OPERATIONS RESEARCH

objects :

1. To train the Students to solve assignment Problems, transportation Problem

2. To train the Students in network Problems

UNIT-I

OPERATIONS RESEARCH : Introduction - Basics of OR - OR's decision Making - Role of computers in OR - Linear Programing formulations & graphical Solution of two Variables - Canonical & Standard forms of LPP

UNIT-II

Simplex method : Simplex method for <, =, > constraints - Chave's method of Penalties - Two phase Simplex Method

UNIT-III

Trans Portation Problem : Transportation algorithm Degeneracy algorithm - Degeneracy in Transportation Problem, Unbalanced Transportation Problem - Assignment algorithm - unbalanced Assignment Problem

UNIT - IV

Sequencing Problem : Processing of Jobs through two machines - processing of n Jobs through 3 Machines - processing of two Jobs through m Machines

UNIT - V

Networks : Network - Fulkerson's rule - Measure activity PERT Computation - CPM computati - Resource Scheduling.

Text Book

1. Man mohan & Gupta, Operations Researc - Sultan Chand Publishers, New Delhi.

Reference(s)

1. Premkumar Gupta and D.S. Hira. Operations Research : An Introduction S Chand and Co. Ltd, New Delhi.

2. Handy A. Taha, operations Research (7th Edi) Mc Millan Publishing company New Delhi 1982

Transportation Model

Introduction :

Transportation deals with the transportation of a commality (single Product) from Soures (origins or Supply or capacity centres) to 'n' destinations (sinks or demand or requirement centres)

Its assumed that

i) level of Supply at each soures and the amout of demand at each desions and

ii) The unit transportation cost at Commodity from each Soures to each destination are Known its assumed that the cost, of transportation iii) its linear

The objective is to determine the amout to be shifted from each Soures to destinations Such that the total transportation cast is minimum

Mathematical formulation of a transportation Problem

Let as assume that there are 'm' Soues and 'n' destinations let a be

Method : 1 North West Corner rule

Step-1

The first assignment is made in the cell occupying the upper left hand (north-west) corner of the transportation table the maximum possible amount is alloted there (ie) $x_{11} = \min\{a_1, b_1\}$

Case (i) :

If min $\{a_1, b_1\} = a_1$ then put $x_{11} = a_1$ decrease b_1 by a_1 and move vertically to the 2nd row (ie) to the cell (2,1) cross o of the first row.

Case (ii) :

If num $\{a_1, b_1\} = b_1$ then $x_{11} = b_1$ and decrease a_1 by b_1 and have horizontally right (i.e) to the cell (1,2) cross out the first column

Case (iii) :

If num $\{a_1, b_1\} = a_1 = b_1$ then put $x_{11} = a_1 = b_1$ and more diagonally to the cell (2,2) cross out the first row and first coloumn

Step 2 :

Repeat the procedure untill all the aim requirements are satistied.

Problems:

Solve the following T.P problem using north-west corner rule (NWC)

					Supply
2	11	10	3	7	4
1	4	7	2	1	8
3	9	4	8	12	9
Demand ~~3~~	3	4	5	6	

Sol

Since $\Sigma a_i = \Sigma b_j = 21$ The T.P problem is balanced

$\boxed{3}2$	11	10	3	7	~~4~~ 1
1	4	7	2	1	8
3	9	4	8	12	9
~~3~~	3	4	5	6	

$\boxed{1}11$	10	3	7	~~1~~
4	7	2	1	8
9	9	8	12	9
~~3~~ 2	4	5	6	

$\Rightarrow$

$\boxed{2}4$	7	2	1	~~8~~ 6
9	4	8	12	9
~~2~~	4	5	6	

$\Rightarrow$

$\boxed{4}7$	2	1	~~6~~ 2
4	8	12	9
~~4~~	5	6	

$\boxed{2}2$	1	~~2~~
8	12	9
~~5~~ 3	6	

$\Rightarrow$

$\boxed{3}8$	12	~~9~~ 6
~~3~~	6	

$\Rightarrow$

$\boxed{6}12$	~~6~~
~~6~~	

$\boxed{3}2$	$\boxed{1}11$	10	3	7
1	$\boxed{2}4$	$\boxed{4}7$	$\boxed{2}2$	1
3	9	4	$\boxed{3}8$	$\boxed{6}12$

Here m+n-1 = 5+3-1 = 7 no. of allocations

Hence the T.P is basic feasible solution

The Transportation test

RS (2×3)+(1×11)+(2×11)+(4×7)+(2×2)+(8×3)

+(12×6)

$\boxed{\text{R.S } 153}$

Problem-2

Solve the fallowing T.P problem using North-west corner rule (NWC)

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

Sol

Since $\Sigma a_i = \Sigma b_j = 30$ The T.P Problem is balanced

$\boxed{7}$ 1	2	6	7
$\boxed{3}$ 0	$\boxed{9}$ 4	2	12
3	$\boxed{1}$ 1	[illegible] 5	11
10	10	10	

Here m+n-1 = 3+3-1 = 5 no. of allocations

Hence the T.P is best feasible Solution

The T.P test

R.S (7×1)+(3×0)+(9×4)+(1+1)+(10×5)

R.S. 94

step 1 :

Find the difference (penalty) between the smallest and next smallest cost in each in each row (coloumn) and write then in brackets against the corresponding rows (coloumns)

Step 2 :

Identity the row colowmn with largest Penalty, If a tie occurs breath the arbitary choose the cell with smallest cost in that selected row (or) coloumn and allocate cross out the satisfied row (or) coloumn and go to step (3)

Step 3 :

Again compute the column and row penalities for the reduced transportation table and then go to . step (2). Repeat the procedure untill the rim requirements are satisfied

1. Find the initial basic feasible solution for the fallowing T.P Least cost method (LCM)

1	2	1	4	30
3	3	2	1	50
4	2	5	9	20
20	40	30	10	

Sol

Since $\Sigma a_i = \Sigma b_j = 100$

The given T.P is balanced

$\boxed{20}$ 1	2	$\boxed{19}$ 1	4
3	$\boxed{20}$ 3	$\boxed{20}$ 2	$\boxed{10}$ 1
4	$\boxed{20}$ 2	5	9

Here $m+n-1 = 3+4-1 = 6$ no. of allocations

Hence the T P is basic feasible solution

Transportation cost

= RS (20×1)+(10×1)+(20×3)+(20×2)+(1×10)+

(20×2)

R.S. 180

2. LCM

A	B	C	D	E	
4	1	2	6	9	100
6	4	3	5	7	120
5	2	6	4	8	120
40	50	70	90	90	

Sol

4	$\boxed{50}$ 1	$\boxed{50}$ 2	6	9	~~100~~ 50
$\boxed{10}$ 6	4	$\boxed{30}$ 3	5	$\boxed{90}$ 7	~~120~~ ~~100~~ 90
$\boxed{60}$ 5	2	6	$\boxed{90}$ 4	8	120 ~~30~~
~~40~~ 10	~~50~~	~~70~~ ~~30~~	~~90~~	~~90~~	

Here $m+n-1 = 3+5-1 = 7$ no. of allocations

The T.P is basic feasible solution

The Transportation cost

RS (50×1)+(50×2)+(10×6)+(20×3)+(90×7)

+(30×5)+(90×4)

R.S 1410

3. LCM:

7	3	2	2
2	1	3	3
3	4	6	5
4	1	5	

Sol

Since $\Sigma a_i = \Sigma b_j = 10$ T.P is balanced

7	3	$\boxed{2}$ 2	$\cancel{2}$	0
$\boxed{2}$ 2	$\boxed{1}$ 1	3	$\cancel{3}$	$\cancel{2}$
$\boxed{2}$ 3	4	$\boxed{3}$ 6	$\cancel{5}$	$\cancel{3}$
$\cancel{4}$	$\cancel{1}$	$\cancel{5}$		
$\cancel{2}$		3		

Here $m+n-1 = 3+3-1 = 5$ no. of allocations

The T.P is basic feasible solution

T.P cost

$= (1\times1)+(2\times2)+(2\times2)+(2\times3)+(3\times6)$

$=$ R.S 33

1. Find the initial basic ~~feasil~~ feasible solution for the following T.P by VAM

A	B	C	D	
11	13	17	14	250
16	18	14	10	300
21	24	13	10	400
200	225	275	250	

Q1

//

Since $\Sigma a_i = \Sigma b_j = 950$

The Given T.P is balanced

$\boxed{200}$ 11	13	17	14	250	(2)
16	18	14	10	300	(4)
21	24	13	10	400	(3)
200	225	275	250		
(5)	(5)	(1)	(0)		

$\boxed{50}$ 13	17	14	50	(1)
16	14	10	300	(4)
24	13	10	400	(3)
225	275	250		
(5)	(1)	(0)		

$\boxed{175}$ 18	14	10	~~300~~ 125	(4)
24	13	10	400	(3)
175	275	250		
(6)	(1)	(0)		

[illegible] 14	10	125	(4)
13	10	400	(3)
275	250		
(1)	(0)		

		275
$\boxed{275}$ 13	$\boxed{125}$ 10	400 (3)
275	125	

$\boxed{200}$ 11	$\boxed{50}$ 13	17	14	250
16	$\boxed{175}$ 18	14	$\boxed{125}$ 10	300
21	24	$\boxed{275}$ 13	$\boxed{125}$ 10	400
200	225	275	250	

Here $m+n-1 = 3+4-1 = 6$ no. of allocations

The T.P is basic feasible Solution

T.P cost

$= R.S\ (11 \times 200) + (13 \times 50) + (18 \times 175) + (10 \times 125) +$

$(13 \times 275) + (10 \times 125)$

R.S. 12,075

2

8	12	18	9	70
30	25	35	40	

Sol

Since $\sum a_i \neq \sum b_j$

The Given T.P in unbalanced.

11	20	㉟ 7	⑮ 8	0	50	(7)	(1)	(3)	(3)
21	16	20	⑩ 12	㉚ 0	40	(12)	(4)	(4)	(0)
㉚ 8	㉕ 12	18	⑮ 9	0	70	(8)	(1)	(1)	(1)
30	25	35	40	30					
(3)	(4)	(11)	(1)	(0)					
(3)	(4)	(11)	(1)						
(3)	(4)	(0)	(1)						
(3)	(8)	(0)	(1)						
(3)	(0)	(0)	(1)						

11	20	35 7	15 8	0	50
21	16	20	10 12	30 0	40
30 8	25 12	18	15 9	0	70
30	25	35	40	30	

Here m+n−1 = 5+3−1 = 7 no. of allocations

The T.P is basic feasible Solution

T.P cost R.S (7×35) + (8×15) + (10×12) + (0×30) + (30×

+ (25×12) + (9×15)

R.S 1160.

2 ~~[illegible]~~ Problem Sum in TP

15

10	20	5	7	10
13	9	12	8	20
4	5	7	9	30
14	7	1	0	40
3	12	5	19	50
60	60	20	10	

i) North west corner (NWC)

ii) least cost method (LCM)

iii) VAM (Vogel's approximation method)

i) NWC

$^{10}10$	20	5	7	10
$^{20}13$	9	12	8	20
$^{30}4$	5	7	9	30
14	$^{40}7$	1	0	40
$^{0}3$	$^{20}12$	$^{20}5$	$^{10}19$	50
60	60	20	10	

Since $\sum a_i = \sum b_j = 150$ The Given T.P is balanced

Now $m+n-1 = 5+4-1 = 8 =$ no of allocation

The T.P is basic feasible solution

T.P cost

Rs (10×10)+(20×13)+(30×4)+(40×7)+(20×12)+(20×5)+(10×19)

~~Rs [illegible]~~

Rs 1290

14

ii) LCM

10	^{10}20	5	7	10
13	^{20}9	12	8	20
^{10}4	^{20}5	7	9	30
14	^{10}7	^{20}1	^{10}0	40
^{50}3	12	5	19	50
60	60	20	10	

Here $m+n-1 = 5+4-1 \neq 8$ no. of allocation

The T.P is basic feasible solution

The T.P cost

R.S $(10\times20)+(20\times9)+(10\times4)+(20\times5)+(10\times7)$
$(50\times3)+(20\times1)+(10\times0)$

R.S 760

iii) VAM

10	20	5	7	10	(2)
13	9	12	8	20	(1)
4	5	7	9	30	(1)
14	7	1	$\boxed{10}$ 0	~~40~~ 30	(1)
3	12	5	19	50	(2)
60	60	20	~~10~~		
(1)	(2)	(4)	(7)		

10	20	5	10	(5)
13	9	12	30	(3)
4	5	7	30	(1)
14	7	20 1	~~20~~ 10	(6)
3	12	5	50	(2)
60	60	~~20~~ 10		
(1)	(2)	(4)		

10 10	~~20~~	10	(10)
13	9	20	(4)
4	5	50	(1)
16	7	10	(7)
3	7	50	(4)
60	60		
(1)	(2)		

13	9	30	(4)
4	5	30	(1)
16	7	10	(7)
3	12	40	(8)
~~60~~ 60			
(1)	(2)		

30 16 ~~30~~

~~30~~ 5 30

10 7 ~~10~~

10 10	20	5	7
13	20 9	12	8
4	30 5	7	9
14	10 7	20 1	10 0
60 3	5 12	5	19

m+n-1 = 5+4-1 = 8 no of allocation

non basic feasible solution

TP cost

R.S (10×10) + (9×20) + (5×30) (10×7) + (20×1)

(10×0) + (60×3) + (12×5)

R.S 670

Problem

21	16	25	13	11	[illegible]
17	18	14	23	13	
32	27	18	41	19	
6	10	12	15		

Sol

Since Σai = Σbj = 43

The T.P is balanced

[illegible]

m+n-1 = 3+4-1 = 6 no of allocation

The T.P is basic feasible solution

The T.P cost

= (11×13) + (17×6) + (7×18) + (4×23) + (7×27) + (

= 796

For occupaid calls

(1, 4) (2, 1) (2, 2) (2, 4) (3, 2) (3, 3)

$\boxed{C_{ij} = U_i + V_j}$ $\boxed{U_2 = 0}$

$C_{21} = U_2 + V_1$

$17 = 0 + V_1$

$\boxed{V_1 = 17}$

$C_{22} = U_2 + V_2$

$18 = 0 + V_2$

$\boxed{V_2 = 18}$

$C_{24} = U_2 + V_4$

$23 = 0 + V_4$

$\boxed{V_4 = 23}$

$C_{32} = U_3 + V_2$

$27 = U_3 + 18$

$27 - 18 = U_3$

$\boxed{U_3 = 9}$

$C_{33} = U_3 + V_3$

$18 = 9 + V_3$

$18 - 9 = V_3$

$\boxed{V_3 = 9}$

$C_{14} = U_1 + V_4$

$13 = U_1 + 23$

$13 - 23 = U_1$

$\boxed{U_1 = -10}$

For unoccupaid calls:

(1, 1) (1, 2) (1, 3) (2, 3) (3, 1) (3, 4)

$\boxed{d_{ij} = C_{ij} - (U_i + V_j)}$

$d_{11} = C_{11} - (U_1 + V_1)$

$= 21 - (-10 + 17)$

$d_{11} = 21 - 7 = 14$

$d_{12} = C_{12} - (U_1 + V_2)$

$= 16 - (-10 + 18)$

$= 16 - 8 = 8$

$\boxed{d_{12} = 8}$

$d_{13} = C_3 - (U_1 + V_3)$

$= 25 - (-10 + 9)$

$= 25 + 1 = 26$

$d_{23} = C_{23} - (U_2 + V_3)$

$= 14 - (0 + 9)$

$= 5$

$d_{31} = C_{31} - (U_3 + V_1$

$= 32 - (9 + 17)$

$= 32 - 26$

$\boxed{d_{31} = 6}$

$d_{34} = C_{34} - (U_3 + V$

$= 41 - 19$

$d_{34} = 41 - 32 = 9$

Since all $d_{ij} > 0$ the solution optimal & unique

The T.P is basic feasible Solution

T.P cost $\boxed{R.S\ 796}$

1. Find the Optimal Solution following T.P Problem using least cost method

4	1	2	6	9
6	4	3	5	7
5	2	6	4	8

Sol

Since $\sum a_{ij} = \sum b_j = 340$

the T.P is balanced

4	1 $\boxed{50}$	2 $\boxed{50}$	6	9
$\boxed{10}$ 6	4	3 $\boxed{20}$	5	$\boxed{90}$ 7
$\boxed{30}$ 5	2	6	$\boxed{90}$ 4	8

$m+n-1 = 5+3-1 = 7$ no. of allocations

The Solution non-degenerate

The T.P cost

$RS\ (1\times50)+(2\times50)+(3\times20)+(4\times90)+(5\times70)+(6\times1$

(7×90)

RS 1410

For occupaid cells

(1,2) (1,3) (2,1) (2,3) (2,5) (3,1) (3,4)

$\boxed{u_2 = 0}$

$C_{ij} = u_i + v_j$

$C_{21} = u_2 + v_1$

$6 = 0 + v_1$

$\boxed{v_1 = 6}$

$C_{23} = u_2 + v_3$

$3 = 0 + v_3$

$\boxed{v_3 = 3}$

$C_{25} = u_2 + v_5$

$7 = 0 + v_5$

$\boxed{v_5 = 7}$

$C_{31} = u_3 + v_1$

$5 = u_3 + 6$

$5 - 6 = u_3$

$\boxed{u_3 = -1}$

$C_{34} = U_3 + V_4$

$4 = -1 + V_4$

$\boxed{V_4 = 5}$

$C_{13} = U_1 + V_3$

$2 = U_1 + 3$

$\boxed{U_1 = -1}$

$C_{12} = U_1 + V_2$

$1 = -1 + V_2$

$\boxed{V_2 = 2}$

For Unoccupied Cells

$d_{ij} = C_{ij} - (U_i + V_j)$

$d_{11} = C_{11} - (U_1 + V_1)$

$= 4 + 1 - 6$

$\boxed{d_{11} = -1}$

$d_{14} = C_{14} - (U_1 + V_4)$

$= 6 + 1 - 5$

$\boxed{d_{14} = 2}$

$d_{15} = C_{15} - (U_1 + V_5)$

$= 9 + 1 - 7 = 3$

$\boxed{d_{15} = 3}$

$d_{22} = C_{22} - (U_2 + V_2)$

$= 4 - 0 - 2 = 2$

$d_{24} = C_{24} - (U_2 + V_4)$

$= 5 - 0 - 5 = 0$

$d_{32} = C_{32} - (U_3 + V_2)$

$= 2 + 1 - 2 = 1$

$d_{33} = C_{33} - (U_3 + V_3)$

$= 6 + 1 - 3 = 4$

$d_{35} = C_{35} - (U_3 + V_5)$

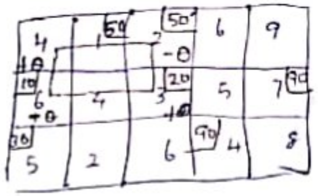
$= 8 + 1 - 7 = 2$

$d_{ij} > 0$

Since $d_{11} = -1$ we draw a closing loop consisting of horizontal and vertical lines begining and ending at the cell (1,1)

$\theta = \min\{50, 20, 10\}$

$\theta = 10$



$\boxed{10}$ 4	1 $\boxed{50}$	2 $\boxed{40}$	4	[illegible]
6	4	3 $\boxed{30}$	5	$\boxed{70}$ 7
$\boxed{30}$ 5	2	6	$\boxed{90}$ 4	8

T.P cost

= 5 (4×10) + (1×50) + (2×40) + (3×30) + (7×70) + (3×

+ (90×4)

= 51400

For occupaid calls

$u_1 = 0$

$c_{11} = u_1 + v_1$

$4 = 0 + v_1$

$\boxed{v_1 = 4}$

$c_{12} = u_1 + v_2$

$1 = 0 + v_2$

$\boxed{v_2 = 1}$

$c_{13} = u_1 + v_3$

$2 = 0 + v_3$

$\boxed{v_3 = 2}$

$c_{23} = u_2 + v_3$

$3 = u_2 + 2$

$\boxed{u_2 = 1}$

$c_{25} = u_2 + v_5$

$7 = 1 + v_5$

$\boxed{v_5 = 6}$

$c_{31} = u_3 + v_1$

$5 = u_3 + 4$

$\boxed{u_3 = 1}$

$c_{34} = u_3 + v_4$

$4 = 1 + v_4$

$\boxed{v_4 = 3}$

For un occupaid calls

$d_{22} = c_{22} - (u_2 + v_2)$

$= 4 - 1 - 1 = 2$

$d_{24} = c_{24} - (u_2 + v_4)$

$= 5 - 1 - 3 = 1$

$d_{32} = c_{32} - (u_3 + v_2)$

$= 2 - 1 - 1 = 0$

$d_{33} = c_{33} - (u_3 + v_3)$

$= 6 - 1 - 2 = 3$

$d_{35} = c_{35} - (u_3 + v_5)$

$= 8 - 1 - 6 = 1$

1 Solve the following assignment problem

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ A & 8 & 4 & 2 & 6 & 1 \\ B & 0 & 9 & 5 & 5 & 4 \\ C & 3 & 8 & 9 & 2 & 6 \\ D & 4 & 3 & 1 & 0 & 3 \\ E & 9 & 5 & 8 & 9 & 5 \end{array}$$

Sol

Since the no of rows = no of columns the given cost matrix the assignment problem is balanced

Row minimum

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

column minimum

$$\begin{pmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{pmatrix}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ A & 7 & 3 & \cancel{0} & 5 & (0) \\ B & (0) & 9 & 4 & 5 & 4 \\ C & 1 & 6 & 6 & (0) & 4 \\ D & 4 & 3 & (0) & \cancel{0} & 3 \\ E & 4 & (0) & 2 & 4 & \cancel{0} \end{array}$$

A→5, B→1, C→4, D→3, E→2

= 1+0+2+1+5

= 9 units of cost

3. Find the n.p of Salesman to Various districts which field maximum Profit

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ A & 16 & 10 & 14 & 11 \\ B & 14 & 11 & 15 & 15 \\ C & 13 & 15 & 13 & 12 \\ D & 13 & 12 & 14 & 15 \end{array}$$

Sol

Since this is maximization problem It can be converted into an equivalent minimization problem by subtracting all cost elements

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 2 & 5 & 1 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

Row minimum

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ A & (0) & 6 & 2 & 5 \\ B & 1 & 4 & (0) & \times \\ C & \times & (0) & 2 & 3 \\ D & 2 & 3 & 1 & (0) \end{array}$$

A→1 B→3 C→2 D→4

= 16 +15 +15+15

= 61 units of cost

4) Find the optimum Solution

$$\begin{pmatrix} 9 & 22 & 58 & 1 & 19 \\ 43 & 76 & 72 & 56 & 63 \\ 41 & 28 & 91 & 37 & 45 \\ 74 & 42 & 27 & 49 & 39 \\ 36 & 11 & 57 & 22 & 25 \end{pmatrix}$$

Sol

Since no. of ~~allocation~~ rows = no. of Columns

So A P in balanced

Row minimum:

$$\begin{pmatrix} 0 & 13 & 49 & 2 & 10 \\ 0 & 35 & 29 & 7 & 20 \\ 13 & 0 & 63 & 9 & 17 \\ 47 & 15 & 0 & 22 & 12 \\ 25 & 0 & 16 & 11 & 14 \end{pmatrix}$$

Column minimum

$$\begin{pmatrix} 0 & 13 & 49 & (0) & 0 \\ (0) & 35 & 29 & 5 & 10 \\ 13 & (0) & 63 & 7 & 7 \\ 47 & 15 & (0) & 20 & 2 \\ 25 & 0 & 46 & 9 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 17 & 49 & (0) & 0 \\ (0) & 39 & 29 & 5 & 10 \\ 9 & (0) & 59 & 3 & 3 \\ 47 & 19 & (0) & 20 & 2 \\ 21 & 0 & 42 & 5 & (0) \end{pmatrix}$$

$J_1 \to M_4$ $J_2 \to M_1$ $J_3 \to M_2$ $J_4 \to M_3$ $J_5 \to M_5$

$= 11+43+28+27+25$

$= 134$ unit of cost

5) Solve the travelling Salesmen Problem

$$\begin{pmatrix} - & 46 & 16 & 40 \\ 41 & - & 50 & 40 \\ 82 & 32 & - & 60 \\ 40 & 40 & 36 & - \end{pmatrix}$$

Sol

$$\begin{pmatrix} \infty & 46 & 16 & 40 \\ 41 & \infty & 50 & 40 \\ 82 & 32 & \infty & 60 \\ 40 & 40 & 36 & \infty \end{pmatrix}$$

Row minimum

$$\begin{pmatrix} \infty & 30 & 0 & 24 \\ 1 & \infty & 10 & 0 \\ 50 & 0 & \infty & 25 \\ 4 & 4 & 0 & \infty \end{pmatrix}$$

Column minimum

$$\begin{pmatrix} \infty & 30 & (0) & 24 \\ (0) & \infty & 10 & 0 \\ 49 & (0) & \infty & 25 \\ 3 & 4 & 0 & \infty \end{pmatrix} \Rightarrow \begin{pmatrix} \infty & 27 & (0) & 21 \\ 0 & \infty & 13 & (0) \\ 49 & (0) & \infty & 25 \\ (0) & 1 & 0 & \infty \end{pmatrix}$$

A → C B → D C → B D → A

= 16 + 40 + 32 + 40

= 128 units of cost

Unit-IV

Sequencing problem

Definitions.

i) Total elapsed time (t) is time between starting the first job and completing the last one

ii) Idle time on a machine is the time the machine remains idle during the total elapsed time

iii) no passing rule in a sequencing problem

If each of the n jobs is to be processed through two machines m_1 & m_2 in the order m_1, m_2 then this rule means that each job will go to the machine m_1 first and then to machine m_2

Step-wise Procedure for determining the optimal sequence for n jobs on 2 machines · 25

(Johnson's Method):

Let $A_{11}, A_{21}, A_{31}, \ldots A_{n1}$ be processing times of n jobs on machines 1 and let $A_{12}, A_{22}, A_{32} \ldots A_{n2}$ be the processing time of n jobs on machines-2

Step 1

Find the min (A_{i1}, A_{i2}) $i = 0, 1, 2 \ldots n$

Step 2

(a) if this minimum be A_{i1} for $i = 1$ process the i^{th} job first

(b) If this minimum be A_{m2} for some $i = m$ process the m^{th} job last of all

Step 3

a) If there is a time (i.e.) $A_{i1} - A_{m2}$ process the i^{th} job first and m^{th} job in the last

(b) If the tie for the minimum occurs among the A_{i1}-s, choose the job corresponding to the minimum of A_{i1}-s and process it first of all

(c) If the tie for minimum of A_{i2}-s choose the job corresponding to the minimum of A_{i1}-s and process it in the last

Step 4:

Cancel the jobs already assigned and repeat Step 1 to 3 until all the Jobs have been assigned

1. There are 5 jobs Each of which is to be processed through two machines M_1, M_2 in the order M_1, M_2 processing hours are as follows 26

Job	1	2	3	4	5
M_1	3	8	5	7	4
M_2	4	10	6	5	8

determine the optimum sequence for the 5 Jobs and minimum total ellapsed time find also Idle time of the Machines M_1 & M_2

Sol

Job	1	2	3	4	5
M_1	3	8	5	7	4
M_2	4	10	6	5	8

optimum sequence is

1	5	3	2	4

Job	M_1		M_2		Idle time	
	Time in	Time out	Time in	Time OUT	M_1	M_2
1	0	3	3	7	0	3
5	3	7	7	15	0	0
3	7	12	15	21	0	0
2	12	20	21	31	0	0
4	20	27	31	36	0	0

Total elapsed time = 36 hours

Total Idle time = M_1 = 36 - 27 = 9 hrs

Total Idle time M_2 = 3 hrs

m1		m4					
Time in	Time out	Time in	Time out	Time in	Time out	Time in	Time out
17	25	25	30	30	57	37	52
26	45	45	51	51	66	60	75
57	65	65	65	65	79	79	103
74	77	77	81	51	89	103	114

$M_4 = (114-81) + 25 + 15 + 14 + 7$

$= 96$ Hrs

$M_5 = (114-89) + 30 + 14 + 8 + 2$

$= 79$ hrs

$M_6 = 37 + 8 + 1$

$= 46$ Hrs

28

2

Job	M_1	M_2	M_3	M_4
A	13	5	7	14
B	12	6	9	19
C	9	7	8	15
D	8	5	6	15

Sol

	A	B	C	D
H	34	21	27	46
K	29	18	22	49

The optimum sequence is

B	D	A	C

Job	M_1		M_2		M_3		M_4		m_1	m_2	m_3	m_4
	Time In	Time out	Time In	Time out	Time In	Time out	Time in	Time out				
B	0	8	8	14	14	21	21	26	0	5	14	21
D	8	22	22	41	41	56	56	71	0	5	20	30
A	22	35	41	53	56	65	71	79	0	0	0	0
C	35	42	53	61	65	73	79	85	0	0	0	0

The minimum elapsed time = 85 hrs

Idle time for $m_1 = 85 - 42 = 43$ hrs

$m_2 = (85-61) + 8 + 8$

$= 40$ hrs

$m_3 = (85-73) + 14 + 20$

$= 46$ hrs

$m_4 = 21 + 30$

$= 51$ hrs

UNIT-V

Scheduling by PERT and CPM

Introduction :

A Project is defined as a combination of interrelated activities all of which must be excuted on ae creation order to achive a set goal.

(*) Programme Evalution Review Technique and Critical Path method are two of the many network techniques which are widely use for Planning, Shedaling and controlling large complex Projects.

The Three main managerial functions for any Project are

1) Planning
2) Scheduling
3) control

planning :

The Phase involves a listing of tasks or jobs that must be performed complete a Project under considerable

Scheduling :

This Phase in voling the laying out of the actual activities of the projects in logical sequence of time in which they have to be Performed

men and matrial requirments as wellas the expected completion

time of each activity at each stage
of the projects are also determined

Control

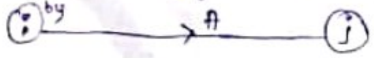
This phase consists of reviewing the progress of the project whether the actual performance is according to planned schedule and finding the reason for difference if any b/w the shedule and performance

Basic terminologsis:

Activity is at ask or an item of work to be done in a project an activity continous resources like time labours etc,

An activity is represented by an arrow with a node (event) at the begining and a node at the end indecating the start and termination (finish) of the activity Nodes are denoted by circles. Since this is a logical Since is a logical diagram. lenth or shape of the arrow has no meaning. The direction Indicates the progress of the activity.

For Example If is the activity whose Initia node is a i and the termination node is j the it is denoted diagrammatically



4. Delete the activities originating from the start node, 2, 3 .. in step 3

5. Number all the resulting new start nodes without any predecessor next to the last number in step(3)

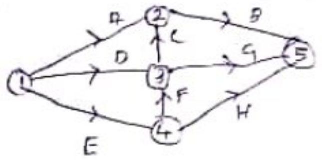
6. Repeat the process until the terminal node without any successor activity is reached and number this terminal node suitably 5)

——— x ——— x ———

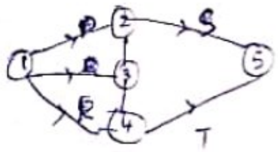
Problem:

① construct the network for the project whose activities and their relationships are as given below.

Activities = A, D, E can start simultaneously
Activities = B, C > A , G, F > D, C , H > E, F



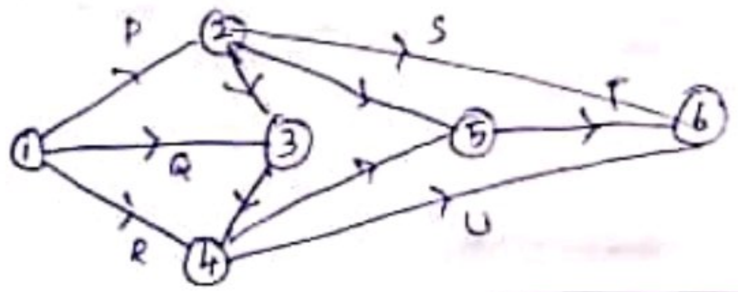
② If there are 5 activities P, Q, R, S, T such that P, Q, R have no predecessors ~~such that P, Q, R have no~~ but S & T have immediate predecessors P, Q and Q, R respectively. represent this situations by a networks



Draw the networks for the Project

and their Precedence relationship and given below

Activitiy	P	Q	R	S	T	U
Predecessors	-	-	-	P,Q	P,R	Q,R



Critical Path (CPM)

Path connecting the first initial node to the very last terminal node, of longest duration in any project network is called the critical path

Critical path plays a very important role in project scheduling problems

Floats:

Total floats of an activity (T.F) is defined as the difference between the lastest finish and the

earliest finish of the activity (or) the differenc
between the latest start and the earlist
start of the activity

Total float of an activity i-j = $(LF)_{ij} - (EF)_{ij}$

(or)

$= (LS)_{ij} - (ES)_{ij}$

Note:

(L-E) of an event of i-j is called the even

Free float of an activity (F.F) is that portio
the total float which can be used for resched
that activities of that activity.

Free Float an activity i-j = Total float of i-j - (L-E
the event j

= total float of i-j - slack of head event j

= total float of i-j - slack of the head even

where,

L = latest occurence

E = Earliest occurence

Obviously Free Float ≤ total float for any activity.

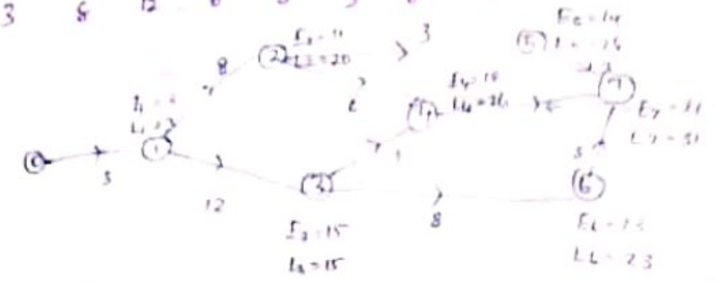
Activity (i) (j)	Duration (in weeks) (n)	Earliest			
		Start Ei	finish Ej = Ei+n	start Li = Lj−n	finish Lj − n
1 - 2	8	0.	8	0	8
1 - 3	7	0.	7	8	15
1 - 5	12	0.	12	9	21
2 - 3	4	8	12	11	15
2 - 4	10	8	18	8	18
3 - 4	3	12	15	15	19
3 - 5	5	12	17	16	21
3 - 6	10	12	22	15	25
4 - 6	7	18	25	18	25
5 - 6	4	17	21	21	25

1. Calculate the ~~first~~ total float and Independent float for the Project whose, activities are given below

Activity:	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration: in week	8	7	12	4	10	3	5	10	7	4

A Same Problem Sum float

: 0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
3	8	12	6	3	3	8	5	3	8



Activity (i)-(j)	Duration (n)	Earlist Start (Ei)	Earlist Finish Ej = Ei - n	Latest Start Li = Lj - n	Latest Finish Lj	Total Float Li - Ei
0-1	3	0	3	0	3	0
1-2	8	3	11	12	26	9
1-3	12	3	15	3	15	0
2-4	6	11	17	20	21	9
2-5	3	11	14	25	28	14
3-4	3	15	18	23	26	8
3-6	8	15	23	15	23	0
4-7	5	18	23	26	31	8
5-7	3	14	17	28	31	14
6-7	8	23	31	23	31	0

Free Float TF - (L-E) event j	Independent Float FF - (Li-Ei) event i
0	0
0	-9
0	0
1	-7
0	-14
0	-8
0	0
8	8
14	14
6	0

(Li - Ei)

[illegible]

i) compute the expected standard deviation of the project length σ_e and calculate the standard normal deviate

36

$T_s - T_E$ where,

σ_e

T_s → specified or scheduled time to complete the project

T_E → normal expected project duration

σ_e → expected duration standard deviation of the project length

ii) using (7) one can estimate the probability of completing the project with in a specified time using the normal curve (area table

3

Difference between PERT and CPM

PERT	CPM
i) PERT was developed in a brand new R● and D project it had to consider and deal with the uncertainties associated with such projects. Thus the project duration is regarded as a random variable and there fore probability are calculated so as to characterise it.	i) CPM was developed for conventional project like construction project which consists of well known routine task whose resource requirement and duration were known with certainty
ii) Emphasis is given to important stages of task rather than the activities required to be performed to reach a particular event or task in the analysis of network (i.e)	ii) CPM is suited to established or trade off for the optimum balancing between schedule time and cost of the project
PERT network difference between PERT and CPM	

ii) PERT is usually for Projects in which time estimates are uncertain (e.g) R+D activities which are usally non-repetitive

iii) CPM is used for the project involving well known activities repetitive in nature. However the distinction between PERT and CPM is most lts tonced

iv) PERT helps in identifying critical areas a project so that suitable necessary adjustment may be made to meet the Scheduled completion date of the project

[redacted]

N Kavitha 27/02/2020

Problems

1 Construct the network for the project whose activities and true three time estimates of these activities are given below compute

a) Expected duration of each activity

b) Expected variance of " "

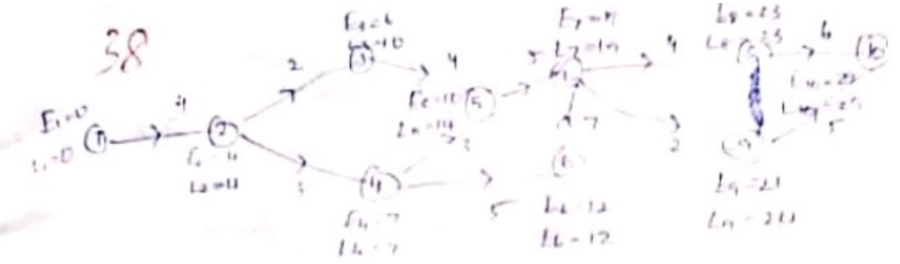
c) Expected " " the project length.

activity	t_o	t_m	t_p
1-2	3	4	5
2-3	1	2	3
2-4	2	3	4
3-5	3	4	5
4-5	1	3	5
4-6	3	5	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7-9	1	2	3
8-10	4	6	8
9-10	3	5	7

$t_e = \frac{t_o + 4t_m + t_p}{6}$

$t_e = \frac{t_o + 4t_m + t_p}{6}$

$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$



				(a)	(b)
Activity	to	tm	tp	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	[illegible]	4	5	4	1/9 = 0.11
2-3	1	2	3	2	1/9 = 0.11
2-4	2	3	4	3	1/9 = 0.11
3-5	3	4	5	4	1/9 = 0.11
4-5	1	3	5	3	4/9 = 0.44
4-6	3	5	7	5	1/9 = 0.11
5-7	4	5	6	5	1/9 = 0.11
6-7	6	7	68	7	4/9 = 0.44
7-8	2	4	86	4	1/9 = 0.11
7-9	1	2	63	2	4/9 = 0.44
8-10	4	6	8	6	4/9 = 0.45
9-10	3	5	7	5	

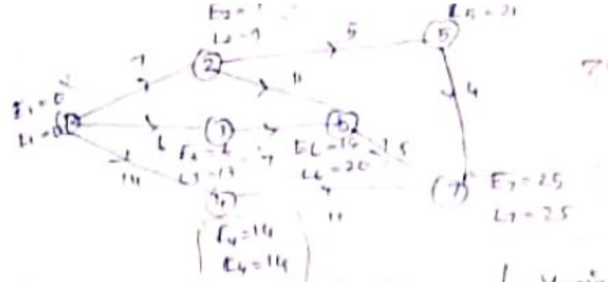
(c) Expected Variance of Project length

= Sum of the expected variances of all the critical activities

$= \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{15}{9} = \frac{5}{3} = 1.67.$

1. A Project Consists of the following activities and time activities

Activity	Least time to	Greatest Time tp	tm
1-2	3	15	6
1-3	2	14	5
1-4	6	30	12
2-5	2	9	5
2-6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5



Activity	t_0	t_p	t_m	Duration $t_e = \frac{t_0 + 4t_m + t_p}{6}$	Variance $\sigma_e^2 = \left(\frac{t_p - t_0}{6}\right)^2$
1-2	3	15	6	7	4
1-3	2	14	5	6	4
1-4	6	30	12	14	16
2-5	2	8	5	5	1
2-6	5	17	11	11	4
3-6	3	15	6	7	4
4-7	3	27	9	11	16
5-7	1	7	4	4	1
6-7	2	8	5	5	1

The critical Path

1-4-7

= 25 days.

Expected variance of the project length = Sum of the expected variance of all the critical durations

= 16 + 16 = 32

σ_e = Standard durations of the project length $= \sqrt{32} = 4\sqrt{2} = 5.656 = \sigma_e$

$$Z = \frac{T_S - T_E}{\sigma_E} = \frac{27 - 25}{5.656} = \frac{2}{5.656} = 0.35$$

Probability of the project will be completed in 27 days $P(T_S \leq 27) = P(Z \leq 0.35)$

$= 0.6368 \Rightarrow 63.7\%$

UNIT-I

Introduction :

The new approach to Systematic and scientific Study of the operations of the System is called operations research (OR)

Linear Programming problem (LPP) :

many business are con-cerned with a problem of planning activity In each case there are limited resources and your problem is to In make such a use of these resources so as to yield the maximum production (or) maximum profit This type of problem is called LPP

mathematical formulation of LPP :

If x_j $(j=1,2\ldots n)$ are the n decision variables of the Problem and the System is subject to m constraints the general mathematical model can be written in the form. optimize $z=f(x_1,x_2\ldots x_n)$

Subject to,

$$g_i(x_1,x_2\ldots x_n) \le, =, \ge b_i(x_1,x_2,\ldots x_n)$$

and

$$x_1,x_2\ldots x_n \ge 0$$

Graphical Method :

Linear programming problems involving only two variables can be effectively solved by a graphical method which provides a pictorial representation of the Problems and solutions.

1. Solve the following the LPP by graphical Method

$\max z = 3x_1 + 2x_2$

Subject to $-2x_1 + x_2 \le 1$

$x_1 \le 2$

$x_1 + x_2 \le 3$ and

$x_1, x_2 \ge 0$

Sol:

$-2x_1 + x_2 = 1$

$x_1 = 2$

$x_1 + x_2 = 3$

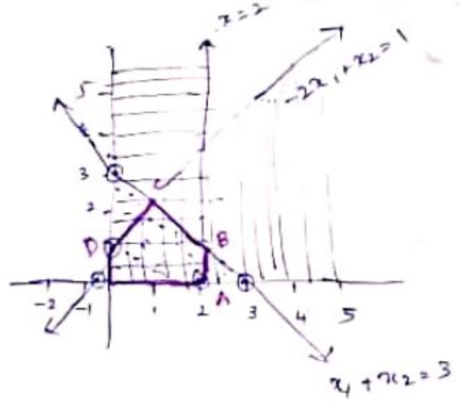
$x_1 = 0, x_2 = 0$

$-2x_1 + x_2 = 1$

x_1	0	-0.5
x_2	1	0

$x_1 + x_2 = 3$

x_1	0	3
x_2	3	0



Here feasible Solution

O(0,0), A(2,0), D(0,1)

Find B

$x_1 = 2$

$x_1 + x_2 = 3$

$2 + x_2 = 3$

$\therefore x_2 = 1$

B(2,1)

$-3x = -2$

$x_1 = 2/3$

$x_1 + x_2 = 3$

$2/3 + x_2 = 3$

$x_2 = 3 - 2/3$

$x_2 = 7/3$

$\boxed{C(2/3, 7/3)}$

A(0,0)	0
B(2,0)	6
[illegible](2,1)	8
C(2/3, 7/3)	[illegible]
D(0,1)	[illegible]

max z = 8

$x_1 = 2$ $x_2 = 1$

1. max $z = 5x_1 + 8x_2$

Subject $15x_1 + 10x_2 \leq 180$

$10x_1 + 20x_2 \leq 200$

$15x_1 + 20x_2 \leq 210$ and $x_1, x_2 \geq 0$

Sol:

$15x_1 + 10x_2 = 180$

$10x_1 + 20x_2 = 200$

$15x_1 + 20x_2 = 210$

$x_1 = 0, x_2 = 0$

$15x_1 + 10x_2 = 180$

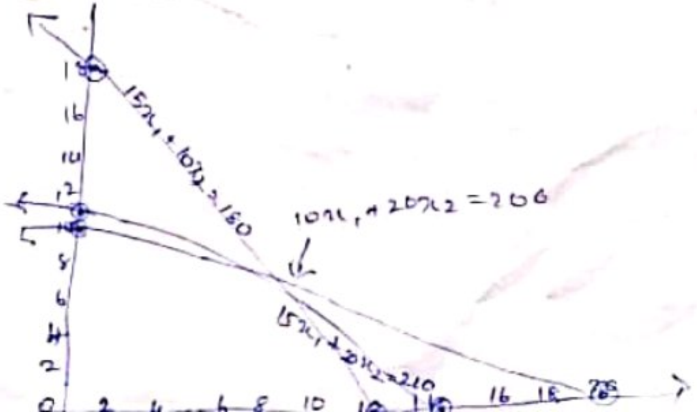
x_1	0	12
x_2	18	0

$10x_1 + 20x_2 = 200$

x_1	0	20
x_2	10	0

$15x_1 + 20x_2 = 210$

x_1	0	14
x_2	10.5	0



Here feasible solution

O(0,0) A(12,0) D(0,10)

Find B

$15x_1 + 10x_2 = 180$

$15x_1 + 20x_2 = 210$

$(-) \quad (-) \quad (-)$

$-10x_2 = -30$

$\boxed{x_2 = 3}$

$15x_1 + 10(3) = 180$

$15x_1 = 180 - 30$

$15x_1 = 150$

$\boxed{x = 10}$

$\boxed{B(10,3)}$

Find C

$10x_1 + 20x_2 = 200$

$15x_1 + 20x_2 = 210$

$-5x_1 = -10$

$\boxed{x_1 = 2}$

$10x_1 + 20x_2 = 200$

$20 + 20x_2 = 200$

$20x_2 = 180$

$\boxed{x_2 = 9}$

Vertex	Value of z $\max z = 5x_1 + 8x_2$
O(0,0)	0
A(12,0)	60
B(10,3)	74
C(2,9)	$\boxed{82}$
D(0,10)	80

import mistake. to Exam write

Max $z = 82$

$x_1 = 2 \quad x_2 = 7$

3. Solve the problem (LPP) graphically

$\max z = 100x_1 + 40x_2$

Subject to $5x_1 + 2x_2 \leq 1000$

$3x_1 + 2x_2 \leq 900$

$x_1 + 2x_2 \leq 500$

$x_1, x_2 \geq 0$

Sol

12 8/10 44

$5x_1 + 2x_2 = 1000$

$3x_1 + 2x_2 = 900$

$x_1 + 2x_2 = 500$

$x_1 = 0, x_2 = 0$

$5x_1 + 2x_2 = 1000$

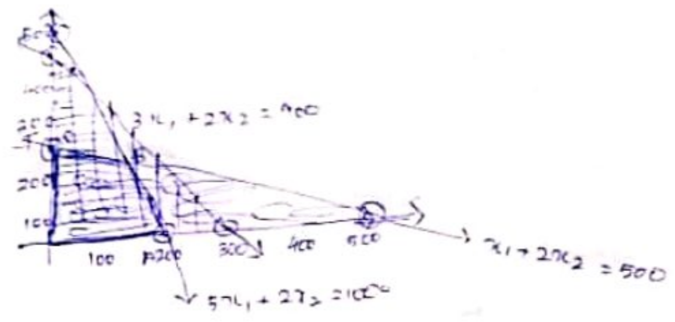
x_1	0	200
x_2	500	0

$3x_1 + 2x_2 = 900$

x_1	0	300
x_2	450	0

$x_1 + 2x_2 = 500$

x_1	0	500
x_2	250	0



Here feasible solution

O(0,0) A(200,0) C(0,250)

To find B

$$5x_1 + 2x_2 = 1000$$
$$x_1 + 2x_2 = 500$$
$$4x_1 = 500$$
$$x_1 = \frac{500}{4}$$
$$\boxed{x_1 = 125}$$
$$125 + 2x_2 = 500$$
$$2x_2 = 500 - 125$$
$$2x_2 = 375$$
$$x_2 = \frac{375}{2}$$
$$\boxed{x_2 = 187.5}$$

B(125, 187.5)

Vertices	Value of z max $z = 100x_1 + 40x_2$
O(0,0)	0
A(200,0)	20,000
B(125,187.5)	20,000
C(0,250)	10,000

Here the maximum value of z occurs at two vertices A & B

any point on the line joining A and B will also give the same maximum value of z

1. Solve the Problem LPP graphically

max $Z = 4x_1 + 3x_2$

Subject to

$x_1 - x_2 \geq -1$

$-x_1 + x_2 \leq 0$

$x_1, x_2 \geq 0$

Sol

$x_1 - x_2 = -1$

$-x_1 + x_2 = 0$

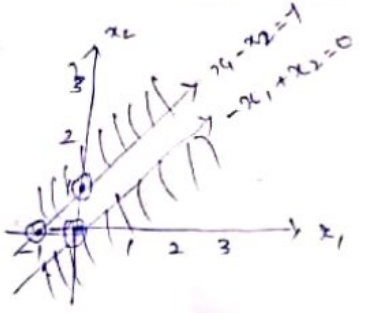
$x_1 = 0 \quad x_2 = 0$

$x_1 - x_2 = -1$

x_1	0	-1
x_2	1	0

$-x_1 + x_2 = 0$

x_1	0	0
x_2	0	0



There being no point (x_1, x_2) common to both the Shaded regions so the Problem cannot be prove

Hence the Problem have no feasible Solution.