unit - I.

ODE / PDE / LAPLACE TRANSFORMS

Ordinary differential Equation of first order but of higher degree. Equations solvable for ± solvable for dy/dx. clairauts form (simple cases only). Linear equations with constant coefficients. Finding Particular integrals in the cases of e^{kx}, $\sin(kx)$, $\cos(kx)$ (where k is a constant), x^k where k is a positive integer, and $e^{kx} f(x)$ where $f(x)$ is any function of x (only problems in all the above - No proof needed for any formula?

unit - II.

Formation of Partial differential equations by eliminating constant and by elimination of arbitary functions - definition of generals, Particular & complete solutions - Singular integral (Geomatrical meaning not required).. Solutions of first order equations in the standard forms - $f(p, q) = 0$, $f(x, p, q) = 0$, $f(x, p, q) = 0$.

AND VECTOR

$f(z, p, q) = 0$, $f_1(x, p) = f_2(y, q)$, $z = xp + f(p, q)$ Lagranges Method of solving P_p where P, Q, R are functions of x, y, z – (Geometrical meaning is not of needed) (Only Problems in all the alone no proof needed for any formula).

Unit - III

Laplace transform - Definition - L(L (cos at), L (sin (at)). $L(t^n)$, where a positive integer. Basic theorems Laplace transform (formula only) - $L(e^{s}$ $L(e^{st} \sin bt)$, $L[e^{-st} f(t)]$, $-L[f(t)]$, $L[f''(t)]$, $L[f'(t)]$ -

Unit - IV

Inverse Laplace Transforms related to the alone standard forms solving second order ODE with Constant coefficients using Laplace Transforms.

TEXT BOOK

1. S. Narayanan, Differential equations, S. Viswanathan publishes - 1996

2. S. Narayanan ; T.K. Manikavasagam Pillai - Calculus vol-II S. Viswanathan Publishes Pvt. Ltd - 2003.

3. M.L. Khanna, Differential calculus Jaiprakash and CO - meerut - 2004.

Unit - I

Differential equation of first order and Highest Degree

The general form of the first order differential but of n^{th} equation degree is given by

$$p^n + P_1 p^{n-1} + P_2 p^{n-2} + \dots + P_{n-1} p + P_n = 0$$

where $P_1, P_2, P_3, \dots P_n$ are function of x. y are constant. where $P = \frac{dy}{dx}$

1. Solve differential equation $\left(\frac{dy}{dx}\right)^2 - 5\left(\frac{dy}{dx}\right)$ [illegible]

$\frac{dy}{dx} = P$

$P^2 - 5P + 6 = 0$

$(P-3)(P-2) = 0$

$P = 2, 3$

$\boxed{P = 2}$

$\frac{dy}{dx} = 2$

$dy = 2dx$

$\int$ing on both sides

$\int dy = 2\int dx$

$y = 2x + c$

$y - 2x - c = 0$

$\boxed{P = 3}$

$\frac{dy}{dx} = 3$

$dy = 3dx$

$\int$ing on both sides

$\int dy = 3\int dx$

$y = 3x + c$

$y - 3x - c = 0$

The required equation

$y - 2x - c = 0 \qquad y - 3x - c = 0$

$(y - 2x - c)(y - 3x - c) = 0$

2. [illegible]

[illegible]

[illegible]

$P = 4, \quad P = 3$

$\frac{dy}{dx} = 4 \qquad \frac{dy}{dx} = 3$

$dy = 4dx \qquad dy = 3dx$

$\int$ing on both sides

$\int dy = 4\int dx \qquad \int dy = 3\int dx$

$y = 4x + c \qquad y = 3x + c$

$y - 4x - c = 0 \qquad y - 3x - c = 0$

$(y - 4x - c)(y - 3x - c) = 0$

3. $P^2 - 2P - 3 = 0$

$(P-3)(P+1) = 0$

$P = 3, \quad P = -1$

$\frac{dy}{dx} = 3 \qquad \frac{dy}{dx} = -1$

$dy = 3x \qquad dy = -dx$

$y = 3x + c \qquad y = -1(x) + c$

$y - 3x - c = 0 \qquad y + x - c = 0$

$(y - 3x - c)(y + x - c) = 0$

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6. $p^2+6p+8=0$

8
$4 \quad 2$
6

$p^2+6p+8=0$

$(p+4)(p+2)=0$

$p=-4 \qquad p=-2$

$\frac{dy}{dx}=-4 \qquad \frac{dy}{dx}=-2$

$dy=-4dx \quad dy=-2dx$

$\int$ ing on both sides

$\int dy=-4\int dx \qquad \int dy=-2\int dx$

$y=-4x+c \qquad y=-2x+c$

$y+4x-c=0 \qquad y+2x-c=0$

$(y+4x-c)(y+2x-c)=0$

7. Solve the PE is $4p^2-8p+3=0$

$4p^2-8p+3=0$

12
$-6 \quad -2$
$\frac{-6}{4}=\frac{3}{2} \quad \frac{2}{4}=\frac{1}{2}$
-8

$(p-3/2)(p-1/2)=0$

$p=3/2 \qquad p=+1/2$

$\frac{dy}{dx}=3/2 \qquad \frac{dy}{dx}=\frac{1}{2}$

$2dy=3dx \qquad 2dy=dx$

$\int$ ing on both sides

$2\int dy=3\int dx \qquad 2\int dy=dx$

$2y=3x+c \qquad 2y=x+c$

$2y-3x-c=0 \qquad 2y-x-c=0$

$(2y-3x-c)(2y-x-c)=0$

2. Solve $x = 1 - \frac{p}{\sqrt{p^2+1}}$

Given $x = 1 - \frac{p}{\sqrt{p^2+1}} \rightarrow ①$ $\quad \frac{u}{v} = \frac{vu}{v}$

Diff w.r.t. 'y'

$(\sqrt{x} = \frac{1}{2}$

$$\frac{dx}{dy} = 0 - \frac{\left[\sqrt{p^2+1}\,\frac{dp}{dy} - p\,\frac{1}{2\sqrt{p^2+1}}\,2p\,.\frac{dp}{dy}\right]}{p^2+1}$$

$$= \frac{-\frac{dp}{dy}\left[\sqrt{p^2+1} - \frac{p^2}{\sqrt{p^2+1}}\right]}{p^2+1}$$

$$\frac{dx}{dy} = \frac{2\,\frac{dp}{dy}\left[\frac{p^2+1-p^2}{\sqrt{p^2+1}}\right]}{p^2+1}$$

$$= -\frac{dp}{dy}\left[\frac{1}{\sqrt{p^2+1}} \times \frac{1}{p^2+1}\right]$$

$\frac{1}{2}+1 = \frac{3}{2}$

$$= -\frac{dp}{dy}\left[\frac{1}{(p^2+1)^{3/2}}\right]$$

$$\frac{1}{p} = -\frac{dp}{dy}\left[\frac{1}{(p^2+1)^{3/2}}\right]$$

from ①,

$$x = 1 - \frac{p}{\sqrt{p^2+1}}$$

$$(x-1) = \frac{-p}{\sqrt{p^2+1}}$$

squaring on both sides

$$(x-1)^2 = \frac{p^2}{p^2+1}$$

$$(x-1)^2 = \frac{(p^2+1)-1}{p^2+1}$$

$$\therefore (x-1)^2 = 1 - \frac{1}{p^2+1}$$

from ④,

$$(x-1)^2 = 1 - (y-c)^2$$

$$\therefore (x-1)^2 + (y-c)^2 = 1$$

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Type I.

(a) The roots can be real and equal $m_1 = m_2 = m$

$$\therefore y = e^{mx}(Ax+B)$$

If any three roots are equal $m_1 = m_2 = m_3 = m$

$$\therefore y = e^{mx}(Ax^2+Bx+C)$$

(b) The roots can be real and Imaginary.

$$m = \alpha \pm i\beta$$

$$\therefore y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$$

(c) The roots can be real and distinct

$$\therefore m_1 \neq m_2$$

$$\therefore y_c = Ae^{m_1 x} + Be^{m_2 x}$$

Particular Integral

$y_p\, x = e^{ax}$, then the particular integral is $P.I = \frac{1}{\phi(a)} e^{ax}$

~~Solve $(D^2-4D+13)=0$~~

$$= \frac{4 \pm \sqrt{-36}}{2}$$

~~$(\sqrt{-1}=i)$~~

$$= \frac{4 \pm i\sqrt{36}}{2} \qquad (\sqrt{-1}=i)$$

$$= \frac{4+6i}{2}$$

$m = 2 \pm 3i \qquad (\because m = \alpha \pm i\beta)$

$\alpha = 2 \;;\; \beta = 3$

$\therefore Y_c = Ae^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$Y_c = Ae^{2x}(A\cos 3x + B\sin 3x)$

Particular Integral

$$P.I = \frac{1}{D^2-4D+13}e^{2x}$$

$$= \frac{1}{(2)^2-4(2)+13}e^{2x}$$

$$= \frac{1}{4-8+13}e^{2x} = \frac{1}{17-4}e^{2x}$$

$$= \frac{1}{9}e^{2x}$$

$Y = Y_c + Y_p$ or $Y = CF + PI$

$Y = Ae^{2x}(A\cos 3x + B\sin 3x) + \frac{e^{2x}}{9}$

2. Solve $(D^2+6D+5)y = e^{2x}$

$m^2+6m+5 = 0$

$(m+1)(m+5) = 0$

$m+1=0$	$m+5=0$
$m=-1$	$m=-5$

$m = -1, -5$

$\therefore Y_c = Ae^{m_1x} + Be^{m_2x}$

$Y_c = Ae^{-x} + Be^{-5x}$

Particular Integral

$$PI = \frac{1}{D^2+6D+5}e^{2x}$$

$$= \frac{1}{4+12+5}e^{2x}$$

$$Y_p = \frac{1}{21}e^{2x}$$

$Y = Y_c + Y_p$

$\therefore Y = Ae^{-x} + Be^{-5x} + \frac{1}{21}e^{2x}$

5. Solve $(D^3-3D^2+4D-2)y=e^x$

Given, $(D^3-3D^2+4D-2)y=e^x$

∴ the auxillary equation is

$$m^3-3m^2+4m-2=0$$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 4 & -2 \\ & 0 & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

$a=1,\ b=-2,\ c=2$

$m+1=0$

$m=-1$

$m^2-2m+2=0$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{+2\pm\sqrt{4-4(1)(2)}}{2}$$

$$=\frac{2\pm\sqrt{4-8}}{2}$$

$$=\frac{2\pm\sqrt{4}}{2} = \frac{2\pm 2i}{2}$$

$\boxed{m=1\pm i}$ $m=\alpha\pm i\beta$

$\alpha=1,\ \beta=1$

$y_c = Ae^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y_c = Ae^{x}(A\cos x + B\sin x)$

$$y_p = \frac{1}{D^3-3D^2+4D-2}e^x$$

$$=\frac{1}{1-3+4-2}e^x$$

$$=\frac{1}{5-5}e^x$$

$$=\frac{1}{0}e^x.$$

$$=\frac{x}{3D^2-6D+4}e^x$$

$= xe^x$

$= xe^x$

$y = y_c + y_p$

$= Ae^x(A\cos x + B\sin x) + xe^x$

4. solve $(D^2-D)y = 12e^x$

Given $(D^2-D)y = 12e^x$

$m(m-1) = 0$

$m_1 = 0$ ~~$m_2=0$~~

$\boxed{m=0}$, $\boxed{m=1}$

$y_c = Ae^{m_1x} + Be^{m_2x}$

$y_c = Ae^{0x} + Be^{x}$

$y_c = A + Be^{x}$

$y_p = \frac{1}{D^2-D} 12e^x$

$= \frac{1}{1-1} 12e^x$

$= \frac{1}{0} 12e^x$

$= \frac{x12e^x}{2D-1}$

$= \frac{12xe^x}{2-1}$

$\frac{1}{P} = 12xe^x$

$Y = y_c + y_p$

$Y = A + Be^x + 12xe^x$

4. $(3D^2-D-14)y = 13e^{2x}$ ↓ $Y = Ae^{-2x} + Be^{7x/3} - \frac{13e^{2x}}{4}$

1. $(D^2-2D+1)y = e^{4x} + 5e^{0x} \to Y = e^x(Ax+B) + \frac{1}{9}e^{4x} + 5$

2. $\frac{d^2y}{dx^2} - 16\frac{dy}{dx} + 3y = e^{5x} \to Y = Ae^{..} + Be^{..} + \frac{x}{14}e^{5x}$

3. $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = e^{4x} \to Y = e^{4x}(Ax+B) + x^2\frac{e^{4x}}{2}$

5. Solve $(D^2-2D+1)y = e^{4x}+5$

Given $(D^2-2D+1)y = e^{4x}+5$

the auxiliary eqn is

$m^2-2m+1 = 0$

$(m-1)(m-1) = 0$

$m = 1, 1$

$m = 1$

$y_c = e^{mx}(Ax+B)$

$y_c = Ae^{x}(Ax+B)$

∴ particular integral

$y_p = \frac{1}{D^2-2D+1} \cdot e^{4x} + \frac{1}{D^2-2D+1} \cdot 5e^{0x}$

$= \frac{1}{16-8+1} e^{4x} + \frac{1}{1} 5e^{0x}$

$y_p = \frac{e^{4x}}{9} + 5$

$y_p = \frac{e^{4x}+45}{9}$

$\boxed{y = y_c + y_p}$

$y = Ae^{x}(Ax+B) + \frac{e^{4x}+45}{9}$

6. Solve $5\frac{d^2y}{dx^2} - 16\frac{dy}{dx} + 3y = e^{3x}$

Given $(5D^2 - 16D+3)y = e^{3x}$

The auxiliary equation is

$5m^2 - 16m + 3 = 0$

$(m-3)(5m-1) = 0$

$m = 3, \frac{1}{5}$

$y_c = Ae^{3x} + Be^{x/5}$

$y_c = Ae^{3x} + Be^{x/5}$

The particular integral

$y_p = \frac{1}{5D^2 - 16D + 3} \cdot e^{3x}$

$= \frac{x}{10D - 16} \cdot e^{3x}$

$= \frac{x}{10(3) - 16} \cdot e^{3x}$

$y_p = \frac{xe^{3x}}{14}$

$y = y_c + y_p$

$y = Ae^{3x} + Be^{x/5} + \frac{xe^{3x}}{14}$

7. Solve $\frac{d^2y}{dx^2} - \frac{8dy}{dx} + 16y = e^{4x}$

Given $(D^2 - 8D + 16)y = e^{4x}$

The auxiliary equation is

$m^2 - 8m + 16 = 0$

$(m-4)(m-4) = 0$

$m = 4, 4$

$m = 4$

$y_c = Ae^{4x}(Ax+B)$

The particular integral

$y_p = \frac{1}{D^2 - 2D + 16} e^{4x}$

$= \frac{x}{2D - 8} e^{4x}$

$y_p = \frac{x^2}{2} \cdot e^{4x}$

$y = y_c + y_p$

$y = Ae^{4x}(Ax+B) + x^2 \cdot \frac{e^{4x}}{2}$

8. Solve $(3D^2 - D - 14)y = 13e^{2x}$

Given $(3D^2 - D - 14)\, y = 13e^{2x}$

the auxiliary equation is

$$3m^2 - m - 14 = 0$$

$$(m+2)(3m-7) = 0$$

$$m = -2, \frac{7}{3}$$

$$y_c = Ae^{-2x} + Be^{7/3 x}$$

the particular integral is

$$y_P = \frac{1}{3D^2 - D - 14} 13e^{2x}$$

$$= \frac{1}{3(4) - (2) - 14} 13e^{2x}$$

$$= \frac{1}{12 - 16} 13e^{2x}$$

$$y_P = -\frac{1}{4} 13e^{2x}$$

$$y = y_c + y_p$$

$$y = Ae^{-2x} + Be^{7/3 x} - \frac{13e^{2x}}{4}$$

Type - II

1. Solve $(D^2 + 4D + 4)\, y = e^{-3x} + \cos x$

Given $(D^2 + 4D + 4)\, y = e^{-3x} + \cos x$

the auxiliary equn is

$$m^2 + 4m + 4 = 0$$

$$(m+2)(m+2) = 0$$

$m+2 = 0$ | $m+2 = 0$

$m = -2$ | $m = -2$

$$m_1 = m_2$$

the complementary function

$$y_c = Ae^{mx}(Ax+B)$$

$$y_c = Ae^{-2x}(Ax+B)$$

particular Integral

$$y = y_c + y_{P_1} + y_{P_2}$$

$$y_{P_1} = \frac{1}{D^2 + 4D + 4}$$

$$= \frac{1}{1} e^{-3x} \quad \left(\begin{array}{l} D^2 = a^2 \\ D^2 = -1^2 \end{array} \right)$$

$$y_{P_1} = e^{-3x} \qquad a = 1$$

$$y_{P_2} = \frac{1}{D^2 + 4D + 4} \cos x$$

2. Solve $(D^2+3D+2)y = e^{-2x} + \sin x$

Given $(D^2+3D+2)y = e^{-2x} + \sin x$

The auxiliary equation is

$m^2+3m+2 = 0$

$(m+1)(m+2) = 0$

$m+1=0 \quad | \quad m+2 = 0$

$m = -1 \quad | \quad m = -2$

$m = -1, -2$

The complementary function

$y_c = Ae^{m_1 x} + Be^{m_2 x}$

$y_c = Ae^{-x} + Be^{-2x}$

Particular Integral

$y = y_c + y_{p_1} + y_{p_2}$

$y_{p_1} = \dfrac{1}{D^2+3D+2} e^{-2x}$

$= \dfrac{1}{4-6+2} e^{-2x}$

$= \dfrac{1}{0} e^{-2x}$

$= \dfrac{x e^{-2x}}{2D+3} \quad = \dfrac{x e^{-2x}}{-4+3}$

$= \dfrac{x e^{-2x}}{-1} = -x e^{-2x}$

D^2+3D+2

$D^2 = -a^2 = -1$

$= \frac{1}{-1+3D+2} \sin x$

$= \frac{1}{3D+1} \sin x$

$= \frac{1}{3D+1} \times \frac{3D-1}{3D-1} \sin x$

$= \frac{(3D-1) \sin x}{9D^2-1}$ $\quad D^2=-1$

$= \frac{3D(\sin x) - \sin x}{-9-1}$

$y_{P_2} = \frac{3\cos x - \sin x}{-10}$

$y = y_c + y_{P_1} + y_{P_2}$

$= Ae^{-x} + Be^{-2x} - xe^{-2x}$

$+ \frac{3\cos x - \sin x}{-10}$

5. solve $(D^2-4D+4)y = e^{2x} + \cos 2x$

Given $(D^2-4D+4)y = e^{2x} + \cos 2x$

the auxiliary equation is

$m^2 - 4m + 4 = 0$

$(m-2)(m-2) = 0$

$m-2=0 \mid m-2=0$

$m = 2 \mid m = 2$

$m_1 = m_2$

$m = 2, 2$

the complementary function

$y_c = Ae^{mx}(Ax+B)$

$y_c = Ae^{2x}(Ax+B)$

particular integral

$y = y_c + y_{P_1} + y_{P_2}$

$y_{P_1} = \frac{1}{D^2-4D+4} e^{2x}$

$= \frac{1}{4-8+4} e^{2x}$

$= \frac{1}{0} e^{2x}$

$= \frac{xe^{2x}}{2D-4}$

$= \frac{xe^{2x}}{4-4}$

$= \frac{xe^{2x}}{0}$

$= \frac{x^2e^{2x}}{2}$

$y = y_c + y_p$

$= Ae^{0x}(A\cos ax + B\sin ax)$

$- \frac{x\cos x}{2a}$

5\. Solve $(D^2+a^2)y = \cos ax$

Given $(D^2+a^2)y = \cos ax$

the auxiliary equation is

$m^2 + a^2 = 0$

$m^2 = -a^2$

$m = \pm ai$

the complementary function is

$y_c = Ae^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y_c = Ae^{0x}(A\cos ax + B\sin ax)$

particular integral

$y_p = \frac{1}{D^2+a^2}\cos ax$ $\quad D^2 = -a^2$

$= \frac{1}{-a^2+a^2}\cos ax$

$= \frac{1}{0}\cos ax$

$= \frac{x}{2a}\cos ax$ $\quad \left(\frac{1}{D} = \int\right)$

$= \frac{x}{2}\int\cos ax$

$= \frac{x}{2}\left(\frac{\sin ax}{a}\right)$

$y_p = \frac{x\sin ax}{2a}$

$y = y_c + y_p$

$y = Ae^{0x}(A\cos ax + B\sin ax) + \frac{x\sin ax}{2a}$

6\. Solve $(D^2+4)y = \cos 2x$

Given $(D^2+4)y = \cos 2x$

The auxiliary equation is

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i$ $\quad (\alpha + i\beta)$

$\alpha = 0, \ \beta = 2$

the complementary function is

$y_c = Ae^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y_c = Ae^{0x}(A\cos 2x + B\sin 2x$

particular integral

$$y_p = \frac{1}{D^2+4} \quad \left(\begin{matrix} D = -2^2 \\ D^2 = -4^2 \end{matrix} \right)$$

$$= \frac{1}{-4+4} \cos 2x$$

$$= \frac{1}{0} \cos 2x$$

$$= \frac{x \cos 2x}{2D}$$

$$= \frac{x}{2} \int \cos 2x$$

$$= \frac{x}{2} \cdot \frac{\sin 2x}{2}$$

$$= \frac{x \sin 2x}{4}$$

$$y_p = \frac{x \sin 2x}{4}$$

$$y = y_c + y_p$$

$$y = Ae^{0x}(A\cos 2x + B\sin 2x) + \frac{x \sin 2x}{4}$$

7 Solve $(D^2-4)y = \sin 2x$

Given $(D^2-4)y = \sin 2x$

the auxilliary equation is

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm 2$$

the complementary function is

$$y_c = Ae^{m_1 x} + Be^{m_2 x}$$

$$y_c = Ae^{-2x} + Be^{2x}$$

particular Integral

$$y = y_c + y_{p_1} + y_{p_2}$$

$$y_p = \frac{1}{D^2-4} \sin 2x$$

$$= \frac{1}{(-4)-4} \sin 2x$$

$$= \frac{1}{-8} \sin 2x$$

$$y_p = \frac{\sin 2x}{-8}$$

$$y = y_c + y_p$$

$$= Ae^{-2x} + Be^{2x} + \frac{\sin 2x}{-8}$$

8 Solve $(D^2-4)y = \sin^2 x$

Given, $(D^2-4)y = \sin^2 x$

The auxiliary equation is

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm 2$$

The complementary function is

$$y_c = Ae^{m_1 x} + Be^{m_2 x}$$

$$y_c = Ae^{-2x} + Be^{2x}$$

Particular integral

$$y_p = \frac{1}{D^2-4} \sin^2 x$$

$$= \frac{1}{D^2-4}\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{2}\frac{1}{D^2-4} - \frac{1}{D^2-4}\frac{1}{2}\cos 2x$$

$(D^2=-4)$

$$= \frac{1}{2}\frac{1}{D^2-4} e^{0x} - \frac{1}{2}\frac{1}{D^2-4}\cos 2x$$

$$= \frac{1}{2}\left(\frac{1}{-4}\right) e^{0x} - \frac{1}{2}\frac{1}{-4-4}\cos 2x$$

$$= \frac{-1}{8} e^{0x} - \frac{1}{\cancel{2}}\frac{1}{-8}\cos 2x$$

$$y_p = \frac{-1}{8} e^{0x} + \frac{1}{16}\cos 2x$$

9. Solve $(D^2+6D+8)y = \cos^2 x$

Given $(D^2+6D+8)y = \cos^2 x$

The auxiliary equation is

$$m^2+6m+8 = 0$$

$$(m+2)(m+4) = 0$$

$$m = -2, -4$$

The complementary function

$$y_c = Ae^{m_1 x} + Be^{m_2 x}$$

$$y_c = Ae^{-2x} + Be^{-4x}$$

Particular integral

$$y_p = \frac{1}{D^2+6D+8}\cos^2 x$$

$$= \frac{1}{D^2+6D+8}\left(\frac{1+\cos 2x}{2}\right)$$

$$= \frac{1}{2}\frac{1}{D^2+6D+8} e^{0x} + \frac{1}{D^2+6D+8}\cos 2x$$

$(D^2=-4)$

$$= \frac{1}{2}\cdot\frac{1}{8} + \frac{1}{2}\frac{1}{-4+6D+8}$$

$$= \frac{1}{16} + \frac{1}{2}\frac{1}{6D+4}\cos 2x$$

$$y = y_c + y_p$$

$$= \frac{1}{16} + \frac{1}{2}\frac{1}{6D+4}\times\frac{6D-4}{6D-4}\cos 2x$$

$$= \frac{1}{16} + \frac{1}{2}\cdot\frac{(6D-4)\cos 2x}{36D^2-16} \quad D^2=-4$$

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$-144-16$

$$= \frac{1}{16} + \frac{1}{2} - \frac{12\sin 2x - 4\cos 2x}{-160}$$

$Y = Y_c + Y_p$

$$Y = Ae^{-2x} + Be^{-4x} + \frac{1}{16} + \frac{1}{2} - \frac{12\sin 2x - 4\cos 2x}{-160}$$

Type-III :-

1. Solve $(D^2+D+1)y = x^2$

Given $(D^2+D+1)y = 0$

the auxiliary equation is

$m^2+m+1 = 0$

$a=1, b=1, c=1$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{3}i}{2}$$

$\alpha = -\frac{1}{2}, \ \beta = \frac{\sqrt{3}}{2}$

the complementary function is

$Y_c = Ae^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$$Y_c = Ae^{-1/2 x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right)$$

$(1+x)^{-1} = 1 - x + x^2 - x^3.$

$(1-x)^{-1} =$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= (1-(D^2+D) + (D^2+D)^2 - (D^2+D)^3 + \ldots)x^2$

$= (1-D)x^2$

$= x^2 - D(x^2)$

$Y_p = x^2 - 2x$

$Y = Y_c + Y_p$

$$Y = Ae^{-1/2 x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right) + x^2 - 2x$$

2. Solve $(D^2-2D+3)y = x^3$

Given $(D^2-2D+3)y = x^3$

the auxiliary equation is

$m^2 - 2m + 3 = 0$

$a=1, b=-2, c=3$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4-4\times 3}}{2}$$

$$= \frac{2 \pm \sqrt{4-12}}{2}$$

$= \frac{e}{2D}\left[1 \quad \frac{}{2D}\right]$

$= \frac{e^x}{2D}\left[1 + \frac{D^3+3D^2}{2D}\right]^{-1} x$

$(a+b)^3 =$

$\cancel{a^3+3a^2b+3ab^2}$

$a^3 + 3a^2b + 3ab^2 + b^3$

$= \frac{e^x}{2D}\left[1 - \left(\frac{D^3+3D^2}{2D}\right)\right] x$

$\left((1+x)^{-1} = 1 - x + x^2 - x^3 \ldots\right)$

$= \frac{e^x}{2D}\left[1 - \frac{D^3}{2D} - \frac{3D^2}{2D}\right] x$

$= \frac{e^x}{2D}\left[1 - \frac{1}{2}D^2 - \frac{3}{2}D\right] x$

$= \frac{e^x}{2D}\left[x - \frac{1}{2}D^2(x) - \frac{3}{2}D(x)\right]$

$= \frac{e^x}{2D}\left[x - \frac{1}{2}(0) - \frac{3}{2}D\right]$

$= \frac{e^x}{2D}\left(\int x\,dx - \frac{3}{2}\int dx\right)$

$= \frac{e^x}{2D}\left[x - \frac{3}{2}\right]$

$= \frac{e^x}{2}\left[\frac{x^2}{2} - \frac{3}{2}x\right]$

The general solution is

$y = Ae^{0x} + Be^{x} + Ce^{x} \frac{e^x}{2}\left[\frac{x^2}{2} - \frac{3}{2}x\right]$

7 Solve $(D^4 - 2D^3 + D^2)y = x^3$

Given that $(D^4 - 2D^3 + D^2)y = x^3$

The auxilliary equation is

$m^4 - 2m^3 + m^2 = 0$

$(m-1)(m-1) = 0$

$m^2 = 0$

$m^2 = 0$

$m = 1, 1$

The complementary function is

$Y_c = (Ax+B) + (Cx+D)e^x$

particular integral

$Y_P = \frac{1}{D^4 - 2D^3 + D^2} x^3$

$= \frac{1}{D^2 + D^4 - 2D^3} x^3$

$= \frac{1}{D^2 + D^4 - 2D^3} x^3$

$= \frac{1}{D^2}\left(1 - (2D - D^2)^{-1} x^3\right)$

$= \frac{1}{D^2}\left[(1 + 2D - D^2 + (2D - D^2)^2 + (2D - D^2)^3\right] x^3$

$= \frac{1}{D^2}\left[1 + 2D - D^2 + 4D^2 + D + 4D^3 + 8D^3\right] x^3$

$= \frac{1}{D^2}\left[x^3 + 2x\,x^3 - D^2x^3 - x^2 - D^2x^3 + 4Dx^3 - 4D^3x^3 + 8D^3x^3\right]$

$= \frac{96}{4D^2}\left[x^2 - \frac{1}{4}D^2(x^2)\right]$

$= \frac{96}{4D^2}\left[x^2 - \frac{1}{4}(2)\right]$

$= \frac{24}{D^2}\left[x^2 - \frac{1}{2}\right]$

$= 24\left[\frac{1}{D^2}x^2 - \frac{1}{D^2}(1/2)\right]$

$= 24\left[\frac{x^4}{12} - \frac{x^2}{4}\right] \Rightarrow 24\left(\frac{x^3 - 3x}{12}\right)$

$= 2(x^4 - 3x^2)$

$Y_P = 2x^2(x^2 - 3)$

$\therefore$ the general solution is

$Y = Y_C + Y_P$

$Y = e^{0x}(A\cos 2x + B\sin 2x) + 2x^2(x^2 - 3)$

9. Solve $(D^2+1)y = x^4$

Given $(D^2+1) = x^4$

the auxiliary equation is $m^2+1=0$

$m^2 = -1$

$m = \pm i$

$(\alpha = 0,\ \beta = 1)$

The complementary function is

$Y_C = e^{0x}(A\cos x + B\sin x)$

particular integral.

$Y_P = \frac{1}{D^2+1}x^4$

$= \frac{1}{(1+D^2)}x^4$

$= (1+D^2)^{-1}x^4$

$= [1 - D^2 + (D^2)^2 - (D^2)^3]x^4$

$= [1 - D^2 + D^4]x^4$

$= x^4 - D^2(x^4) + D^4(x^4)$

$Y_P = x^4 - 12x^2 + 24$

The general solution is

$Y = Y_C + Y_P$

$Y = e^{0x}(A\cos x + B\sin x) + x^4 - 12x^2 + 24$

Solve $(D^2-5D+6)y = x^2$

Given $(D^2-5D+6)y = x^2$

The auxiliary equation is

$$m^2-5m+6=0$$

$$(m-2)(m-3)=0$$

$$m = 2, 3$$

The complementary function is

$$y_c = Ae^{2x}+Be^{3x}$$

Particular Integral

$$y_p = \frac{1}{(D^2-5D+6)}x^2$$

$$= \frac{1}{6\left(1+\frac{D^2-5D}{6}\right)}x^2$$

$$= \frac{1}{6}\left(1+\frac{D^2-5D}{6}\right)^{-1}x^2$$

$$= \frac{1}{6}\left[1-\left(\frac{D^2-5D}{6}\right)+\left(\frac{D^2-5D}{6}\right)^2\right]$$

$$= \frac{1}{6}\left[1-\frac{1}{6}(D^2-5D+\frac{1}{36}(D^2-5D)^2\right]x^2$$

$$= \frac{1}{6}\left[1-\frac{1}{6}(D^2-5D+\frac{1}{36}(D^4+25D^2-10D^3)\right]x^2$$

$$= \frac{1}{6}\left[1-\frac{1}{6}D^2+\frac{5}{6}D+\frac{25}{36}D^2\right]x^2$$

$$= \frac{1}{6}\left[x^2-\frac{1}{6}D^2(x^2)+\frac{5}{6}D(x^2)+\frac{25}{36}D^2(x^2)\right]$$

$$= \frac{1}{6}\left[x^2-\frac{1}{6}(2)+\frac{5}{6}(2x)+\frac{25}{36}(2)\right]$$

$$= \frac{1}{6}\left[x^2-\frac{1}{3}+\frac{5x}{3}+\frac{25}{18}\right]$$

$$= \frac{1}{6}\left[x^2+\frac{5x}{3}+\frac{19}{18}\right]$$

The general solution is

$$y = y_c+y_p$$

$$y = Ae^{2x}+Be^{3x}+\frac{1}{6}\left[x^2+\frac{5x}{3}+\frac{19}{18}\right]$$

11. Solve $(D^4+8D^2+16)y = 16x+16$

Given $(D^4+8D^2+16)y = 16x+16$

The auxiliary equation is $m^4+8m^2+16=0$

$$(m^2+4)^2=0$$

$$(m^2+4)(m^2+4)=0$$

$$m = \pm 2i$$

The complementary function is

$$y_c = e^{0x}((Ax+B)\cos 2x + (Cx+D)\sin 2x)$$

$$y_p = \frac{1}{D^4+8D^2+16}(16x+16)$$

$$= \frac{1}{16\left[\frac{11D^4+8D^2}{16}\right]} 16x+10$$

$$= \frac{1}{16}\left[1+\frac{11D^4+8D^2}{16}\right]^{-1} 16x+10$$

$$= \frac{1}{16}\left[1-\left(\frac{D^4+8D^2}{16}\right)\right] 16x+10$$

$$= \frac{1}{16}\left[16x+10\right]$$

$$= \frac{16x}{16}+\frac{10}{16} = x+\frac{5}{8}$$

$$y_p = \frac{8x+5}{8}$$

the General solution is

$$y = [(Ax+B)\cos 2x + (Cx+D)\sin 2x] + \frac{8x+5}{8}$$

10. Solve $(D^2-5D+6)y = x^2+3$

Given $(D^2-5D+6)y = x^2+3$

the auxilliary equation is

$$m^2-5m+6=0$$

$$(m-2)(m-3)=0$$

$$m = 2,3$$

$$y_c = Ae^{2x}+Be^{3x}$$

$$y_p = \frac{1}{D^2-5D+6} x^2+3$$

$$= \frac{1}{6\left(1+\frac{D^2-5D}{6}\right)} x^2+3$$

$$= \frac{1}{6}\left[1+\frac{D^2-5D}{6}\right]^{-1} x^2+3$$

$$= \frac{1}{6}\left[1-\frac{D^2-5D}{6}+\frac{(D^2-5D)^2}{6}\right] x^2+3$$

$$= \frac{1}{6}\left[1-\frac{D^2}{6}-\frac{5D}{6}+\frac{1}{36}(D^4+25D^2-10D^3)\right] x^2+3$$

$$= \frac{1}{6}\left[1-\frac{D^2}{6}-\frac{5}{6}D+\frac{25}{36}D^2\right] x^2+3$$

$$= \frac{1}{6}\left[(x^2+3)+\frac{1}{6}D^2(x^2+3)-\frac{5}{6}D(x^2+3)+\frac{25}{36}D^2(x^2+3)\right]$$

$$= \frac{1}{6}\left[x^2+3-\frac{1}{6}(2)-\frac{5}{6}(2x)+\frac{25}{36}(2)\right]$$

$$= \frac{1}{6}\left[x^2-\frac{5}{3}x+3-\frac{1}{3}+\frac{25}{18}\right]$$

$$= \frac{1}{6}\left[x^2-\frac{5}{3}x+\frac{54-6+25}{18}\right]$$

$$= \frac{1}{6}\left[x^2-\frac{5}{3}x+\frac{73}{18}\right]$$

$$= \frac{1}{6}\left[\frac{18x^2-30x+73}{18}\right]$$

$$y_p = \frac{1}{108}\left[18x^2-30x+73\right]$$

the general solution is

$$y = [(Ax+B)\cos 2x + (Cx+D)\sin 2x] + \frac{8x+5}{8}$$

11) Solve $(D^3-3D^2-6D+8)y = x$

Given $(D^3-3D^2-6D+8)y = x$

The auxiliary equation is

$m^3-3m^2-6m+8 = 0$

$(m-1)$ is a factor

$m-1=0$, $m=1$

$m^2-2m-8=0$

$(m+2)(m-4)=0$

$m=-2,4$

The complementary function is

$y_c = Ae^{x}+Be^{-2x}+Ce^{4x}$

particular integral is

$$y_p = \frac{1}{D^3-3D^2-6D+8}x$$

$$= \frac{1}{8\left[1-\frac{D^3-3D^2-6D}{8}\right]}x$$

$$= \frac{1}{8}\left[1+\frac{D^3-3D^2-6D}{8}\right]^{-1}x$$

$$= \frac{1}{8}\left[1-\left(\frac{D^3-3D^2-6D}{8}\right)\right]x$$

$$= \frac{1}{8}\left[1+\frac{6}{8}D\right]x$$

$$= \frac{1}{8}\left[x+\frac{6}{8}D(x)\right]$$

$$= \frac{1}{8}\left[x+\frac{6}{8}(1)\right] = \frac{1}{8}\left[x+\frac{3}{4}\right]$$

the general solution is

$y = Ae^{x}+Be^{-2x}+Ce^{4x}+\frac{1}{8}\left(x+\frac{3}{4}\right)$

2) Solve $(D^2+4D+4)y = e^{-x}\sin 2x$

Given $(D^2+4D+4)y = e^{-x}\sin 2x$

The auxiliary equation is

$m^2+4m+4 = 0$

$(m+2)(m+2)=0$

$m=-2,-2$

The complementary function is

$y_c = (Ax+B)e^{-2x}$

particular integral

$$y_p = \frac{1}{D^2+4D+4}e^{-x}\sin 2x$$

$$= e^{-x}\frac{1}{(D-1)^2+4(D-1)+4}\sin 2x$$

$$= e^{-x}\frac{1}{D^2-2D+1+4D-4+4}\sin 2x$$

$$= e^{-x}\frac{1}{D^2+2D+1}\sin 2x$$

$$= e^{-x}\frac{1}{-4+2D+1}\sin 2x$$

$$= e^{-x}\frac{1}{2D-3}\sin 2x$$

$$= e^{-x}\frac{(2D+3)}{(2D-3)(2D+3)}$$

$$= e^{-x}\frac{(2D+3)}{4D^2-9}\sin 2x$$

$$= e^{-x}\frac{2D(\sin 2x)+3\sin 2x}{4(-4)-9}$$

$$= e^{-x}\left(\frac{2(\cos 2x-2)+3\sin 2x}{-16-9}\right)$$

$$y_p = e^{-x}\frac{(4\cos 2x+3\sin 2x)}{-25}$$

the general solution is

$$y = (Ax+B)e^{-2x}+e^{-x}\left(\frac{4\cos 2x+3\sin 2x}{-25}\right)$$

13. Solve $(D^2-4D+3)y = e^{-x}\cos 2x$

Given $(D^2-4D+3)y = e^{-x}\cos 2x$

the auxilliary equation is

$$m^2-4m+3=0$$
$$(m-1)(m-3)=0$$
$$m = +1, 3$$

the complimentary function is

$$y_c = Ae^{x}+Be^{3x}$$

particular integral

$$y_p = \frac{1}{D^2+4D+3}e^{-x}\cos 2x$$

$$= e^{-x}\frac{1}{D^2-4D+3}\cos 2x$$

$$= e^{-x}\frac{1}{(D-1)^2-4(D-1)+3}\cos 2x$$

$$= e^{-x}\frac{1}{D^2-2D+1-4D+4+3}\cos 2x$$

$$= e^{-x}\frac{1}{D^2-6D+8}\cos 2x$$

$$= e^{-x}\frac{1}{-4-6D+8}\cos 2x$$

$$= e^{-x}\frac{1}{-6D+4}\cos 2x$$

$$= e^{-x}\frac{(-6D-4)\cos 2x}{(-6D+4)(-6D-4)}$$

$$= e^{-x}\frac{(-6D-4)\cos 2x}{(-6D+4)(-6D-4)}$$

$$= e^{-x}\frac{(-6D-4)\cos 2x}{36D^2-16}$$

$$= e^{-x}\frac{-6D\cos 2x-4\cos 2x}{36(-4)-16}$$

$$= e^{-x}\frac{-6(-\sin 2x-2-4\cos 2x)}{-160}$$

the general solution is

$$= e^{-x}\frac{(12\sin 2x-4\cos 2x)}{160}$$

$$y = Ae^{x}+Be^{3x}-e^{-x}\left(\frac{3\sin 2x-\cos 2x}{40}\right)$$

$$= e^{-x}\frac{4(3\sin 2x-\cos 2x)}{160\ 40}$$

$$= e^{-x}\frac{(3\sin 2x-\cos 2x)}{-40}$$

17. Solve $(D^2-2D+1)y = x^2 e^{3x}$

The auxilliary equation is

$m^2-2m+1=0$

$(m-1)(m-1)=0$

$m = 1,1$

$y_c = (Ax+B)e^{x}$

particular integral

$y_p = \frac{1}{D^2-2D+1} x^2 e^{3x}$

$= e^{3x} \frac{1}{(D+3)^2-2(D+3)+1} x^2$

$= e^{3x} \frac{1}{D^2+9+6D-2D-6+1} x^2$

$= e^{3x} \frac{1}{4\left[1+\frac{D^2+4D}{4}\right]} x^2$

$= \frac{e^{3x}}{4} \left[1+\frac{D^2+4D}{4}\right]^{-1} x^2$

$= \frac{e^{3x}}{4} \left[1-\left(\frac{D^2+4D}{4}\right)+\left(\frac{D^2+4D}{4}\right)^2\right]$

$= \frac{e^{3x}}{4}\left[1-\frac{1}{4}D^2-\frac{4}{4}D+\frac{1}{16}(D^4+16D^2 + 8D^3)\right]$

$= e^{3x}\left[1-\frac{1}{4}D^2-D+\frac{16}{16}D^2\right]x^2$

$= \frac{e^{3x}}{4}\left[x^2-\frac{1}{4}D^2(x^2)-D(x^2)+D^2(x^2)\right]$

$= \frac{e^{3x}}{4}\left[x^2-\frac{1}{4}(2)-2x+2\right]$

$= \frac{e^{3x}}{4}\left[x^2-2x-\frac{1}{2}+2\right]$

$y_p = \frac{e^{3x}}{4}\left[x^2-2x+\frac{3}{2}\right]$

The general solution is

$y = (Ax+B)e^{x} + \frac{e^{3x}}{4}\left[x^2-2x+\frac{3}{2}\right]$

~~Type~~

UNIT-III

Laplace Transforms

Definition.

$f(t)$ be a function of a variable t which is defined for all positive values of t.

Let s be the real constant.

If $\int_0^\infty e^{-st} f(t)\,dt$ exists and the equation be $F(s)$, then $F(s)$ is called the Laplace transform of $f(t)$.

It is denoted by

$L[f(t)] = \int_0^\infty e^{-st}[f(t)]\,dt = F(s)$

Type

UNIT-III

Laplace Transforms

Definition.

$f(t)$ be a function of a variable 't' which is defined for all positive values of t.

Let s be the real constant.

If $\int_0^{\infty} e^{-st} f(t)\,dt$ exist and this equation is $F(s)$ that $F(s)$ is called the Laplace transform of $f(t)$.

It is denoted by

$$L[f(t)] = \int_0^{\infty} e^{-st} [f(t)]\,dt = F(s)$$

Formula :-

S.N.	F(t)	f(t)
1.	e^{at}	$\frac{1}{s-a}$
2.	e^{-at}	$\frac{1}{s+a}$
3.	$\sin at$	$\frac{a}{s^2+a^2}$
4.	$\cos at$	$\frac{s}{s^2+a^2}$
5.	$\sinh at$	$\frac{a}{s^2-a^2}$
6.	$\cosh at$	$\frac{s}{s^2-a^2}$
7.	k	k/s
8.	1	$1/s$
9.	t^n	$\frac{n!}{s^{n+1}}$

2. prove that $L[e^{-at}] = \frac{1}{s+a}$

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$L[e^{-at}] = \int_0^\infty e^{-st} e^{-at}\,dt$$

$$= \int_0^\infty (e^{-t(s+a)})\,dt$$

$$= \left(\frac{e^{-t(s+a)}}{-s+a}\right)_0^\infty$$

$$= -\frac{1}{s+a}[0-1] = \frac{1}{s+a}$$

$$= \frac{1}{s+a}$$

Hence proved.

3. prove that $L[\cos at] = \frac{s}{s^2+a^2}$

proof.-

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^\infty e^{-st} \cos at\,dt$$

$$= \text{Real part of } \int_0^\infty e^{-st} e^{iat}\,dt$$

$\left\{ e^{i\theta} = \cos\theta + i\sin\theta \right\}$ $= \text{R.P of } L[e^{iat}]$

$$= \text{R.P of } \frac{1}{s-ia}$$

$$= \text{R.P of } \frac{1}{s-ia} \times \frac{s+ia}{s+ia}$$

$$= \text{R.P of } \frac{s+ia}{s^2+a^2} \quad (i^2=-1)$$

$$L(\cos at) = \text{R.P of } \frac{s}{s^2+a^2}$$

1. Find $L(e^{2t}+3e^{-4t})$

$L(e^{2t}+3e^{-4t}) = L(e^{2t})+3L(e^{-4t})$

$= \frac{1}{s-2} + 3\left(\frac{1}{s+4}\right)$

$= \frac{s+4+3(s-2)}{(s-2)(s+4)}$

$= \frac{s+4+3s-6}{s^2-2s+4s-8}$

$= \frac{4s-2}{s^2+2s-8}$

2. Find $L(3e^{5t}+5\cos t)$

Given, $L(3e^{5t}+5\cos t)$

$L(3e^{5t}+5\cos t) = 3L(e^{5t})+5L(\cos t)$

$= 3\left(\frac{1}{s-5}\right)+5\left(\frac{s}{s^2+1}\right)$

$= \frac{3}{s-5} + \frac{5s}{s^2+1} = \frac{3s^2+3+5s^2-25s}{s^3+s-5s^2-5}$

$= \frac{8s^2-25s+3}{s^3-5s^2+s-5}$

3. Find $L(\sin^2 2t)$

$L(\sin^2 2t) = L\left(\frac{1-\cos 4t}{2}\right)$

$= L\left(\frac{1}{2}\right) - \frac{1}{2}L(\cos 4t)$ $\left[\begin{array}{l}\sin^2\theta = \frac{1-\cos 2\theta}{2}\\ \cos^2\theta = \frac{1+\cos 2\theta}{2}\end{array}\right]$

$= \frac{1}{2}L(1) - \frac{1}{2}\left[\frac{s}{s^2+16}\right]$ $(a=4)$

$= \frac{1}{2}\times\frac{1}{s} - \frac{1}{2}\left[\frac{s}{s^2+16}\right]$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+16}\right]$

$= \frac{1}{2}\left[\frac{s^2+16-s^2}{s^3+16s}\right]$

$= \frac{1}{2}\left[\frac{16}{s^3+16s}\right]$

$L(\sin^2 2t) = \frac{8}{s^3+16s}$

4. Find $L(\cos^2 3t)$

$L(\cos^2 3t) = L\left(\frac{1+\cos 6t}{2}\right)$

$= \frac{1}{2}L(1)+\frac{1}{2}L(\cos 6t)$

$= \frac{1}{2}\times\frac{1}{s}+\frac{1}{2}\left(\frac{s}{s^2+36}\right)$

$= \frac{1}{2}\left[\frac{1}{s}+\frac{s}{s^2+36}\right]$

$= \frac{1}{2}\left[\frac{s^2+36+s^2}{s^3+36s}\right] = \frac{1}{2}\left[\frac{2s^2+36}{s^3+36s}\right]$

$= \frac{1}{2}\left[\frac{2(s^2+18)}{s^3+36s}\right] = L(\cos^2 3t) = \frac{s^2+18}{s^3+36s}$

5. Find $L[\sin(\omega t+\alpha)]$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$= L[\sin\omega t \cos\alpha + \cos\omega t \sin\alpha]$

$= L[\sin\omega t \cos\alpha) + L(\cos\omega t \sin\alpha)$

$= \cos\alpha\, L(\sin\omega t) + \sin\alpha\, L(\cos\omega t)$

$= \cos\alpha\left(\frac{\omega}{s^2+\omega^2}\right) + \sin\alpha\left(\frac{s}{s^2+\omega^2}\right)$

$= \frac{\cos\alpha\,\omega + \sin\alpha\, s}{s^2+\omega^2}$

6. Find $L(\sin 3t \cdot \sin 2t)$

$\sin A \sin B = \frac{\cos(A-B)-\cos(A+B)}{2}$

$L(\sin 3t \cdot \sin 2t) = \frac{1}{2}L(\cos t) - \frac{1}{2}L(\cos 5t)$ $\begin{pmatrix}A=3\\B=2\end{pmatrix}$

$2\sin A \sin B = \cos A - \cos B$

$= \frac{1}{2}\left[\frac{s}{s^2+1}\right] - \frac{1}{2}\left[\frac{s}{s^2+25}\right]$

$= \frac{1}{2}\left[\frac{s}{s^2+1} - \frac{s}{s^2+25}\right]$

$= \frac{1}{2}\left[\frac{s^3+25s-s^3-s}{(s^2+1)(s^2+25)}\right]$

$= \frac{1}{2}\left[\frac{24(s)}{(s^2+1)(s^2+25)}\right]$

7. Find $L[(\sinh 6t) + 3e^{-5t} + \cos 5t]$

$= L(\sinh 6t) + 3L(e^{-5t}) + L(\cos 5t)$

$= \frac{6}{s^2-36} + 3\left(\frac{1}{s+5}\right) + \frac{s}{s^2+25}$

$= \frac{6}{s^2-36} + \frac{3}{s+5} + \frac{s}{s^2+25}$

$$= \frac{6(s+5)(s^2+25) + 3(s^2-36)(s^2+25) + s(s^2-36)(s+5)}{(s^2-36)(s+5)(s^2+25)}$$

$$= \frac{[6s+30](s^2+25) + (3s^2-108)(s^2+25) + (s^3-36s)(s+5)}{(s^2-36)(s+5)(s^2+25)}$$

$$= \frac{6s^3+150s+30s^2+750+3s^4+75s^2-108s^2-2700+s^4+5s^3-36s^2-180s}{(s^2-36)(s+5)(s^2+25)}$$

$$= \frac{4s^4+11s^3-39s^2-30s-1950}{(s^2-36)(s+5)(s^2+25)}$$

8 Solve $L(t^{1/2})$

$$L(t) = L(t^{n})$$

$$= \frac{\Gamma(\frac{1}{2}+1)}{s^{1/2+1}}$$

$$= \frac{\frac{1}{2}\Gamma\frac{1}{2}}{s^{3/2}}$$

$$= \frac{\sqrt{\pi}}{2 s^{3/2}}$$

Formula

$\Gamma(n+1) = n\Gamma(n)$

$\Gamma 1 = 1$

$\Gamma\frac{1}{2} = \sqrt{\pi}$

9. Solve $L(t^{5/2})$

$$L(t^{5/2}) = \frac{\Gamma(\frac{5}{2}+1)}{s^{5/2+1}}$$

$$= \frac{\frac{5}{2}\Gamma\frac{5}{2}}{s^{7/2}}$$

$$= \frac{\frac{5}{2}\Gamma(\frac{3}{2}+1)}{s^{7/2}}$$

$$= \frac{\frac{5}{2}\cdot\frac{3}{2}\Gamma\frac{3}{2}}{s^{7/2}}$$

$$= \frac{\frac{5}{2}\cdot\frac{3}{2}\Gamma(\frac{1}{2}+1)}{s^{7/2}}$$

$$= \frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2}}{s^{7/2}}$$

$$= \frac{15\sqrt{\pi}}{8 s^{7/2}}$$

Type III ➤

First shifting theorem

If the laplace transform of $f(t)$ is $F(s)$ then the laplace transform of $e^{-at} f(t)$ is $F(s+a)$

(i.e) $F(s+a)$

(i.e) $L(e^{-at}) f(t) = F(s+a)$

Proof :-

Given $L[e^{-at} f(t)] = \int_0^\infty e^{-st} e^{-at} f(t)\,dt$

$\cancel{L[f(t)] = \int_0^\infty}$

$$= \int_0^\infty e^{-t(s+a)} f(t)\,dt$$

$$= F(s+a)$$

$$= F(s),\ s \to s+a$$

1. Find $L(e^{-3t}\sin^2 t)$

Given, $L(e^{-3t}\sin^2 t)$

$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = F(s)$.

$f(t) = \sin^2 t$

$L(\sin^2 t) = L\left(\frac{1-\cos 2t}{2}\right)$

$= \frac{1}{2}\left[L(1) - \frac{1}{2}L(\cos 2t)\right]$

$= \frac{1}{2} \times \frac{1}{s} - \frac{1}{2}\left(\frac{s}{s^2+4}\right)$

$= \frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+4}\right\}$

Replace s by $s+3$.

$= \frac{1}{2}\left\{\frac{1}{s+3} - \frac{s+3}{(s+3)^2+4}\right\}$

$= \frac{1}{2}\left\{\frac{1}{s+3} - \frac{s+3}{s^2+6s+9+4}\right\}$

$= \frac{1}{2}\left\{\frac{1}{s+3} - \frac{s+3}{s^2+6s+13}\right\}$

2. Find $L(e^t \cosh 2t + \frac{1}{2}\sinh 2t)$

Given,

$L(e^t \cosh 2t + \frac{1}{2}\sinh 2t)$

$= L(e^t \cosh 2t) + \frac{1}{2} L(\sinh 2t)$

$= L e^t (\cosh 2t)$

$f(t) = \cosh 2t$

$L f(t) = L(\cosh 2t) = \frac{s}{s^2-4}$

Replace $s \to s-1$

$L(e^t \cosh 2t) = \frac{s-1}{(s-1)^2-4} \to ①$

$\frac{1}{2} L(\sinh 2t)$

$\therefore f(t) = \sinh 2t$

$L f(t) = \frac{1}{2} L(\sinh 2t)$

$= \frac{1}{2} \cdot \frac{2}{s^2-4}$

$= \frac{1}{s^2-4} \to ②$

① + ② we get

$= \frac{s-1}{(s-1)^2-4} + \frac{1}{s^2-4}$

5 Find $L(e^{-2t}\cos^2 t)$

$f(t) = \cos^2 t$

$L[f(t)] = L[\cos^2 t]$

$= L\left(\frac{1+\cos 2t}{2}\right)$

$= \frac{1}{2}[L(1) + L(\cos 2t)]$

$= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$

Replace s by $s+2$

$= \frac{1}{2}\left[\frac{1}{s+2} + \frac{(s+2)}{(s+2)^2+4}\right]$

$= \frac{1}{2}\left[\frac{2(s+2)^2+4}{(s+2)(s^2+4s+2s+8)}\right]$

Here, $f(t) = t^{n}$

$L[f(t)] = L[t^n] = \frac{n!}{s^{n+1}}$

Replace s by $s+4$

$L[f(t)] = \frac{n!}{(s+4)^{n+1}}$

Transform of derivatives

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

Now, $L\{f'(t)\} = \int_0^\infty e^{-st}\,dt$

$u = e^{-st}$ $\quad dv = f'(t)\,dt$

$du = -se^{-st}\,dt$ $\quad v = f(t)$

$L[f'(t)] = \left[e^{-st} f(t) \right]_0^\infty + s\int_0^\infty f(t) e^{-st}\,dt$

$= -f(0) + sL\{f(t)\}$

$L f'(t) = sL\{f(t)\} - f(0)$

$L\{f''(t)\} = \int_0^\infty e^{-st} f''(t)\,dt$

$u = e^{-st}$ $\quad dv = f''(t)\,dt$

$du = -se^{-st}$ $\quad v = f'(t)$

$= \left[e^{-st} f'(t) \right]_0^\infty + \int_0^\infty f'(t)\, s e^{-st}\,dt$

$= [-f'(0)] + s\int_0^\infty f'(t) e^{-st}\,dt$

$= -f'(0) + sL\{f'(t)\}$

Theorem of Integral:-

If $f(t)$ is continuous and of exponential order then the Laplace transform of $\int_0^t f(t)\,dt$

$L\int_0^t f(t)\,dt = \frac{1}{s} L f(t)$

Proof:-

$G(t) = \int_0^t f(t)\,dt$

$G'(t) = f(t)$

$G(0) = f(0) = 0$

We know that

$L[f'(t)] = sL[f(t)] - f(0)$

$L[G'(t)] = sL[G(t)] - 0$

$L[f(t)] = sL\left[\int_0^t f(t)\,dt\right] - 0$

$\frac{1}{s}L[f(t)] = L\int_0^t f(t)\,dt$

Change of scale

Result-1:-

If $L[f(t)] = F(S)$

then $L[f(at)] = \frac{1}{a} F(\frac{s}{a})$

Proof.-

$L[f(t)] = \int_0^\infty e^{-st} f(t) dt$

$L[f(at)] = \int_0^\infty e^{-st} f(at) dt$

put $at = u$ | $t = u/a$

$a dt = du$

$dt = \frac{du}{a}$

$= \int_0^\infty e^{-s(u/a)} f(u) \frac{du}{a}$

$= \frac{1}{a}\int_0^\infty e^{-s u/a} f(u) du$

change the dummy variable of integration.

$= \frac{1}{a}\int_0^\infty e^{-s t/a} f(t) dt$

$= \frac{1}{a} F(s/a) \quad [\frac{1}{a} F(s/a \times a)]$

Replace s by s/a

$= \frac{1}{a} F(s)$

Result-2:-

If $L[f(t)] = F(s)$

then $L[f(t/a)] = a F(as)$

Proof:

$L[f(t)] = \int_0^\infty e^{-st} f(t) dt$

$L[f(t/a)] = \int_0^\infty e^{-st} f(t/a) dt$

$= \int_0^\infty e^{-s(ua)} f(u) a du$ | $t/a = u$, $t = ua$

$= a\int_0^\infty e^{-s(ua)} f(u) du$ | $dt = a du$

change the dummy variable of integration

$= a\int_0^\infty e^{-sat} f(t) dt$

$= a F(as)$

$= a F(s)$

Type: IV

Derivatives and integrals of transform:

$L[f(t)] = F(s)$

then

$L[t f(t)] = -F'(s)$

we know that,

$L[f(t)] = \int_0^\infty e^{-st} f(t) dt = F(s)$

$F(s) = d/s [\int_0^\infty e^{-st} f(t) dt]$

$= \int_0^\infty \frac{d}{ds}(e^{-st}) f(t) dt$

$L[t\,f(t)] = -F'(s)$

1. Find $L[t\cos 3t]$

$L[f(t)] = F(s)$

$f(t) = \cos 3t$

$L[f(t)] = L(\cos 3t) = \frac{s}{s^2+9}$ $[a = 3]$

$= F(s)$

$\frac{u}{v} = \frac{u'v - uv'}{v^2}$

$F(s) = \frac{s}{s^2+9}$

$F'(s) = \frac{(s^2+9)(1) - s(2)(s)}{(s^2+9)^2}$

$= \frac{s^2+9-2s^2}{(s^2+9)^2}$

$F'(s) = \frac{-s^2+9}{(s^2+9)^2}$

$-F'(s) = \frac{s^2-9}{(s^2+9)^2}$

2. Find $L[t\sin^2 t]$

$L(f(t)) = F(s)$

$f(t) = \sin^2 t$

$L[f(t)] = \sin^2 t$

$L[f(t)] = L[\sin^2 t]$

$= L\left[\frac{1-\cos 2t}{2}\right]$

$= \frac{1}{2}\left[L(1) - L(\cos 2t)\right]$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$

$= \frac{1}{2}\left[\frac{s^2+4-s^2}{s(s^2+4)}\right]$

$F(s) = \frac{1}{2}\left(\frac{4}{s^3+4s}\right)$

$F'(s) = \frac{1}{2}\left[\frac{(s^3+4)0 - 4(3s^2+4)}{(s^3+4s)^2}\right]$

$F'(s) = \frac{1}{2}\left[\frac{-12s^2-16}{(s^3+4s)^2}\right]$

$-F'(s) = \frac{1}{2}\left(\frac{12s^2+16}{(s^3+4s)^2}\right)$

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Type V :-

1. Using laplace transform evaluate the following integral $\int_0^\infty t\, e^{2t} \sin 3t$

Given $\int_0^\infty t e^{2t} \sin 3t \, dt$

$L\left[\int_0^\infty t\, e^{2t} \sin 3t\, dt\right]$

$L\left[t \sin 3t\right]$

$f(t) = \sin 3t$

$L[f(t)] = L[\sin 3t] = \frac{3}{s^2+9} = F(s)$

$\therefore F(s) = \frac{3}{s^2+9} = \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2}$

$= \frac{-6s}{(s^2+9)^2} \Rightarrow F'(s) = \frac{6}{(s^2+9)^2}$

Replace s by $s-2$

$= \frac{6(s-2)}{((s-2)^2+9)^2}$

$= \frac{6s-12}{(s^2-4s+4+9)^2}$

$= \frac{6s-12}{(s^2-4s+13)^2}$

2. Find $L\left[t e^{-t} \sin t\right]$

$f(t) - [\sin t]$

$L[f(t)] = L(\sin t) = \frac{1}{s^2+1}$

replace s by $s+1$

$= \frac{1}{(s+1)^2+1} = \frac{1}{s^2+1+2s+1}$

$F(s) = \frac{1}{s^2+2s+2}$

$F'(s) = \frac{(s^2+2s+2)(0) - (1)(2s+2)}{(s^2+2s+2)^2}$

$= \frac{-2s-2}{(s^2+2s+2)^2}$

$-F'(s) = \frac{2s+2}{(s^2+2s+2)^2}$

5. Find $\int_0^\infty t e^{-3t} \cos 2t \, dt$

Given $\int_0^\infty t e^{-3t} \cos 2t\, dt$

$L(t \cos 2t)$

$f(t) = \cos 2t$

$L[f(t)] = L[\cos 2t] = \frac{s}{s^2+4}$

$F(s) = \frac{s}{s^2+4}$ (u) (v)

$F'(s) = \frac{(s^2+4)(1) - s(2s)}{(s^2+4)^2}$

$= \frac{s^2+4-2s^2}{(s^2+4)^2} = \frac{-s^2+4}{(s^2+4)^2}$

$-F'(s) = \frac{s^2-4}{(s^2+4)^2}$

$$-F'(s) = \frac{(s+3)\ldots}{((s+3)^2+4)^2}$$

$$= \frac{s^2+2(s)(3)+(3)^2-4}{s^2+2(s)(3)+(3)^2+4+4}$$

5. Find $L[te^{-t}\cosh 2t]$

$f(t) = \cosh 2t$

$L[f(t)] = L[\cosh 2t]$

$= \frac{s}{s^2+4} = F(s)$

$$F'(s) = \frac{(s^2-4)(1)-s(2s)}{(s^2-4)^2}$$

$$= \frac{-2s^2-s^2-4}{(s^2-4)^2}$$

$$-F'(s) = \frac{4s-s^2+4}{(s^2-4)^2}$$

Replace s by $s+1$

$$L[te^{-t}\cosh 2t] = \frac{+4(s+1)}{[(s+1)^2-4]^2}$$

$$= \frac{4s+4}{(s^2+1+2s-4)^2}$$

$$= \frac{-s^2-4}{(s^2-4)^2} = \frac{s^2+4}{(s^2-4)^2} = \frac{(s+1)^2+4}{((s+1)^2-4)^2}$$

$$= \frac{s^2+1+2s+4}{(s^2+1+2s-4)^2} = \frac{s^2+2s+5}{(s^2+2s-3)^2}$$

Type VI

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$$

6. Find the Laplace transform of

$$L\left[\frac{e^{-at}-e^{-bt}}{t}\right]$$

Given $L\left[\frac{e^{-at}-e^{-bt}}{t}\right]$

$$= L\left(\frac{f(t)}{t}\right) = \int_s^\infty F(s)\,ds$$

$f(t) = e^{-at} - e^{-bt}$

$L[f(t)] = L[e^{-at} - e^{-bt}]$

$= L[e^{-at}] - L[e^{-bt}]$

$= \frac{1}{s+a} - \frac{1}{s+b} = F(s)$

$$= \int_s^\infty \left(\frac{1}{s+a} - \frac{1}{s+b}\right) ds$$

$$= \int_s^\infty \frac{ds}{s+a} - \int_s^\infty \frac{ds}{s+b}$$

$$= \left[\log(s+a) - \log(s+b)\right]_s^\infty$$

$L[f(t)] = [\log \ldots$

$= 0 - (\log \frac{s+a}{s+1})$

$= \log \left[\frac{s+1}{s+a}\right]$

2. Find $L\left[\left(\frac{\sin t}{t}\right)^2\right]$

Given $L\left[\left(\frac{\sin t}{t}\right)^2\right]$

$L\left(\frac{f(t)}{t}\right) = \int_0^\infty F(s)\,ds$

$f(t) = \sin^2 t$

$L[f(t)] = L(\sin^2 t)$

$= \frac{1}{2}\{L(1) - L(\cos 2t)\}$

$= \frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+4}\right\} = F(s)$

$\therefore L\left(\frac{\sin^2 t}{t^2}\right) = \int_s^\infty \int_s^\infty \frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+4}\right\} ds$

$= \frac{1}{2}\int_s^\infty \int_s^\infty \left(\frac{ds}{s} - \frac{s\,ds}{s^2+4}\right)$

$= \frac{1}{2}\int_s^\infty \int_s^\infty \left(\frac{ds}{s} - \frac{dt}{2t}\right) ds$

$= \frac{1}{2}\int_s^\infty \int_s^\infty \left(\frac{ds}{s} - \frac{dt}{2t}\right) ds$

$= \frac{1}{2}\int_s \left[\log s - \frac{1}{2}\log(s^2+4)\right]_s^\infty ds$

$= \frac{1}{2}\int_s \left[\log\left(\frac{s}{s^2+4}\right)\right]_s^\infty ds$

$t = s^2+4$

$2s\,ds = dt$

$s\,ds = \frac{dt}{2}$

$= \frac{1}{2}\int_s^\infty \left[\log\left(\frac{s}{s^2+4}\right)\right]_s^\infty ds$

$= \frac{1}{2}\int_0 \left[0 - \log\left(\frac{s}{s^2+4}\right)\right]_s^\infty ds$

$= \frac{1}{2}\int_0^\infty \log \frac{s^2+4}{s}\,ds$

Laplace transform of special functions:-

Unit step function (or) heavy side's function:

1) Prove that $L[u_a(t)] = \frac{e^{-as}}{s}$

Proof:-

$L[U_a(t)] = \frac{e^{-as}}{s}$

$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$L[u(t)] = \int_0^\infty e^{-st} u_a(t)\,dt$

$= \int_0^\infty e^{-st} U_a(t)\,dt$

$= \int_0^\infty e^{-st} U_a(t)\,dt$

$= 0 + \int_a^\infty e^{-st}(1)\,dt$

$= \left[\frac{-e^{-st}}{s}\right]_a^\infty$

$= \frac{e^{-\infty}}{s} + e^{-as}$

$L[u_a(t)] = \frac{e^{-as}}{s}$

Initial value theorem [I V T]

statement.

If $f(t)$ and $f'(t)$ are laplace transforms and $L[f(t)] = F(s)$ then

$$\underset{t \to 0}{Lt}\, f(t) = \underset{s \to \infty}{Lt}\, sF(s)$$

Proof:

$$L[f'(t)] = sL[f(t)] - f(0)$$

$$\underset{s \to \infty}{Lt}\, L[f'(t)] = \underset{s \to \infty}{Lt}\, [sL[f(t)] - f(0)]$$

$$LHS \Rightarrow \underset{s \to \infty}{Lt}\, L[f'(t)] = \underset{s \to \infty}{Lt} \int_0^\infty e^{-st} f'(t)\, dt$$

$$= \int_0^\infty \underset{s \to \infty}{Lt}\, e^{-st} f'(t)\, dt$$

$[\because e^{-\infty} = 0]$

$$\underset{s \to \infty}{Lt}\, L[f'(t)] = 0$$

$$0 = \underset{s \to \infty}{Lt}\, [sL f(t)] - f(0)$$

$$\underset{s \to \infty}{Lt}\, sL[f(t)] - \underset{s \to \infty}{Lt}\, f(0) = 0$$

$$\underset{s \to \infty}{Lt}\, [L f(t)] = \underset{t \to 0}{Lt}\, f(t)$$

Final value theorem [F.V.T]

statement

If $f(t)$ and $f'(t)$ are laplace transforms and $L[f(t)] = F(s)$ then $\underset{t \to \infty}{Lt}\, f(t) = \underset{s \to 0}{Lt}\, sF(s)$.

proof:

$$L[f'(t)] = sL[f(t)] - f(0)$$

$$\underset{s \to 0}{Lt}\, L[f'(t)] = \underset{s \to 0}{Lt} \int_0^\infty e^{-st} f'(t)\, dt$$

$$= \int_0^\infty f'(t)\, dt = [f(t)]_0^\infty$$

$$LHS \quad f(\infty) - f(0)$$

$$= \underset{t \to \infty}{Lt}\, f(t) - f(0)$$

$$\underset{t \to \infty}{Lt}\, f(t) - f(0) = \underset{s \to 0}{Lt}\, s.F(s) - f(0)$$

$$Lt\, f(t) = \underset{s \to 0}{Lt}\, sF(s)$$

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1. Verify [illegible]

$\underset{s\to\infty}{Lt}\ sF(s) = \underset{t\to 0}{Lt}\ f(t)$

Given: $f(t) = 2 + 3\cos t$

$L[f(t)] = L[2 + 3\cos t] = 2L[1] + 3L[\cos t]$

$= \frac{2}{s} + \frac{3s}{s^2+1}$

$= \frac{2s}{s} + \frac{3s^2}{s^2(1+1/s^2)}$ [illegible]

$\underset{s\to\infty}{Lt}\ sF(s) = \underset{s\to\infty}{Lt}\left(2 + \frac{3}{(1+1/s^2)}\right)$

$= 2 + \frac{3}{1+\frac{1}{\infty}} = 2+3$

$\underset{s\to\infty}{Lt}\ sF(s) = 5 \rightarrow ①$

R.H.S $= \underset{t\to 0}{Lt}\ f(t)$

$= \underset{t\to 0}{Lt}\ (2 + 3\cos t)$

$= 2 + 3\cos 0 \qquad \cos(0) = 1$

$= 2 + 3(1)$

$= 5 \rightarrow ②$

① = ②

$\underset{s\to\infty}{Lt}\ s.F(s) = \underset{t\to\infty}{Lt}\ f(t)$

2. Verify [illegible] for $f(t) = t^2 e^{-3t}$

$\underset{s\to 0}{Lt}\ sF(s) = \underset{t\to\infty}{Lt}\ f(t)$

To find: $\underset{s\to 0}{Lt}\ sF(s)$ $\qquad \left[\because t^2 = \frac{2}{s^3}\right]$

$L[f(t)] = L[t^2 e^{-3t}]$ $\left[\because t^2 = \frac{2}{s^3}\right]$

$= \frac{2}{(s+3)^3} = F(s)$

$sF(s) = s\,\frac{2}{s^3 + 3s^2\cdot 3 + 3s\cdot 9 + 27}$

$= \frac{2s}{s[s^2 + 9s + 27 + \frac{27}{s}]} = 0 \rightarrow ①$

$\underset{t\to\infty}{Lt}\ f(t) = \underset{t\to\infty}{Lt}\ t^2 e^{-3t}$

$= 0 \rightarrow ②$

① = ②

$\underset{s\to 0}{Lt}\ sF(s) = \underset{t\to\infty}{Lt}\ f(t)$

UNIT

Initial value theorem [I V T]

statement.

If $f(t)$ and $f'(t)$ are laplace transforms and $L[f(t)] = F(S)$ then

$$\underset{t \to 0}{Lt}\, f(t) = \underset{s \to \infty}{Lt}\, sF(s)$$

Proof:

$$L[f'(t)] = SL[f(t)] - f(0)$$

$$\underset{s \to \infty}{Lt}\, L[f'(t)] = \underset{s \to \infty}{Lt}\,[SL[f(t)] - f(0)]$$

$$LHS \Rightarrow \underset{s \to \infty}{Lt}\, L[f'(t)] = \underset{s \to \infty}{Lt} \int_0^\infty \cancel{[f(t)] - f(0)}\, f'(t)\,dt$$

$$= \int_0^\infty \underset{s \to \infty}{Lt}\, e^{-st} f'(t)\,dt$$

$[\because e^{-\infty} = 0]$

$$\underset{s \to \infty}{Lt}\, L[f'(t)] = 0$$

$$0 = \underset{s \to \infty}{Lt}\,[SL\, f(t)] - f(0)$$

$$\underset{s \to \infty}{Lt}\, SL[f(t)] - \underset{s \to \infty}{Lt}\, f(0) = 0$$

$$\underset{s \to \infty}{Lt}\,[L f(t)] = \underset{t \to 0}{Lt}\, f(t)$$

theorem [F.V T]

statement

If $f(t)$ and $f'(t)$ are laplace transforms and $L[f(t)] = F(S)$ then $\underset{t \to \infty}{Lt}\, L(t) = \underset{s \to 0}{Lt}\, sF(s)$.

Proof:

$$L[f'(t)] = SL[f(t)] - f(0)$$

$$\underset{s \to 0}{Lt}\, L[f'(t)] = \underset{s \to 0}{Lt} \int_0^\infty e^{-st} f'(t)\,dt$$

$$= \int_0^\infty f'(t)\,dt = [f(t)]_0^\infty$$

$$LHS \quad f(\infty) - f(0)$$

$$= \underset{t \to \infty}{Lt}\, f(t) - f(0)$$

$$\underset{t \to \infty}{Lt}\, f(t) - f(0) = \underset{s \to 0}{Lt}\, s.F(s) - f(0)$$

$$Lt\, f'(t) = \underset{s \to 0}{Lt}\, sF(s)$$

1. Verify I.V.T for f(t) = 2+3 cos t

$\underset{s\to\infty}{Lt}\ s\,F(s) = \underset{t\to 0}{Lt}\ f(t)$

Given, $f(t) = 2+3\cos t$

$L[f(t)] = L[2+3\cos t] = 2L[1] + 3L[\cos t]$

$= \frac{2}{s} + \frac{3s}{s^2+1}$

$= \frac{2s}{s} + \frac{3s^2}{s^2(1+1/s^2)} = 2 + \frac{3}{1+1/s^2}$

$\underset{s\to\infty}{Lt}\ s\,F(s) = \underset{s\to\infty}{Lt}\left(2+\frac{3}{(1+1/s^2)}\right)$

$= 2+\frac{3}{1+\frac{1}{\infty}} = 2+3$

$\underset{s\to\infty}{Lt}\ sF(s) = 5 \to ①$

R.H.S $= \underset{t\to 0}{Lt}\ f(t)$

$= \underset{t\to 0}{Lt}\ (2+3\cos t)$

$= 2+3\cos 0$ $\qquad \cos(0) = 1$

$= 2+3(1)$

$= 5 \to ②$

① = ②

$\underset{s\to\infty}{Lt}\ s.F(s) = \underset{t\to 0}{Lt}\ f(t)$

2. Verify F.V.T for $f(t) = t^2 e^{-3t}$

$\underset{s\to 0}{Lt}\ s\,F(s) = \underset{t\to\infty}{Lt}\ f(t)$ $\qquad \left[\because t^2 = \frac{2}{s^3}\right]$

To find: $\underset{s\to 0}{Lt}\ s\,F(s)$

$L[f(t)] = L[t^2 e^{-3t}]$ $\qquad \left[\because t^2 = \frac{2}{s^3}\right]$

$= \frac{2}{(s+3)^3} = F(s)$

$sF(s) = s\,\frac{2}{s^3+3s^2\cdot 3+3\cdot s\cdot 9+27}$

$= \frac{2s}{s\left[s^2+9s+27+\frac{27}{s}\right]} = 0 \to ①$

$\underset{t\to\infty}{Lt}\ f(t) = \underset{t\to\infty}{Lt}\ t^2 e^{-3t}$

$= 0 \to ②$

① = ②

$\underset{s\to 0}{Lt}\ s\,F(s) = \underset{t\to\infty}{Lt}\ f(t)$

UNIT-IV

Inverse Laplace Transforms:-

Definition

If $L(f(t)) = F(S)$, then $f(t)$ is called inverse Laplace transform of $F(s)$. It is denoted by $f(t) = L^{-1}F(s)$.

Note:-

* If k is constant, then
$L^{-1}k\,F(s) = k\,L^{-1}[F(s)]$

~~* If $L^{-1}[F(s)] + [G(s)] =$~~

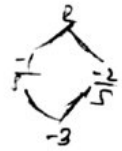
* If $L^{-1}F[F(s) + G(s)] = L^{-1}[F(s)] + L^{-1}(G(s))$

If a and b are constant then $L^{-1}[a\,F(s) + b\,G(s)]$
$= a\,L^{-1}[F(s)] + b\,L^{-1}G(s)]$

Type-I: Method of partial fraction

1. Find $L^{-1}\left(\frac{s-5}{s^2-3s+2}\right)$

s^2-3s+2
$(s-1)(s-2) = 0$



$L^{-1}\left[\frac{s-5}{(s-1)(s-2)}\right]$

$\frac{s-5}{(s-1)(s-2)} = \frac{A}{(s-1)} + \frac{B}{(s-2)}$

$s-5 = A(s-2) + B(s-1)$

Put $s = 2$
$2-5 = A(0) + B(2-1)$
$-3 = B$
$\boxed{B = -3}$

Put $s = 1$
$1-5 = A(1-2) + B(0)$
$-4 = -A$
$\boxed{A = 4}$

$L(e^{at}) = \frac{1}{s-a}$

$\frac{s-5}{s^2-3s+2} = \frac{4}{s-1} - \frac{3}{s-2}$

$L^{-1}\left[\frac{s-5}{s^2-3s+2}\right] = L^{-1}\left(\frac{4}{s-1}\right) - L^{-1}\left(\frac{3}{s-2}\right)$
$= 4L^{-1}\left(\frac{1}{s-1}\right) - 3L^{-1}\left(\frac{1}{s-2}\right)$
$= 4(e^{t}) - 3(e^{2t})$

2. Find $L^{-1}\left[\frac{s^2-s+2}{s(s+2)(s-3)}\right]$

$$\frac{s^2-s+2}{s(s+2)(s-3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s-3} \rightarrow (1)$$

$$s^2-s+2 = A(s+2)(s-3) + B(s)(s-3) + C(s)(s+2)$$

Put $s=0$

$$2 = -6A + B(0) + C(0)$$

$$\therefore A = \frac{2}{-6} \quad \boxed{A = -\frac{1}{3}}$$

Put $s = -2$

$$(-2)^2+(+2)+2 = A(-2+2)(-2-3) + B(-2)(-2-3) + C(-2)(-2+2)$$

$$4+4 = A(0) + B(-2)(-5) + C(0)$$

$$8 = A(0) + B(+10)$$

$$8 = B(10)$$

$$8 \quad B = \frac{1}{10} = \frac{8^4}{10_5}$$

$$\boxed{B = \frac{4}{5}}$$

Put $s = 3$

$$(3)^2-3+2 = A(3+2)(3-3) + B(3)(3-3) + C(3)(3+2)$$

$$9-1 = A(0) + B(0) + C(15)$$

$$\boxed{C = \frac{8}{15}}$$

$A = -\frac{1}{3}$, $B = \frac{4}{5}$, $C = \frac{8}{15}$ in (1)

$$\frac{s^2-s+2}{s(s+2)(s-3)} = \frac{-1/3}{s} + \frac{4/5}{s+2} + \frac{8/15}{s-3}$$

$$= -\frac{1}{3}\left(\frac{1}{s}\right) + \frac{4}{5}\left(\frac{1}{s+2}\right) + \frac{8}{15}\left(\frac{1}{s-3}\right)$$

$$L^{-1}\left[\frac{s^2-s+2}{s(s+2)(s-3)}\right] = -\frac{1}{3}L^{-1}\left(\frac{1}{s}\right) + \frac{4}{5}L^{-1}\left(\frac{1}{s+2}\right) + \frac{8}{15}L^{-1}\left(\frac{1}{s-3}\right)$$

$$= -\frac{1}{3}(1) + \frac{4}{5}(e^{-2t}) + \frac{8}{15}(e^{3t})$$

3. Find $L^{-1}\left(\frac{s}{(s^2+a^2)(s^2+b^2)}\right)$

$$\frac{s}{(s^2+a^2)(s^2+b^2)} = \frac{As+B}{s^2+a^2} + \frac{Cs+D}{s^2+b^2}$$

$$s = (As+B)(s^2+b^2) + (Cs+D)(s^2+a^2)$$

$$s = As^3 + Asb^2 + Bs^2 + Bb^2 + Cs^3 + Csa^2 + Ds^2 + Da^2$$

~~$s = As^3 + A$~~

Equalizing the coefficient of s^3

$$0 = A + C$$

$$A = -C \ ; \ C = -A$$

Equality the coefficient of s^2

$$0 = B + D \Rightarrow B = -D, \ D = -B$$

Equality the coefficient of s, $\therefore A = \frac{1}{b^2-a^2}$

$$1 = Ab^2 + Ca^2 \Rightarrow 1 = Ab^2 - Aa^2$$

$$1 = A(b^2-a^2)$$

Find $L^{-1}[1/(s^2+a^2)(s^2+b^2)]$

$S = (AS+B)(s^2+b^2) + (CS+D)(s^2+a^2)$

$S = (DS+F)(S^2+b^2) + (CS+D)(S^2+a^2)$

$S = AS^3 + AS b^2 + BS^2 + CS^3 + CSa^2 + DS^2 + Da^2$

Equating the coefficient of s^3

$0 = A + C$

$A = -C$

$C = -A$

Equating the coefficient of s^2

$0 = B + D$

$-B = D$

$D = -B$

Equating the coefficient of S

$1 = Ab^2 + Ca^2$

$1 = Ab^2 - Aa^2$

$= A(b^2 - a^2)$

$A = \frac{1}{b^2-a^2}$

Equating the coefficient of constant

$0 = Bb^2 + Da^2$

$= Bb^2 - Ba^2$

$= B(b^2-a^2)$

$\boxed{B = 0}$, $\boxed{D = 0}$

$A = \frac{1}{b^2-a^2}$, $C = \frac{1}{a^2-b^2}$

$$\frac{S}{(s^2+a^2)(s^2+b^2)} = \frac{AS+B}{s^2+a^2} + \frac{CS+D}{s^2+b^2}$$

$$= \frac{\left(\frac{1}{b^2-a^2}\right)s}{s^2+a^2} + \frac{\left(\frac{1}{a^2-b^2}\right)s}{s^2+b^2}$$

$$= \frac{\left(\frac{1}{a^2-b^2}\right)s}{s^2+b^2} - \frac{\left(\frac{1}{a^2-b^2}\right)s}{s^2+a^2}$$

$$= \frac{1}{a^2-b^2}\left[\frac{s}{s^2+b^2} - \frac{s}{s^2+a^2}\right]$$

$$L^{-1}\left[\frac{s}{(s^2+a^2)(s^2+b^2)}\right] = \frac{1}{a^2+b^2}\left[L^{-1}\left(\frac{s}{s^2+b^2}\right) - L^{-1}\left(\frac{s}{s^2+a^2}\right)\right]$$

$$= \frac{1}{a^2-b^2}\left[\cos bt - \cos at\right]$$

Find $L^{-1}\left[\dfrac{14s+10}{49s^2+28s+13}\right]$ $(a+b)^2 = a^2+b^2+2ab$

$49s^2+28s+13 = 49s^2+28s+4-4+13$

$= (7s+2)^2+9$

$= (7s+2)^2+3^2$

$= L^{-1}\left[\dfrac{14s+10}{(7s+2)^2+3^2}\right]$

$= L^{-1}\left[\dfrac{14s+10}{49[(s+2/7)^2+(3/7)^2]}\right]$

$= L^{-1}\left[\dfrac{14s+10}{49[(s+2/7)^2+(3/7)^2]}\right]$

$= L^{-1}\left[\dfrac{14(s+2/7)+6}{49(s+2/7)^2+(3)^2}\right]$

$= L^{-1}\left(\dfrac{14(s+2/7)}{49(s+2/7)^2+3^2}\right) + L^{-1}\left(\dfrac{6}{49(s+2/7)^2+(3)^2}\right)$

$= \dfrac{14}{49}L^{-1}\left(\dfrac{(s+2/7)}{(s+2/7)^2+(3/7)^2}\right) + L^{-1}\left(\dfrac{6}{(s+2/7)^2+(3/7)^2}\right)$

$= \dfrac{14}{49}L^{-1}\dfrac{(s+2/7)}{(s+2/7)^2+(3/7)^2} + \dfrac{6}{49}L^{-1}\dfrac{1}{(s+2/7)^2+(3/7)^2}$

multiply & divided by 3/7

$= \dfrac{14}{49}e^{-2/7}\cos(3/7)t + \dfrac{6}{49}\times\dfrac{7}{3}\left(\dfrac{3/7}{(s+2/7)^2+(3/7)^2}\right)$

$= \dfrac{14}{49}e^{-2/7}\cos(3/7)t + \dfrac{2}{7}[e^{-2/7}\sin(3/7)t]$

$L[e^{-at}f(t)] = F(s+a)$

$e^{-at}L^{-1}[f(s)] = L^{-1}F(s+a)$

1. Find $L^{-1}\left[\dfrac{5s-13}{s^2-6s+15}\right]$

Take

$s^2-6s+15 = s^2-6s+9+6$

$= (s-3)^2+6$

$L^{-1}\left[\dfrac{5s-13}{(s-3)^2+6}\right] = L^{-1}\left[\dfrac{5s-13}{(s-3)^2+(\sqrt{6})^2}\right]$

multiply and divided by 3, addition and subtraction by s

$= L^{-1}\left[\dfrac{5[(s-3)+3]}{(s-3)^2+(\sqrt{6})^2}\right]$

$= L^{-1}\left[\dfrac{5[(s-3)+3]-13}{(s-3)^2+(\sqrt{6})^2}\right]$

$= L^{-1}\left[\dfrac{5(s-3)+15-13}{(s-3)^2+(\sqrt{6})^2}\right]$

$= L^{-1}\left[\dfrac{5(s-3)+2}{(s-3)^2+(\sqrt{6})^2}\right]$

$= L^{-1}\left[\dfrac{5(s-3)}{(s-3)^2+(\sqrt{6})^2}\right] + L^{-1}\left[\dfrac{2}{(s-3)^2+(\sqrt{6})^2}\right]$

$$= 5L^{-1}\left[\frac{s-3}{(s-3)^2+(\sqrt{6})^2}\right] + 2L^{-1}\left[\frac{1}{(s-3)^2+(\sqrt{6})^2}\right]$$

$$\cancel{=5L^{-1}}$$

$$= 5\left[\cos\sqrt{6}\,t\right] e^{3t} + \frac{2}{\sqrt{6}}\left[\frac{\sqrt{6}}{(s-3)^2+(\sqrt{6})^2}\right]$$

$$= 5e^{3t}\left[\cos\sqrt{6}t + \frac{2}{\sqrt{6}} e^{3t}\sin\sqrt{6}t\right]$$

$$= e^{3t}\left[5\cos\sqrt{6}t + \frac{2}{\sqrt{6}}\sin\sqrt{6}t\right]$$

$$= e^{3t}\left[5\cos\sqrt{6}t + \frac{2}{\sqrt{6}}\sin\sqrt{6}t\right]$$

4. Find $L\,[49s^2+28s\ldots$

$49s^2+28s+13 = 49s^2+28s+4-4+13$

$= (7s+2)^2+9 \quad (a+b)^2 = a^2+2ab+b^2$

$= (7s+2)+3^2$

$L^{-1}\left[\frac{14s+10}{49s^2+28s+13}\right] = L^{-1}\left[\frac{14s+10}{(7s+2)^2+(3)^2}\right]$

$= L^{-1}\left[\frac{14s+10}{49\left[(s+2/7)^2+(3)^2\right]}\right]$

$= L^{-1}\left[\frac{14s+4-4+10}{49\left[(s+2/7)^2+(3)^2\right]}\right]$

$= L^{-1}\left[\frac{14(s+2/7)+6}{49\left[(s+2/7)^2+(3)^2\right]}\right]$

$= L^{-1}\frac{14\,(s+2/7)}{49\left[(s+2/7)^2+(3)^2\right]} + L^{-1}\left(\frac{6}{49\left[(s+2/7)^2+(3)^2\right]}\right.$

$= \frac{1}{49}L^{-1}\frac{14\,(s+2/7)}{(s+2/7)^2+(3/7)^2} + \frac{1}{49}L^{-1}\left(\frac{6}{(s+2/7)^2+(3/7)^2}\right.$

$= \frac{14}{49}L^{-1}\frac{(s+2/7)}{(s+2/7)^2+(3/7)^2} + \frac{6}{49}L^{-1}\frac{1}{(s+2/7)^2+(3/7)^2}$

$= \frac{14}{49}e^{-2/7}\cos(3/7)t + \frac{6^2}{49} \times \frac{1}{3}\left(\frac{3/7}{(s+2/7)^2+(3/7)^2}\right)$

$= \frac{14}{49}e^{-2/7}\cos(3/7) + \frac{2}{7}\left[e^{-2/7}\sin(3/7)\,t\right]$

$L\left[e^{-at}f(t)\right] = F(s+a)$

$e^{-at}\;L^{-1}\left[f(s)\right] = L^{-1}F(s+a)$

1. Find $L^{-1}\left[\frac{5s-13}{s^2-6s+15}\right]$

Take $s^2-6s+15 = s^2-6s+9+6$

$= (s-3)^2+6$

$L^{-1}\left[\frac{5s-13}{(s-3)^2+6}\right] = L^{-1}\left[\frac{5s-13}{(s-3)^2+(\sqrt6)^2}\right]$

mul & div by 3 add & sub by 3

$= L^{-1}\left[\frac{5\left[(s-3)+3\right]-13}{(s-3)^2+(\sqrt6)^2}\right]$

$= L^{-1}\left[\frac{5(s-3)+15-13}{(s-3)^2+(\sqrt6)^2}\right] = L^{-1}\left[\frac{5(s-3)+2}{(s-3)^2+(\sqrt6)^2}\right]$

$= L^{-1}\left[\frac{5(s-3)}{(s-3)^2+(\sqrt6)^2}\right] + L^{-1}\left[\frac{2}{(s-3)^2+(\sqrt6)^2}\right]$

$= 5L^{-1}\left[\frac{s-3}{(s-3)^2+(\sqrt6)^2}\right] + 2L^{-1}\left[\frac{1}{(s-3)^2+(\sqrt6)^2}\right]$

$= 5\left[\cos\sqrt6\,t\right]e^{3t} + \frac{2}{\sqrt6}\left[\frac{\sqrt6}{(s-3)^2+(\sqrt6)^2}\right]$

$= 5e^{3t}\left[\cos\sqrt6\,t + \frac{2}{\sqrt6}e^{3t}\sin\sqrt6\,t\right]$

$= e^{3t}\left[5\cos\sqrt6\,t + \frac{2}{\sqrt6}\sin\sqrt6\,t\right]$

$= e^{3t}\left[5\cos\sqrt6\,t + \frac{2}{\sqrt6}\sin\sqrt6\,t\right]$

2. Find $L^{-1}\left[\frac{1}{(s+1)(s^2+2s+5)}\right]$

$$\frac{1}{(s+1)(s^2+2s+5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2s+5} \quad \text{--- (1)}$$

$1 = A(s^2+2s+5) + (Bs+C)(s+1)$

$= As^2+2sA+5A+Bs^2+Bs+Cs+C$

Equating the coeff of s^2

$0 = A+B \Rightarrow A = -B \Rightarrow \boxed{B=-A}$

Equating the coefficient s

$0 = 2A+B+C$

$0 = 2(B)+B+C$

$0 = -B+C$

$0 = 2A+B+C$

$= 2A-A+C$

$= A+C$

$C = -A$

Equating the constant term

$1 = 5A+C \Rightarrow 1 = 5A-A \Rightarrow 4A=1 \Rightarrow \boxed{A=\frac{1}{4}}$

substitute A, B, C values to eqn (1)

$$\frac{1}{(s+1)(s^2+2s+5)} = \frac{1/4}{(s+1)} + \frac{-\frac{1}{4}s+(-\frac{1}{4})}{s^2+2s+5}$$

$= L^{-1}\left[\frac{1}{(s+1)(s^2+2s+5)}\right]$

$= \frac{1}{4}L^{-1}\left[\frac{1}{s+1}\right] - \frac{1}{4}\left[\frac{s+1}{s^2+2s+5}\right]$

$= \frac{1}{4}L^{-1}\left[\frac{1}{s+1}\right] - \frac{1}{4}\left(\frac{s+1}{s^2+2s+4+1}\right)$

$= \frac{1}{4}L^{-1}\left(\frac{1}{s+1}\right) - \frac{1}{4}\left[\frac{s+1}{(s+1)^2+(2)^2}\right]$

$= \frac{1}{4}e^{-t} - \frac{1}{4}e^{-t}\cos 2t$

$= \frac{1}{4}e^{-t}(1-\cos 2t) = \frac{1}{2}e^{-t}\left(\frac{1-\cos 2t}{2}\right)$

$= \frac{1}{2}e^{-t}\sin^2 t$

$\therefore L^{-1}\frac{1}{(s+1)(s^2+2s+5)} = \frac{e^{-t}}{2}\sin^2 t$

3. Find $L^{-1}\frac{1}{(s+a)^n}$

$L^{-1}\left[\frac{1}{(s+a)^n}\right] = e^{-at} L^{-1}\left[\frac{1}{s^n}\right]$

$= e^{-at} L^{-1}\left[\frac{(n-1)!}{(n-1)!\,s^n}\right]$

$= \frac{e^{-at}}{(n-1)!}\left[L^{-1}\frac{(n-1)!}{s^n}\right]$

$= \frac{e^{-at}}{(n-1)!}t^{n-1}$

$\therefore L^{-1}\left(\frac{1}{(s+a)^n}\right) = \frac{e^{-at}}{(n-1)!}t^{n-1}$

Type - III

$L^{-1}[s(F(s))] = f'(t)$ if $f(0)=0$

$L^{-1}[s^2 F(s)] = f''(t)$ if $f(0) = f'(0) = 0$

1. Find $L^{-1}\left[\frac{s}{(s+3)^2+4}\right]$

$L^{-1}\left[\frac{s}{(s+3)^2+4}\right]$

$F(s) = \left[\frac{s}{(s+3)^2+4}\right]$

$L^{-1}[s(F(s))] = f'(t)$

$f(t) = L^{-1}[f(s)] = L^{-1}\left[\frac{1}{(s+3)^2+(2)^2}\right]$

Multiply & divide by 2

$F(t) = \frac{e^{-3t}}{2}L^{-1}\left[\frac{2}{s^2+a^2}\right] = \frac{e^{-3t}}{2}[\sin 2t]$

$F'(t) = \frac{d}{dt}(d\,f(t)) = \frac{d}{dt}\left[\frac{e^{-3t}}{2}\sin 2t\right]$

$= \frac{1}{2}\left[e^{-3t}\sin 2t\right]$

$= \frac{1}{2}\left[e^{-3t}\cos 2t(2) + \sin 2t\, e^{-3t}(-3)\right]$

$= \frac{1}{2}\left[2e^{-3t}\cos 2t - 3\sin 2t\, e^{-3t}\right]$

2. Find $L^{-1}\left[\frac{s^2}{(s-2)^3}\right]$

$F(s) = \left[\frac{1}{(s-2)^3}\right]$

$f(t) = L^{-1}[F(s)] = L^{-1}\left[\frac{1}{(s-2)^3}\right]$

$= e^{2t}\left[\frac{1}{(s)^3}\right] = \frac{e^{2t}}{2} L^{-1}\left[\frac{2!}{s^{2+1}}\right]$

$f(t) = \frac{e^{2t}}{2} t^2$

$f'(t) = \frac{d}{dt}[f(t)] = \frac{1}{2}[e^{2t} t^2]$

$= \frac{1}{2}[e^{2t}(2t) + t^2 e^{2t}(2)]$

$f'(t) = \frac{1}{2}(2)[e^{2t} t + e^{2t} t^2]$

$f'(t) = [e^{2t} t + e^{2t} t^2]$

$f''(t) = \frac{d}{dt}[f'(t)]$

$= e^{2t}(1) + t e^{2t}(2) + e^{2t}(2t) + t^2 e^{2t}(2$

$= e^{2t} + 2te^{2t} + 2te^{2t} + 2t^2 e^{2t}$

$f''(t) = e^{2t} + 4te^{2t} + 2t^2 e^{2t}$