# M-2 Syllabus

Algebra - Analytical geometry. (3D) And Trignometry.

## Unit-I

Binomial, Exponential & Logarithmic series (formula only) summation & approximation related problems only

## Unit-2 :-

Non-singular, Symmetric, Skew Sym, Orthogonal, Hermitian, Skew Hermitian, & Unitary matrices - characteristic equation, eigen values, eigen vector - Cayley Hamelton theorem (no. proof).

## Unit-3 :-

Finding the shortest distance between two skew lines and Equation of the plane containing them - condition for coplanarity - Eq. of. Sphere, Tangent plane, plane sec. of. Sphere, Finding the centre and radius of the

Circle . Intersection sphere . Through circle of Intersection, (only problem)

Unit-4 :-

Expension $\sin n\theta$, $\cos n\theta$, $\tan n\theta$, (n- being a +ve integer)

Expansion of $\sin^n\theta$, $\cos^n\theta$, $\sin^n\theta\cos^m\theta$ in. a series of sines and cosines of multiples of $\theta$ ($\theta$- given in radians)

-Expansion of $\sin\theta$, $\cos\theta$, $\tan\theta$ in terms of powers. of $\theta$ (problems).

Unit-5 :-

Euler's formula for $e^{i\theta}$ - Definition of hyperbolic funct- formula in valving hyp. funct. - Rel. b/w hyperbolic & Circular fun. Exp. of. $\sinh x$, $\cosh x$, $\tanh x$, in power of x - Expansion of Inverse hyperbolic func. $\sinh^{-1}x$, $\cosh^{-1}x$, $\tanh^{-1}x$, Sep. of real & Imaginary parts of $\sin(x+iy)$, $\cos(x+iy)$, $\tan(x+iy)$ & $\sinh(x+iy)$ $\cosh(x+iy)$, $\tanh(x+iy)$

Matrix :-

A matrix is a rectangular array of numbers arranged row & columns.

Eg $\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \end{bmatrix}$

Types :-

i) Row matrix

Any matrix which has only one row is called row matrix.

Eg :- $(1\ 2\ 3\ 4\ 5)_{1\times 5}$

column matrix :-

Any matrix which has only one column is called column matrix.

Eg :- $\begin{bmatrix} 2 \\ 4 \\ 8 \\ 5 \end{bmatrix}_{4\times 1}$

Square Matrix :-

A matrix which has equal number of rows and columns.

## Diagonal Matrix :-

If all the elements of the square matrix are zero expect those on the leading diagonal of matrix is called diagonal matrix.

In other words of

$A = (a_{ij})$ is such that $a_{ij} = 0$ when $i \neq j$ then

Eg $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3}$.

## Scalar Matrix :-

A Diagonal matrix whose diagonal element all are Equal is called Scalar matrix.

Eg $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3}$

## Unit or Identity matrix :-

A Scalar matrix is which each diagonal ~~matrix~~ element is called unit or identity matrix.

It densted by In the matrix of order n.

(Eg) :- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$.

Zero Matrix or Null matrix :-

If the element of a matrix are all zero is called Zero or Null matrix (or). Zero matrix.

It's denotedlly

Eg $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{2\times 4}$.

Triangular Matrix :-

A Square matrix in which all the entries above the main diagonal are zero is called lower triangular matrix. If all the entries below the main diagonal are zero it's called upper Triangular Matrix.

Eg $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 8 \\ 0 & 0 & 9 \end{bmatrix}_{3\times 3}$ upper triangular

$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 8 & 0 \\ 9 & 5 & 6 \end{bmatrix}_{3\times 2}$ → lower Triangular

## negative of matrix :

Let a be any matrix the negative of a matrix Ais -A and is abtained by changing the sign of at the entries of matrix A.

Ex :

$$A = \begin{bmatrix} 4 & -5 \\ 6 & 7 \end{bmatrix} \Rightarrow A = \begin{bmatrix} -4 & 5 \\ -6 & -7 \end{bmatrix}$$

Problems :

1. $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ find A+B

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

(i) $A+B = \begin{bmatrix} 2+1 & 3+2 \\ 1+1 & 4+1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 5 \\ 2 & 5 \end{bmatrix}$

(ii) $A-B = \begin{bmatrix} 2-1 & 3-2 \\ 1-1 & 4-1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$

(iii) $2A+3B = 2\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = 2A$

$$= \begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 3 & 3 \end{bmatrix}$$

$$2A+3B = \begin{bmatrix} 4+3 & 6+6 \\ 2+3 & 8+3 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 5 & 11 \end{bmatrix}$$

(iv) $AB = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+3 & 4+3 \\ 1+4 & 2+4 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 5 & 8 \\ 5 & 6 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ find (i) $A+B$ (ii) $AB$

3. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

(i) $3A + 2B + C$

$3A = 3 \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 6 \\ 9 & 3 & 3 \\ 3 & 6 & 9 \end{bmatrix}$, $2B = 2 \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 6 \\ 8 & 10 & 2 \\ 4 & 2 & 4 \end{bmatrix}$

$3A + 2B = \begin{bmatrix} 3 & 6 & 6 \\ 9 & 3 & 3 \\ 3 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 6 \\ 8 & 10 & 2 \\ 4 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 8 & 12 \\ 17 & 13 & 5 \\ 7 & 8 & 13 \end{bmatrix}$

$3A + 2B + C = \begin{bmatrix} 7 & 8 & 12 \\ 17 & 13 & 5 \\ 7 & 8 & 13 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 10 & 13 \\ 18 & 14 & 6 \\ 9 & 9 & 14 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ it's lower triangular matrix.

quality of matrix :

Two matrix $A = [a_{ij}]$ and $[b_{ij}] = B$ are said to be equal if and only if.

1. Both the matrices A and B are of the same order
2. The corresponding entries in both the matrices A and B are equal.

Ex :
$a_{ij} = b_{ij}$ for all i and j

Transpose of matrix :

The matrix obtained from the given matrix by a interchanging its rows into columns and it's columns into rows is called Transpose of matrix A and it's denoted by $A'$ (or) $A^T$.

Ex :

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3 \times 2}$

$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2 \times 3}$

(iii) $A-B-C$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1-2 & 2-1 & 2-3 \\ 3-4 & 1-5 & 1-1 \\ 1-2 & 2-1 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & -1 \\ -1 & -4 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A-B-C = \begin{bmatrix} -1 & 1 & -1 \\ -1 & -4 & 0 \\ -1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1-1 & 1-2 & -1-1 \\ -1-1 & -4-1 & 0-1 \\ -1-2 & 1-1 & 1-1 \end{bmatrix}$$

$$\therefore A-B-C = \begin{bmatrix} -2 & -1 & -2 \\ -2 & -5 & -1 \\ -3 & 0 & 0 \end{bmatrix}$$

(iv) $AB+BC$

$$AB = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8+4 & 2+10+2 & 3+2+4 \\ 6+4+2 & 3+5+1 & 9+1+2 \\ 2+8+6 & 1+10+3 & 3+2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 13 & 9 \\ 12 & 9 & 12 \\ 16 & 14 & 11 \end{bmatrix}$$

$$BC = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1+6 & 4+1+3 & 2+1+3 \\ 4+5+2 & 8+5+1 & 4+5+1 \\ 2+2+4 & 4+1+2 & 2+1+2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 6 \\ 11 & 14 & 10 \\ 8 & 7 & 5 \end{bmatrix}$$

$$AB+BC = \begin{bmatrix} 14 & 13 & 9 \\ 12 & 9 & 12 \\ 16 & 14 & 11 \end{bmatrix} + \begin{bmatrix} 9 & 8 & 6 \\ 11 & 14 & 10 \\ 8 & 7 & 5 \end{bmatrix}$$

$$\therefore AB+BC = \begin{bmatrix} 23 & 21 & 15 \\ 23 & 23 & 22 \\ 24 & 21 & 16 \end{bmatrix}$$

2. Ans :

(i) $A+B = \begin{bmatrix} 1+2 & 2+1 & 2+3 \\ 3+4 & 1+5 & 1+1 \\ 1+2 & 2+1 & 3+2 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 7 & 6 & 2 \\ 3 & 3 & 5 \end{bmatrix}$

(ii) $AB = \begin{bmatrix} 2+8+4 & 1+10+2 & 3+2+4 \\ 6+4+2 & 3+5+1 & 9+1+2 \\ 2+8+6 & 1+10+3 & 3+2+6 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 14 & 13 & 9 \\ 12 & 9 & 12 \\ 16 & 14 & 11 \end{bmatrix}$$

Symmetric matrix :

A square matrix $A = [a_{ij}]$ is said to be symmetric matrix if $(i,j)^{th}$ elements is same as it's $(j,i)^{th}$ element

(i) $A = A^T$

$a_{ij} = a_{ji}$ for all $i$ and $j$

②, ④ Skew symmetric matrix :

A square matrix $A = [a_{ij}]$ is said to be skew symmetric matrix if $(i,j)^{th}$ elements is nagative of its $(j,i)^{th}$ element.

(ie) $A^{-1} = -A$

(ie) $a_{ij}$ for all $i$ and $j$

Problems :

1. Find AB and BA given that $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 2 & -2 \\ 2 & -1 & -4 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & -1 \\ 1 & 2 & -2 \\ 2 & -1 & -4 \end{bmatrix}$

$AB = \begin{bmatrix} (-4-1+0) & (2-2-0) & (-2+2-0) \\ (-0-2+2) & (0-4-1) & (-0+4-4) \\ (-2+0+2) & (1+0-1) & (-1-0-4) \end{bmatrix}$

$\therefore AB = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

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$$BA = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 2 & -2 \\ 2 & -1 & -4 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (-4+0-1) & (2-2-0) & (-0+1-1) \\ (2+0-2) & (-1-4-1) & (0+2-2) \\ (4-0-4) & (-2+2-0) & (0-1-4) \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$\therefore \quad AB = BA$ .

2. If $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$ and the equation

$A(A-I)(A+2I) = 0$

$$A - I = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 1 \\ 3 & 0 & 3 \\ -5 & 2 & -5 \end{bmatrix}$$

$$A + 2I = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 1 \\ 3 & 3 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

$$(A-I)(A+2I) = \begin{bmatrix} 1 & -3 & 1 \\ 3 & 0 & 3 \\ -5 & 2 & -3 \end{bmatrix} \begin{bmatrix} 4 & -3 & 1 \\ 3 & 3 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-9-5 & -3-9+2 & 1-9-2 \\ 12+0-15 & -9+0+6 & 3+0-6 \\ -20+6+25 & 15+6-10 & -5+0+10 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -10 & -10 \\ -3 & -3 & -3 \\ 11 & 11 & 11 \end{bmatrix}$$

$$A(A-I)(A+2I) = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix} \begin{bmatrix} -10 & -10 & -10 \\ -3 & -3 & -3 \\ 11 & 11 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -20+9+11 & -20+9+11 & -20+9+11 \\ -30-3+33 & -30-3+33 & -30-3+33 \\ 50-6-44 & 50-6-44 & 50-6-44 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

$\therefore A(A-I)(A+2I) = 0$

$\therefore$ Hence the proof.

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(ii) $3A - AB + 2B$

$$3A = \begin{bmatrix} 3 & 6 & 6 \\ 9 & 3 & 3 \\ 3 & 6 & 9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8+4 & 1+10+2 & 3+2+4 \\ 6+4+2 & 3+5+1 & 9+1+2 \\ 2+8+6 & 1+10+3 & 3+2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 13 & 9 \\ 12 & 9 & 12 \\ 16 & 14 & 11 \end{bmatrix}$$

$$2B = 2\begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 6 \\ 8 & 10 & 2 \\ 4 & 2 & 4 \end{bmatrix}$$

$$3A - AB = \begin{bmatrix} 3 & 6 & 6 \\ 9 & 3 & 3 \\ 3 & 6 & 9 \end{bmatrix} - \begin{bmatrix} 14 & 13 & 9 \\ 12 & 9 & 12 \\ 16 & 14 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & -7 & -3 \\ -3 & -6 & -9 \\ -13 & -8 & -2 \end{bmatrix}$$

$$3A - AB + 2B = \begin{bmatrix} -11 & -7 & -3 \\ -3 & -6 & -9 \\ -13 & -8 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 6 \\ 8 & 10 & 2 \\ 4 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -7 & -5 & 3 \\ 5 & 4 & -7 \\ -9 & -6 & 2 \end{bmatrix}$$

(1) 5 mark

3. If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, $A^3 - 3A^2 - A + 9I = 0$

$$A \times A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0+3 & 0+0-3 & 3-0+3 \\ 2+2-1 & 0+1+1 & 6-1-1 \\ 1-2+1 & 0-1-1 & 3+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix}$$

$$A^2 \times A = \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 4-6+6 & 0-3-6 & 12+3+6 \\ 3+4+4 & 0+2-4 & 9-2+4 \\ 0+5-4 & 0-2-5 & 0+2+5 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix}$$

$$\Rightarrow 3A^2 = 3 \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 12 & -9 & 18 \\ 9 & 6 & 12 \\ 0 & -6 & 15 \end{bmatrix}$$

$$A^3 - 3A^2 = \begin{bmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix} - \begin{bmatrix} 12 & -9 & 18 \\ 9 & 6 & 12 \\ 0 & -6 & 15 \end{bmatrix}$$

exist non zero vector $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ such that

$Ax = \lambda x$ and $x$ is called Eigen vector.

corresponding to the eigen value $\lambda$.

1. Problems :

1. Find the Eigen value and Eigen vector

4th 5mark

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

Sol :

characheristic equation $|A-\lambda I| = 0$

$$\therefore \begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} = 0$$

$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

$\therefore \lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

$c_1 = 1+5+1 = 7$

$c_2 = +\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix}$

$= +(5-1) + (1-9) + (1-15)$

$= 4-8+4$

$c_2 = 0$

$c_3 = +1\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix}$

$= 1(5-1) - 1(1-3) + 3(1-15)$

$= 1(4) - 1(-2) + 3(-14)$

$= 4+2-42$

$c_3 = -36$

$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

$\lambda_3 - 7\lambda^2 - 0\lambda + 36 = 0$

$\therefore A^3 - 7A^2 - 0A + 36 = 0$

Eigon value :

$$\begin{array}{c|cccc}
3 & 1 & -7 & 0 & 36 \\
 & 0 & 3 & -12 & -36 \\
\hline
 & 1 & -4 & -12 & \boxed{0}
\end{array}$$

$1\lambda^2 - 4\lambda - 12 = 0$

$$\begin{array}{c|c}
\multicolumn{2}{c}{-12} \\
\hline
2 & -6 \\
\hline
\multicolumn{2}{c}{-4}
\end{array}$$

$\lambda + 2 = 0 \quad \lambda - 6 = 0$

$\lambda = -2, \quad \lambda = 6, \quad \lambda = 3$

$\therefore$ Eigon value $\lambda = -2, 3, 6$

Eigon vector :

$(1-\lambda)x_1 + 1x_2 + 3x_3 = 0 \longrightarrow (1)$

$1x_1 + (5-\lambda)x_2 + 1x_3 = 0 \longrightarrow (2)$

$3x_1 + 1x_2 + (1-\lambda)x_3 = 0 \longrightarrow (3)$

case (i) :

$\lambda = -2$

$(1+2)x_1 + x_2 + 3x_3 = 0$

$1x_1 + (5+2)x_2 + x_3 = 0$

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$$3x_1 + x_2 + (1+2)\,x_3 = 0$$

$$\Rightarrow 3x_1 + x_2 + 3x_3 = 0 \rightarrow ①$$

$$\Rightarrow x_1 + 7x_2 + x_3 = 0 \rightarrow ②$$

$$\Rightarrow 3x_1 + x_2 + 3x_3 = 0 \rightarrow ③$$

consider ①, ②

$$\begin{array}{cccc} 3 & 1 & 3 & 3 \\ 1 & 7 & 1 & 1 \end{array}$$

$$\frac{x_1}{21-1} = \frac{x_2}{1-21} = \frac{x_3}{3-3}$$

$$\frac{x_1}{20} = \frac{x_2}{-20} = \frac{x_3}{0}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 20 \\ -20 \\ 0 \end{bmatrix}$$

case (ii) :

$$\lambda = 3$$

$$(1-3)\,x_1 + x_2 + 3x_3 = 0 \rightarrow ①$$

$$1x_1 + (5-3)\,x_2 + 1x_3 = 0 \rightarrow ②$$

$$3x_1 + 1x_2 + (1-3)\,x_3 = 0 \rightarrow ③$$

$$\Rightarrow -2x_1 + x_2 + 3x_3 = 0$$

$$\Rightarrow 1x_1 + 2x_2 + x_3 = 0$$

$$\Rightarrow 3x_1 + x_2 - 2x_3 = 0$$

consider ①, ②

$$\begin{array}{cccc} -2 & 1 & 3 & -2 \\ 1 & 2 & 1 & 1 \end{array} \qquad \frac{x_1}{-4-1} = \frac{x_2}{1-6} = \frac{x_3}{3+2}$$

$$\frac{x_1}{-5} = \frac{x_2}{-5} = \frac{x_3}{5}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \\ 5 \end{bmatrix}$$

case (iii) :

$\lambda = 6$

$$(1-6)x_1 + x_2 + 3x_3 = 0 \rightarrow ①$$
$$1x_1 + (5-6)x_2 + x_3 = 0 \rightarrow ②$$
$$3x_1 + 1x_2 + (1-6)x_3 = 0 \rightarrow ③$$

$\Rightarrow -5x_1 + x_2 + 3x_3 = 0$

$\Rightarrow x_1 - x_2 + x_3 = 0$

$\Rightarrow 3x_1 + x_2 - 5x_3 = 0$

$$\begin{array}{cccc} -5 & 1 & 3 & -5 \\ 1 & -1 & 1 & 1 \end{array}$$

$$\frac{x_1}{5-1} = \frac{x_2}{1+3} = \frac{x_3}{3+5} \Rightarrow \frac{x_1}{4} = \frac{x_2}{4} = \frac{x_3}{8}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix}$$

$$x = \begin{bmatrix} 20 \\ -20 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $(A-I)\ (A-4I) = 0$

$$A - I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 1-0 & 1-0 \\ 1-0 & 2-1 & 1-0 \\ 1-0 & 1-0 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A - 4I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4 & 1-0 & 1-0 \\ 1-0 & 2-4 & 1-0 \\ 1-0 & 1-0 & 2-4 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$\therefore (A-I)(A-4I) = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2+1+1 & 1+(-2)+1 & 1+1-2 \\ -2+1+1 & 1-2+1 & 1+1-2 \\ -2+1+1 & 1-2+1 & 1+1-2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2+2 & 2-2 & 2-2 \\ -2+2 & 2-2 & 2-2 \\ -2+2 & 2-2 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore (A-I)(A-4I) = 0$

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5. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and the equation $A^2 - 5A - 2I = 0$ using the results detments $A^5$.

X. (2) 5mark

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$5A = 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$2I = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^2 - 5A - 2I = 0$$

$$\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 7-5-2 & 10-10-0 \\ 15-15-0 & 22-20-2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore A^2 - 5A - 2I = 0$

One $A^2 - 5A - 2I = 0$

$$A^2 = 5A + 2I$$

$$(A^2)^2 = A^4$$

$$(A^2)^2 = (5A + 2I)^2$$

$$A^4 = (5A + 2I)(5A + 2I)$$

$$= 25A^2 + 10AI + 10AI + 4I^2$$

$= 25A^2 + 20AI + 4I^2$

$A^4 . A = 25A^3 + 20A^2I + 4AI^2$

$A^5 = \cancel{20}\ 25A^3 + 4AI^2 + 20A^2I$

$A^3 = A^2 . A$

$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 7+30 & 14+40 \\ 15+66 & 30+88 \end{bmatrix}$

$= \begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix}$

$A^3 = \begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix}$

$25A^3 = 25 \begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix} = \begin{bmatrix} 925 & 1350 \\ 2025 & 2950 \end{bmatrix}$

$4AI^2 = 4 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$

$20A^2I = 20 \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} = \begin{bmatrix} 140 & 200 \\ 300 & 440 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 140 & 200 \\ 300 & 440 \end{bmatrix}$

$A^5 = 25A^3 + 4AI^2 + 20A^2I$

$A^5 = \begin{bmatrix} 925 & 1350 \\ 2025 & 2950 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} + \begin{bmatrix} 140 & 200 \\ 300 & 440 \end{bmatrix}$

$A^5 = \begin{bmatrix} 925+4+140 & 1350+8+200 \\ 2025+12+300 & 2950+16+440 \end{bmatrix}$

$\therefore A^5 = \begin{bmatrix} 1069 & 1558 \\ 2337 & 3406 \end{bmatrix}$

$\therefore$ Hence the proof $A^5 = \begin{bmatrix} 1069 & 1558 \\ 2337 & 3406 \end{bmatrix}$

$$= \begin{bmatrix} 4-12 & -9+9 & 21-18 \\ 11-9 & -2-6 & 11-12 \\ 1-0 & -7+6 & 7-15 \end{bmatrix}$$

$$A^3 - 3A^2 = \begin{bmatrix} -8 & 0 & 3 \\ 2 & -8 & -1 \\ 1 & -1 & -8 \end{bmatrix}$$

$$A^3 - 3A^2 - A = \begin{bmatrix} -8 & 0 & 3 \\ 2 & -8 & -1 \\ 1 & -1 & -8 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8-1 & 0-0 & 3-3 \\ 2-2 & -8-1 & -1+1 \\ 1-1 & -1+1 & -8-1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -9 \end{bmatrix}$$

$$A^3 - 3A^2 - A + 9I = \begin{bmatrix} -9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -9 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -9+9 & 0+0 & 0+0 \\ 0 & -9+9 & 0+0 \\ 0+0 & 0+0 & -9+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^3 - 3A^2 - A + 9I = 0$

Hence the Proof.

cauley's Hamitton Thearom : *X*X.X.

Every square matrix A sallsfies own chargechers equation is known as cauley's Hamitton Thearom.

$a_0 + a_1x + a_2x^2 + a_3x^3 \ldots\ldots + a_nx^n$

is charecherstic polynomal of degree n of A.

$a_0I + a_1A + a_2A^2 + \ldots\ldots + a_nA^n = 0$.

Formula $\Rightarrow A - xI = 0$ (or) $A - \lambda I = 0$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-x & 2 \\ 3 & 4-x \end{bmatrix}$

(1.) charecherstic equation and cauley's Hamitton Thearom

5

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

using the cauley's Hamitton thearom

$A - xI = 0$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \Rightarrow \begin{bmatrix} 1-x & 0 \\ 3 & 4-x \end{bmatrix}$

charecherstic polynomal is

$|A - xI| = 0$

$\begin{vmatrix} 1-x & 2 \\ 3 & 4-x \end{vmatrix} = 0$

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$\therefore \lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\boxed{A^3 - 18A^2 + 45A = 0}$

3. Find the characherstic matrix polynomal equation

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

Sol :

$|A - \lambda I| = 0$ Let $A \begin{bmatrix} 1-\lambda & 0 & 2 \\ 0 & 1-\lambda & 2 \\ 1 & 2 & 0-\lambda \end{bmatrix}$

characherstic Polynomal equation.

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 1-\lambda & 2 \\ 1 & 2 & 0-\lambda \end{vmatrix}$$

$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

$c_1 = 1+1+0 = 2$

$c_2 = +\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$= (0-4) + (0-2) + (1-0)$

$= -4 - 2 + 1$

$\boxed{c_2 = -5}$

$c_3 = 1\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} - 0\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$

$= 1(0-4) - 0(0-1) + 2(0-1)$

7. $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ find inverse of matrics.

$$|A - xI| = 0$$

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1-x & 2 \\ 3 & 1-x \end{bmatrix}$$

$$(1-x)(1-x)-6 = 0$$

$$(1-x-x+x^2)-6 = 0$$

$$(1-2x+x^2)-6 = 0$$

$$(1-2x+x^2-6) = 0$$

$$x^2-2x-5 = 0$$

$$A^2-2A-5 = 0$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2+2 \\ 3+3 & 6+1 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 6 & 7 \end{bmatrix}$$

$$2A = 2\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 2 \end{bmatrix}$$

$$A^2-2A-5 = 0$$

$$\begin{bmatrix} 7 & 4 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A^2-2A-5I = 0$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

5. Q. find the characherstic equation of the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$\Rightarrow |A - xI| = 0$

$\Rightarrow \begin{vmatrix} 1-x & 0 & 3 \\ 2 & 1-x & -1 \\ 1 & -1 & 1-x \end{vmatrix}$

again $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$\lambda^3 - \lambda^2 c_1 + \lambda c_2 - c_3 = 0$

$c_1 = 1 + 1 + 1 = 3$

$c_2 = + \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$

6. To find $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ $A^3 - 5A^2 + 8A - 4I = 0$ , $A^4 = ?$

$A^3 - 5A^2 + 8A - 4I = 0 \rightarrow$ correct answer

$A^2 = A \cdot A$

$$= \begin{bmatrix} 1+0-0 & 0+0+0 & -2+0-4 \\ 2+4+0 & 0+4+0 & -4+8+8 \\ 0+0+0 & 0+0+0 & 0+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0+0 & -2+0-12 \\ 6+8+0 & 0+8+0 & -12+16+24 \\ 0+0+0 & 0+0+0 & 0+0+8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -14 \\ 14 & 8 & 28 \\ 0 & 0 & 8 \end{bmatrix}$$

$$5A^2 = 5 \begin{bmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & -30 \\ 30 & 20 & 60 \\ 0 & 0 & 20 \end{bmatrix}$$

$$8A = 8\begin{bmatrix}1 & 0 & -2\\ 2 & 2 & 4\\ 0 & 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}8 & 0 & -16\\ 16 & 16 & 32\\ 0 & 0 & 16\end{bmatrix}$$

$$4I = \begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix}$$

$$\therefore A^3 - 5A^2 = \begin{bmatrix}1 & 0 & -14\\ 14 & 8 & 28\\ 0 & 0 & 8\end{bmatrix} - \begin{bmatrix}5 & 0 & -30\\ 30 & 20 & 60\\ 0 & 0 & 20\end{bmatrix}$$

$$= \begin{bmatrix}-4 & 0 & 16\\ -16 & -12 & -32\\ 0 & 0 & -12\end{bmatrix}$$

$$A^3 - 5A^2 + 8A = \begin{bmatrix}-4 & 0 & 16\\ -16 & -12 & -32\\ 0 & 0 & -12\end{bmatrix} + \begin{bmatrix}8 & 0 & -16\\ 16 & 16 & 32\\ 0 & 0 & 16\end{bmatrix}$$

$$= \begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix}$$

$$A^3 - 5A^2 + 8A - 4I = \begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix} - \begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix} = 0$$

$$A^3 - 5A^2 + 8A - 4I = 0$$

$$A^3 = 5A^2 - 8A + 4I$$

$$A^3 . A = A\left[5A^2 - 8A + 4I\right]$$

$$A^4 = 5A^3 - 8A^2 + 4AI$$

$$5A^3 = 5\begin{bmatrix}1 & 0 & -14\\ 14 & 8 & 28\\ 0 & 0 & 8\end{bmatrix} = \begin{bmatrix}5 & 0 & -70\\ 70 & 40 & 140\\ 0 & 0 & 40\end{bmatrix}$$

$$8A^2 = 8\begin{bmatrix}1 & 0 & -6\\ 6 & 4 & 12\\ 0 & 0 & 4\end{bmatrix} = \begin{bmatrix}8 & 0 & -48\\ 48 & 32 & 96\\ 0 & 0 & 32\end{bmatrix}$$

$$4AI = 4\begin{bmatrix}1 & 0 & -2\\ 2 & 2 & 4\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}4 & 0 & -8\\ 8 & 8 & 16\\ 0 & 0 & 8\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}4 & 0 & -8\\ 8 & 8 & 16\\ 0 & 0 & 8\end{bmatrix}$$

$5A^3 - 8A^2 + 4AI = 0$

$$\Rightarrow \begin{bmatrix}5 & 0 & -70\\ 70 & 40 & 140\\ 0 & 0 & 40\end{bmatrix} - \begin{bmatrix}8 & 0 & -48\\ 48 & 32 & 96\\ 0 & 0 & 32\end{bmatrix} + \begin{bmatrix}4 & 0 & -8\\ 8 & 8 & 16\\ 0 & 0 & 8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-3 & 0 & -22\\ 22 & 8 & 44\\ 0 & 0 & 8\end{bmatrix} + \begin{bmatrix}4 & 0 & -8\\ 8 & 8 & 16\\ 0 & 0 & 8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & 0 & -30\\ 30 & 16 & 60\\ 0 & 0 & 16\end{bmatrix}$$

$$\therefore A^4 = \begin{bmatrix}1 & 0 & -30\\ 30 & 16 & 60\\ 0 & 0 & 16\end{bmatrix}$$

$= (1+1) + (1-3) + (1-0)$

$= (2) + (-2) + 1$

$= \cancel{2} - 2 + 1$

$c_2 = 1$

$$c_3 = 1\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} - 0\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$$

$= 1(1+1) - 0(0) + 3(-2-1)$

$= (2) - 0 + 3(-3)$

$= 2 - 9$

$c_3 = -9$

$\lambda^3 - \lambda^2 3 + \lambda(1) + 9 = 0$

$\lambda^3 - 3\lambda^2 \cancel{+} \lambda + 9 = 0$

$\therefore \quad A^3 - 3A^2 \cancel{+} A + 9 = 0$

This is characherstic equation $\boxed{A^3 - 3A^2 - A + 9 = 0}$

(6) Find the inverse of matrics using the cauley's Hamilton thearom $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

3rd 5mark

Sol :

Given let $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

$\Rightarrow |A - xI| =$

$$\Rightarrow \begin{vmatrix} 1-x & 0 & -2 \\ 2 & 2-x & 4 \\ 0 & 0 & 2-x \end{vmatrix}$$

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Inverse of matrix

$\Rightarrow A^2 \cdot A^{-1} - 2A \cdot A^{-1} - 5A^{-1} = 0$

$\Rightarrow A^2 \cdot \frac{1}{A} - 2AA^{-1} - 5A^{-1} = 0$

$A - 2I - 5A^{-1} = 0$

$-5A^{-1} = 2I - A$

$5A^{-1} = A - 2I$

$\Rightarrow A - 2I$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} - 2 \cancel{\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} - \cancel{\begin{bmatrix} 2 & 4 \\ 6 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} - \cancel{\begin{bmatrix} 2 & 4 \\ 6 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \cancel{\begin{bmatrix} -1 & -2 \\ -3 & 0 \end{bmatrix}} \begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix}$

$5A^{-1} = \begin{bmatrix} -1 & +2 \\ +3 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1/5 & +2/5 \\ +3/5 & -1/5 \end{bmatrix}$.

$\therefore \quad A^{-1} = \begin{bmatrix} -1/5 & +2/5 \\ +3/5 & -1/5 \end{bmatrix}$.

**Eigen values and Eigen vectors:**

**Definition:** ~~x.x.x.x.~~ 10 mark question

Let 'A' be an $n \times n$ matrix Number $\lambda$ is called a eigen value of A. If there

SOL :

① $\Rightarrow \frac{x-3}{+3} = \frac{y-8}{-1} = \frac{z-3}{1} = r_1$ from the equation ①

$\frac{x-3}{+3} = r_1, \quad \frac{y-8}{-1} = r_1, \quad \frac{z-3}{1} = r_1$

$x-3 = 3r_1, \quad y-8 = -1r_1, \quad z-3 = 1r_1$

$x = 3r_1+3, \quad y = -1r_1+8, \quad z = 1r_1+3$

② $\Rightarrow \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} = r_2$ from the equation ②

$\frac{x+3}{-3} = r_2, \quad \frac{y+7}{2} = r_2, \quad \frac{z-6}{4} = r_2$

$x+3 = -3r_2, \quad y+7 = 2r_2, \quad z-6 = 4r_2$

$x = -3r_2 \cancel{+} -3, \quad y = 2r_2-7, \quad z = 4r_2+6$

From the equation ①, ②

$x_1 = 3r_1+3, \quad y_1 = -1r_1+8, \quad z_1 = 1r_1+3 \rightarrow$ (A)

$x_2 = -3r_2-3 \quad y_2 = 2r_2-7 \quad z_2 = 4r_2+6 \rightarrow$ (B)

$\Rightarrow [(x_1-x_2), (y_1-y_2), (z_1-z_2)]$ The deraction ratio formula

$\Rightarrow [(3r_1+3)-(-3r_2-3)], [(-1r_1+8)-(2r_2-7)] [(1r_1+3)-(4r_2+6)]$

$\Rightarrow [[3r_1+3+3r_2+3], [-1r_1+8-2r_2+7] [1r_1+3-4r_2-6]]$

$\Rightarrow 3r_1+3+3r_2+3-1r_1+8-2r_2+7+1r_1+3-4r_2-6$ summa

$\Rightarrow [\underbrace{[3r_1+3r_2+6]}_{x}, \underbrace{[-1r_1-2r_2+15]}_{y}, \underbrace{[1r_1-4r_2-3]}_{z}]$

$\overset{x}{3r_1} \overset{y}{-1r_1} \overset{z}{+1r_1} = 0 \rightarrow$ ① $\rightarrow$ (C)

$\overset{x}{3r_2} \overset{y}{-2r_2} \overset{z}{-4r_2} = 0 \rightarrow$ ② $\rightarrow$ D

$\overset{L}{①} \Rightarrow 3x-1y+1z=0 \Rightarrow 3x-1y+1z=0$

$\overset{D}{②} \Rightarrow 3x-2y-4z=0 \Rightarrow -3x+2y+4z=0$

(C) $\Rightarrow 3(3r_1+3)-1(-1r_1+8)+1(1r_1+3)=0$

(D) $\Rightarrow -3(-3r_2-3)+2(+2r_1+(-7))+4(4r_2+6)=0$

(D) $\Rightarrow -3(-3r_2-3)+2(2r_1-7)+4(4r_2+6)=0$

Ⓒ ⇒ From the equation Ⓒ

Ⓒ ⇒ $3x - 1y + 1z = 0$

$\Rightarrow 3(3r_1 + 3r_2 + 6) - 1(-1r_1 - 2r_2 + 15) + 1(1r_1 - 4r_2 - 3) = 0$

$\Rightarrow 9r_1 + 9r_2 + 18 + 1r_1 + 2r_2 - 15 + 1r_1 - 4r_2 - 3 = 0$

$\Rightarrow 9r_1 + 2r_1 + 9r_2 + 2r_2 - 4r_2 + 18 - 15 - 3 = 0$

$\Rightarrow 11r_1 + 7r_1 = 0 \longrightarrow$ Ⓔ

From the equation Ⓓ

Ⓓ ⇒ $-3x + 2y + 4z = 0$

$\Rightarrow -3(3r_1 + 3r_2 + 6) + 2(-1r_1 - 2r_2 + 15) + 4(1r_1 - 4r_2 - 3) = 0$

$\Rightarrow -9r_1 - 9r_2 - 18 - 2r_1 + 4r_2 + 30 + 4r_1 - 16r_2 - 12 = 0$

$\Rightarrow -9r_1 - 2r_1 + 4r_1 - 9r_2 + 4r_2 - 16r_2 - 18 + 30 - 12 = 0$

$\Rightarrow -7r_1 - 29r_2 = 0 \rightarrow$ Ⓕ

Ⓔ ⇒ $11r_1 + 7r_2 = 0$

Ⓕ ⇒ $-7r_1 - 29r_2 = 0$

Ⓔ × 7 ⇒ $77r_1 + 49r_2 = 0$

Ⓕ × 11 ⇒ $-77r_1 - 319r_2 = 0$

$-270r_2 = 0$

$r_2 = \frac{0}{-270}$

$\boxed{r_2 = 0}$

$r_2 = 0$ ⇒ Ⓔ ⇒ $11r_1 + 7r_2 = 0$

$11r_1 + 7(0) = 0$

$11r_1 + 0 = 0$

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Problems :

1. Find the equation of the sphere which has its of the point (6, -1, 2) and touches the play

$2x - y + 2z - 2 = 0$
$\;a \quad b \quad c \quad d$

Sol :

$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

center $a, b, c$, piont $x, y, z$, radius $= r$

center $(2, -1, 2)$ piont $(6, -1, 2)$ radius $= -2$

$\Rightarrow$

$(6-2)^2 + (-1+1)^2 + (2-2)^2 = (-2)^2$

$\Rightarrow$ formula $r = \pm \left( \dfrac{ax + by + cz + d}{\sqrt{a^2 + b^2 + c^2}} \right)$

$= \pm \left( \dfrac{2(6) + ((-1)(-1) + (2)(2) + (-2)}{\sqrt{2^2 + (-1)^2 + (2)^2}} \right)$

$= \pm \left( \dfrac{12 + 1 + 4 - 2}{\sqrt{4 + 1 + 4}} \right)$

$r = \pm \left( \dfrac{15}{\sqrt{9}} \right) = \pm \dfrac{15}{3} = \pm 5$

$\Rightarrow (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

$\Rightarrow (6-2)^2 + (-1+1)^2 + (2-2)^2 = 5^2$

$\Rightarrow (4)^2 + (0)^2 + (0)^2 = 25$

$\Rightarrow 16 = 25$

$\Rightarrow (x-2)^2 + (y+1)^2 + (z-2)^2 = 5^2$

$\Rightarrow x^2 + 2^2 - 2(2)(x) + y^2 + 1^2 + (2)(1)(y) + (z^2) + (2)^2 - 2(2)(z) = 25$

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## Co-planar lines:

The condition that two given stright line's should be also co-planar. Let the equation of the stright line are

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$$

$$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

Problems:

1. Prove that the lines $\frac{x-3}{2} = \frac{y-2}{-5} = \frac{z-1}{3}$ and $\frac{x-1}{-4} = \frac{y+2}{0} = \frac{z-6}{2}$ are co-planar find the point of intersection and the plan though.

$$\frac{x-3}{2} = \frac{y-2}{-5} = \frac{z-1}{3}, \quad \frac{x-1}{-4} = \frac{y+2}{0} = \frac{z-6}{2}$$

$$(1) \Rightarrow \frac{x-3}{2} = \frac{y-2}{-5} = \frac{z-1}{3} = r$$

$$\frac{x-3}{2} = r, \quad \frac{y-2}{-5} = r, \quad \frac{z-1}{3} = r$$

$x-3 = 2r \quad y-2 = -5r, \quad z-1 = 3r$

$x = 2r+3 \quad y = -5r+2, \quad z = 3r+1 \longrightarrow (1)$

$$(2) \Rightarrow \frac{x-1}{-4} = \frac{y+2}{0} = \frac{z-6}{2} = r_1$$

$$\frac{x-1}{-4} = r_1, \quad \frac{y+2}{0} = r_1, \quad \frac{z-6}{2} = r_1$$

$x-1 = -4r_1 \quad y+2 = 0r_1 \quad z-6 = 2r_1$

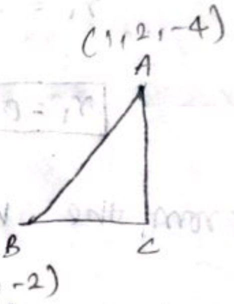
$x = -4r_1+1, \quad y = 0r_1 - 2, \quad z-6 = 2r_1 \quad z = 2r_1+6 \longrightarrow (2)$

$(1) \Rightarrow x = 2r+3, \quad y = -5r+2, \quad z = 3r+1$

$(2) \Rightarrow x = -4r_1+1, \quad y = 0r_1 - 2, \quad z = 2r_1+6$

(2) Find the distance from the point $(1,2,-4)$ from the line $\frac{x-3}{2} = \frac{y-1}{-5} = \frac{z+2}{3}$.

8th smart




$AC^2 = AB^2 - BC^2$

$A(1,2,-4)$ $B = (3,1,-2)$

$\Rightarrow AB^2 = (x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2$

$= (1-3)^2 + (2-1)^2 + (-4+2)^2$

$= (-2)^2 + (1)^2 + (-2)^2$

$= 4+1+4$

$AB^2 = 9$

direction ratio : $(2,-5,3)$ Direction of cosine's

$l = \frac{l}{\sqrt{l^2+m^2+n^2}}$ $m = \frac{m}{\sqrt{l^2+m^2+n^2}}$ $n = \frac{n}{\sqrt{l^2+m^2+n^2}}$

$l = \frac{2}{\sqrt{2^2+(-5)^2+3^2}} = \frac{+2}{\sqrt{38}}$ $m = \frac{-5}{\sqrt{38}}$, $n = \frac{3}{\sqrt{38}}$

$x_1\ y_1\ z_1$ $x_2\ y_2\ z_2$
$(1,2,-4)$ $(3,1,-2)$

$BC = l(x_1-x_2) + m(y_1-y_2) + n(z_1-z_2)$

$BC = \frac{2}{\sqrt{38}}(1-3) + \frac{-5}{\sqrt{38}}(2-1) + \frac{3}{\sqrt{38}}(-4+2)$

$= \frac{2}{\sqrt{38}}(-2) - \frac{5}{\sqrt{38}}(1) + \frac{3}{\sqrt{38}}(-2)$

$= -\frac{4}{\sqrt{38}} - \frac{5}{\sqrt{38}} - \frac{6}{\sqrt{38}} = \frac{-4-5-6}{\sqrt{38}} = \frac{-15}{\sqrt{38}}$

$BC = \frac{-15}{\sqrt{38}}$, $BC^2 = \left(\frac{-15}{\sqrt{38}}\right)^2 \Rightarrow \frac{225}{(\sqrt{38})^2}$ $BC^2 = \frac{225}{38}$

$AC^2 = AB^2 - BC^2$

$= 9 - \frac{225}{38} = \frac{342-225}{38} = \frac{117}{38}$

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② $\Rightarrow -4x+1,\ y=0x_1-2,\ z=2x_1+6$

$x=-4(-1)+1,\ y=0(-1)-2,\ z=2(-1)+6$

$x=+4+1,\ y=0-2,\ z=-2+6$

$x=5,\ y=-2,\ z=4$

$(x,y,z)=(5,-3,4),\ (5,-2,4)$

$$\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0 \quad \begin{vmatrix} 3-1 & 2-(-2) & 1-6 \\ 2 & -5 & 3 \\ -4 & 0 & 2 \end{vmatrix}=0$$

$$\begin{vmatrix} 2 & 4 & -5 \\ 2 & -5 & 3 \\ -4 & 0 & 2 \end{vmatrix}=0$$

$x_1=3,\ y_1=2,\ z_1=1$

$x_2=1,\ y_2=-2,\ z_2=6$

$\Rightarrow (2)(-10)-4(4+12)-5(0-20)=0$

$\Rightarrow -20-4(16)-5(-20)=0$

$\Rightarrow -20-64+100=0$

$\Rightarrow 16$

The co-planar line the plane through the equation.

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 2 & -5 & 3 \\ -4 & 0 & 2 \end{vmatrix}=0$$

$\Rightarrow (x-3)(-10)-(y-2)(4+12)+(z-1)(-20)$

$\Rightarrow 10x+30-16y+32-20z+20=0$

$\Rightarrow -10x-16y-20z+82=0$

$\Rightarrow -10x-16y-20z+82=0 \Rightarrow 10x+16y+20z-82=0$

$\Rightarrow 5x+8y+10z-41=0$

$$\begin{vmatrix} 2 & -9 & -3 \\ -3 & 8 & 2 \\ -4 & 7 & 1 \end{vmatrix} = 0$$

$2(8-14) + 9(-3+8) - 3(-21+32) = 0$

$2(-6) + 9(5) - 3(11) = 0$

$-12 + 45 - 33 = 0$

$45 - 45 = 0$

$= 0$

The co-planar line the plane through the equation,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+1 & y+10 & z-1 \\ -3 & 8 & 2 \\ -4 & 7 & 1 \end{vmatrix} = 0$$

$= (x+1)(8-14) - (y+10)(-3+8) + (z-1)(-21+32)$

$= (x+1)(-6) - (y-10)5 + (z-1)(11) = 0$

$= -6x - 6 - 5y - 50 + 11z - 11 = 0$

$= -6x - 5y + 11z - 11 - 50 - 6 = 0$

$= -6x - 5y + 11z - 67 = 0$

$= 6x + 5y - 11z + 67 = 0$

3. Find the equation of the sphere whose center is $(1,1,1)$ and which passes through the point $(2,0,3)$

$x = x$ $\quad$ $y = y$ $\quad$ $z = z_1$

$2r + 3 = -4r_1 + 1$ $\quad$ $-5r + 2 = -2$ $\quad$ $3r + 1 = 2r_1 + 6$

$2r + 4r_1 = 1 - 3$ $\quad$ $-5r = -2 - 2$ $\quad$ $3r - 2r_1 = 6 - 1$

$2r + 4r_1 = -2 \rightarrow (3)$ $\quad$ $-5r = -4 \rightarrow (4)$ $\quad$ $3r - 2r_1 = 5 \rightarrow (5)$

$(3) \Rightarrow \quad 2r + 4r_1 = -2$

$(5) \Rightarrow \quad 3r - 2r_1 = 5$

$(3) \times 1 \Rightarrow \quad 2r + \cancel{4r_1} = -2$

$(5) \times 2 \Rightarrow \quad 6r - \cancel{4r_1} = 10$

$8r \quad = 8$

$\boxed{r = 8/8}$

$\boxed{r = 1}$

$r = 1$ என்ற (2) ல் பிரதியிட,

$(3) \Rightarrow \quad 2(1) + 4r_1 = -2$

$2 + 4r_1 = -2$

$4r_1 = -2 - 2$

$4r_1 = -4$

$r_1 = -\frac{4}{4}$

$\boxed{r_1 = -1}$

$r = 1, \quad r_1 = -1$ form the equation (1), (2)

$(1) \Rightarrow x = 2r + 3, \quad y = -5r + 2, \quad z = 3r + 1$

$x = 2(1) + 3, \quad y = -5(1) + 2, \quad z = 3(1) + 1$

$x = 2 + 3, \quad y = -5 + 2, \quad z = 3 + 1$

$x = 5, \quad y = -3, \quad z = 4$

$r_1 = 1$ equation from (3).

$-3r + 4r_1 = -2$

$-3r + 4(1) = -2$

$-3r + 4 = -2$

$-3r = -2 - 4$

$-3r = -6$

$r = -6/-3$

$\boxed{r = 2}$

$r = 2$ from equation (1)

$x = -3r - 1$ $\quad$ $y = 8r - 10$ $\quad$ $z = 2r + 1$

$x = -3(2) - 1$ $\quad$ $y = 8(2) - 10$ $\quad$ $z = 2(2) + 1$

$x = -6 - 1$ $\quad$ $y = 16 - 10$ $\quad$ $z = 4 + 1$

$x = -7,$ $\quad$ $y = 6$ $\quad$ $z = 5$

$r_1 = 1$ equation from (2)

$x = -4r - 3$ $\quad$ $y = 7r_1 - 1$ $\quad$ $z = r_1 + 4$

$x = -4(1) - 3$ $\quad$ $y = 7(1) - 1$ $\quad$ $z = 1 + 4$

$x = -4 - 3$ $\quad$ $y = 7 - 1$ $\quad$ $z = 5$

$x = -7$ $\quad$ $y = 6$

$(x_1, y_1, z_1) = (-1, -10, 1)$

$(x_2, y_2, z_2) = (-3, -1, 4)$

$(l_1, m_1, n_1) = (-3, 8, 2)$

$(l_2, m_2, n_2) = (-4, 7, 1)$

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} -1 + 3 & -10 + 1 & 1 - 4 \\ -3 & 8 & 2 \\ -4 & 7 & 1 \end{vmatrix} = 0$$

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$11 r_1 = 0$

$r_1 = \frac{0}{11}$

$\boxed{r_1 = 0}$

From the value $r_1, r_2$ equation Ⓐ, Ⓑ $r_1 = 0, r_2 = 0$

Ⓐ $\Rightarrow x_1 = 3r_1 + 3 \quad y_1 = -1r_1 + 8 \quad z_1 = 1r_1 + 3$

$x_1 = 3(0)+3 \qquad y_1 = -1(0)+8 \qquad z_1 = 1(0)+3$

$x_1 = 0+3 \qquad y_1 = 0+8 \qquad z_1 = 0+3$

$\boxed{x_1 = 3} \qquad \boxed{y_1 = 8} \qquad \boxed{z_1 = 3}$

Ⓑ $\Rightarrow x_2 = -3r_2 - 3 \quad y_2 = 2r_2 - 7 \quad z_2 = 4r_2 + 6$

$x_2 = -3(0)-3 \qquad y_2 = 2(0)-7 \qquad z_2 = 4(0)+6$

$x_2 = \cancel{-0-3} \qquad y_2 = 0-7 \qquad z_2 = 0+6$

$\boxed{x_2 = -3} \qquad \boxed{y_2 = -7} \qquad \boxed{z_2 = 6}$

$(x, y, z) = (3, 8, 3), (-3, -7, 6)$

distance formula:

$\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$\Rightarrow \sqrt{(3-(-3))^2 + (8-(-7))^2 + (3-6)^2}$

$\Rightarrow \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$

$\Rightarrow \sqrt{(6)^2 + (15)^2 + (-3)^2}$

$\Rightarrow = \sqrt{36 + 225 + 9}$

$\Rightarrow = \sqrt{270}$ ,,

2. $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ and $\frac{x+3}{-4} = \frac{y+1}{7\cancel{2}} = \frac{z-4}{1} \Rightarrow$ the point of intersection and the plane through.

Sol :

$$\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2} = r, \quad \frac{x+3}{-4} = \frac{y+1}{\cancel{2}\,7} = \frac{z-4}{1} = r_1$$

$x+1 = -3r, \; y+10 = 8r, \; z-1 = 2r \rightarrow (1)$

$x = -3r-1, \; y = 8r-10, \; z = 2r+1$

$$\frac{x+3}{-4} = \frac{y+1}{\cancel{2}\,7} = \frac{z-4}{1} = r_1$$

$x+3 = -4r_1, \; y+1 = 7r_1, \; z-4 = 1r_1 \rightarrow (2)$

$x = -4r_1 - 3, \; y = 7r_1 - 1, \; z = 1r_1 + 4$

$x = x$

$-3r - 1 = -4r_1 - 3$

$-3r + 4r_1 = 1 - 3$

$-3r + 4r_1 = -2 \rightarrow (3)$

$y = y$

$8r - 10 = 7r_1 - 1$

$8r - 7r_1 = -1 + 10$

$8r - 7r_1 = 9 \rightarrow (4)$

$z = z$

$2r + 1 = r_1 + 4$

$2r - r_1 = 4 - 1$

$2r - r_1 = 3 \rightarrow (5)$

Equation (3), (4)

$(3) \times 8 \Rightarrow -24r + 32r_1 = -16$

$(4) \times 3 \Rightarrow 24r - 21r_1 = 27$

$$11r_1 = 11$$

$$r_1 = 11/11$$

$$\boxed{r_1 = 1}$$

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SOL :

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

centre $=\overset{a,b,c}{(1,1,1)}$ , point $=\overset{x,y,z}{(2,0,3)}$

$$(2-1)^2+(0-1)^2+(3-1)^2=r^2$$

$$(1)^2+(-1)^2+(2)^2=r^2$$

$$1+1+4=r^2$$

$$6=r^2$$

$\sqrt{6}=r$ (r - radius).

$\Rightarrow (x-a)^2+(y-b)^2+(z-c)^2=(\sqrt{6})^2$

$\Rightarrow (x-1)^2+(y-1)^2+(z-1)^2=6$

$\Rightarrow x^2+1^2-2(1)(x)+y^2+1^2-2(1)(y)+z^2+1^2-2(1)(z)=6$

$\Rightarrow x^2+1-2x+y^2+1-2y+z^2+1^2-2z=6$

$\Rightarrow x^2+y^2+z^2-2x-2y-2z+1+1+1=6$

$\Rightarrow x^2+y^2+z^2-2x-2y-2z+3=6$

$\Rightarrow x^2+y^2+z^2-2x-2y-2z=3$

$\Rightarrow x^2+y^2+z^2-2x-2y-2z-3=0$

Formula :

The length of perpendicular form of center point $(x,y,z)$ to the plane $ax+by+cz+d=0$

The length of perpendicular radius is $\Rightarrow$

$$\pm\left(\frac{ax+by+cz+d}{\sqrt{a^2+b^2+c}}\right).$$

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Unit - IV [∵ $\cos^2\theta = 1-\sin^2\theta$
$\sin^2\theta = 1-\cos^2\theta$]

Trignomentry

Expansion :

| | Formula | Result |
|---|---|---|
| 1. | $(\cos n\theta + i\sin n\theta)$ | $(\cos\theta + i\sin\theta)^n$ |
| 2. | $\cos n\theta$ | $\cos^n\theta - \frac{n(n-1)}{2!}\cos^{n-2}\sin^2\theta + \cdots$ (or) $\cos^n\theta - nC_2\cos^{n-2}\sin^2\theta + \cdots$ |
| 3. | $\sin n\theta$ | $n\cos^{n-1}\theta\sin\theta - \frac{n(n-1)(n-2)}{3}\cos^{n-3}\theta\sin^3\theta + \cdots$ (or) $nC_1\cos^{n-1}\theta\sin\theta - nC_3\cos^{n-3}\theta\sin^3\theta + \cdots$ |
| 4. | $\tan\theta$ | $\frac{\sin n\theta}{\cos n\theta}$ |
| 5. | $\tan\theta$ | $\frac{nC_1\cos^{n-1}\theta\sin\theta - nC_3\cos^{n-3}\theta\sin^3\theta + \cdots}{\cos^n\theta - nC_2\cos^{n-2}\theta(\sin^2\theta) + \cdots}$ |
| 6. | $\tan n\theta$ | $\frac{nC_1\cos^{n-1}\theta\sin\theta \; nC_3\cos^{n-3}\theta\sin^3\theta + \cdots}{1 - nC_2\cos^{n-2}\theta\sin^3\theta + \cdots}$ |
| 7. | $\tan n\theta$ | $\frac{nC_1\tan\theta - nC_3\tan^3\theta}{1 - nC_2\tan^2\theta + \cdots}$ |

Expansion of $\tan(A+B+C+1)$

$\cos A + i\sin A = \cos A(1+i\tan A)$

$\cos B + i\sin B = \cos B(1+i\tan B)$

$\cos C + i\sin C = \cos C(1+i\tan C)$

Expansion of formulas :

$(\cos n\theta + i\sin n\theta) = (\cos\theta + i\sin\theta)^n$

$(\cos n\theta + i\sin n\theta) = \cos^n\theta + n\cos^{n-1}\theta\,(i\sin\theta)$

$+ \frac{n(n-1)}{2!}\cos^{n-2}\theta\,(i\sin^2\theta) + \frac{n(n-1)(n-2)}{3!}$

$\cos^{n-3}\theta\,(i\sin 3\theta) + \cdots\cdots$

$[i = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1]$ $i^5 = i$, $i^6 = -1$, $i^7 = -i$

(or)

$= \cos n\theta + nc_1\cos^{n-1}\theta\,(i\sin\theta) + nc_2$

$\cos^{n-2}\theta\,(i\sin^2\theta) + nc_3\cos^{n-3}\theta\,(i\sin^3\theta)$

Problem :

1. Expand $\cos 5\theta$ interms of $\sin\theta$

Sol:

Given there $n = 5$

$(\cos\theta + i\sin 5\theta) = (\cos\theta + i\sin n\theta)^5$

$\Rightarrow \cos n\theta + nc_1\cos^{n-1}\theta\,(i\sin\theta) + nc_2\cos^{n-2}\theta\,(i\sin^2\theta) + nc_3\cos^{n-3}\theta$

$(i\sin 3\theta) + nc_4\cos^{n-4}\theta\,(i\sin^4\theta) + nc_5$

$\cos^{n-5}\theta\,(i\sin^5\theta)$

$\Rightarrow \cos^5\theta + 5c_1\cos^{5-1}\theta\,(i\sin\theta) + 5c_2\cos^3\theta\,(i\sin^2\theta) + 5c_3\cos^2\theta$

$(i\sin 3\theta) + 5c_4\cos^1\theta\,(i\sin^4\theta) + 5c_5\cos^0\theta\,(i\sin^5\theta)$

$$\Rightarrow \cos^5\theta + 5C_1 \cos^4\theta (i\sin\theta)^1 + 5C_2 \cos^3\theta (i\sin\theta)^2 + 5C_3 \cos^2\theta (i\sin\theta)^3 + 5C_4 \cos^1\theta (i\sin\theta)^4 + 5C_5 \cos^0\theta (i\sin\theta)^5$$

$$\Rightarrow \cos^5\theta + i5C_1 \cos^4\theta \sin\theta - 5C_2 \cos^3\theta \sin^2\theta - i5C_3 \cos^2\theta \sin^3\theta + 5C_4 \cos^1\theta \sin^4\theta + i5C_5 \cos^0\theta \sin^5\theta$$

Real part

5θ $\Rightarrow \cos^5\theta - 5C_2 \cos^3\theta \sin^2\theta + 5C_4 \cos^1\theta \sin^4\theta$

$$\cos 5\theta = \cos^5\theta - \frac{5\times4}{2\times1}\cos^3\theta\sin^2\theta + \frac{5\times4\times3\times2}{4\times3\times2\times1}\cos^1\theta\sin^4\theta$$

$$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos^1\theta\sin^4\theta$$

$$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos^1\theta(1-\cos^2\theta)^2$$

$$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos^1\theta(1+\cos^4\theta - 2\cos^2\theta)$$

$$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos^1\theta + 5\cos^5\theta - 10\cos^3\theta$$

$$\cos 5\theta = 10\cos^3\theta - 10\cos^5\theta - 5\cos^1\theta - 5\cos^5\theta + 10\cos^3\theta$$

$$= \cos^5\theta + 10\cos^5\theta + 5\cos^5\theta - 10\cos^3\theta - 10\cos^3\theta + 5\cos^1\theta$$

$$= 16\cos^5\theta - 10\cos^3\theta - 10\cos^3\theta + 5\cos^1\theta$$

$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos^1\theta$$

$$\therefore \cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

2) Expand $\sin 5\theta$ interms of $\cos\theta$.

Solu:

$(\cos 5\theta + i\sin 5\theta) = (\cos\theta + i\sin\theta)^5$

$n = 5$

$= \cos^n\theta + nC_1 \cos^{n-1}\theta\,(i\sin\theta) + nC_2 \cos^{n-2}\theta\,(i\sin^2\theta)$

$+ nC_3 \cos^{n-3}\theta\,(i\sin^3\theta) + nC_4 \cos^{n-4}\theta\,(i\sin^4\theta) +$

$nC_5 \cos^{n-5}\theta\,(i\sin^5\theta).$

$= \cos^5\theta + 5C_1 \cos^4\theta\,(i\sin\theta) + 5C_2 \cos^3\theta\,(i\sin^2\theta)$

$+ 5C_3 \cos^2\theta\,(i\sin^3\theta) + 5C_4 \cos^1\theta\,(i\sin^4\theta) +$

$5C_5 \cos^0\theta\,(i\sin^5\theta).$

$= \cos^5\theta + 5C_1 \cos^4\theta\,(i\sin\theta) + 5C_2 \cos^3\theta\,(i\sin\theta)^2 +$

$5C_3 \cos^2\theta\,(i\sin\theta)^3 + 5C_4 \cos^1\theta\,(i\sin\theta)^4 +$

$5C_5 \cos^0\theta\,(i\sin\theta)^5$

$= \cos^5\theta + i\,5C_1 \cos^4\theta \sin\theta - 5C_2 \cos^3\theta \sin^2\theta -$

$i\,5C_3 \cos^2\theta \sin^3\theta + 5C_4 \cos^1\theta\,(i\sin^4\theta) +$

$i\,5C_5 \cos^0\theta \sin^5\theta.$

Imaginary part.

$= 5C_1 \cos^4\theta \sin\theta - 5C_3 \cos^2\theta \sin^3\theta +$

$5C_5 \cos^0\theta \sin^5\theta.$

$= (5)\cos^4\theta \sin\theta - \frac{5\times \cancel{4}^{2}\times \cancel{3}}{1\times \cancel{2}\times \cancel{3}} \cos^2\theta \sin^3\theta +$

$\cos^0\theta \sin^5\theta$

$= 5\cos^4\theta \sin\theta - 10\cos^2\theta \sin^3\theta + \cos^0\theta \sin^5\theta$

$= 5\cos^4\theta \sin\theta - 10\cos^2\theta \sin^3\theta + \sin^5\theta$

$(\div)\ \sin\theta$

$= 5\cos^4\theta - 10\cos^2\theta \sin^2\theta + \sin^4\theta$

$= 5\cos^4\theta - 10\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$

$= 5\cos^4\theta - 10\cos^2\theta + 10\cos^4\theta + 1 + \cos^4\theta - 2\cos^2\theta$

$= 16\cos^4\theta - 12\cos^2\theta + 1$

2) Expand $\cos 8\theta$ interms of $\sin\theta$.

SOL:

$(\cos 8\theta + i\sin 8\theta) = (\cos\theta + i\sin\theta)^8$

$\Rightarrow \cos^n\theta + nc_1 \cos^{n-1}\theta\,(i\sin\theta) + nc_2 \cos^{n-2}\theta\,(i\sin^2\theta) + \dots$

$\Rightarrow \cos^8\theta + 8c_1 \cos^7\theta (i\sin\theta)^1 + 8c_2 \cos^6\theta (i\sin\theta)^2 + 8c_3 \cos^5\theta (i\sin\theta)^3 + 8c_4 \cos^4\theta (i\sin\theta)^4 + 8c_5 \cos^3\theta (i\sin\theta)^5 + 8c_6 \cos^2\theta (i\sin\theta)^6 + 8c_7 \cos^1\theta (i\sin\theta)^7 + 8c_8 \cos^0\theta (i\sin\theta)^8$

$\Rightarrow \cos^8\theta + i8c_1 \cos^7\theta \sin\theta - 8c_2 \cos^6\theta \sin^2\theta - i8c_3 \cos^5\theta \sin^3\theta + 8c_4 \cos^4\theta \sin^4\theta + i8c_5\cos^3\theta \sin^5\theta - 8c_6 \cos^2\theta \sin^6\theta - i8c_7 \cos^1\theta \sin^7\theta + 8c_8 \sin^8\theta$

$\Rightarrow \cos^8\theta - 8c_2 \cos^6\theta \sin^2\theta + 8c_4 \cos^4\theta \sin^4\theta - 8c_6 \cos^2\theta \sin^6\theta + 8c_8 \sin^8\theta$.

$\Rightarrow \cos^8\theta - \frac{8\times7}{1\times2!}\cos^6\theta \sin^2\theta + \frac{8\times7\times6\times5}{1\times2\times3\times4!}\cos^4\theta \sin^4\theta - \frac{8\times7\times6\times5\times4\times3}{1\times2\times3\times4\times5\times6!}\cos^2\theta \sin^6\theta + 1\sin^8\theta$

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$$(2\cos\theta)^n = (x+\frac{1}{x})^n$$

x.x.x.

[(i) $x^n + nc_1 x^{n-1}\frac{1}{x} + nc_2 x^{n-2}\frac{1}{x^2} + nc_3 x^{n-3}\frac{1}{x^3} + \dots$

(ii) $2^n \cos^n\theta = 2\cos n\theta + nc_1 2\cos(n-2)\theta + nc_2 2(\cos)(n-4)\theta + \dots$

(or)

(iii) $2^{n-1}\cos^n\theta = \cos n\theta + nc_1 \cos(n-2)\theta + nc_2 \cos(n-4)\theta + \dots$]

x.x.

x.x. 10 mark :

x.x.x. (1.) Expand $\cos^6\theta$ and $\cos^5\theta$ in series of cosines of multiple of '$\theta$'.

Sol: (i) $\cos^6\theta$

$x = \cos\theta + i\sin\theta$

$2\cos\theta = (x + 1/x)$

$(2\cos\theta)^n = (x+\frac{1}{x})^n$ , $n=6$

(i) $\Rightarrow x^n + nc_1 x^{n-1}\frac{1}{x} + nc_2 x^{n-2}\frac{1}{x^2} + nc_3 x^{n-3}\frac{1}{x^3} + nc_4 x^{n-4}\frac{1}{x^4} + nc_5 x^{n-5}\frac{1}{x^5} + nc_6 x^{n-6}\frac{1}{x^6}$

$= x^6 + 6c_1 x^5\frac{1}{x} + 6c_2 x^4\frac{1}{x^2} + 6c_3 x^3\frac{1}{x^3} + 6c_4 x^2\frac{1}{x^4} + 6c_5 x^1\frac{1}{x^5} + 6c_6 x^0\frac{1}{x^6}$

$= x^6 + 6x^{\cancel{5}\,4}\frac{1}{\cancel{x}} + \frac{\cancel{6}^{3}\times 5}{1\times\cancel{2}} x^{\cancel{4}\,2}\frac{1}{\cancel{x^2}} + \frac{\cancel{6}^{2}\times 5\times\cancel{4}^{2}}{1\times\cancel{2}\times\cancel{3}} \cancel{x^3}\frac{1}{\cancel{x^3}} + \frac{\cancel{6}^{3}\times 5\times\cancel{4\times 3}}{1\times\cancel{2}\times\cancel{3\times 4}} x^2\frac{1}{x^4_2} + \frac{6\times\cancel{5\times 4\times 3\times 2}}{1\times\cancel{2\times 3\times 4\times 5}} x^1\frac{1}{x^5_4} + 1 x^0\frac{1}{x^6}$

$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

$= (x^6 + \frac{1}{x^6}) + 6(x^4 + \frac{1}{x^4}) + 15(x^2 + \frac{1}{x^2}) + 20$ .

(ii) $2^n \cos^n\theta = 2\cos n\theta + nc_1\, 2\cos(n-2)\theta + nc_2\, 2\cos(n-4)\theta + \dots$

$2^6 \cos^6\theta = 2\cos 6\theta + 6c_1\, 2\cos 4\theta + 6c_2\, 2\cos 2\theta + 20$

$= 2\cos 6\theta + 6 \cdot 2\cos 4\theta + \frac{6\times 5}{1\times 2}\, 2\cos 2\theta + 20$

$= 2\cos 6\theta + 12\cos 4\theta + 15 \cdot 2\cos 2\theta + 20$

$\div 2 \quad = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10.$

(iii) $2^{n-1}\cos^n\theta = \cos n\theta + nc_1 \cos(n-2) + nc_2 \cos(n-4) + \dots$

$2^5 \cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10.$

(ii) $\cos^5\theta$.

Let $x = \cos\theta + i\sin\theta$

$2\cos\theta = x + \frac{1}{x}$

$(2\cos\theta)^n = (x + \frac{1}{x})^n$

$n = 5$

(i) $x^n + nc_1\, x^{n-1}\frac{1}{x} + nc_2\, x^{n-2}\frac{1}{x^2} + nc_3\, x^{n-3}\frac{1}{x^3} + \dots$

$\Rightarrow x^5 + 5c_1\, x^4 \cdot \frac{1}{x} + 5c_2\, x^3 \cdot \frac{1}{x^2} + 5c_3\, x^2 \cdot \frac{1}{x^3} + 5c_4\, x^1 \cdot \frac{1}{x^4}$

$+ 5c_5\, x^0 \cdot \frac{1}{x^5}$

$\Rightarrow x^5 + 5x^3 + \frac{5\times 4}{1\times 2}\, x + \frac{5\times 4\times 3}{1\times 2\times 3} \cdot \frac{1}{x^2} + \frac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4} \cdot \frac{1}{x^3}$

$+ \frac{1}{x^5}$

$\Rightarrow x^5 + 5x^3 + 10x + \frac{10}{x^2} + \frac{5}{x^3} + \frac{1}{x^5}.$

$\Rightarrow x^5 + \frac{1}{x^5} + 5(x^3 + \frac{1}{x^3}) + 10(x + \frac{1}{x})$

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3. Sum the series $1+\frac{3}{4}+\frac{3\times5}{4\times8}+\frac{3\times5\times7}{4\times8\times2}+\dots$

SOL:

$$S = 1+\frac{3}{1!}\left(\frac{1}{4}\right)+\frac{3.5}{2}\left(\frac{1}{4}\right)^2+\frac{3.5.7}{6}\left(\frac{1}{4}\right)^3+\dots$$

$$= 1+\frac{3}{1!}\left(\frac{1}{4}\right)+\frac{3.5}{2!}\left(\frac{1}{4}\right)^2+\frac{3.5.7}{3!}\left(\frac{1}{4}\right)^3+\dots$$

formula:

$$(1-x)^{-p/q} = 1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3+\dots$$

First find p & q value find x

$p=3$ , $q=2$

$\frac{x}{q}=\frac{1}{4}$

$\frac{x}{2}=\frac{1}{4}$

$x=2/4$

$x=1/2$

$$(1-1/2)^{-3/2} = \left(\frac{1}{2}\right)^{-3/2} = 2^{3/2} = 2\sqrt{2}.$$

4. sum of the series $\frac{5}{3.6}+\frac{5.7}{3.6.9}+\frac{5.7.9}{3.6.9.12}+\dots$

SOL:

$$S = \frac{5}{3.6}+\frac{5.7}{3.6.9}+\frac{5.7.9}{3.6.9.12}+\dots$$

$$= \frac{5}{2}\left(\frac{1}{3}\right)^2+\frac{5.7}{6}\left(\frac{1}{3}\right)^3+\frac{5.7.9}{24}\left(\frac{1}{3}\right)^4+\dots$$

$$= \frac{\sin 2h\beta}{\sin 2\alpha}$$

$$= \sin 2h\beta \cdot \frac{1}{\sin 2\alpha}$$

$$= \sin 2h\beta \cdot \operatorname{cosec} 2\alpha$$

$\therefore \tan(A+B) = \sin 2h\beta \cdot \operatorname{cosec} 2\alpha$

Hence proved.

(3.) prove that $\tanh^{-1} x = \frac{1}{2} \log\left(\frac{1+x}{1-x}\right)$

(8)

Sol:

Let $y = \tanh^{-1} x$

$\tanh y = x$

$x = \tanh y$

$x = \frac{\sinh y}{\cosh y}$ $\quad \left[\because \sinh y = \frac{e^y - e^{-y}}{2},\ \cosh y = \frac{e^y + e^{-y}}{2}\right]$

$= \frac{e^y - e^{-y}}{e^y + e^{-y}}$

$\Rightarrow \cosh y \, x = \sinh y$ $\quad \left[\because \frac{\sinh y}{\cosh y} = \frac{e^y - e^{-y}}{\cancel{2}} \times \frac{\cancel{2}}{e^y + e^{-y}}\right]$

$\Rightarrow x(e^y + e^{-y}) = (e^y - e^{-y})$

$\Rightarrow x(e^y + e^{-y}) = (e^y - e^{-y})$

$\Rightarrow xe^y + xe^{-y} = e^y - e^{-y}$

$\Rightarrow xe^{-y} + e^{-y} = e^y - xe^y$

$\Rightarrow e^{-y}(x+1) = e^y(1-x)$

$\Rightarrow \frac{1+x}{1-x} = e^y \cdot e^y$

(ii) $2^n \cos^n \theta = 2\cos n\theta + nc_1\, 2\cos(n-2) + nc_2\, 2\cos(n-4)\theta + \ldots$

$2^5 \cos^5\theta = 2\cos 5\theta + 5c_1\, 2\cos 3\theta + 5c_2\, 2\cos\theta$

$= 2\cos 5\theta + 5 \cdot 2\cos 3\theta + \frac{5\times 4}{1\times 2} \cdot 2\cos\theta$

$= 2\cos 5\theta + 10\cos 3\theta + 10 \cdot 2\cos\theta$

$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$

$\div 2 \quad = \cos 5\theta + 5\cos 3\theta + 10\cos\theta$

(iii) $2^{n-1}\cos^n\theta = \cos n\theta + nc_1 \cos(n-2)\theta + nc_2\cos(n-4)\theta + \ldots$

$2^4\cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta$.

2. Expand $\cos^8\theta$ and $\cos^4\theta$ in series of cosines of multiple of '$\theta$'.

(i) $\cos^8\theta$.

Sol:

let $x = \cos\theta + i\sin\theta$

$2\cos\theta = x + \frac{1}{x}$

$(2\cos\theta)^n = (x + \frac{1}{x})^n$

$n = 8$

(i) $x^n + nc_1 x^{n-1}\frac{1}{x} + nc_2 x^{n-2}\frac{1}{x^2} + nc_3 x^{n-3}\frac{1}{x^3} + \ldots$

$\Rightarrow x^8 + 8c_1 x^7 \frac{1}{x} + 8c_2 x^6 \frac{1}{x^2} + 8c_3 x^5 \frac{1}{x^3} + 8c_4 x^4 \frac{1}{x^4} +$

$8c_5 x^3 \frac{1}{x^5} + 8c_6 x^2 \frac{1}{x^6} + 8c_7 x^1 \frac{1}{x^7} + 8c_8 x^0 \frac{1}{x^8}$

$\Rightarrow x^8 + 8c_1 x^6 + 8c_2 x^4 + 8c_3 x^2 + 8c_4 + 8c_5 \frac{1}{x^2} + 8c_6 \frac{1}{x^4} +$

$8c_7 \frac{1}{x^6} + 8c_8 \frac{1}{x^8}$.

$$\Rightarrow x^8 + 8x^6 + \frac{8\times7}{1\times2}x^4 + \frac{8\times7\times6}{1\times2\times3}x^2 + \frac{8\times7\times6\times5}{1\times2\times3\times4} + \frac{8\times7\times6\times5\times4}{1\times2\times3\times4\times5}$$

$$\frac{1}{x^2} + \frac{8\times7\times6\times5\times4\times3}{1\times2\times3\times4\times5\times6}\frac{1}{x^4} + \frac{8\times7\times6\times5\times4\times3\times2}{1\times2\times3\times4\times5\times6\times7}\frac{1}{x^6}$$

$$+ \frac{1}{x^8}$$

$$\Rightarrow x^8 + 8x^6 + 28x^4 + 56x^2 + 70 + 56\left(\frac{1}{x^2}\right) + 28\left(\frac{1}{x^4}\right) + 8\left(\frac{1}{x^6}\right)$$

$$+ \frac{1}{x^8}$$

$$\Rightarrow x^8 + \frac{1}{x^8} + 8\left(x^6 + \frac{1}{x^6}\right) + 28\left(x^4 + \frac{1}{x^4}\right) + 56\left(x^2 + \frac{1}{x^2}\right) + 70.$$

(ii) $2^n \cos^n\theta = 2\cos n\theta + nC_1\, 2\cos(n-2)\theta + nC_2 \cos(n-4)\theta + \dots$

$$2^8 \cos^8\theta = 2\cos 8\theta + 8C_1\, 2\cos 6\theta + 8C_2\, 2\cos 4\theta + 70$$

$$= 2\cos 8\theta + 8\cdot 2\cos 6\theta + 28\cdot 2\cos 4\theta + 70$$

$$= 2\cos 8\theta + 16\cos 6\theta + 56\cos 4\theta + 70$$

$\div 2$

$$= \cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 35$$

(ii) $2^{n-1}\cos^n\theta = \cos n\theta + nC_1 \cos(n-2)\theta + nC_2 \cos(n-4)\theta + \dots$

$$2^7 \cos^8\theta = \cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + \cancel{35}\ 29$$

(iii) $\cos^4\theta$

Let $x = \cos\theta + i\sin\theta$

$$2\cos\theta = x + \frac{1}{x}$$

$$(2\cos\theta)^n = \left(x + \frac{1}{x}\right)^n$$

$n = 4$

(i) $x^n + nc_1 x^{n-1}\frac{1}{x} + nc_2 x^{n-2}\frac{1}{x^2} + nc_3 x^{n-3}\frac{1}{x^3} + \dots$

$\Rightarrow x^4 + 4c_1 x^3 \frac{1}{x} + 4c_2 x^2 \frac{1}{x^2} + 4c_3 x^1 \frac{1}{x^3} + 4c_4 x^0 \frac{1}{x^4}$

$\Rightarrow x^4 + 4x^2 + 6 + 4\frac{1}{x^2} + \frac{1}{x^4}$

$\Rightarrow x^4 + \frac{1}{x^4} + 4\left(x^2 + \frac{1}{x^2}\right) + 6.$

(ii) $2^n \cos^n\theta = 2\cos n\theta + nc_1\, 2\cos(n-2)\theta + nc_2\, 2\cos(n-4)\theta + \dots$

$2^4 \cos^4\theta = 2\cos 4\theta + 4c_1\, 2\cos 2\theta + 4c_2\, 2\cos(0)\theta + 6$

$= 2\cos 4\theta + 4.\, 2\cos 2\theta + 6.\, 2\cos(0)\theta + 6$

$= 2\cos 4\theta + 8\cos 2\theta + 12\cos(0)\theta + 6$

$\div 2 \quad = \cos 4\theta + 4\cos 2\theta + 6\cos(0)\theta + 3$

(iii) $2^{n-1}\cos^n\theta = \cos n\theta + nc_1 \cos(n-2)\theta + nc_2 \cos(n-4)\theta + \dots$

$= \cos 4\theta + 4\cos 2\theta + 6\cos(0)\theta + 3.$

# Unit - V

## Hyperpolic function :

Formula :

1. $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$

2. $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$

3. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \frac{x^n}{n!}$

2 mark

Eulers formula :

1. $e^{i\theta} = \cos\theta + i\sin\theta$

2. $e^{-i\theta} = \cos\theta - i\sin\theta$

3. $\cos\theta = e^{i\theta} + e^{-i\theta}/2$

4. $\sin\theta = e^{i\theta} - e^{-i\theta}/2i$

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$$\left[\frac{e^x+e^{-x}}{2}\right]^2-\left[\frac{e^x-e^{-x}}{2}\right]^2=1$$

$$\frac{(e^x+e^{-x})^2}{4}-\frac{(e^x-e^{-x})^2}{4}=1$$

$$\Rightarrow \frac{1}{4}\ (e^x+e^{-x})^2-(e^x-e^{-x})^2=1$$

$$\Rightarrow \frac{1}{4}\left[(e^x)^2+(e^{-x})^2+2e^x e^{-x}\right]-\left[(e^x)^2+(e^{-x})^2-2e^x e^{-x}\right]$$

$$\Rightarrow \frac{1}{4}\left[e^{2x}+e^{-2x}+2e^x e^{-x}\right]-\left[e^{2x}+e^{-2x}-2e^x e^{-x}\right]$$

$$\Rightarrow \frac{1}{4}\left[e^{2x}+e^{-2x}+2e^x e^{-x}-e^{2x}-e^{-2x}+2e^x e^{-x}\right]$$

$$\Rightarrow \frac{1}{4}\ \cancel{[4]}\ [2e^x e^{-x}+2e^x e^{-x}]=\frac{1}{4}[4e^x e^{-x}]=\frac{1}{4}[4e^{-x+x}]$$

$$\Rightarrow \frac{1}{4}[4(e)^0]=\frac{1}{4}[4(1)]=1$$

$\therefore \cosh x^2-\sinh x^2=1$

Hence proved.

(2) If $\tan A=\tan\alpha\tanh\beta$, $\tan B=\cot\alpha\tanh\beta$ prove that $\tan(A+B)=\sinh 2\beta\ \operatorname{cosec} 2\alpha$ ※※※※ ※※

Sol :

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\cdot\tan B}$$

$$=\frac{\tan\alpha\tanh\beta+\cot\alpha\tanh\beta}{1-\tan\alpha\tanh\beta\cdot\cot\alpha\tanh\beta}$$

$$=\frac{\tanh\beta(\tan\alpha+\cot\alpha)}{1-\tanh^2\beta(\tan\alpha\cdot\cot\alpha)}$$

1. $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$

Given $n = 6$

$(\cos 6\theta + i\sin 6\theta) = (\cos\theta + i\sin\theta)^6$

$\cos 6\theta = \cos^n\theta + nc_1 \cos^{n-1}\theta\,(i\sin\theta) + nc_2 \cos^{n-2}\theta\,(i\sin^2\theta) +$
$nc_3 \cos^{n-3}\theta\,(i\sin^3\theta) + nc_4 \cos^{n-4}\theta\,(i\sin^4\theta) +$
$nc_5 \cos^{n-5}\theta\,(i\sin^5\theta) + nc_6 \cos^{n-6}(i\sin^6\theta)$

$= \cos^6\theta + 6c_1 \cos^5\theta\,(i\sin\theta) + 6c_2\cos^4\theta\,(i\sin^2\theta) +$
$6c_3 \cos^3\theta\,(i\sin^3\theta) + 6c_4 \cos^{\cancel{n-4}}\theta\,(i\sin^4\theta) +$
$6c_5 \cos^1\theta\,(i\sin^5\theta) + 6c_6 \cos^0\theta\,(i\sin^6\theta)$

$= \cos^6\theta + i6c_1\cos^5\theta\sin\theta - 6c_2\cos^4\theta\sin^2\theta - i6c_3$
$\cos^3\theta\sin^3\theta + 6c_4\cos^2\theta\sin^4\theta + i6c_5\cos^1\theta$
$\sin^5\theta - 6c_6\cos^0\theta\,(\sin^6\theta)$

$\cos 6\theta \rightarrow$ Real part

$\cos 6\theta = \cos^6\theta - 6c_2\cos^4\theta\sin^2\theta + 6c_4\cos^2\theta\sin^4\theta$
$- 6c_6\cos^0\theta\,(\sin^6\theta)$

$= \cos^6\theta - \frac{\overset{3}{\cancel{6}}\times 5}{1\times \cancel{2}_1}\cos^4\theta\sin^2\theta + \frac{\overset{3}{\cancel{6}}\times 5\times\cancel{4}\times\cancel{3}}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}}$
$\cos^2\theta\sin^4\theta - 1(1)\sin^6\theta$

$= \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6$

$= \cos^6\theta - 15\cos^4\theta(1-\cos^2\theta) + 15\cos^2\theta(1-\cos^2\theta)^2$
$- \sin^6\theta$

$$= \cos 6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta\,(1-\cos^2\theta)^2 - \sin^6\theta$$

$$= \cos 6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta\,(1 - 2\cos^2\theta + \cos^4\theta) - \sin^6\theta$$

$$= \cos 6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta - 30\cos^4\theta + 15\cos^6\theta - \sin^6\theta$$

$$= \cos 6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta - 30\cos^4\theta + 15\cos^6\theta - (1-\cos^2\theta)^3$$

$a^3 - 3ab^2 + 3a^2b - b^3$

$$= \cos 6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta - 30\cos^4\theta + 15\cos^6\theta - 1 \mp 3(1)(\cos^2\theta)^2 + 3(1)(\cos^2\theta)\,(-(\cos^2\theta)^3)$$

$$= \cos 6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta - 30\cos^4\theta + 15\cos^6\theta - 1 \mp 3\cos^4\theta + 3\cos^2\theta + \cos^6\theta$$

$$= 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

$$\therefore \cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$

Hence proved.

$$\frac{x}{q} = \frac{1}{6}$$

$$\frac{x}{3} = \frac{1}{6}$$

$$x = \frac{3}{6}$$

$$\boxed{x = 1/2}$$

$$(1-1/2)^{-2/3} = \left(\frac{1}{2}\right)^{-2/3} = 2^{2/3} = (4)^{1/3}$$

$$(1-1/2)^{-2/3} = (4)^{1/3}$$

6. Sum of series $1 + \frac{1}{3} + \frac{1 \cdot 3}{3 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{3 \cdot 6 \cdot 9} + \ldots$

Sol:

$$S = 1 + \frac{1}{3} + \frac{1 \cdot 3}{2}\left(\frac{1}{3}\right)^2 + \frac{1 \cdot 3 \cdot 5}{3!}\left(\frac{1}{3}\right)^3 + \ldots$$

Formula:

$$(1-x)^{-p/q} = 1 + \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 +$$

$$p = 1, \quad q = 2$$

$$\frac{x}{q} = \frac{1}{3}$$

$$\frac{x}{2} = \frac{1}{3}$$

$$x = \frac{2}{3}$$

$$(1-x)^{-1/2} = \left(-\frac{1}{3}\right)^{-1/2} = (3)^{1/2} = \sqrt{3}$$

$$C = \frac{75}{21}$$

$$\boxed{C = \frac{25}{7}}$$

sub in $A = \frac{20}{77}$, $C = \frac{25}{7}$ ② Equation

② ⇒ $7A - 5B - 7C = 0$

$$\cancel{7}\left(\frac{20}{\cancel{77}}\right) - 5B - \cancel{7}\left(\frac{25}{\cancel{7}}\right) = 0 \Rightarrow \frac{20}{11} - 5B - 25 = 0$$

$$\frac{20 - 55B - 275}{11} = 0$$

$$20 - 55B - 275 = 0 \Rightarrow 20 - 55B - 275 = 0$$

$$20 - 275 = 55B$$

$$B = -\frac{225}{55}$$

$$\boxed{B = -\frac{51}{11}}$$

sub in partial fraction

$$\frac{3-7x^2}{(1-3x)(2x+3)(x+2)} = \frac{20}{77}(1-3x)^{-1} - \frac{51}{33}\left(1+\frac{2x}{3}\right)^{-1} + \frac{25}{14}\left(1+\frac{x}{2}\right)^{-1}$$

$$= \frac{20}{77}\left[1+3x+3^2x^2+\dots+3^n x^n\right] - \frac{17}{11}\ 1 - \frac{2x}{3} + \frac{2^2x^2}{3^2}$$

$$+\dots+(-1)^n \frac{2^n x^n}{3^n}+\dots\Big] + \frac{25}{14}\left[1-\frac{x}{2}+\frac{x^2}{2}+\dots+\frac{(-1)^n x^n}{2^n}+\dots\right]$$

⇒ ∴ $x^n$ coefficients

$$= \frac{20}{77}3^n - \frac{17}{11}(-1)^n\frac{2^n}{3^n} + \frac{25}{14}\frac{(-1)^n}{2^n}.$$

4. If $\cosh u = \sec\theta$ show that $u = \log\tan(\pi/4 + \frac{\theta}{2})$

(11) Sol:

Given $\cosh u = \sec\theta$

$$u = \frac{\sec\theta}{\cosh}$$

$$u = \cosh^{-1}(\sec\theta) \quad [\because \tfrac{1}{8} = \log]$$

$$= \log(\sec\theta + \sqrt{\sec^2\theta - 1})$$

$$= \log(\sec\theta + \tan\theta)$$

$$= \log\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \log\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$$= \log \frac{1-\cos(\pi/2+\theta)}{\sin(\pi/2+\theta)}$$

$$= \log \frac{2\sin^2\theta\,(\pi/4+\theta/2)}{2\sin(\pi/4+\theta/2)\cos(\pi/4+\theta/2)}$$

$$= \log\tan(\pi/4+\theta/2)$$

Hence proved.

5. $2\sinh x \cosh x = \sin 2hx$ (or) $\sinh 2x$

(12) proof :

$$2\sinh x\cosh x = 2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x+e^{-x}}{2}\right)$$

$$= 2\tfrac{1}{2}(e^x - e^{-x})\left(\frac{e^x+e^{-x}}{2}\right)$$

$$= \tfrac{1}{2}(e^x - e^{-x})(e^x + e^{-x})$$

$$= \tfrac{1}{2}[e^{2x} - e^{-2x}]$$

$\Rightarrow \cos^8\theta - 28\cos^6\theta\sin^2\theta + 70\cos^4\theta\sin^4\theta - 28\cos^2\theta\sin^6\theta + \sin^8\theta$

$\Rightarrow (\cos^2\theta)^4 - 28(\cos^2\theta)^3\sin^2\theta + 70(\cos^2\theta)^2\sin^4\theta - 28\cos^2\theta\sin^6\theta + \sin^8\theta$

$\Rightarrow (1-\sin^2\theta)^4 - 28\underline{(1-\sin^2\theta)^3}\sin^2\theta + 70(1-\sin^2\theta)^2\sin^4\theta - 28(1-\sin^2\theta)\sin^6\theta + \sin^8\theta$

$\Rightarrow$ formula:

$$(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)(n-2)(n-3)x^4}{4!} +$$

$(a-b)^3 = a^3 - 3ab^2 + 3a^2b - b^3 \Rightarrow 1 - 3(1)(\sin^2\theta)^2 + 3(1)(\sin^2\theta) - 1(\sin^2\theta)^3$

$\Rightarrow 1 - 3\sin^4\theta + 3\sin^2\theta - \sin^6\theta$

Powers of sines and cosines of '$\theta$'.

Interms of function of multiple of '$\theta$'

formulas | Result's:

$\cos\theta + i\sin\theta = x$

$\cos\theta - i\sin\theta = 1/x$

$x + \frac{1}{x} = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta$

$\boxed{x + \frac{1}{x} = 2\cos\theta}$

$x - \frac{1}{x} = \cancel{\cos\theta} + i\sin\theta - \cancel{\cos\theta} + i\sin\theta$

$\boxed{x - \frac{1}{x} = 2i\sin\theta}$

Formula:

Expansion of $\cos n\theta$ when $n$ is a power integral

$2\cos\theta = x + \frac{1}{x}$

$$-4 = A(-3)^2 + B(0)(-3) + C(0)$$

$$-4 = 9A$$

$$\boxed{A = -4/9}$$

$x^2$ coefficient on Both side

Put $A = -\frac{4}{9}$ $B = \frac{4}{9}$

$$\frac{x-2}{(x+2)(x-1)^2} = \frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$= \frac{-4}{9(x+2)} + \frac{4}{9(x-1)} - \frac{1}{3(x-1)^2}$$

$$= \frac{-4}{9 \times 2\left(\frac{1+x}{2}\right)} + \frac{4}{9}(1-x)^{-1} - \frac{1}{3}(1-x)^{-2}$$

$$= \frac{-4}{18}\left(\frac{1+x}{2}\right)^{-1} - \frac{4}{9}(1-x)^{-1} - \frac{1}{3}(1-x)^{-2}$$

formulas :

$$\Big[ \because (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$(1-x)^{n} = 1 + nx + \frac{n(n+1)x^2}{2!} + \frac{n(n+1)(n+2)x^3}{3!} + \dots$$

$$(1+x)^{-n} = 1 - nx + \frac{n(n+1)x^2}{2!} - \frac{n(n+1)(n+2)x^3}{3!} + \dots \Big]$$

$$= -\frac{2}{9}\left[1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots + \frac{(-1)^n x^n}{2^n}\right] - \frac{4}{9}\left[1 + x + x^2 + x^3 \dots + x\right]$$

$$-\frac{1}{3}\left[1 + 2x + 3x^2 + 4x^3 + \dots + (n+1)x^n + \dots\right]$$

$x^n$ co-effcient

$$= -\frac{2}{9}(-1)^n \frac{1}{2^n} - \frac{4}{9} - \frac{1}{3}(n+1)$$

$$= \frac{e^{2x} - e^{-2x}}{2} \qquad \left[\because \sinh x = \frac{e^{x} - e^{-x}}{2}\right]$$

$$= \sinh 2x$$

$$2 \sinh x \cosh x = \sinh 2x$$

Hence proved

(6.) $\cosh^2 x + \sinh^2 x = \cosh 2x$.

Sol :

$$\cosh^2 x + \sinh^2 x = \left(\frac{e^{x} + e^{-x}}{2}\right)^2 + \left(\frac{e^{x} - e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}(e^{x} + e^{-x})^2 + \frac{1}{4}(e^{x} - e^{-x})^2$$

$$\left[\because \cosh x = \frac{e^{x} + e^{-x}}{2}\right]$$

$$= \frac{1}{4}[e^{2x} + e^{-2x} + 2e^{x}e^{-x} + e^{2x} e^{-2x} - 2e^{x}e^{-x}]$$

$$= \frac{1}{4}[2e^{2x} + 2e^{-2x}]$$

$$= \frac{2}{4}[e^{2x} + e^{-2x}]$$

$$= \frac{1}{2}[e^{2x} + e^{-2x}]$$

$$= \frac{e^{2x} + e^{-2x}}{2}$$

$$\therefore \cosh^2 x + \sinh^2 x = \cosh 2x$$

Hence proved.

$$= -\frac{2}{9}\frac{(-1)^n}{2^n} - \frac{4}{9} - \frac{3}{9}(n+1)$$

$$= \frac{1}{9}\left[\frac{(-1)^{n+1}}{2^{n-1}} - 4 - 3n - 3\right]$$

$$= \frac{1}{9}\left[\frac{(-1)^{n+1}}{2^{n-1}} - 7 - 3n\right]$$

2. To find the coefficients of $x^n$ in the expansion of

$$\frac{3-7x^2}{(1-3x)(2x+3)(x+2)}$$

sol :

$$\frac{3-7x^2}{(1-3x)(2x+3)(x+2)} = \frac{A}{(1-3x)} + \frac{B}{(2x+3)} + \frac{C}{x+2}$$

$$\frac{3-7x^2}{(1-3x)(2x+3)(x+2)} = \frac{A(2x+3)(x+2)+B(1-3x)(x+2)+C(1-3x)(2x+3)}{(1-3x)(2x+3)(x+2)}$$

$$\Rightarrow 3-7x^2 = A(2x+3)(x+2) + B(1-3x)(x+2) + C(1-x3)(2x+3)$$

$$= A(2x^2+4x+3x+6) + B(x+2-3x^2-6x) + C(2x+3-6x^2-9x)$$

Equatate $x^2$ coefficient on both sides

$$-7 = 2A - 3B - 6C$$

$$2A - 3B - 6C = -7 \longrightarrow (1)$$

Equate $x$ co-efficients

$$0 = 7A - 5B - 7C$$

$$7A - 5B - 7C = 0 \longrightarrow (2)$$

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$ax^2+by^2+c=0$ method

$a=1,\ b=-2x \quad c=1$

$e^y \Rightarrow \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$e^y \Rightarrow \dfrac{-(-2x) \pm \sqrt{(-2x)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow \dfrac{2x \pm \sqrt{4x^2-4}}{2}$

$\Rightarrow \dfrac{2x \pm 2\sqrt{x^2-1}}{2}$

$e^y \Rightarrow x \pm \sqrt{x^2-1}$

$e^y$ is always positive

$\therefore e^y = x + \sqrt{x^2-1}$

Taking log on both sides

$\log e^y = \log x \pm \sqrt{x^2+1}$

$\therefore \log e^y = \log x + \sqrt{x^2-1}$

(3) Solve $y = \sin h^{-1} x$

Sol:

$\sin hy = x$ $\quad [\because \sin hy = \dfrac{e^y - e^{-y}}{2}$

$x = \sin hy$ $\quad e^{-y} = \dfrac{1}{e^y}]$

$x = \dfrac{e^y - e^{-y}}{2}$ $\quad\Big|\quad 2x = e^y - 1/e^y$

$2x = \dfrac{(e^y)^2 - 1}{e^y}$

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$$\Rightarrow \frac{(1+x)}{(1-x)} = e^{2y}$$

Taking 'log' on both sides

$$\log e^{2y} = \log \frac{(1+x)}{(1-x)}$$

$$2y = \log \frac{(1+x)}{(1-x)}$$

$$y = \frac{1}{2} \log \frac{(1+x)}{(1-x)}$$

Hence proved

2. prove that $\cosh^{-1} x = \log (x + \sqrt{x^2-1})$

A sol:

Let $y = \cosh^{-1} x$

$\cosh^{-1} x = y$

$x = \cosh y$ $\left[\because \cosh y = \frac{e^{y} + e^{-y}}{2}\right.$

$\left. e^{1/y} = e^{-y} \right]$

$$x = \frac{e^{y} + e^{-y}}{2}$$

$$2x = e^{y} + e^{-y}$$

$$2x = e^{y} + \frac{1}{e^{y}}$$

$$2x = \frac{(e^{y})^2 + 1}{e^{y}}$$

$$e^{y} \cdot 2x = (e^{y})^2 + 1$$

$$(e^{y})^2 - 2x e^{y} + 1 = 0$$

$$e^{2y} - 2x e^{y} + 1 = 0$$

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$$S = \frac{5}{2!}\left(\frac{1}{3}\right)^2 + \frac{5.7}{3!}\left(\frac{1}{3}\right)^3 + \frac{5.7.9}{4!}\left(\frac{1}{3}\right)^4 + \dots$$

$$(1+3S) = 1 + \left(\frac{1}{3}\right) + \frac{3.5}{2!}\left(\frac{1}{3}\right)^2 + \frac{3.5.7}{3!}\left(\frac{1}{3}\right)^3 + \frac{3.5.7.9}{4!}\left(\frac{1}{3}\right)^4 + \dots$$

formula :

$$(1-x)^{-p/q} = 1 + \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 + \dots$$

where $p = 3, \quad q = 2$

$p + q = 5$ $\qquad \frac{x}{q} = \frac{1}{3}$

$3 + q = 5$ $\qquad \frac{x}{2} \Rightarrow x = 2/3$

$q = 5 - 3$

$\boxed{q = 2}$

$$(1-2/3)^{-3/2} = \left(\frac{3-2}{3}\right)^{-3/2} = \left(\frac{1}{3}\right)^{-3/2} = 3^{3/2}$$

$$= (3^3)^{1/2} = \sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

$$= \sqrt{3} - 2/3$$

5. Find the sum of infinity of the following series

$$1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$$

SOL:

$$S = 1 + \left(\frac{2}{6}\right) + \frac{2.5}{2}\left(\frac{2}{6}\right)^2 + \frac{2.5.8}{6}\left(\frac{2}{6}\right)^3 + \dots$$

$$S = 1 + \frac{2}{1!}\left(\frac{2}{6}\right) + \frac{2.5}{2!}\left(\frac{2}{6}\right)^2 + \frac{2.5.8}{3!}\left(\frac{2}{6}\right)^3 + \dots$$

formula:

$$(1-x)^{-p/q} = 1 + \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 + \dots$$

$p = 2 \qquad q = 3$

$$= \frac{\tanh\beta\,(\tan\alpha + \cot\alpha)}{1 - \tanh^2\beta\left(\tan\alpha \cdot \frac{1}{\tan\alpha}\right)} \qquad \left[\because \cot\alpha = \frac{1}{\tan\alpha}\right.$$

$$\tan = \frac{\sin}{\cos}$$

$$= \frac{\tanh\beta\,(\tan\alpha + \cot\alpha)}{1 - \tanh^2\beta} \qquad \cot = \frac{\cos}{\sin}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cosh^2\theta - \sinh^2\theta = 1$$

$$= \frac{\frac{\sinh\beta}{\cosh\beta}\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)}{1 - \frac{\sinh^2\beta}{\cosh^2\beta}} \qquad \therefore \sin 2A = 2\sin A\cos A$$

$$\left.\frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$$

$$= \frac{\frac{\sinh\beta}{\cosh\beta}\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha \cdot \cos\alpha}\right)}{\frac{\cosh^2\beta - \sinh^2\beta}{\cosh^2\beta}}$$

$$= \frac{\frac{\sinh\beta}{\cosh\beta}\left(\frac{1}{\sin\alpha \cdot \cos\alpha}\right)}{\frac{1}{\cosh^2\beta}}$$

$$= \frac{\sinh\beta}{\cosh\beta} \cdot \frac{\cosh^2\beta}{1} \cdot \frac{1}{\sin\alpha \cdot \cos\alpha}$$

$$= \frac{\sinh\beta \cdot \cosh\beta}{\sin\alpha \cdot \cos\alpha}$$

$$= \frac{\frac{1}{2}\sinh 2\beta}{\frac{1}{2}\sin 2\alpha}$$

$$= \frac{\frac{1}{\cancel{2}}\sinh 2\beta}{\sin 2\alpha} \cdot \frac{\cancel{2}}{1}$$