 - S_2 = 3$

$6x_1 + 7x_2 - S_3 = 5$

$x_1, x_2, \&\ S_1, S_2, S_3 \geq 0$

2. $Max\ z = 5x_1 + 7x_2$

Subject to $x_1 + x_2 \leq 8$

$3x_1 + 4x_2 \geq 3$

$6x_1 + 7x_2 \geq 5$ & $x_1, x_2 \geq 0$

soln:

$Max\ z^* = +5x_1 + 7x_2 + 0S_1 + 0S_2 + 0S_3$

Subject to $\leq$ → Add → slack variable

$\geq$ → sub → surplus variable.

$x_1 + x_2 + S_1 = 8$

$3x_1 + 4x_2 - S_2 = 3$

$6x_1 + 7x_2 - S_3 = 5$

x_1, x_2 & $S_1, S_2, S_3 \geq 0$

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Canonical from:

① From the following LPP Canonical form:

$$\text{Maximize } z = 2x_1 + 3x_2 + x_3$$

subject to $4x_1 - 3x_2 + x_3 \leq 6$

$x_1 + 5x_2 - 7x_3 \geq -4$

and $x_1, x_3 \geq 0$, x_2 is unrestricted.

As x_2 is unrestricted.

$x_2 = x_2' - x_2''$ → Formula

where $x_2', x_2'' \geq 0$.

∴ The LPP becomes

$$\text{Max } z = 2x_1 + 3x_2 + x_3$$

$$= 2x_1 + 3x_2' - 3x_2'' + x_3$$

Subject to.

$$4x_1 - 3x_2' + 3x_2'' + x_3 \leq 6$$

$$x_1 + 5x_2' - 5x_2'' - 7x_3 \geq -4$$

and $x_1, x_2'; x_2'', x_3 \geq 0$

Convert the ~~sound~~ second constaints into $\leq$ type by multiplying both sides by (-1)

now the LPP becomes

$$\text{Max } z = 2x_1 + 3x_2' - 3x_2'' + x_3$$

subject to

$$4x_1 - 3x_2' + 3x_2'' + x_3 \le 6$$

$$-x_1 - 5x_2' + 5x_2'' + 7x_3 \le 4$$

$$x_1, x_2', x_2'', x_3 \ge 0$$

which is the required Canonical form.

(2) From express the following LPP Canonical Form

$$\text{maximize } z = 2x_1 + x_2 + 4x_3$$

subject to

$$-2x_1 + 4x_2 \le 4$$

$$x_1 + 2x_2 + x_3 \ge 5.$$

$$2x_1 + 3x_3 \le 2$$

and $x_1, x_2 \ge 0$ x_3 is unrestricted.

Solution:

As x_3 is unrestricted.

$$x_3 = x_3' - x_3''$$

where ~~$x_3' - x_3 = x_3$~~ $x_3', x_3'' \ge 0$

∴ The LPP becomes

$$\text{Max } z^* = 2x_1 + x_2 + 4x_3$$

$$= 2x_1 + x_2 + 4x_3' - 4x_3''$$

Subject to

$-2x_1 + 4x_2 \le 4$

$x_1 + 2x_2 + x_3' - x_3'' \ge 5$

$2x_1 + 3x_3' - 3x_3'' \le 2$

and $x_1, x_3'; x_3''; x_3 \ge 0$

Convert the tird Contain into $\le$ type by multiplying both sides by (-1)

now the Lpp becomes.

$$\max z = 2x_1 + x_2 + 4x_3$$
$$= 2x_1 + x_2 + 4x_3' - 4x_3''$$

subject to

$$-2x_1 + 4x_2 \le 4$$
$$x_1 + 2x_2 + x_3 \ge 5$$
$$2x_1 + 3x_3 \le 2$$

$$-2x_1 + 4x_2 \le 4.$$
$$-x_1 + 2x_2 - x_3' + x_3'' \le 5$$
$$2x_1 + 3x_3' - 3x_3'' \le 2$$

$$x_1, x_2, x_3', x_3'' \ge 0$$

which is the required Cannonical form.

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Mathamatical formulation of an LPP :

i) Max ≤ , ii) Mini ≥

Ex : 1.

A Manufacture producos two types of Model A & model B. Each A Model required ~~foro~~ fours of grainding and two hours of polising where as each B Model required two hours of graeiding & Five hours of polishing. A Manufactur has to grainder and three polishes each graincler work for 40 hours per week & each polished works for 60 hours per week. profit on Model A is Rs 3 as Model B is RS 4 what ever is produced in a week is sold ing. Mark how should the manufacture allocate his production Capacity to the two type of Models so that he may make the Maximum profit week Formulate the problem as an LPP.

Soln :

Let x_1 & x_2 be no of units of Model A & B

	Model A	Model B	hours
Grainder	4	2	40 hours
Policies	2	5	60 hours
profit	Rs 3	RS 4	

2G & 3P From the LPP format

$$\max \cdot z = 3x_1 + 4x_2$$

subject to,

$$4x_1 + 2x_2 \leq 2 \times 40 \text{ hour}.$$

$$2x_1 + 5x_2 \leq 3 \times 60 \text{ hour}$$

$$x_1, x_2 \geq 0$$

Ex : 2

A Manufacture has 3 machine A, B, C with which he producer 3 different Article P, Q, R, the different machine time require per Articles the amount of the time avaliable in any week an each machine and the estimate profits per article are furnished in the following table:

Article	A	B	C	profit pen Article.
P	8	4	2	20
Q	2	3	0	6
R	3	0	1	8
Avaliable machine hours	250	150	50	

sol:

Let x_1, x_2, x_3 be the no of units of caticles P, Q, R

The required LPP is.

$$\max z = 20x_1 + 6x_2 + 8x_3$$

subject to,

$$8x_1 + 2x_2 + 3x_3 \leq 250$$

$$4x_1 + 3x_2 + 0x_3 \leq 150$$

$$2x_1 + 0x_2 + 1x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0.$$

Ex: 3.

A firm can produce three type of cloth set A, B, C three kinds of wool are required for it say red wool, green wool and blue wool, one unit length of type A cloth requires two meters of red wool, three meters of blue wool, one unit Length of types B cloth requires three meters of red wool, two meters of green wool. and two maters of blue wool, one unit length of types C cloth requires five meters of green wool and four meters of blue wool. The firm as a stock of Eight meters of red wool. ten meters of green wool. and fifteen meters of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs 3, Types A cloths Rs. 5 and that type C cloth is Rs 4. Determine have the firm use the avaliable material so as to maximize the total Income from the finished cloth Formulate these problem as an Lpp.

soln:

Let x_1, x_2, x_3 be the number of units of cloth A, B, C respectively. To be produced or manufactured. The given data Can be represented as in the following table.

Types of cloth

wool	A	B	C	stock
Red	2m	3m	-	8m
Green	-	2m	5m	10m
blue	3m	2m	4m	15
Income Rate	Rs 3	Rs 5	Rs 4	

$\max z = 3x_1 + 5x_2 + 4x_3$ [object]

subject to,

$$2x_1 + 3x_2 + 0x_3 \le 8$$

$$0x_1 + 2x_2 + 5x_3 \le 10$$

$$3x_1 + 2x_2 + 4x_3 \le 15$$

$$x_1, x_2, x_3 \ge 0.$$

Ex : 4

A person requires 10, 12, 12 units of chemicals A, B, C respectively for his garden. A Liquid product contains 5, 2, 1 units of A, B, C respectively per 1 Jar bottle. A dry product contains 1, 2, 4; unit of A ;B and C per pocket. If the liquid product sells for RS 3 per Jar and the dry product sells for RS 2 per pocket How many of each should be purchased in order to minimize the cost and meet the requirement formulate the problem as an LPP.

Sol :

Let x_1, x_2 be the no of units of Liquid products and dry product to be purchased respectively.

product

CHEMICALS		x_1 liquid	x_2 DRY	Required chemicals
	A	5	1	10
	B	2	2	12
	C	1	4	12
	cost	RS. 3	RS. 2	

The required LPP is

$$\min z = 3x_1 + 2x_2$$

Subject to,

$$5x_1 + x_2 \geq 10$$

$$2x_1 + 2x_2 \geq 12$$

$$x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

Theory:

Mathamatical Formulation of an lpp.

Step: 1

define all decision variables and specify the Units of Mesurement step 2 :

Determine whether the objective function is to be maximized or minimized. Then express it as a linear function of decision variable multiplied by the profit or cost consideration.

step 3 :

Formulate all the constraints impossed by the resource available the and express them as linear equality or ~~ire~~ inequality in terms of decision variables.

UNIT - IV

Assignment problem :

(1) Solve the following A.P :

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline A & 8 & 4 & 2 & 6 & 1 \\ B & 0 & 9 & 5 & 5 & 4 \\ C & 3 & 8 & 9 & 2 & 6 \\ D & 4 & 3 & 1 & 0 & 3 \\ E & 9 & 5 & 8 & 9 & 5 \end{array}$$

Soln :

The cost matrix of the given A P is balanced.

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
A & 8 & 4 & 2 & 6 & 1 \\
B & 0 & 9 & 5 & 5 & 4 \\
C & 3 & 8 & 9 & 2 & 6 \\
D & 4 & 3 & 1 & 0 & 3 \\
E & 9 & 5 & 8 & 9 & 5
\end{array}
$$

No. of Rows = No. of Coloumns.

5 = 5.

i) Row minima.

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
A & 7 & 3 & 1 & 5 & 0 \\
B & 0 & 9 & 5 & 5 & 4 \\
C & 1 & 6 & 7 & 0 & 4 \\
D & 4 & 3 & 1 & 0 & 3 \\
E & 4 & 0 & 3 & 4 & 0
\end{array}
$$

ii) column minima

$$
\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5 \\
\hline
7 & 3 & 0 & 5 & 0 \\
0 & 9 & 4 & 5 & 4 \\
1 & 6 & 6 & 0 & 4 \\
4 & 3 & 0 & 0 & 3 \\
4 & 0 & 2 & 4 & 0
\end{array}
$$

The optimal schedule $A \to 5$, $B \to 1$ $C \to 4$, $D \to 3$, $E \to 3$

$= 1 + 0 + 2 + 1 + 5$

$= 9$ unit of Cost.

2)
$$\begin{array}{c|cccc} & I & II & III & IV \\ \hline A & 10 & 5 & 13 & 15 \\ B & 3 & 9 & 18 & 3 \\ C & 10 & 7 & 3 & 2 \\ D & 5 & 11 & 9 & 7 \end{array}$$

Soln:

The cost of Matrix of the given AP is

$$\begin{pmatrix} 10 & 5 & 13 & 15 \\ 3 & 9 & 18 & 3 \\ 10 & 7 & 3 & 2 \\ 5 & 11 & 9 & 7 \end{pmatrix}$$

No of rows = No of columns

The AP is balanced

i) Row minima

$$\begin{pmatrix} 5 & 0 & 8 & 10 \\ 0 & 6 & 15 & 0 \\ 8 & 5 & 1 & 0 \\ 0 & 6 & 4 & 2 \end{pmatrix}$$

ii) Columns Minima

$$\begin{pmatrix} 5 & 0 & 7 & 10 \\ 0 & 6 & 14 & 0 \\ 8 & 5 & 0 & 0 \\ 0 & 6 & 3 & 2 \end{pmatrix}$$

3. Solve the following A.P

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline A & 4 & 7 & 3 & 7 \\ B & 8 & 2 & 5 & 5 \\ C & 4 & 9 & 6 & 9 \\ D & 7 & 5 & 4 & 8 \end{array}$$

The cost of Matrix Ap is

$$\begin{pmatrix} 4 & 7 & 3 & 7 \\ 8 & 2 & 5 & 5 \\ 4 & 9 & 6 & 9 \\ 7 & 5 & 4 & 8 \end{pmatrix}$$

No. of rows $\neq$ No. of columns
The Ap is unbalanced

$$\begin{pmatrix} 4 & 7 & 3 & 7 & 0 & 0 \\ 8 & 2 & 5 & 5 & 0 & 0 \\ 4 & 9 & 6 & 9 & 6 & 0 \\ 7 & 5 & 4 & 8 & 0 & 0 \\ 6 & 3 & 5 & 4 & 0 & 0 \\ 6 & 8 & 7 & 3 & 0 & 0 \end{pmatrix}$$

No. of rows = No. of Columns
The Ap is balanced.

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i) Row minima

$$\begin{pmatrix} 4 & 7 & 3 & 7 & 0 & 0 \\ 8 & 2 & 5 & 5 & 0 & 0 \\ 4 & 9 & 6 & 9 & 0 & 0 \\ 7 & 5 & 4 & 8 & 0 & 0 \\ 6 & 3 & 5 & 4 & 0 & 0 \\ 6 & 8 & 7 & 3 & 0 & 0 \end{pmatrix}$$

ii) ~~Row~~ column minima

$$\begin{pmatrix} 4 & 7 & 3 & 7 & 0 & 0 \\ 8 & 2 & 5 & 5 & 0 & 0 \\ 4 & 9 & 6 & 9 & 0 & 0 \\ 7 & 5 & 4 & 8 & 0 & 0 \\ 6 & 3 & 5 & 4 & 0 & 0 \\ 6 & 8 & 7 & 3 & 0 & 0 \end{pmatrix}$$

$$\begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{pmatrix} \cancel{0} & 5 & (0) & 4 & 0 & 0 \\ 4 & (0) & 2 & 2 & 0 & 0 \\ (0) & 7 & 3 & 6 & \cancel{0} & 0 \\ 3 & 3 & 1 & 5 & (0) & 0 \\ 2 & 1 & 2 & 1 & 0 & (0) \\ 2 & 6 & 4 & 0 & \cancel{0} & \cancel{0} \end{pmatrix}$$

The optimal schedule
A -> 3, B -> 2, C -> 1,
D -> 5, E -> 0, F => 3

cost = 3 + 2 + 4 + 3
= 12 unit of cost

Maximization case is Assignment problem

1) Find the Assignment of Salesman to various districts which will yield Maximum profit

districts

$$\text{Salesman} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 16 & 10 & 14 & 11 \\ 14 & 11 & 15 & 15 \\ 15 & 15 & 13 & 12 \\ 13 & 12 & 14 & 15 \end{array}$$

Soln: The cost Matrix of the given AP is

$$\begin{pmatrix} 16 & 10 & 14 & 11 \\ 14 & 11 & 15 & 15 \\ 15 & 15 & 13 & 12 \\ 13 & 12 & 14 & 15 \end{pmatrix}$$

Maxima = 16

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 2 & 5 & 1 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

Row Minima

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{pmatrix}$$

column Minima

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} (0) & 6 & 2 & 5 \\ 1 & 4 & (0) & \cancel{0} \\ \cancel{0} & (0) & 2 & 3 \\ 2 & 3 & 1 & (0) \end{pmatrix}$$

The optimal schedule is => A -> (1)
B -> 3 C -> 2
D -> 4

cost = 16 + 15 + 15 + 15 = 61 unit of cost

solve the Assignment problem for Maximization given profit Matrix

Machine

$$\begin{array}{c|cccc} & P & Q & R & S \\ \hline A & 51 & 53 & 54 & 50 \\ B & 47 & 50 & 48 & 50 \\ C & 49 & 50 & 60 & 61 \\ D & 63 & 64 & 60 & 60 \end{array}$$

$$\begin{pmatrix} 51 & 53 & 54 & 50 \\ 47 & 50 & 48 & 50 \\ 49 & 50 & 60 & 61 \\ 63 & 64 & 60 & 60 \end{pmatrix}$$

$$\begin{array}{cccc} \cancel{4} & \cancel{3} & \cancel{6} & \cancel{10} \end{array}$$

$$\begin{pmatrix} 13 & 11 & 10 & 14 \\ 17 & 14 & 16 & 14 \\ 15 & 14 & 4 & 3 \\ 1 & 0 & 4 & 4 \end{pmatrix}$$

Row Mima.

$$\begin{pmatrix} 3 & 1 & 0 & 4 \\ 3 & 0 & 2 & 0 \\ 12 & 11 & 1 & 0 \\ 1 & 0 & 4 & 4 \end{pmatrix}$$

column

$$\begin{pmatrix} 2 & 1 & 0 & 4 \\ 2 & 0 & 2 & 0 \\ 11 & 11 & 1 & 0 \\ 0 & 0 & 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & (0) & 4 \\ 2 & (0) & 2 & \cancel{0} \\ 11 & 11 & 1 & (0) \\ (0) & 0 & 4 & 4 \end{pmatrix}$$

The optimal schedules = A -> 3 B -> 2

C -> 4 D -> 1

cost = 54 + 50 + 61 + 63

= 104 + 124

= 228

UNIT - 4 - Theory

Introduction:

The assignment problem can be stated in the form of $m \times n$ matrix (c_{ij}) called a cost matrix (or) Effectiveness matrix where c_{ij} is the cost matrix of assigning ith mechine on the j^{th} job.

$$
\begin{array}{c|cccccc}
 & & & \text{Jobs} & & & \\
 & 1 & 2 & 3 & \cdots & n \\
\hline
\text{Mechines } 1 & c_{11} & c_{12} & c_{13} & \cdots & c_{1n} \\
2 & c_{21} & c_{22} & c_{23} & \cdots & c_{2n} \\
3 & c_{31} & c_{32} & c_{33} & \cdots & c_{3n} \\
\vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
m & c_{m1} & c_{m2} & c_{m3} & \cdots & c_{mn}
\end{array}
$$

Assignment Algorithm (or) Hungarian method.

First check weather the number of rows is eqyal to the number of columns. If it is so, the assignment problem is said to be balanced. Then proceed to step 1. If it is not balanced then it should be balanced before applying the algorithm. The method of balancing is discussed.

step : 1

suppract the smallest cost elements of each row from all the elements in the row of the given Cost matrix. see, the each row Contrins atleast one row.

step : 2

supract the smallest cost element of each column from all the elements in the collumn of the resulting cost matrix obtained by step 1

step : 3 [Assigning the zero]

a) Examine the rows successifely untill a row with exactly one unmarked zero is found make an assignment to this single unmarked zero by encircled zero. as these will not be Considered for any future assignment. Continue in this way. until all the rows have been examined.

b) Examine the Columns Successively untill a Coloumm with exactly one unm--arked zero is found. make an assignment to this single unmarked zero by encircling it and Cross any other zero in its row. Continue untill all the Columns have been examized.

step 4 : [Apply optimal test]

a) If each row and each Column Contain exactly on encircled zero then the Current assignment is optimal.

b) If atleast one row column contain is without an assignment. (i.e), If there is atleast one row/column is without one encircled zero] then then the current assignment is not optimal.

step: 5

Cover all the zeros by drawing a minimum ~~nub~~ numbers of straight lines as follows.

a) Mark (v) the rows that do not have assignment.

b) Mark (v) the columns (not already marked) that have zero in market rows

c) Mark (v) the rows (not already Marked) that have assignment in market columns.

d) Repeak (b) and (c) until no more making is reqired.

Draw lines through all unmarked rows and Marked columns. if the number of there lines is equal to the order column of the matrix. then it is an optimum solution. otherwise not

step: 6

Determine the smallest cost element not Convered by straights lines.

subtract this smallest cost element from all the uncovered elements and add this to all those element which are lying in the Intersection of three straight lines and do not change the ramaining elements and do not which. lio on the straight lines.

step: 7

Repeat steps (1) to (6) untill on optimum assignment is attained.

Note: 1 In Case Some rows or coloumns contain more than one zero, encircle any ~~un~~ unmarked zero arbitraily and. Cross all. other zero is left unmarked or encircled.

note: 2 The above assignment problem is only minization problems.

note: 3 If the given assignment problem is of maximization type convert it to a minimization assignment problem by $max z = -min(-z)$ and multiply all the given Cost element by -1 in the cost ~~mark~~ matrix and the solve by assignment algorithym.

Note: 4

Some times, a final. Cost matrix Contains more than required number of zeros at independent position the Implies that there is more than one optimal solution (multiple optimal solutions with the same optimum assignment Cost

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Maximization Cost in Assignment problems:

2M

2) In an assignment problem we may have to deal with minimization of an objective function) For example, we may have to assign persons to jobs in such a way that the total profit is maximized. The maximization problem has to be converted into an equivalent minimization problem and them solved by the usival Hungarian method.

The conversion of the maximization problem into an equivalant minimization problem can be done by anyone of the following method.

3 2M (i) since $\max z = -\text{Main}(-z)$. multiply all the cost an of the cost matrix by -1

ii) subtract all the cost elements C_{ij} of the cost matrix from the highest cost elements in that cost matrix.)

2M

2) Travelling salesman problem:

(The problem of finding the shortest distance (or) minimum time or minimum cost) if the salesman stails from his head quarters and press through each city under this jurisdiction exactly once and returns to the head quarters is called the travelling sales man problem or a Travelling salesperson problem.

1) soln the following A.P Travelling sales man

$$\begin{array}{c|ccccc} & M_1 & M_2 & M_3 & M_4 & M_5 \\ J_1 & 9 & 22 & 58 & 11 & 19 \\ J_2 & 43 & 78 & 72 & 50 & 63 \\ J_3 & 41 & 28 & 91 & 37 & 45 \\ J_4 & 74 & 42 & 27 & 49 & 39 \\ J_5 & 36 & 11 & 57 & 22 & 25 \end{array}$$

Soln :

The Cost Matrix of AP in

$$\begin{pmatrix} 9 & 22 & 58 & 11 & 19 \\ 43 & 78 & 72 & 50 & 63 \\ 41 & 28 & 91 & 37 & 45 \\ 74 & 42 & 27 & 49 & 39 \\ 36 & 11 & 57 & 22 & 25 \end{pmatrix}$$

Since, No of Rows = No of Columns

The Ap is balanced.

Row minima

$$\begin{pmatrix} 0 & 13 & 49 & 2 & 10 \\ 0 & 35 & 29 & 7 & 20 \\ 13 & 0 & 63 & 9 & 17 \\ 47 & 15 & 0 & 22 & 12 \\ 25 & 0 & 46 & 11 & 14 \end{pmatrix}$$

Column Minima

$$\begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \\ 47 & 15 & 0 & 20 & 2 \\ 25 & 0 & 46 & 9 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \end{pmatrix}$$

+4 -4

$$\begin{pmatrix} \cancel{x} & 17 & 49 & (0) & \cancel{x} \\ (0) & 39 & 29 & 5 & 10 \\ 9 & (0) & 69 & 3 & 3 \\ 47 & 19 & (0) & 20 & 2 \\ 21 & \cancel{x} & 42 & 5 & (0) \end{pmatrix}$$

The optimal schedule is
$J_1 \to M_4$, $J_2 \to M_1$; $J_3 \to M_2$
$J_4 \to M_3$ $J_5 \to M_5$

The Cost $= 11 + 43 + 28 + 27 + 25$
$= 134$ unit of Cost

solve the following Travelling Salesman problem

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & - & 46 & 16 & 40 \\ B & 41 & - & 50 & 40 \\ C & 82 & 32 & - & 60 \\ D & 40 & 40 & 36 & - \end{array}$$

16 + 40 + 32 + 36
56 + 68
68
124

Soln:

The cost Matrix AP is
No. of rows = No. of column
The AP is balanced

Row Minima

$$\begin{pmatrix} \alpha & 30 & 0 & 24 \\ 1 & \alpha & 10 & 0 \\ 50 & 0 & \alpha & 28 \\ 4 & 4 & 0 & \alpha \end{pmatrix}$$

Column Minima

$$\begin{pmatrix} \alpha & 30 & 0 & 24 \\ 0 & \alpha & 10 & 0 \\ 49 & 0 & \alpha & 28 \\ 3 & 4 & 0 & \alpha \end{pmatrix}$$

$$\begin{pmatrix} \alpha & 30 & (0) & 24 \\ (0) & \alpha & 10 & \cancel{(0)} \\ 50 & (0) & \alpha & 28 \\ 3 & 4 & \cancel{0} & \alpha \end{pmatrix} \qquad \begin{pmatrix} \alpha & 30 & (0) & 24 \\ (0) & \alpha & 10 & (0) \\ 49 & (0) & \alpha & 28 \\ (3) & 4 & \cancel{0} & \alpha \end{pmatrix}$$

$$\begin{matrix} \alpha & 27 & 0 & 21 \\ 0 & \alpha & 13 & 0 \\ 53 & 0 & \alpha & 31 \\ 0 & 1 & 0 & \alpha \end{matrix} \qquad \begin{pmatrix} \alpha & 27 & \cancel{0} & 21 \\ \cancel{0} & \alpha & 13 & (0) \\ 50 & (0) & \alpha & 28 \\ 3 & 4 & (0) & \alpha \end{pmatrix}$$

$$\begin{pmatrix} \alpha & 27 & (0) & 21 \\ \cancel{(0)} & \alpha & 13 & (0) \\ 49 & (0) & \alpha & 28 \\ (0) & 4 & \cancel{0} & \alpha \end{pmatrix}$$

The optimal schedule is A -> C B -> D
C -> B D -> A

The optimal cost = 16 + 40 + 32 + 40
= 128 unit cost

3.

$$\begin{array}{c|cccc} & M_1 & M_2 & M_3 & M_4 \\ \hline J_1 & 5 & 7 & 11 & 6 \\ J_2 & 8 & 5 & 9 & 6 \\ J_3 & 4 & 7 & 10 & 7 \\ J_4 & 10 & 4 & 8 & 3 \end{array}$$

The cost Matrix AP is
No. of rows = No. of coloumns
The AP is balanced

Row minima :

$$\begin{bmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{bmatrix}$$

Column Minima:

$$\begin{bmatrix} 0 & 2 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 7 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} (0) & 2 & 2 & 1 \\ 3 & (0) & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 7 & 1 & 1 & (0) \end{bmatrix}$$

$$\begin{bmatrix} (0) & 2 & 2 & 1 \\ 3 & (0) & 0 & 1 \\ 0 & 1 & (0) & 1 \\ 7 & 1 & 1 & (0) \end{bmatrix}$$

$= J_1 \to M_1, J_2 \to M_2 \to J_3 \to M_3$

$J_4 \to M_4$

$= 5 + 5 + 10 + 3$

$= 23$ unit of Cost

Minimize

1) solve the following Travelling Salesman problem so as to Minimize the cost per cycle

$$\begin{array}{cc|ccccc} & & & & \text{To} & & \\ & & A & B & C & D & E \\ \text{From} & A & - & 3 & 6 & 2 & 3 \\ & B & 3 & - & 5 & 2 & 3 \\ & C & 6 & 5 & - & 6 & 4 \\ & D & 2 & 2 & 6 & - & 6 \\ & E & 3 & 3 & 4 & 6 & - \end{array}$$

The cost Matrix Ap is given by

$$\begin{pmatrix} \alpha & 3 & 6 & 2 & 3 \\ 3 & \alpha & 5 & 2 & 3 \\ 6 & 5 & \alpha & 6 & 4 \\ 2 & 2 & 6 & \alpha & 6 \\ 3 & 3 & 4 & 6 & \alpha \end{pmatrix}$$

No of rows = No of column
The Ap is balanced.

Row minima.

$$\begin{pmatrix} \alpha & 1 & 4 & 0 & 1 \\ 1 & \alpha & 3 & 0 & 1 \\ 2 & 1 & \alpha & 2 & 0 \\ 0 & 0 & 4 & \alpha & 4 \\ 0 & 0 & 1 & 3 & \alpha \end{pmatrix}.$$

column minima.

$$\begin{pmatrix} \alpha & 1 & 3 & 0 & \cancel{0}1 \\ 1 & \alpha & 2 & 0 & \cancel{0}1 \\ 2 & 1 & \alpha & 0 & 0 \\ 0 & 0 & 3 & \alpha & 4 \\ 0 & 0 & 0 & 1 & \alpha \end{pmatrix}$$

Row minima :

$$\begin{bmatrix} 0 & & 2 & 6 \\ 3 & 0 & 4 & \\ 0 & 3 & & \\ 7 & 1 & & \alpha \end{bmatrix}$$

Column Mi

$$\begin{pmatrix} & 2 & (0) & \cancel{\alpha} \\ & 1 & \cancel{\alpha} & \cancel{\alpha} \\ 1 & \alpha & 3 & (0) \\ (0) & 3 & \alpha & 4 \\ \cancel{\alpha} & (0) & 4 & \alpha \end{pmatrix}$$

A->D , B->A, C->E, D->B F->C.

A->B B->A C->E - C

cost = 2+3+4+2+4
= 15 unit of cost.

Minimum Cost Unit = 1

$$\begin{pmatrix} \alpha & \cancel{\alpha} & 2 & (0) & \cancel{\alpha} \\ \cancel{(0)} & \alpha & (1) & \cancel{\alpha} & \cancel{0} \\ 2 & 1 & \alpha & 3 & (0) \\ \cancel{\alpha} & (0) & 3 & \alpha & 4 \\ (0) & \cancel{0} & \cancel{(0)} & 4 & \alpha \end{pmatrix}.$$

A->B ; B->C ; C->E, D-B , E->A

A->D->B->C->E->A D->B E->A

The cost = 2+5+4+2+3
= 16 unit of cost

i) Solve the following Assignment problem with aiming minimization of Cost.

machine — places

	I	II	III	IV	V
A	15	10	25	25	10
B	1	8	10	20	2
C	8	9	17	20	10
D	14	10	25	27	15
E	10	8	25	27	12

The cost by Matrix A p is given by.

$$\begin{bmatrix} 15 & 10 & 25 & 25 & 10 \\ 1 & 8 & 10 & 20 & 2 \\ 8 & 9 & 17 & 20 & 10 \\ 14 & 10 & 25 & 27 & 15 \\ 10 & 8 & 25 & 27 & 12 \end{bmatrix}$$

No of Row = No of column
The AP is balanced.

Row minima

$$\begin{bmatrix} 5 & 0 & 15 & 15 & 0 \\ 0 & 7 & 9 & 19 & 1 \\ 0 & 1 & 9 & 12 & 2 \\ 4 & 0 & 15 & 17 & 5 \\ 2 & 0 & 17 & 19 & 4 \end{bmatrix}$$

column minima

$$\begin{bmatrix} \cancel{14}5 & 0 & 6 & 3 & 0 \\ 0 & 7 & 0 & 7 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 4 & 0 & 6 & 5 & 5 \\ 2 & 0 & 8 & 7 & 4 \end{bmatrix}$$

1 2 0

$$\begin{bmatrix} 3 & \cancel{0} & 4 & 1 & (0) \\ \cancel{0} & 9 & (0) & 7 & 1 \\ \cancel{0} & 3 & \cancel{0} & (0) & 2 \\ 2 & (0) & 4 & 3 & 5 \\ (0) & \cancel{0} & 6 & 5 & 4 \end{bmatrix}$$

The optimal schedule is

$A_1 \to E_5$; $B_2 \to C_3 \to C_3 \to D_4$

$D_4 \to B_2$, $E_5 \to A_1$.

$= 10 + 10 + 20 + 10 + 10$

$= 60$.

$$\begin{bmatrix} 3 & \cancel{0} & 4 & 1 & (0) \\ \cancel{0} & 9 & (0) & 7 & 1 \\ \cancel{0} & 3 & \cancel{0} & (0) & 2 \\ 2 & \cancel{0} & 4 & 3 & 5 \\ \cancel{0} & (0) & 6 & 5 & 4 \end{bmatrix}$$

$= A \to E$, $B \to C$, $C \to D$, $D \to A$, $E \to B$

$= E \to A \to E \to B \to C \to D \to A \to E$

$= 10 + 10 + 20 + 14 + 8$

$= 62$

Model of AP :

i) assignment of employees to machines
ii) assignment of operators to jobs.
iii) effectiveness of teachers and subjects
iv) allocation of Machines for optimum utilization of space
v) salesman to different sales ares
vi) clerks to various counters

Any basic feasiable solution of an Assignment problem consists (2n - 1) variables of which the (n - 1) variables are ~~feci-tities~~ zero. n is number of Jobs or number of facilities ... now as the problem forms one to one basic or one jobs is to be assigned to one facilility or machine

The assignment problem is a particular case of transportation problem in which a number of operations are to be assigned to an equal number of operator, where each operator performs only one operation. The objective is to maximize overall profit or minimize overall cost for a given assignment schedule