UNIT-I

ALGEBRAIC EQUATION

Defn:

Any Equation which is purely a polynomial in x is known as Algebraic equation

Ex:

i) $ax^2 + bx + c = 0$

ii) $5x^3 - 6x^2 + 7x + 2 = 0$

TRANSCENDAL EQUATION (or) (Trigonometry Equ)

Defn:

If an equation contains logarithmic Exponential function then it is known as trigonometric equation

Ex:

i) $x^3 - 3\cos x = 0$

ii) $e^x - 3\sin x = 0$

iii) $3\log x + 2 = 0$

Result:

If $f(x)$ is continuous function from $x = a$ and $x = b$ and if $f(a), f(b)$ are of opposite since the equation $f(x) = 0$ will have atleast one real root bebetween a & b.

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Fundamental Theorem (or) Intermediate value theorem.

Bisection method:

step 1: Let the given eqn be $f(x)$

step 2: Find a and b, So that $f(a)$ and $f(b)$ are opposite sign.

step 3: Calculate $x_0 = \frac{a+b}{2}$

step 4: Find x_0

step 5: If $f(x_0)$ is +ve then the root lies between a and x_0

~~step 6:~~

If $f(x)$ is -ve. The root lies between x_0 and b

step 6: Continue the interation untill the desired root is obtained.

Find the root of the equation.

$x^3-4x-9=0$ correct the three decimal places using bisection method.

Let $f(x) = x^3-4x-9$

$f(1) = -12 = -VE$

$f(2) = -9 = -VE$

$f(3) = 6 = +VE$

Since $f(2)$ & $f(3)$ are opposite signs

the root lies b/w 2 & 3

S.NO	a -VE	b +VE	$x_0 = \frac{a+b}{2}$	$f(x_0)$	Remark
1.	2	3	2.5	-3.375	$f(x_0)$ = -VE The root lies x_0 & b
2.	2.5	3	2.750	0.797	$f(x_0)$ = +VE The root lies b/w a & x_0
3.	2.5	2.750	2.625	-1.412	$f(x_0)$ = -VE The root lies b/w x_0 & b
4.	2.625	2.750	2.688	-0.330	$f(x_0)$ = -VE The root lies b/w x_0 & b
5.	2.688	2.750	2.719	0.225	$f(x_0)$ = +VE The root lies b/w a & x_0
6.	2.688	2.719	2.704	~~-0.0[illegible]~~ -0.045	$f(x_0)$ = -VE The root lies b/w x_0 & b
7.	2.704	2.719	2.712	0.099	$f(x_0)$ = +VE The root lies b/w a & x_0
8.	2.704	2.712	2.708	0.026	$f(x_0)$ = +VE The root lies b/w a & x_0

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9.	2.704	2.708	2.706	-0.009	$f(x_0)$ = -VE The root lies b/w x_0 & b
10.	2.706	2.708	2.707	0.008	$f(x_0)$ = +VE The root lies b/w a & x_0
11.	2.706	2.707	2.707	0.008	

The required root is 2.707.

2. Solve $x^3 - 9x + 1 = 0$

Soln:

Let $f(x) = x^3 - 9x + 1$

$f(1) = -7 = -VE$

$f(2) = -9 = -VE$

$f(3) = 1 = +VE$

S.No	a -VE	b +VE	$x_0 = \frac{a+b}{2}$	$f(x_0)$	Remark
1.	2	3	2.5	-5.875	$f(x_0)$ = -VE The root lies b/w x_0 & b
2.	2.5	3	2.750	-2.953	$f(x_0)$ = -VE The root lies b/w x_0 & b
3.	2.750	3	2.875	-1.111	$f(x_0)$ = -VE The root lies b/w x_0 & b

4.	2.875	3	2.938	-0.082	$f(x_0) = -VE$. The root lies b/w x_0 & b
5.	2.938	~~2.969~~ 3	2.954.	0.451	$f(x_0) = +VE$
6.	2.938	2.969	2.954.	0.191	The root lies b/w
7.	2.938	2.954	2.946	0.054	a & x_0
8.	2.938	2.946	2.942	-0.014	$f(x_0) = -VE$. The root lies b/w x_0 & b
9.	2.942	2.946	2.944	0.020	$f(x_0) = +VE$
10.	2.942	2.944	2.943	0.003	The root lies b/w
11.	2.942	2.943	2.943	0.003	a & x_0

The required root is 2.943

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Newton Raphson method:

Step 1 : Let the given equation be $f(x)$

Step 2 : Find a & b. So that $f(a)$ & $f(b)$

Step 3 : $f(a)$ and $f(b)$ are opposite sign.

Step 4 : Calculate $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Step 5 : Repeat step 4 & using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Step 6 : Repeat the procedure ~~with~~ untill the desired root is obtained.

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1. Use Newton Raphson method to find the root of the equn $x^3 - 2x - 5 = 0$

Soln:

$f(x) = x^3 - 2x - 5$

$f(1) = -6$ (-ve)

$f(2) = -1$ (-ve)

$f(3) = 16$ (+ve)

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$a = 2 \quad b = 3$

$x_0 = \frac{a+b}{2} = \frac{2+3}{2} = 5/2 = 2.5$

S.No	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
1.	2.5	5.6250	16.7500	2.1642
2.	2.1642	0.8082	12.0513	2.0971
3.	2.097	0.0285	11.1935	2.0946
4.	2.0946	0.0005	11.1620	2.0946
5.	2.0946	0.0005	11.1620	2.0946

The required root is 2.0946.

2. Find the root of $x - e^{-x}$ using newton Raphson method.

Soln:

$f(x) = x - e^{-x}$

$f(0) = 0 - e^{-0} = 0 - 1.000 = (-ve)$

$f(1) = 1 - e^{-1} = 0.6321 = (+ve)$

The root lies between 0 and 1

$a = 0 \quad b = 1$

$f(x) = x - e^{-x}$

$f'(x) = 1 - e^{-x}(-1) = 1+e^{x}$

$x_0 = \frac{a+b}{2}$

$= \frac{0+1}{2} = 1/2$

$\boxed{x_0 = 0.5}$

S.No	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
1.	0.5	-0.2703	1.6703	0.5618
2.	0.5618	-0.0084	1.5702	0.5618
3.	0.5671	-0.0001	1.5672	0.5672
4.	0.5672	0.0001	1.5671	0.5671

$\therefore$ The required root is 0.5671.

3. Find the root of $x\sin x + \cos x = 0$ using newton Rapson method.

Soln:

$f(x) = x\sin x + \cos x$

$f(2) = 1.402 = (+ve)$

$f(3) = -0.5666 = (-ve)$

The root lies between 2 and 3

$a = 2$, $b = 3$

$f'(x) = x\cos x + \cancel{\sin x} - \cancel{\sin x}$

$f'(x) = x\cos x$

$x_0 = \frac{a+b}{2}$

$= \frac{2+3}{2}$

$= 5/2$

$\boxed{x_0 = 2.5}$

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S.No	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$
1.	2.5	0.6950	-2.0029	2.8470
2.	2.8470	-0.1303	-2.7244	2.7992
3.	2.7992	-0.0021	-2.6367	2.7984
	2.7984	0.0000	-2.6352	2.7984

∴ The required root is 2.7984

4. Find the root of x^3-5x+3 using Newton Rapson method.

Soln:

$f(x)=x^3-5x+3$

$f(1)=-1$ (-ve)

$f(2)=1$ (+ve)

The root lies between 1 & 2

$f(x)=x^3-5x+3$ $a=1, b=2$

$f'(x)=3x^2-5$

$x_0=\frac{a+b}{2}=\frac{1+2}{2}=3/2$ $x_0=1.5$

S.No	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$
1.	1.5	-1.1250	1.7500	2.1429
2.	2.1429	2.1257	8.7761	1.9007
3.	1.9007	0.3631	5.8381	1.8385
4.	1.8385	0.0248	5.1402	1.8343
5.	1.8343	0.0003	5.0940	1.8342
6.	1.8342	-0.0002	5.0929	1.8342

Newton's Forward and Backward method

(or)

Newton Interpolation method

1) Newton forward formula:

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

where $x = x_0 + ph$

$$\therefore p = \frac{x - x_0}{h}$$

2) Newton Backward formula:

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 p_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

where

$x = x_n + ph$

$$\therefore p = \frac{x - x_n}{h}$$

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1) If x : 75 80 85 90 find $y(79)$

y : 246 202 118 40

Soln.

S.No	x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
1	75	246	$202-246 = -44$	$-84+44 = -40$	
2	80	202	$118-202 = -84$		$6+40 = 46$
3	85	118		$-78+84 = 6$	
4	90	40	$40-118 = -78$		

$$P = \frac{x - x_0}{h} \qquad x = 79, \quad x_0 = 75$$

$$h = 5.$$

$$= \frac{79 - 75}{5}$$

$$P = \frac{4}{5}$$

$$\boxed{P = 0.8}$$

Newton forward formula

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

$$y(79) = 246 + (0.8)(-44) + \frac{(0.8)(0.8-1)}{2}(-40) + \frac{(0.8)(0.8-1)(0.8-2)}{6}(46)$$

$$= 246 - 35.200 + \left(\frac{-0.160}{2}\right)(-40) + \left(\frac{0.192}{6}\right)(46)$$

$$= 246 - 35.2 + 3.200 + 1.472$$

$$\boxed{y(79) = 215.472}$$

2) Find the population of the town in the senses was given below

Estimate the population for the year 1895 and 1925

year :	1891	1901	1911	1921	1931
population :	46	66	81	93	101

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Soln:

S.No	x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
1.	1891	46	$66-46 = 20$	$15-20 = -5$	$-3+5 = 2$	$-1-2 = -3$
2.	1901	66	$81-66 = 15$	$12-15 = -3$	$-4+3 = -1$	
3.	1911	81	$93-81 = 12$	$8-12 = -4$		
4.	1921	93	$101-93 = 8$			
5.	1931	101				

$p = \frac{x - x_0}{h}$ $\quad x = 1895 \quad x_0 = 75$

$h = 10$

$= \frac{1895 - 1891}{10} = 4/10$

$\boxed{p = 0.4}$

Newton's forward formula

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 +$$

$$\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y(1895) = 46 + (0.4)(20) + \frac{(0.4)(0.4-1)}{2}(-5) + \frac{(0.4)(0.4-1)(0.4-2)}{6}$$

$$(2) + \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{24}(-3)$$

$$= 46 + 8 + 0.6 + 0.128 + 0.125$$

$$y(1895) = 54.853$$

~~Newtons backward~~

$x = x_n + ph$

$1925 = 1931 + p(10)$

$1925 - 1931 = p10$

$\frac{-6}{10} = p$

$\boxed{p = -0.6}$

Newton's backward formula

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 p_n +$$

$$\frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$y(1925) = 101 + (-0.6)(8) + \frac{(-0.6)(-0.6-1)}{2}(-4) +$$

$$\frac{(-0.6)(-0.6-1)(-0.6-2)}{6}(-1) + \cancel{(-0.6)(-0.6-1)} +$$

$$\frac{(-0.6)(-0.6-1)(-0.6-2)(-0.6-3)}{24} \times (-3)$$

$$= 101 - 4.800 + \frac{0.960}{2} + \frac{0.336}{6} + \frac{2.419}{24}$$

$$\boxed{y(1925) = 96.837}$$

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3. Find the construct newton forward interpolation formula. Find y(5)

Soln:

x : 4 6 8 10

y : 1 3 8 16

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
4	1	3-1=2		
			5-2=3	
6	3	8-3=5		3-3=0
8	8		8-5=3	
		16-8=8		
10	16			

$x = x_0 + ph$

$5 = 4 + p(2)$

$5 - 4 = 2p$

$1 = 2p$

$p = 1/2$

$\boxed{p = 0.5}$

Newton's Forward formula

$$y(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 1 + (0.5)2 + \frac{(0.5)(0.5-1)}{2}(3) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(0)$$

$$= 1 + 1 - \frac{0.750}{2} + 0$$

$$= 2 - 0.375$$

$\boxed{y(5) = 1.625}$

Lagranges Interpolation method

Let $y_0, y_1, y_2 \dots y_n$ be the values of $f(x)$ at $x_0, x_1, x_2 \dots x_n$. Then an interpolation polynomial $f(x)$ is given by

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\dots(x_0-x_n)}(y_0) +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)}(y_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)}(y_2) +$$

$$\dots\dots + \dots\dots +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})}(y_n)$$

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1) For the data given below. Find the value of $f(x)$ corresponding to $x=6$

x	1	2	7	8
y	1	5	5	4

Soln:

Here $x_0 = 1,\ x_1 = 2,\ x_2 = 7,\ x_3 = 8,$

$y_0 = 1,\ y_1 = 5,\ y_2 = 5,\ y_3 = 4$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}(y_0) +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}(y_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}(y_2) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}(y_3)$$

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)}(1) + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)}(3$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)}(5) +$$

$$\frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)}(4)$$

$$= \frac{8}{-42} + \frac{50}{30} + \frac{200}{30} + \frac{-80}{42}$$

$$= -0.190 + 1.667 + 6.667 - 1.905$$

$$\boxed{f(6) = 6.239}$$

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Iteration method:

Let $f(x)=0$ be given equation whose roots are to be determined. In this interation method, first we write the given equation in the form $x=f(x)$.

Let $x=x_0$ be an initial approximation x_1 is given by $x_1=p(x_0)$ root α.

The second, third .... etc. approximation are given by

$$x_2=p(x_1)$$
$$x_3=p(x_2)$$
$$\vdots$$
$$x_n=p(x_{n-1})$$

Here x_n is the n^{th} iteration and the value of x_{n_1} gives the root of the given equation at the n^{th} iteration.

Sufficient condition for convergence of iteration

Let $x=\alpha$ be a root of the equation $f(x)=0$ which is equivalent to $x=\phi(x)$. Let I be any iteration containing the root α.

If $|\phi'(x)| < 1$ for all x in I then the sequence of approximation is chosen in I.

1) Find the root of the equation

$x^3 + x^2 - 1 = 0$

Soln:

Given $f(x) = x^3 + x^2 - 1$

$f(0) = -1$ (-ve)

$f(1) = 1^3 + 1^2 - 1$

$f(1) = 1$ (+ve)

$a = 0 \quad b = 1$

$x = \frac{a+b}{2} = \frac{0+1}{2} = \frac{1}{2}$

$\boxed{x = 0.5}$

$x = \phi(x)$

$x^3 + x^2 = 1$

$x^2 [x+1] = 1$

$x^2 = \frac{1}{x+1}$

$\therefore x = \frac{1}{\sqrt{x+1}} \quad (\sqrt{1} = 1)$

$\phi(x) = \frac{1}{\sqrt{x+1}}$ $\left(\frac{d}{dx}(x^n) = n x^{n-1}\right)$

$= \frac{1}{(x+1)^{1/2}} = (x+1)^{-1/2}$

$\phi'(x) = -\frac{1}{2}(x+1)^{-1/2 - 1}$

$\phi'(x) = -\frac{1}{2}(x+1)^{-3/2}$

$$|\phi'(x)| = \left|-\tfrac{1}{2}(x+1)^{-3/2}\right|$$

$$= \left|-\tfrac{1}{2}(0.5+1)^{-3/2}\right|$$

$$= \left|-\tfrac{1}{2}(1.5)^{-3/2}\right|$$

$$= \left|-\tfrac{1}{2}(0.544)\right|$$

$$= |-0.272|$$

$$= |0.272| < 1$$

$$\phi(x) = (x+1)^{-1/2}$$

$$x_1 = (0.5+1)^{-1/2} = (1.5)^{-1/2} = 0.816$$

$$x_2 = (0.816+1)^{-1/2} = (1.816)^{-1/2} = 0.742$$

$$x_3 = (0.742+1)^{-1/2} = (1.742)^{-1/2} = 0.758$$

$$x_4 = (0.758+1)^{-1/2} = (1.758)^{-1/2} = 0.754$$

$$x_5 = (1.754)^{-1/2} = 0.755$$

$$x_6 = (1.755)^{-1/2} = 0.755$$

$\boxed{\text{The root is } 0.755}$

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2) Using Iteration method $x = (5-x)^{1/3}$

Sol: Given $x = (5-x)^{1/3}$

$f(x) = (5-x)^{1/3}$

$f(0) = (5-0)^{1/3} = 1.710$ (+ve)

$f(1) = (5-1)^{1/3} = (4)^{1/3} = 1.587$ (+ve)

$f(-1) = (5+1)^{1/3} = (6)^{1/3} = 1.817$ (+ve)

$f(2) = (5-2)^{1/3} = (3)^{1/3} = 1.442$ (+ve)

$f(-2) = (5+2)^{1/3} = (7)^{1/3} = 1.913$ (+ve)

$f(3) = (5-3)^{1/3} = (2)^{1/3} = 1.260$ (+ve)

$f(-3) = (5+3)^{1/3} = (8)^{1/3} = 2$ (+ve)

$f(4) = (5-4)^{1/3} = (1)^{1/3} = 1$ (+ve)

$f(-4) = (5+4)^{1/3} = (9)^{1/3} = 2.080$ (+ve)

$f(5) = (5-5)^{1/3} = (0)^{1/3} = 0$ (+ve)

$f(-5) = (5+5)^{1/3} = (10)^{1/3} = 2.154$ (+ve)

$f(6) = (5-6)^{1/3} = (-1)^{1/3} = -1$ (-ve)

The root lies between -5 & 6

$a = -5$, $b = 6$

$x_0 = \frac{a+b}{2} = \frac{-5+6}{2} = 1/2 = 0.5$

$\phi(x) = (5-x)^{1/3}$ $\left(\frac{d}{dx}(x)^n = n x^{n-1}\right)$

$|\phi(x)| = \left|\frac{1}{3}(5-x)^{-2/3}\right|$

$= \left|\frac{1}{3}(5-0.5)^{-2/3}\right|$

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$$= \left| \frac{1}{3} (1.651) \right|$$

$$= 0.550$$

$$|\phi'(x)| < 1$$

$\phi(x_1) = (5 - 0.5)^{1/3} = 1.651$

$\phi(x_2) = (5 - 1.651)^{1/3} = 1.496$

$\phi(x_3) = (5 - 1.496)^{1/3} = 1.519$

$\phi(x_4) = (5 - 1.519)^{1/3} = 1.516$

$\phi(x_5) = (5 - 1.516)^{1/3} = 1.516$

The roots are 1.516.

UNIT-2

NUMERICAL DIFFERENTIATION:

Derivate using Newton's forward differents formula

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots$$

where

$$p = \frac{x - x_0}{h}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \ldots\right]$$

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

Newton's interpolation Backward Formula

$$y(x) = y_0 + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n \ldots$$

where, $p = \frac{x - x_n}{h}$

$$\frac{dy}{dx}_{x = x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \ldots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x = x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n \ldots\right]$$

1) From the following table value of x and y find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ prove $x=1.2$

x	1.0	1.2	1.6	1.8	2.0	2.2	1.4
y	2.7183	3.3201	6.0496	7.3891	7.3891	9.0250	4.0552

soln:

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$	$\Delta^6 y_0$
1.0	2.7183	0.1333	0.1333	0.0294			
1.2	3.3201	0.7351			0.0067	0.0013	
1.4	4.0552		0.1627	0.0361			0.0001
		0.8978			0.0080		
1.6	4.9530	1.0966	0.1988	0.0441		0.0014	
1.8	6.0496		0.2429		0.0094		
		1.3395		0.0535			
2.0	7.3891		0.2964				
		1.6359					
2.2	9.0250						

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 \dots\right]$$

$$= \frac{1}{0.2}\left[0.7351 - \frac{1}{2}(0.1627) + \frac{1}{3}(0.0361) - \frac{1}{4}(0.0080) + \frac{1}{5}(0.0014)\right]$$

$$= \frac{1}{0.2}\left[0.7351 - 0.0814 + 0.0120 + 0.0003\right]$$

$$= \frac{1}{0.2}\left[0.6640\right]$$

$$= 3.3200$$

$$\frac{dy}{dx} = 3.3200$$

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$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

$$= \frac{1}{(0.2)^2}\left[0.1627 - 0.0361 + \frac{11}{12}(0.0080) - \frac{5}{6}(0.0014)\right]$$

$$= \frac{1}{0.0400}\left[0.1627 - 0.0361 + 0.0073 - 0.0012\right]$$

$$= \frac{1}{0.0400}\left[0.1327\right]$$

$$= 3.3175$$

$$\frac{d^2y}{dx^2} = 3.3175$$

2) From the following table of values x and y find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.05$

x	1	1.05	1.10	1.15	1.20	1.25	1.30
y	1	0.02470	1.04881	1.07238	1.09544	1.11803	1.140

Soln:

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$	$\Delta^6 y_0$
1	1						
		0.02470					
1.05	0.02470		-0.00059				
		0.02411		0.00005			
1.10	1.04881		-0.00054		-0.00002		
		0.02357		0.00003		0.00003	
1.15	1.07238		-0.00051		0.00001		-0.0006
		0.02306		0.00004			
1.20	1.09544		-0.00047		-0.00002	-0.00003	
		0.02259		0.00002			
1.25	1.11803		-0.00045				
		0.02214					
1.30	1.14017						

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 \ldots\ldots\right]$$

$$= \frac{1}{0.05}\left[(0.02411) - \frac{1}{2}(-0.00054) + \frac{1}{3}(0.00003) - \frac{1}{4}(0.00001)\right]$$

$$= \frac{1}{0.05}\left[0.02411 + 0.00027 + 0.0001 + 0 - 0.0001\right]$$

$$= \frac{1}{0.05}\left[0.02438\right]$$

$$\therefore \frac{dy}{dx} = 0.48760$$

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

$$= \frac{1}{0.00250}\left[(-0.00054) - 0.00003 + \frac{11}{12}(0.00001) - \frac{5}{6}(-0.00003)\right]$$

$$= \frac{1}{0.00250}\left[-0.00054 - 0.00003 + 0.00001 + 0.0003\right]$$

$$= \frac{1}{0.00250}\left[-0.00053\right]$$

$$\therefore \frac{d^2y}{dx^2} = 0.21200$$

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Trapezoidal rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[y_0 + 2\left[y_1 + y_2 + y_3 + \dots\right] + y_n\right]$$

where $h = x_1 - x_0,\ x_2 - x_1$

Simphson's $\frac{1}{3}$rd rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + \dots) + y_n\right]$$

Simphon's $\frac{3}{8}$rd rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + \dots) + 2(y_3 + y_6 + \dots) + y_n\right]$$

---

problem:

1. Find the value of $\int_3^7 x^2 \log x\,dx$ by taking four steps by.

   i) Traperzodial value

   ii) Simphon's $\frac{1}{3}$rd & $\frac{3}{8}$rd rule.

Soln:

$$\int_3^7 x^2 \log x\,dx$$

$\boxed{h=1}$

x	3	4	5	6	7
y	4.294	9.633	17.474	28.013	41.410

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i) Traperzoidal rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right].$$

$$\int_3^7 x^2 \log_x x\, dx = \frac{1}{2}\left[4.294 + 2(9.633 + 17.474 + 28.013) + 41.410\right]$$

$$= \frac{1}{2}\left[155.944\right]$$

$$\int_3^7 x^2 \log x\, dx = 77.972$$

ii) simphon's $\frac{1}{3}$ rd rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 \ldots) + 2(y_2 + y_4 + y_6 + \ldots) + y_4\right]$$

$$= \frac{1}{3}\left[231.236\right]$$

$$\int_3^7 x^2 \log x^2\, dx = 77.079.$$

iii) simphon's $\frac{3}{8}$ rd rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4) + 2(y_3 + y_6) + y_4\right]$$

$$= \frac{3}{8}\left[4.294 + 3(9.633 + 17.474) + 2(28.013) + 41.410\right]$$

$$= \frac{3}{8}\left[183.051\right]$$

$$\int_3^7 x^2 \log x\, dx = 68.644.$$

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2. Find the value of $\int_1^7 x^3\, dx$ by taking fourstep

Soln:

x	1	2	3	4	5	6	7
y	1	8	27	64	125	216	343

Here $h = x_1 - x_0 = 1$ $\boxed{h=1}$

i) Trapezodial rule:

$$\int_{x_0}^{x_n} y\, dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 \dots) + y_n\right]$$

$$= \frac{1}{2}\left[1 + 2(8 + 27 + 64 + 125 + 216) + 343\right]$$

$$= \frac{1}{2}\left[1,224\right]$$

$$\int_1^7 x^3 dx = 612$$

ii) Simphon's $\frac{1}{3}$rd rule:

$$\int_{x_0}^{x_n} y\, dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 \dots) + 2(y_2 + y_4 + y_6) + y_n\right]$$

$$= \frac{1}{3}\left[1 + 4(8 + 64 + 216) + 2(27 + 125 + 343)\right]$$

$$= \frac{1}{3}\left[1800\right]$$

$$\int_1^7 x^3\, dx = 600$$

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iii) simphon's 3/8 rd rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5, \dots) + 2(y_3 + y_0) + y_n\right]$$

$$= \frac{3}{8}\left[1 + 3(8 + 27 + 125 + 216) + 2(64) + 343\right]$$

$$= \frac{3}{8}\left[1600\right]$$

$$\boxed{\int_{1}^{7} x^3\,dx = 600}$$

1. Solution to linear system of equation by using matrix method and Iteration method:

Find the solution to the linear system by using of the following method.

i) Gauss - Elimination method

ii) Gauss - Jacobi method

iii) Gauss - Seidal method

problem:

1) Solve the following equn by Gauss Elimination method $x+3y=4$, $3x-2y=1$.

Soln:

The matrix formula

$$Ax = B$$

$$A = \begin{pmatrix} 1 & 3 \\ 3 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\boxed{AB} \sim \begin{bmatrix} 1 & 3 & 4 \\ 3 & -2 & 1 \end{bmatrix} \quad R_2 \rightarrow R_2 - 3R_1$$

$$\sim \begin{pmatrix} 1 & 3 & 4 \\ 0 & -11 & -11 \end{pmatrix}$$

$\therefore x+3y=4 \quad - (1)$

$-11y = -11$

$11y = 11$

$\boxed{y=1}$

sub $y=1$ in (1)

$$x+3(1)=4$$
$$x+3=4$$
$$x=4-3$$
$$\boxed{x=1}$$

$\therefore \boxed{x=1}\ \boxed{y=1}$

---

2) Solve the following by Gauss elimination method $x+y+z=6$, $2x-y+z=3$, $x+3y-z=4$

Soln:

The matrix Formula

$$Ax=B$$

$$A=\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{pmatrix} \qquad B=\begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix}$$

$$[AB]\sim\begin{pmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & 3 & -1 & 4 \end{pmatrix} \quad \cancel{R_2 \to R_2 - 2R_1} \quad \cancel{R_3 \to R_3 - R_1}$$

$$[AB]\sim\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & -1 & -9 \\ 0 & 2 & -2 & -2 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \\ \cancel{R_3 \to 3R_3 + 2R_2} \end{matrix}$$

$$\sim\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & -1 & -9 \\ 0 & 0 & -8 & -24 \end{pmatrix} \quad R_3 \to 3R_3 + 2R_2$$

$x+y+z=6 \rightarrow (1)$

$-3y-z=-9 \rightarrow (2)$

$\cancel{-}8z=\cancel{-}24 \rightarrow (3)$

$$\boxed{z=3}$$

Sub $z=3$ in (2)

$-3y-3=-9$

$-3y=-9+3$

$\cancel{-}3y=\cancel{-}6$

$$\boxed{y=2}$$

Sub $z=3$, $y=2$ in (1)

$x+2+3=6$

$x+5=6$

$x=6-5$

$$\boxed{x=1}$$

$\therefore$ The soln.

$$\boxed{x=1, y=2, z=3}$$

4) Solve the elimination method

$$x_1 - x_2 + x_3 = 1$$

$$-3x_1 - 2x_2 = -6$$

$$2x_1 - 5x_2 + 4x_3 = 5$$

Soln:

The matrix for

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & -2 & -2 \\ 2 & -5 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ -6 \\ 5 \end{pmatrix}$$

$$(A, B) = \begin{pmatrix} 1 & -1 & 1 & 1 \\ -3 & -2 & -2 & -6 \\ 2 & -5 & 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & -5 & 1 & -3 \\ 0 & -3 & 2 & 3 \end{pmatrix} \begin{array}{l} \\ R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$= \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & -5 & 1 & -3 \\ 0 & 0 & -7 & -24 \end{pmatrix} R_3 \to 3R_2 - 5R_3$$

$$x_1 - x_2 + x_3 = 1 \to (1)$$

$$-5x_2 + x_3 = -3 \to (2)$$

$$\cancel{-}7x_3 = \cancel{-}24 \; - (3)$$

$$\boxed{x_3 = 24/7}$$

(3) Sub (2)

$$-5x_2 + \left(24/7\right) = -3$$

$$-5x_2 = -3 - \left(24/7\right)$$

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$$-5x_2 = \frac{-21-24}{7}$$

$$\cancel{+}5x_2 = \cancel{+}\frac{45}{7}$$

$$x_2 = \frac{\cancel{45}^{9}}{\cancel{5}\times 7}$$

$$\boxed{x_2 = 9/7}$$

① Sub ② and ③.

$$x_1 - x_2 + x_3 = 1$$

$$x_1 - (9/7) + (24/7) = 1$$

$$x_1 + 15/7 = 1$$

$$x_1 = 1 - 15/7$$

$$= \frac{7-15}{7}$$

$$\boxed{x_1 = 8/7}$$

Therefore. $\boxed{x_1 = 8/7}$ $\boxed{x_2 = 9/7}$ $\boxed{x_3 = \frac{24}{7}}$

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2) Solve the following system of eqn by using Gauss seidal method.

$8x - 3y + 2z = 20, \ 4x + 11y - z = 33$

$6x + 3y + 12z = 35$

Soln:

Given $8x - 3y + 2z = 20$

$8x = 20 + 3y - 2z$

$x = \frac{1}{8}[20 + 3y - 2z] \quad -(1)$

$4x + 11y - z = 33$

$11y = 33 - 4x + z$

$y = \frac{1}{11}[33 - 4x + z] \rightarrow (2)$

$6x + 3y + 12z = 35$

$12z = 35 - 6x - 3y$

$z = \frac{1}{12}[35 - 6x - 3y] \rightarrow (3)$

1st Iteration:

Let $y = 0 \quad z = 0$

$x = \frac{1}{8}[20 + 3(0) - 2(0)]$

$= \frac{20}{8} = 2.5$

$y = \frac{1}{11}[33 - 4(2.5) + 0] = 2.091$

$z = \frac{1}{12}[35 - 6(2.5) - 3(2.091)] = 1.144$

2nd Iteration:

$$x = \frac{1}{8}\left[20 + 3(2.09\blacksquare) - 2(1.144)\right] = 2.998$$

$$y = \frac{1}{11}\left[33 - 4(2.998) + (1.144)\right] = 2.014$$

$$z = \frac{1}{12}\left[35 - 6(2.998) - 3(2.014)\right] = 0.914$$

3rd Iteration:

$$x = \frac{1}{8}\left[20 + 3(2.014) + 2(0.914)\right] = 3.027$$

$$y = \frac{1}{11}\left[33 - 4(3.027) + (0.914)\right] = 1.982$$

$$z = \frac{1}{12}\left[35 - 6(3.027) - 3(1.982)\right] = 0.908$$

4th Iteration:

$$x = \frac{1}{8}\left[20 + 3(1.982) - 2(0.908)\right] = 3.016$$

$$y = \frac{1}{11}\left[33 - 4(3.016) + (0.908)\right] = 1.986$$

$$z = \frac{1}{12}\left[35 - 6(3.016) - 3(1.986)\right] = 0.912$$

5th Iteration:

$$x = \frac{1}{8}\left[20 + 3(1.986) - 2(0.912)\right] = 3.017$$

$$y = \frac{1}{11}\left[33 - 4(3.017) + (0.912)\right] = 1.986$$

$$z = \frac{1}{12}\left[35 - 6(3.017) - 3(1.986)\right] = 0.912$$

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6th Iteration

$$x = \frac{1}{8}[20+3(1.986)-2(0.912)] = 3.017$$

$$y = \frac{1}{11}[33-4(3.017)+(0.912)] = 1.986$$

$$z = \frac{1}{12}[35-6(3.017) - 3(1.986) = 0.912$$

Solution:

$x = 3.017$

$y = 1.986$

$z = 0.912$

Gauss Jacobi method of Itreation (or)

Gauss Jacobi method:-

*) Consider the system of equations:

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots = b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n1}x + a_{n2}x_2 + \ldots\ldots\ldots\ldots + a_{nn}x_n = b_n$$

*) We assume that co.efficient matrix of this system is diagonally dominant. The above ~~equation~~ equations can be rewritten as

$$x_1 = \frac{1}{a_{11}}[b_1 - a_{12}x_2 - a_{13}x_3 - \ldots\ldots - a_{1n}x_n]$$

$$x_2 = \frac{1}{a_{22}}[b_2 - a_{21}x_1 - a_{23}x_3 \ldots\ldots a_{2n}x_n]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$x_n = \frac{1}{a_{nn}}[b_n - a_{n1}x_1 - a_{n2}x_2 \ldots\ldots a_{n\,n-1}x_n]$$

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*) We start with the initial value for the variables $x_1, x_2 \ldots\ldots x_n$ to be $x_1^{(0)}, x_2^{(0)} \ldots\ldots x_n^{(0)}$

*) Using 3 value in (1) (2) .... (n) respectily we get $x_1, x_2 \ldots\ldots x_n$

*) Putting $x_1 = x_1, x_2 = x_2 \ldots\ldots x_n = x_n$ in (1) (2) .... (n) respectively we get the next approximations $x_1, x_2, \ldots\ldots x_n$

*) In general of the value of $x_1, x_2 \ldots x_n$ in the r^{th} iteration are $x_1^{(r)}, x_2^{(r)} \ldots\ldots x_n^{(r)}$

Then

$$x_1^{(r+1)} = \frac{1}{a_{11}} \left[b_1 - a_{12} x_1^{(r)} - a_{13} x_2^{(r)} \ldots\ldots - a_{1n} x_n^{(r)} \right]$$

$$x_2^{(r+1)} = \frac{1}{a_{22}} \left[b_2 - a_{21} x_1^{(r)} - a_{23} x_2^{(r)} \ldots\ldots - a_{2n} x_n^{(r)} \right]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$x_n^{(r+1)} = \frac{1}{a_{nn}} \left[b_n - a_{n1} x_1^{(r)} - a_{n2} x_2^{(r)} \ldots\ldots - a_{nn} x_n^{(r)} \right]$$

Problems

1) Solve the following system of equation by Gauss Jacobi method.

$10x - 5y - 2z = 3,\ 4x - 10y + 3z = -3$

$x + 6y + 10z = -3$

Soln:

$10x - 5y - 2z = 3$

$10x = 3 + 5y + 2z$

$x = \frac{1}{10}[3 + 5y + 2z] \rightarrow (1)$

$4x - 10y + 3z = -3$

$-10y = -3 - 4x - 3z$

$10y = 3 + 4x + 3z$

$y = \frac{1}{10}[3 + 4x + 3z] \rightarrow (2)$

$x + 6y + 10z = -3$

$10z = -3 - x - 6y$

$\therefore z = \frac{1}{10}[-3 - x - 6y] \rightarrow (3)$

I^{st} Iteration:

Let $x = 0, y = 0, z = 0$

$x = \frac{1}{10}[3 + 5(0) + 2(0)] = 0.3$

$y = \frac{1}{10}[3 + 4(0) + 3(0)] = 0.3$

$z = \frac{1}{10}[-3 - (0.3) - 6(0.3)] = -0.510$

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3^{rd} Iteration

$$x = \frac{1}{10}\left[3 + 5(0.330) + 2(-0.510)\right] = 0.363$$

$$y = \frac{1}{10}\left[3 + 4(0.330) + 3(-0.510)\right] = 0.303$$

$$z = \frac{1}{10}\left[-3 - 0.390 - 6(0.330)\right] = -0.537$$

4^{th} Iteration

$$x = \frac{1}{10}\left[3 + 5(0.303) + 2(-0.537)\right] = 0.344$$

$$y = \frac{1}{10}\left[3 + 4(0.363) + 3(-0.537)\right] = 0.284$$

$$z = \frac{1}{10}\left[-3 - 0.363 - 6(0.303)\right] = -0.518$$

5^{th} Iteration

$$x = \frac{1}{10}\left[3 + 5(0.284) + 2(-0.518)\right] = 0.338$$

$$y = \frac{1}{10}\left[3 + 4(0.344) + 3(-0.518)\right] = 0.282$$

$$z = \frac{1}{10}\left[-3 - 0.344 - 6(0.284)\right] = -0.505$$

6^{th} Iteration

$$x = \frac{1}{10}\left[3 + 5(0.282) + 2(-0.505)\right] = 0.340$$

$$y = \frac{1}{10}\left[3 + 4(0.338) + 3(-0.505)\right] = 0.284$$

$$z = \frac{1}{10}\left[-3 - (0.338) - 6(0.282)\right] = -0.503$$

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7^{th} Iteration

$$x = \frac{1}{10}[3 + 5(0.284) + 2(0.503)] = 0.341$$

$$y = \frac{1}{10}[3 + 4(0.340) + 3(-0.503)] = 0.285$$

$$z = \frac{1}{10}[-3 - (0.340) - 6(0.284)] = -0.504$$

8^{th} Iteration:

$$x = \frac{1}{10}[3 + 5(0.285) + 2(-0.504)] = 0.342$$

$$y = \frac{1}{10}[3 + 4(0.341) + 3(-0.504)] = 0.285$$

$$z = \frac{1}{10}[-3 - (0.341) - 6(0.285)] = -0.505$$

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UNIT-III

Taylor's Series

consider the first order differential equation $\frac{dy}{dx} = y' = f(x, y) \longrightarrow (1)$

with the Initial condition $y(x) = y_0$

differentiating (1) w.r. to 'x'.

we get $\frac{d^2y}{dx^2} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} y'$

(i.e) $y'' = f(x) + f_y y'$

Differentiating successfully we can obtain

$y''', y^{iv} \ldots\ldots$

Putting $x = x_0$ and $y = y_0$ we get $y_0', y_0'', y_0''' \ldots\ldots$

The Taylor's series expansion of $y(x)$

about $x = x_0$ is given by

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \ldots\ldots$$

$$= y_0 + \frac{(x-x_0)}{1!} y_0' + \frac{(x-x_0)^2}{2!} y_0'' + \ldots\ldots - (2)$$

Substituting the value of $y_0, y_0', y_0'' \ldots\ldots$

we can obtain $y(x)$ for all values of x

for which (2) converges

Let $x_1 = x_0 + h$ and

$$y(x_1) = y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \ldots\ldots$$

Once y_1 is known we can completed $y_1', y_1'', y_1''' \ldots$ from ① & ② etc.

Then y can be expanded is a Taylor's series about $x = x_1$ and we have

$$\boxed{y(x_1+h) = y(x_2) = y_2 = y_1 + \frac{h}{1!}y_1' + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \ldots}$$

1) Solve $y' = x+y$; $y(0) = 1$ by Taylor's series method. the values y at $x = 0.1$ and $x = 0.2$

Soln:

Given $y' = x+y$ $\quad x_0 = 0$ $\quad y_0 = 1$ $\quad h = 0.1$

$y'' = 1 + y'$ $\qquad y_0' = x_0 + y_0 = 0+1 = 1$

$y''' = 0 + y'$ $\qquad y_0'' = 1 + y_0' = 1+1 = 2$

$y''' = y''$ $\qquad y_0''' = y_0''$

$y^{IV} = y'''$ $\qquad y_0''' = 2$

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0'''$$

$$= 1 + (0.1)(1) + \frac{(0.1)^2}{2}(2) + \frac{(0.1)^3}{6}(2)$$

$$= 1 + 0.1 + 0.01 + 0.0032$$

$$\boxed{y_1 = 1.11033}$$

$x_1 = 0.1$, $y_1 = 1.1103$. $h = 0.1$

$$y_2 = y(0.2) = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1'''$$

$$y_1' = x_1 + y_1 = 0.1 + 1.1103 = 1.2103$$

$$y_1'' = 1 + y_1' = 1 + 1.2103$$

$$y_1'' = 2.2103$$

$$y_1''' = y_1'' = 2.2103$$

$$y_2 = y(0.2) = 1.1103 + \frac{(0.1)}{1}(1.2103) + \frac{(0.1)^2}{2}(2.2103)$$

$$+ \frac{(0.1)^3}{6}(2.2103)$$

$$= 1.1103 + 0.12103 + 0.0110515 + 0.0037$$

$$\boxed{y_2 = y(0.2) = 1.024275}$$

2) Using Taylor's series method Find y at $x = 0.1$ correct is four decimal places from $\frac{dy}{dx} = x^2 - y$ then $y(0) = 1$ with $h = 0.1$ compute terms up to x^4.

Soln:

Given $\frac{dy}{dx} = y' = x^2 - y$ $\quad x_0 = 0 \quad y_0 = 1 \quad x_1 = 0.1$

$h = 0.1$

$y' = x^2 - y$	$y_0' = x_0^2 - y_0 = (0)^2 - 1 = -1$
$y'' = 2x - y'$	$y_0'' = 2x_0 - y_0' = 2(0) - (-1)$
	$= 0 + 1 = 1$
$y''' = 2 - y''$	$y_0''' = 2 - y_0'' = 2 - 1$
$y^{iv} = 0 - y'''$	$y_0''' = 1 \quad y_0^{iv} = -1$

$y^{iv} = \ldots$

By Taylor's series:

$$y_1 = y(0.1) = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0''$$

$$= 1 + \frac{(0.1)}{1}(-1) + \frac{(0.1)^2}{2}(1) + \frac{(0.1)^3}{6}(1) + \frac{(0.1)^4}{24}(-1)$$

$$= 1 - 0.1 + 0.0505 + 0.000167 - 0.00004$$

$$\boxed{y(0.1) = 0.905163}$$

Euler's method

1) Using Euler's method find $y(0.2)$ and $y(0.4)$ $y(0.6)$ from $\frac{dy}{dx} = x+y$, $y(0) = 1$ with $h = 0.2$

Soln:

$y(x) = n$

$y(0) = 1$ $y_0 = 1$

$x_0 = 0$

Given $f(x, y) = x + y$

$x_0 = 0 \quad y_0 = 1 \quad h = 0.2$

$x_1 = 0.2 \quad x_2 = 0.4$

By Euler's Algorithm

$$y_1 = y_0 + h f(x_0, y_0)$$

$$= 1 + (0.2)(x_0 + y_0)$$

$$= 1 + (0.2)(0+1)$$

$y(0.2) = 1.2$

$$= 1 + 0.2 = 1.2$$

$$\boxed{y_1 = 1.2}$$

By Euler's Algorithm

$y_2 = y_1 + h\, f(x_1, y_1)$

$= y_1 + h\,[x_1 + y_1]$

$= 1.2 + (0.2)\,[0.2 + 1.2]$

$= 1.2 + (0.2)\,[1.4]$

$= 1.2 + 0.28$

$\boxed{y_2 = 1.48}$

By Euler's Algorithm

$y_3 = y_2 + h\, f(x_2\ y_2)$

$y(0.6) = y_2 + h\,[x_2 + y_2]$

$= 1.48 + 0.2\,[0.4 + 1.48]$

$= 1.48 + (0.2)(1.88)$

$= 1.48 + 0.376$

$\boxed{y(0.6) = 1.856}$

2) Using Euler's method solve $y' = x + y + xy$ $y(0) = 1$ compute at $x = 0.1$ by taking $h = 0.05$

Soln:

By Euler's Algorithm Here $x_0 = 0$ $y_0 = 1$ $h = 0.05$

$y_1 = y_0 + h f(x_0\, y_0)$

$= y_0 + h\,(x_0 + y_0 + x_0 y_0)$

$= 1 + (0.05)\,[0 + 1 + (0 \times 1)]$

$= 1 + (0.05)(1)$

$\boxed{y_1 = 1.05}$

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$x_1 = 0.1 \quad y_1 = 1.05$

By Euler's Algorithm

$y_2 = y_1 + hf(x_1, y_1)$

$= 1.05 + (0.05)(x_1 + y_1 \; x_1 y_1)$

$= 1.05 + (0.05)\left[(0.1 + 1.05) + (0.1)(1.05)\right]$

$= 1.05 + (0.05)(1.255)$

$= 1.05 + 0.057625$

$\boxed{y(0.1) = 1.11275}$

Milne's method

milne's predictor and corrector method formula

$y_{n+1,P} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$

$y_{n+1,C} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$

1) Given $\frac{dy}{dx} = x^3 + y$ $y(0) = 2$. The values of ~~y~~

$y(0.2) = 2.073$ $y(0.4) = 2.452$ $y(0.6) = 3.023$

are got by R-K method of order four. Find

$y(0.8)$ by Milne's predictor and corrector

method.

Soln: Given

$h = 0.2$

$x_0 = 0$ $y_0 = 2$

$x_1 = 0.2$ $y_1 = 2.073$

$x_2 = 0.4$ $y_2 = 2.452$

$x_3 = 0.6$ $y_3 = 3.023$

$y' = \frac{dy}{dx} = x^3 + y$

$y_1' = x_1^3 + y_1 = (0.2)^3 + 2.073 = 2.081$

$y_2' = x_2^3 + y_2 = (0.4)^3 + 2.452 = 2.516$

$y_3' = x_3^3 + y_3 = (0.6)^3 + 3.023 = 3.239$

$y_{3+1} = P = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$

$P = 2 + \frac{4(0.2)}{3}\left[2(2.081) - 2.516 + 2(3.239)\right]$

$= 2 + \frac{0.8}{3}[8.124]$

$= 2 + 2.1664$

y_4 $\boxed{P = 4.1664}$

$$y_{n+1,C} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

$n=3$

$$y_4 = C = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$y'_4 = x_4^3 + y_4 = (0.8)^4 + 4.1664 = 4.6784$$

$$y_4 = 2.452 + \frac{0.2}{3}\left[2.516 + 4(3.239) + 4.6784\right]$$

$$\boxed{y_4 = 3.79536}$$

Runge ~~Kutta~~ 2nd order method

$$y_1 = y_0 + \Delta y$$

where

$$\Delta y = \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + h, y_0 + k_1)$$

Runge ~~Kutta~~ 4th order method:

$$y_1 = y_0 + \Delta y$$

$$\Delta y = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

---

1. Solve apply for 2nd order R-K method $y' = x + y$ when $x = 0.2$ with the initial condition $y(0) = 1$, $h = 0.1$

Soln: Given $y' = x + y$ $x = 0.2$ $y(0) = 1$

$h = 0.1$ $x_0 = 0$ $y_0 = 1$

$f(x, y) = x + y$

$y_1 = y_0 + \Delta y$

$\Delta y = \frac{1}{2}(k_1 + k_2)$

$k_1 = hf(x_0 y_0)$

$k_2 = hf(x_0 + h, y_0 + k_1)$

$k_1 = hf(x_0, y_0)$

$= h(x_0 + y_0)$

$= (0.1)(0+1)$

$= (0.1)(1)$

$\boxed{k_1 = 0.1}$

$k_2 = hf[(0+0.1)\ (1+0.1)]$

$= hf((0.1)(1.1))$

$= (0.1)((0.1) + 1.1)$

$= (0.1)(1.2)$

$\boxed{k_2 = 0.12}$

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$\Delta y = \frac{1}{2}(K_1 + K_2)$

$= \frac{1}{2}((0.1) + (0.12))$

$= \frac{1}{2}(0.22)$

$\boxed{\Delta y = 0.11}$

$y_1 = y_0 + \Delta y$

$= 1 + 0.11$

$\boxed{y_1 = 1.11}$

$x_1 = x_0 + h = 0 + 0.1$

$= 0.1$

$y_2 = y_1 + \Delta y$

$\Delta y = \frac{1}{2}(K_1 + K_2)$

$K_1 = hf(x_1, y_1)$

$K_1 = hf((0.1)(1.11))$

$= (0.1)((0.1) + 1.11)$

$= (0.1)(1.210)$

$\boxed{K_1 = 0.121}$

$K_2 = hf(x_1 + h, y_1 + k_1))$

$= hf((0.1) + 0.1)(1.11 + 0.121)$

$= hf((0.2)(1.231))$

$= (0.1)[0.2 + 1.231]$

$= (0.1)(1.431)$

$\boxed{K_2 = 0.143}$

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$\therefore \Delta y = \frac{1}{2}(K_1 + K_2)$

$= \frac{1}{2}((0.121) + 0.143)$

$\Delta y = \frac{1}{2}(0.264)$

$= 0.132$

$y_2 = y_1 + \Delta y$

$= 1.11 + 0.132$

$\boxed{y_2 = 1.242}$

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