Analytical Geometry - 3D

Objectives:

1. To study 3 dimentional Cartesian Co-ordinates system

2. To Enable the students to develop their skill 3 dimensions.

UNIT- I

Co-ordinates in space - direction Cos of a line in space angle between lines in space equation of a plane in normal form. Angle between planes - Distance of a plane from a point.

UNIT - II

Straight lines in space lines of intersection of planes - plane Containing a line coplanar lines - skew lines and shortest distance between skew lines - length of the perpendicular from point to line

UNIT - III

General equation of a sphere section of sphere by plane - tangent Condition of tangency system of spheres as generated by two spheres - system of sphere generated by a sphere and plane

UNIT - IV

The equation of surface Cone - interaction of straight line and quadric Cone tangent plane and normal

UNIT - V

Condition for plane to tuch the quadric Cone. angle between the lines in which the plane cuts the Cone. Condition that the Cone has three mutually perpendicular generators. Central quadries intersection of a line and quadric tangents and tangents plane Conditions for the plane to touch the Conicoid.

BOOKS FOR STUDY :-

1. Shanthi Narayanan and mittal P.K. Analytical Solid Geometry 16th Edition S. chand & Co., New delhi

2. Narayanan and Manickavasagam, pillay T.K. Treatment as Analytical Geometry. S. Viswanathan (Printers & publishers) Pvt. Ltd.,

5m U.Q

Show that the equation of a right circle Cone whose Vertex is O (origin) axis OZ and semi vertical angle α is $x^2+y^2 = z^2 \tan^2\alpha$

$$\text{Cone} : \frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

$$\cos\alpha = \frac{l(x)+m(y)+n(z)}{\sqrt{l^2+m^2+n^2}\sqrt{x^2+y^2+z^2}}$$

Since the point P line in the given Sphere $ax^2 + by^2 + cz^2 = 1$ and the plane $l_1 x + m_1 y + n_1 z = P$

$$\Rightarrow a(dl)^2 + b(dm)^2 + c(dn)^2 = 1$$

$$\Rightarrow a\lambda^2 l^2 + b\lambda^2 m^2 + c\lambda^2 n^2 = 1$$

$$\Rightarrow \lambda^2(al^2 + bm^2 + cn^2) = 1 \quad — (1)$$

and $l_1(dl) + m_1(\lambda m) + n_1(\lambda n) = P$

$$\Rightarrow \lambda(ll_1 + mm_1 + nn_1) = P \quad — (2)$$

Eliminating λ from (1) & eq (2) we get

$$(2) \Rightarrow d = \frac{P}{ll_1 + mm_1 + nn_1}$$

$$(1) \Rightarrow \left(\frac{P}{ll_1 + mm_1 + nn_1}\right)^2 \cdot (al^2 + bm^2 + cn^2) = 1$$

$$\Rightarrow \left(\frac{P^2}{ll_1 + mm_1 + nn_1}\right)(al^2 + b^2m + cn^2) = 1$$

$$\Rightarrow (al^2 + bm^2 + cn^2) = \frac{(ll_1 + mm_1 + nn_1)^2}{P^2}$$

The equation of the Cone is $ax^2 + by^2 + cz^2 = 1$

$$= \frac{(l_1 x + m_1 y + n_1 z)^2}{P^2}$$

3. Find the equation of right Circular cone whose vertex is at the Origin whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has Vertical angle.

Soln: Let the generator of the right Circular Cone line.

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n} \longrightarrow (1)$$

given ax is $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ — (2)

Given that the Vertical of angle of the right Circular Cone is 30°

(ie) $\frac{60^\circ}{2} = 30^\circ$

Since the angle between (1) & (2) is 30°

$$\cos 30^\circ = \frac{l(1)+m(2)+n(3)}{\sqrt{l^2+m^2+n^2}\sqrt{1^2+2^2+3^2}}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{l+2m+3n}{\sqrt{l^2+m^2+n^2}\sqrt{14}}$$

$$\Rightarrow \frac{3}{4} = \frac{(l+2m+3n)^2}{l^2+m^2+n^2\ (7)}$$

$\Rightarrow 21(l^2+m^2+n^2) = 2(l+2m+3n)^2$

$\Rightarrow 21l^2+21m^2+21m^2 = 2(l^2+4m^2+9n^2+4lm+12mn+6ln)$

$\Rightarrow 21l^2+21m^2+21m^2-2l^2-8m^2-18m^2-8lm-24mn-12ln=0$

$\Rightarrow 19l^2+13m^2+3n^2-8lm-8lm-24mn-12ln=0.$

The equation of the right Circular Cone is

$19x^2+13y^2+3z^2-8xy-24yz-12xz=0.$

4. Find the equation of the Cone whose Vertex is $(1, 2, 3)$ and which passes through the Circular $x^2+y^2+z^2=4$, $x+y+z=1$

Let the generator of the Cone be

$$\frac{x-1}{l} = \frac{y-2}{m} = \frac{z-3}{n}$$

Let $\frac{x-1}{l} = \frac{y-2}{m} = \frac{z-3}{n} = \lambda$

$x = \lambda l + 1$, $y = \lambda m + 2$, $z = \lambda n + 3$

$\Rightarrow$ Any point on the line OP is the from $(\lambda l+1, \lambda m+2, \lambda n+3)$

Since P lies in the line OP

$\Rightarrow P = (\lambda l + 1, \lambda m+2, \lambda n+2$

Also P lies in the circle $x^2+y^2+z^2=4$

$x+y+z=1$

$\Rightarrow (\lambda l+1)^2 + (\lambda m+2)^2 + (\lambda n+3)^2 = 4$

$\Rightarrow \lambda^2 l^2 + 1 + 2\lambda l + \lambda^2 m^2 + 4 + 4\lambda m + \lambda^2 n^2 + 9 + 6\lambda n$

$\Rightarrow \lambda^2(l^2+m^2+n^2) + \lambda(2l+4m+6n) + 10 = 0$ — (1)

and $(\lambda l+1) + (\lambda m+2) + (\lambda n+3) = 1$

$\Rightarrow \lambda(l+m+n) + 1+2+3 = 0$

$\Rightarrow \lambda(l+m+n) + 5 = 0$ — (2)

Eliminating λ from (1) & (2)

we get

(2) $\Rightarrow \lambda = \frac{-5}{l+m+n}$

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$$① \Rightarrow \left(\frac{-5}{l+m+n}\right)^2 (l^2+m^2+n^2) + \left(\frac{-5}{l+m+n}\right)(2l+4m+6n)+10 = 0$$

$$\Rightarrow \frac{25(l^2+m^2+n^2) - 5(2l+4m+6n)(l+m+n) + 10(l+m+n)^2}{(l+m+n)^2} = 0$$

$$\Rightarrow 25(l^2+m^2+n^2) - 5(2l+4m+6n)(l+m+n) + 10(l+m+n)^2 = 0$$

$$\Rightarrow 25l^2 + 25m^2 + 25n^2 - 5(2l^2 + 2lm + 2lm + 4lm + 4m^2 + 6nm + 2ln + 4mn + 6n^2) + 10(l+m+n)^2 = 0$$

$$\Rightarrow 25l^2 + 25m^2 + 25n^2 - 10l^2 - 10lm - 10lm - 10lm - 20lm - 20m^2 - 30mn - 10ln - 20mn - 30n^2 + 10(l+m+n)^2 = 0$$

$$\Rightarrow 25l^2 + 25m^2 + 25n^2 - 10l^2 - 10lm - 10ln - 20lm - 20m^2 - 30mn - 10ln - 20mn - 30n^2 + 10(l^2+m^2+n^2+2lm+2mn+2ln) = 0$$

$$\Rightarrow 25l^2 + 25m^2 + 25n^2 - 10l^2 - 10lm - 10ln - 20ml - 20m^2 - 20mn - 30nl - 30mn - 30n^2 + 10l^2 + 10m^2 + 10n^2 + 20lm + 20mn + 20ln = 0$$

$$\Rightarrow 25l^2 + 15m^2 + 5n^2 - 10lm - 20ln - 30mn = 0$$

$\therefore$ The equation required of the Cone

$$\Rightarrow 5(x-1)^2 + 3(y-2)^2 + (z-3)^2 - 2(x-1)(y-2) - 4(x-1)(z-3) - 6(y-2)(z-3) = 0$$

$$= 5(x^2 - 1 - 2x) + 3(y^2 + 4 - 4y) + (1^2 + 9 - 6z) - 2[xy - 2x - y + z) - 4(xz - 3x - z + 3] - 6(yz - 3y - 2z + 6] = 0$$

$$\Rightarrow 5x^2 + 5 - 10x + 3y^2 - 112 - 12y + z^2 + 9 - 6z - 2xy + 2xy + 2x + 2y - 4 - 4x + 12x + 4x - 12 - 6yz + 8y + 12z - 36 = 0$$

$$\Rightarrow 5x^2 + 3y^2 + z^2 - 2xy + 4xz - 6yz + 6x + 8y + 10z - 26 = 0$$

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(5). 10m.

Show that the plane $3x+2y+z=k$. touches the ellipsed $3x^2+4y^2+z^2=20$. if $k=\pm 10$ & Find P.Q

Soln. Given: the ellipsed is $3x^2+4y^2+z^2=20$

$\Rightarrow \frac{3}{20}x^2+\frac{4}{20}y^2+\frac{1}{20}z^2=1 \Rightarrow \frac{3}{20}x^2+\frac{4}{20}y^2+\frac{1}{20}z^2=1$

★ The Condition for the plane $3x+2y+z=k$. touch the ellipsed $\frac{3}{20}x^2+\frac{1}{5}y^2+\frac{1}{20}z^2=1$

$$\frac{3^2}{3/20}+\frac{2^2}{1/5}+\frac{1^2}{1/20}=k^2$$

$$\Rightarrow 9\times\frac{20}{3}+4\times\frac{5}{1}+1\times\frac{20}{1}=k^2$$

$$\Rightarrow 60+20+20=k^2$$

$$\Rightarrow 100=k^2 \quad (\text{when } k=\pm 10)$$

$$= k=\sqrt{100}$$

$$\cancel{100}=100 \quad \therefore k=\pm 10$$

The plane $3x+2y+z=k$ touch the ellipsed $\frac{3}{20}x^2+\frac{1}{5}y^2+\frac{1}{20}z^2=0$.

when $k=\pm 10$. The tangent plane So the Conicoid. $\frac{3}{20}x^2+\frac{1}{5}y^2+\frac{1}{20}z^2=1$ are.

$$3x+2y+1z=0, \quad 3x+2y+1z=-10$$

★ Finding the Contact point of the tangent plane $3x+2y+1z=-10$ to the Conicoid

$$\frac{3}{20}x^2+\frac{1}{5}y^2+\frac{1}{20}z^2=1$$

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* Let the plane $3x+2y+z=0$ touches the Conicoid at the point (x_1, y_1, z_1).

=> The equation of tangent plane to the Conicoid $\frac{3}{20}x^2+\frac{1}{5}y^2+\frac{1}{20}z^2=1$ at (x_1, y_1, z_1) is

$$\frac{3}{20}xx_1+\frac{1}{5}yy_1+\frac{1}{20}zz_1=1.$$

Equation represents the same plane :.

$$\frac{\frac{3}{20}x_1}{3}=\frac{1/5\, y_1}{2}=\frac{1/20\, z_1}{1}=\frac{1}{10}$$

$$\Rightarrow \frac{x_1}{20}=\frac{y_1}{10}=\frac{z_1}{20}=\frac{1}{10}$$

$$\Rightarrow \frac{x_1}{20}=\frac{1}{10},\quad \frac{y_1}{10}=\frac{1}{10};\quad \frac{z_1}{20}=\frac{1}{10}$$

$$\Rightarrow x_1=20,\quad y_1=1,\quad z_1=2.$$

* Finding the content point of tangent plane $3x+2y+1z=-10$ to the Conicoid.

$$\frac{3}{20}x^2+\frac{1}{5}y^2+\frac{1}{20}z^2=1.$$

* Let the plane $3x+2y+1z=-10$. touches the Conicoid at the point (x_2, y_2, z_2)

=> The equation of the tangent plane to the Conicoid.

$$\frac{3}{20}x^2+\frac{1}{5}y^2+\frac{1}{20}z^2=1 \text{ at } (x_2, y_2, z_2)$$

$$\frac{3}{20}xx_2+\frac{1}{5}yy_2+\frac{1}{20}zz_2=1.$$

Equation represents the same plane.

$$\Rightarrow \frac{\frac{3}{20}x_2}{3} = \frac{\frac{1}{15}y_2}{2} = \frac{\frac{1}{20}z_2}{1} = -\frac{1}{10}$$

$$\Rightarrow \frac{x_2}{20} = \frac{y_2}{10} = \frac{z_2}{20} = \frac{-1}{10}$$

$$\Rightarrow \frac{x_2}{20} = \frac{-1}{10} ; \frac{y_2}{10} = \frac{-\cancel{1}2}{20} ; \frac{z_2}{20} = \frac{-1}{10}$$

$= x_2 = -2$, $y_2 = -1$, $z_2 = -2$.

Distance b/w $P(2,1,2)$ & $Q(-2,-1,-2)$

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$= \sqrt{(-2-2)^2+(-1-1)^2+(-2-2)^2}$$

$$= \sqrt{(-4)^2+(-2)^2+(-4)^2}$$

$$= \sqrt{16+4+16}$$

$$= \sqrt{36}$$

$PQ = 6$

Hence proved.

6. 5m

Find the equation of the cone with the Vertex and the base curve in which the Surface $ax^2+by^2+cz^2=1$ is cut by the $l_1x+m_1y+n_1z=p$

Soln: Let the generator of the cone by

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

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The Co-ordinator of the point where the generator meets the base curve are the form $(\lambda l, \lambda m, \lambda n)$.

When λ is given by the equation.

$$\lambda^2 (al^2 + bm^2 + cn^2) = 1 \longrightarrow (1)$$

and $\lambda (ll_1 + mm_1 + nn_1) = p \longrightarrow (2)$

$$al^2 + bm^2 + cn^2 = \frac{(l l_1 + m m_1 + n n_1)^2}{p^2}.$$

Hence the equation of the Curve.

$$ax^2 + by^2 + cz^2 = \frac{(l_1 x + m_1 y + n_1 z)^2}{p^2}.$$

(4). Intersection of a Straight line and quadratic Cone :-

* Let the equation of the quadratic Cone be.

$$f(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0.$$

when.

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

The Co-ordinates of any point of the line.

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} \longrightarrow (1)$$

and given by.

$$(x + \lambda l ; y + \lambda m , z + \lambda n).$$

If the line meet the Cone, the parameter of the intersecting point are given by

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$$\Rightarrow a(x_1+\lambda l)^2 + b(y_1+\lambda m)^2 + c(z_1+\lambda n)^2 + 2f(y_1+\lambda m)(z_1+\lambda n) + 2g(z_1+\lambda n)(x_1+\lambda l) + 2h(x_1+\lambda l)(y_1+\lambda m) = 0$$

(8) Find the condition for the equation.

$$F(x,y,z) = ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0.$$

to represents a cone. show that co-ordinates of the vector satisfies the equation

$$\frac{\partial F}{\partial x}=0, \quad \frac{\partial f}{\partial y}=0, \quad \frac{\partial F}{\partial z}=0, \quad \frac{\partial F}{\partial t}=0$$

where it's used to more $F(x,y,z)$ homogeneous and equated to unity after differentiation.

Soln

If the orgin shified to the vertex to transformed equation at the cone homogeneous.

(ie). If has no x, y, z and constant terms.

Let (α, β, γ) be the co-ordinates of the vertex

Vertex shifting the orgin to (α, β, γ) the equating of the cone transform into

$$\Rightarrow a(x+\alpha)^2+b(y+\beta)^2+c(z+\gamma)^2+2f(y+\beta)(z+\gamma)+2g(z+\gamma)(x+\alpha)+2h(x+\alpha)(y+\beta)+2u(x+\alpha)+2v(y+\beta)+2w(z+\gamma)+d=0$$

* The equation represents by a cone with vertex at the orgin if it homogenous.

Co-efficients of $x \pm 2a\alpha + 2h\beta + 2g\gamma = 2u = 0$

(ie) $\frac{\partial}{\partial \alpha} F(\alpha, \beta, \gamma) = 0$.

Similarly terms in the transformed equation of the Cone.

$$= a\alpha^2 + b\beta^2 + c\gamma^2 + 2f\beta\gamma + 2g\gamma\alpha + 2h\alpha\beta + 2u\alpha + 2v\beta + 2w\gamma + d = 0$$

$$= 2(a\alpha + h\beta + g\gamma + w + \beta(h\alpha + g\beta + f\gamma + u) + \gamma(g\alpha + f\beta + c\gamma + w)$$
$$+ u\alpha + v\beta + w\gamma + d.$$

$$= \frac{1}{2}\frac{\partial}{\partial f}\left[a\alpha^2 + b\beta^2 + c\gamma^2 + 2f\beta\gamma + 2g\gamma\alpha + 2h\alpha\beta + 2u\alpha t + 2v\beta t + 2w\gamma t + dt^2\right.$$

Hence the $\frac{\partial}{\partial F} F = 0$ where t is used to make $F(\alpha, \beta, \gamma)$ homogeneous and it equated to Unity after differentiation

$$u\alpha + v\beta + w\gamma + d = 0$$

Eliminating α, β, γ from the equation we get the Condition.

$$\begin{vmatrix} a & h & g & u \\ h & b & f & v \\ g & f & c & w \\ u & v & w & d \end{vmatrix} = 0.$$

The Coordinates (α, β, γ) at the Vertex of the Cone given by the equation.

$$\frac{\partial F}{\partial x} = 0, \quad \frac{\partial F}{\partial \beta} = 0, \quad \frac{\partial F}{\partial \gamma} = 0 \text{ and } \frac{\partial F}{\partial t}.$$

where t is made to used $F(\alpha, \beta, \gamma)$ homogeneous and is equated to Unity after differentiation

(6) Find the Condition for the equation $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ to represent a right circular cone obtain the equation of the axis and the vertical angle of the cone.

Soln. Let the equation of the axis be.

$$\frac{x}{p}=\frac{y}{q}=\frac{z}{r} \longrightarrow (1)$$

and Semi Vertical angle be α

Let one of the generators of the cone be,

$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n} \longrightarrow (2)$$

The angle b/w (1) & (2) is α for all values of l, m, n.

$$\cos\alpha = \frac{lp+mq+nr}{\sqrt{l^2+m^2+n^2}\sqrt{p^2+q^2+r^2}}$$

Squaring & Simplifying we get,

$$(l^2+m^2+n^2)(p^2+q^2+r^2)\cos^2\alpha-(lp+mq+nr)^2=0$$

(ie) $l^2\left[(p^2+q^2+r^2)\cos^2\alpha-p^2\right]+m^2\left[(p^2+q^2+r^2)\cos^2\alpha-q^2\right]$

$$+n^2\left[(p^2+q^2+r^2)\cos^2\alpha-r^2\right]-2qrmn-2rpnm-2pqnm=0$$

Hence the equation of the Curve is

$$x^2\left[(p^2+q^2+r^2)\cos^2\alpha-p^2\right]+y^2\left[(p^2+q^2+r^2)\cos^2\alpha-q^2\right]+z^2\left[(p^2+q^2+r^2)\cos^2\alpha-r^2\right]-2pryz-2rpzx-2pqxy=0$$

This is equivalent to

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0 \longrightarrow (3)$$

Hence Comparing Co-efficient of $x^2, y^2, z^2, xy, yz, zx$, we have.

$$\frac{(p^2+q^2+r^2)\cos^2\alpha - p^2}{a} = \frac{(p^2+q^2+r^2)\cos^2\alpha - q^2}{b}$$

$$\frac{(p^2+q^2+r^2)\cos^2\alpha - r^2}{c} = \frac{-qr}{f} = \frac{-rp}{g} = \frac{-pq}{h}$$

Let each ratio be $-\frac{pqr}{k}$

$$p = \frac{k}{f}, \quad q = \frac{k}{g}, \quad r = \frac{k}{h}$$

$$\frac{(p^2+q^2+r^2)\cos^2\alpha - p^2}{a} = \frac{-pqr}{k}$$

Substituting the values of p, q, r in this equation.

we get,

$$\frac{k^2\left(\frac{1}{f^2}+\frac{1}{g^2}+\frac{1}{h^2}\right)\cos^2\alpha - \frac{k^2}{f^2}}{a} = \frac{-k^2}{fgh}$$

(ie) $$fgh\left(\frac{1}{f^2}+\frac{1}{g^2}+\frac{1}{h^2}\right)\cos^2\alpha = \frac{gh-af}{f}$$

Taking the second and third expression, insted of the first, we get

$$fgh\left(\frac{1}{f^2}+\frac{1}{g^2}+\frac{1}{h^2}\right)\cos^2\alpha = \frac{hf-bg}{g} \quad (2)$$

and $$fgh\left(\frac{1}{f^2}+\frac{1}{g^2}+\frac{1}{h^2}\right)\cos^2\alpha = \frac{fg-ch}{h} \quad (3)$$

from (1) and (2) & (3) we get

$$fx = gy = hz$$

The semi vertical angle is obtained from eqn (1), (2) & (3) ~~(4)~~ ~~tangent plane~~ ~~ar~~

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is $\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c} = p^2$.

5m

(1) Find the equation of the two tangent plane of the ellipsed $2x^2+2y^2+z^2 = 2$. which passes through the line $z=0$, $x+y=10$.

Soln.

Ellipsed $2x^2 + 2y^2 + z^2 = 2$

line $x+y = 10$

$x + y + k(z) = 10$ —— (1)

$2x^2 + 2y^2 + z^2 = 2$

$x^2 + y^2 + \frac{z^2}{2} = 1$ —— (2)

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$$\frac{(1)^2}{1} + \frac{(1)^2}{1} + \frac{k^2}{1/2} = (10)^2$$

$$1+1+2k^2 = 100$$

$$2+2k^2 = 100$$

$$2k^2 = 100-2$$

$$2k^2 = 98$$

$$k^2 = 98/2$$

$$k^2 = 49$$

$$k = \sqrt{49}$$

$$k = \pm 7$$

$$x+y+7z = 10$$

$$x+y-7z = 10$$

(2). Find the equation of two tangent plane of the ellipsed $7x^2-3y^2-z^2=-21$ which passes through the line $z=3$, $7x-6y+9=0$

Ellipsed $7x^2-3y^2-z^2 = -21$

$$z=3$$

$$7x-6y+9=0$$

$$7x-6y=-9$$

$$7x-6y+k(z-3) = -9$$

$$7x-6y+kz = 3k-9.$$

$$7x^2-3y^2-z^2 = -21$$

$$\frac{7}{-21}x^2 - \frac{3}{-21}y^2 - \frac{z^2}{-21} = 1$$

$$\frac{1}{-3}x^2 + \frac{1}{7y}y^2 + \frac{z^2}{21} = 1 \quad \text{— (2)}$$

$$\frac{(7)^2}{-1/3} + \frac{(-6)^2}{1/7} + \frac{k^2}{1/21} = (3k-9)^2$$

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$49 \times (-3) + 36 \times (7) + 21(k^2) = 9k^2 + 81 - 54k$.

$-147 + 252 + 21k^2 - 9k^2 - 81 + 54k = 0$.

$12k^2 + 54k + 24 = 0$

$2k^2 + 9k + 4 = 0$.

$7x - 6y - \frac{1}{2}z = 3k\left(\frac{-1}{2}\right) - 9$

$7x - 6y - 4z = 3k(4) - 9$.

$7x - 6y - \frac{1}{2}z = 3\left(\frac{-1}{2}\right)9$

$7x - 6y - \frac{1}{2}z = -\frac{3}{2} - 9$.

$7x - 6y - \frac{z}{2} = \frac{-3-18}{2}$

$7x - 6y - \frac{z}{2} = \frac{21}{2}$

$7x - 6y - \frac{z}{2} = \frac{21}{2}$.

$7x - 6y - 4z = 3(4) - 9$.

$7x - 6y - 4z = 12 - 9$

$7x - 6y - 4z = 4$.

(3) Elipse : $3x^2 + 4y^2 + z^2 = 20$

Line $z = 10$.

line . $3x + 2y = 1\cancel{0}$.

Elipse : $3x^2 + 4y^2 + z^2 = 20$.

$z = 10$

$3x + 2y = 1\cancel{0}$

$3x + 2y + k(z - 10) = 1$

$3x + 2y + kz - 10k = 1$

$3x + 2y + kz = 10k + 1$ — (1)

$3x^2 + 4y^2 + z^2 = 20$

$$\frac{3}{20}x^2 + \frac{4}{20}y^2 - \frac{z^2}{20} = 1$$

$$\frac{3}{20}x^2 + \frac{1}{5}y^2 - \frac{z^2}{20} = 1 \quad \text{— (2)}$$

$$\cancel{9} \; \frac{3}{1/20}x^2 + \frac{1}{1/5}y^2 - \frac{k^2}{1/20} = 10k - 1$$

$$\frac{(3)^2}{3/20}x^2 + \frac{(1)^2}{1/5}y^2 - \frac{k^2}{1/20} = (10k - 1)^2$$

$$\frac{20(3)^2}{3}x^2 + 5(1)^2$$

UNIT - V

1. Condition for the plane $lx + my + nz = 0$ to touch the quadratic Cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

Soln Let (x_1, y_1, z_1) be the point of Contact the tangent plane at (x_1, y_1, z_1) is $x(ax_1 + hy_1 + gz_1) + y(hx_1 + by_1 + fz_1)$

$$+ z(gx_1 + fy_1 + cz_1) = 0$$

This is identical with the plane $lx + my + nz = 0$

$$\therefore \frac{ax_1 + hy_1 + gz_1}{l} = \frac{hx_1 + by_1 + fz_1}{m} = \frac{gx_1 + fy_1 + cz_1}{n}$$

If each Ratio of K,

$$ax_1 + hy_1 + gz_1 - Kl = 0 \quad \text{--- (1)}$$

$$hx_1 + by_1 + fz_1 - Km = 0 \quad \text{--- (2)}$$

$$gx_1 + fy_1 + cz_1 - Kn = 0 \quad \text{--- (3)}$$

Since (x_1, y_1, z_1) lies on $lx + my + nz = 0$

$$lx_1 + my_1 + nz_1 = 0 \quad \text{--- (4)}$$

Eliminating x_1, y_1, z_1 from question (i) (ii) (iii) (iv)

we get.

$$\begin{vmatrix} a & h & g & l \\ h & b & f & m \\ g & f & c & n \\ l & m & n & 0 \end{vmatrix} = 0.$$

Simplify we get.

$$Al^2 + Bm^2 + Cn^2 + 2fmn + 2Gnl + 2Hlm = 0 \quad \text{--- (5)}$$

where A, B, C, f, G, H are the Contactors of a, b, c, g, f, h in the determinant

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

$+2hxy=$

Multiplying ① by A, ② by H and ③ by G and adding we get

tangent

$\Delta x_1 = k(Al + Hm + Gn)$

$by_1 + fz_1)$

Since $\Delta = \Delta a + Hh + Gg$, $Ah + Hb + Gf = 0$.

$=0$

$Ag + Hf + Gc = 0$

$=0$

Similarly $\Delta y_1 = k(Hl + Bm + fn)$

$\Delta z_1 = k(Gl + fm + Cn)$

Hence the point of Contact is given by the equation

$$\frac{x_1}{Al + Hm + Gn} = \frac{y_1}{Hl + Bm + fn} = \frac{z_1}{Gl + Fm + Cn}$$

From condition ⑤

It can be seen that $\frac{x}{l} + \frac{y}{m} + \frac{z}{n}$ which is perpendicular to the plane.

(iii) (iv)

$lx + my + nz = 0$ at the Origin is a generator of the Cone.

$$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0$$

In the determinat's $= \begin{vmatrix} A & H & G \\ h & B & F \\ G & F & C \end{vmatrix}$

we get,

$A' = BC - F^2 = a\Delta'$, $F' = GH - AF = f\Delta'$

$B' = CA - G^2 = b\Delta'$, $G' = HF - BG = g\Delta^2$

$C' = AB - H^2 = c\Delta'$, $H' = FG - CH = h\Delta'$

Hence the perpendicular to the tangent plane to the cone (5) Generate the cone.

$$A'x^2 + B'y^2 + C'z^2 + 2F'yz + 2H'xy = 0$$

(ie) $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ — (7)

The cone (5) and (7) are said to be

B. Reciprocal.

V.V. Import.

(1) ~~Show~~ ~~Th~~

(2) Find the equation of tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contains the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{72}$.

Soln. The line is the Inferection of the planes $72x - 32y = $

$9x - 4y = 0$ and $2\,7y - 72y = 0$; $3y - 8z = 0$

Hence any plane passing through this line is of the form,

$$9x - 4y + K(3y - 8z) = 0.$$

ie;

$$9x + y(3\lambda - 4) - 8\lambda z = 0 \rightarrow (1)$$

The line touches the cone

$$9x^2 + 4y^2 + 16z^2 = 0 \quad (2)$$

Hence the normal to the plane

$$\frac{x}{9} = \frac{y}{3\lambda - 4} = \frac{z}{8\lambda} \quad (3)$$

is a generator of the Reciprocal cone of the cone (C_2)

Equation of the Reciprocal cone of (C_2) is

$$\frac{x^2}{9} - \frac{y^2}{4} + \frac{z^2}{16} = 0 \quad \text{--- (4)}$$

(3) is a generator of the cone (4).

$$\frac{9^2}{9} - \frac{(3\lambda - 4)^2}{4} + \frac{(-8\lambda)^2}{16} = 0$$

Simplify we get $7\lambda^2 + 24\lambda + 20 = 0$

ie ; $\lambda = -2$ (or) $\frac{-10}{7}$.

Hence the equation of the planes are

$9x - 10y + 16z = 0$, $63x - 58y + 80z = 0$.

(5). The angle b/w the lines in which the plane $ux + vy + wz = 0$ cuts the cone.

Example : 4

Find the eqn to the cone through the co-ordinates axis and the lies in which the plane $lx + my + nz = 0$ cuts the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

soln :

let the eqn of the cone passing through the co-ordinates axes by

$$Fyz + Gzx + Hxy = 0$$

Eliminating b/w

$lx + my + nz = 0$ and

$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

$lx + my = -nz$

$\frac{lx + my}{n} = -z$

$z^2 = \frac{(lx + my)^2}{n^2}$

we get

$$ax^2 + by^2 + \frac{c(lx + my)^2}{n^2} - \frac{2fy(lx + my)}{n} - \frac{2gx(lx + my)}{n} + 2hxy = 0$$

(ie) $x^2(an^2 + cl^2 - 2gln) + \dots + y^2(cm^2 + bn^2 - 2fmn) = 0$

similary eliminates b/w

$lx+my+nz=0$

$Fyz+Gzx+Hxy=0$

$\therefore \frac{Fy(lx+my)}{n} - \frac{Gx(lx+my)}{n} + Hxy = 0$

$Fylx + Fmy^2 \quad - Glx^2 + Gmxy$

$Glx^2 + \dots + Fmy^2 = 0$

since the two cones have common generators, we get

$$\frac{an^2+cl^2-2gln}{Gl} = \frac{cm^2+bn^2-2fmn}{Fm}$$

similary elimates 'x' we get condition

$$\frac{bl^2+am^2-2hml}{Hm} = \frac{an^2+cl^2-2gnl}{Gn}$$

$$\frac{an^2+cl^2-2ghn}{Gnl} = \frac{bl^2+am^2-2hlm}{Hlm}$$

$$= \frac{cm^2+bn^2-2fmn}{Fmn}$$

Hence

$$\frac{F}{l(cm^2+bn^2-2fmn)} = \frac{G}{m(cl^2+an^2-2gln)} = \frac{H}{n(am^2+bl^2-2hln)}$$

Hence the equation of the required cone is

$$l(cm^2+bn^2-2fmn)yz + m(cl^2+an^2-2gln)zx + n(am^2+bl^2-2hlm) = 0$$

Repeated Q.

5) Find the equation of the tangent planes to $x^2+y^2+4z^2=1$ which intersects in the line whose equation are $12x-3y-5=0$, $z=1$.

Any plane which passes through the line is given by

$$12x-3y-5+\lambda(z-1)=0$$

(i.e) $12x-3y+\lambda z-(\lambda+5)=0$ — ①

Let this plane touch the conicoid (x_1, y_1, z_1)

The eqn of the tangent plane at (x_1, y_1, z_1) is

$$x^2+y^2+4z^2=1 \quad [ax^2+by^2+cz^2=1]$$

(α, β, γ)

Eq. ① & ② represent the same plane

$$\frac{x_1}{12}=\frac{y_1}{-3}=\frac{4z_1}{\lambda}=\frac{1}{\lambda+5}$$

$$x_1=\frac{12}{\lambda+5}, \quad y_1=\frac{-3}{\lambda+5}, \quad z_1=\frac{\lambda}{4(\lambda+5)}$$

Since (x_0, y, z) lies on the conicoid.

$$x_1^2+y_1^2+4z_1^2=1$$

$$\left(\frac{12}{\lambda+5}\right)^2+\left(\frac{-3}{\lambda+5}\right)+4\left(\frac{\lambda}{4(\lambda+5)}\right)^2=1$$

(ie) $3\lambda^2+40\lambda-512=0$, $\lambda=80$, $\gamma=\frac{-64}{3}$

Hence the eqn at the tangent plane are

$$12x-3y-5+8(z-1)=0$$

$$12-3y-5 \;\&\; \frac{64}{3}(z-1)=0$$

(ie) $12x-3y+8z-13=0$ and $36x-9y-64z+49=0$

Central Quadrics

Definition :-

If $P(x_1, y_1, z_1)$ lies on the Surface

$$Ax^2 + By^2 + Cz^2 = 1 \quad \text{--- (1)}$$

$Q(-x_1, -y_1, -z_1)$ also lies the Surface and O the Origin is the midpoint of PQ

Hence all chords of (1) which pass through (1) are bisected at O.

For this reason (1) is called a diameter.

If the point (x_1, y_1, z_1) lies on the Surface (1) it can easily be (x_1, y_1, z_1) lies on the Surface.

Hence the Surface is Symmetrical about each of the Co-ordinates planes.

These three planes of Symmetry are called the principal planes the ~~area~~ axes of Co-ordinates the principal axes.

1). Find the locus of the point of intersection at three multually perpendicular tangent planes to the central Conicoid $ax^2 + by^2 + cz^2 = 1$

Soln :

The Eqn. of three tangent plane

$$l_r x + m_r y + n_r z = \left\{ \frac{l_r^2}{a} + \frac{m_r^2}{b} + \frac{n_r^2}{c} \right\}^{1/2}$$

Let (x_1, y_1, z_1) be the co-ordinates point these

planens.

then,

$$l x_1 + m_1 y_1 + n_1 z_1 = \left(\frac{l_1^2}{a} + \frac{m_1^2}{b} + \frac{n_1^2}{c}\right)^{1/2} \quad (1)$$

$$l_2 x_2 + m_2 y_2 + n_2 z_2 = \left(\frac{l_2^2}{a} + \frac{m_2^3}{b} + \frac{n_2^2}{c}\right)^{1/2} \quad (2)$$

$$l_3 x_1 + m_3 y_2 + n_2 z_3 = \left(\frac{l_3^2}{a} + \frac{m_3^2}{b} + \frac{n_3^2}{c}\right)^{1/2} \quad (3)$$

These plane's are mutually right angles

The directions are mutually perpendicular as to

planes are respective.

(l_1, m_1, n_1) (l_2, m_2, n_2) (l_3, m_3, n_3) these lines

are mutually right angles.

by considery the lines ox, oy, oz directions

Cosines (l_1, l_2, l_3) (m_1, m_2, m_3) (n_1, n_2, n_3)

$l_1^2 + l_2^2 + l_3^2 = 1$, $m_1^2 + m_2^2 + m_3^2 = 1$

$n_1^2 + n_2^2 + n_3^2 = 1$

ox, oy, oz mutually right angles

$$l_1 m_1 + l_2 m_2 + l_3 m_3 = 0$$

$$m_1 n_1 + m_2 n_2 + m_3 n_3 = 0$$

$$n_1 l_1 + n l_2 + n_3 l_3 = 0.$$

Squaring (1), (2), (3) //

UNIT - I

ANGLE BETWEEN TWO LINES

Definition :-

The angle b/w two non co-planer (ie) non-intersecting lines is and angle b/w two intersecting lines drawn from any point parallel to each ~~to~~ of the given lines.

Direction Co-Sines of a line :-

Let α, β, γ be the angle which any line makes with the two direction of the co-ordinates area the $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called the direction co-sines of the given lines and are generally the direction co-sines of a given line and are generally denoted by l, m, n respectively.

Relation between direction of Cosines. ~~Statement~~.

Statement :-

If l, m, n are the direction Cosine's of line then,

$$l^2+m^2+n^2 = 1.$$

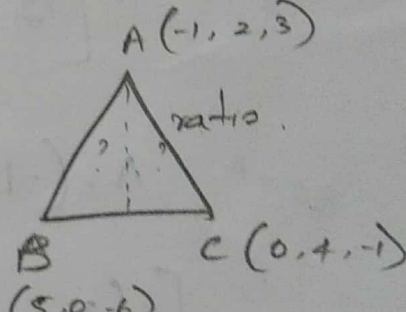
(ie). The Sum of the square of the direction Co-sine's of eventhly line is one the line :

① The optic vertical of a triangle ABC are the Point $(-1,2,3)$ $(5,0,-6)$ and $(0,4,-1)$ respectively.

Determine the direction ratio of the bisector of angle BAC.

Sln

$A\ (\overset{x_1}{-1}, \overset{y_1}{2}, \overset{z_1}{-3})$ $B = (\overset{x_2}{5}, \overset{y_2}{0}, \overset{z_2}{-6})$



$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$= \sqrt{(5+1)^2+(0-2)^2+(-6+3)^2}$$

$$= \sqrt{36+4+9}$$

$$= \sqrt{49}$$

$$AB = 7$$

$$AC = (\overset{x_1}{-1}, \overset{y_1}{2}, \overset{z_1}{-3})\ C = (\overset{x_2}{0}, \overset{y_2}{4}, \overset{z_2}{-1})$$

$$= \sqrt{(0+1)^2+(4-2)^2+(-1+3)^2}$$

$$= \sqrt{1+4+4}$$

$$= \sqrt{9}$$

$$AC = 3$$

AB, AC = ration = 7 : 3

BC side ratio 7 : 3

$a \quad b$

Ratio $7 : 3$

$B\,(\overset{x_1}{5}, \overset{x_2}{0}, \overset{x_3}{-6}) \quad C\,(\overset{y_1}{0}, \overset{y_2}{4}, \overset{y_3}{-1})$

Som

$$= \left[\frac{a(y_1) + b(x_1)}{a+b}, \frac{a(y_2) + b(x_2)}{a+b}, \frac{a(y_3) + b(x_3)}{a+b}\right]$$

$$= \left[\frac{7(0) + 3(5)}{10}, \frac{7(4) + 3(0)}{10}, \frac{7(-1) + 3(-6)}{10}\right]$$

$$= \left[\frac{15}{10}, \frac{28}{10}, \frac{-25}{10}\right]$$

$$BC = \left[\frac{3}{2}, \frac{14}{5}, \frac{-5}{2}\right]$$

$BC \times A \quad A = (-1, 2, -3)$

$$= \left[\left(\frac{3}{2} - (-1)\right), \left(\frac{14}{5} - (2)\right), \left(\frac{-5}{2} - (-3)\right)\right]$$

$$= \left[\left(\frac{3}{2} + 1\right), \left(\frac{14}{5} - 2\right), \left(\frac{-5}{2} + 3\right)\right]$$

$$= \left[\frac{3+2}{2}, \frac{14-10}{5}, \frac{-5+6}{2}\right]$$

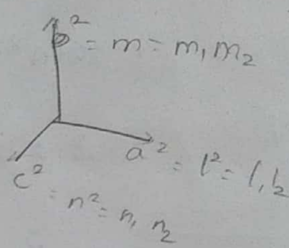
$$BAC = [2.5, 0.8, 0.5]$$

BAC bisector angle.

U.Q. 10 m

1. Show that the straight lines whose the direction cosines are given by the equation $al + bm + cn = 0$, $ul^2 + vm^2 + wn^2 = 0$ are perpendicular (or) parallel according as $a^2(v+w) + b^2(w+u) + c^2(u+v) = 0$

(or) $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$



$al + bm + cn = 0$ — (1)

$ul^2 + vm^2 + wn^2 = 0$ — (2)

$al + bm + cn = 0$

$al = -bm - cn$

$al = -(bm + cn)$

$l = \frac{-(bm + cn)}{a}$ $l^2 = \frac{(bm + cn)^2}{a^2}$

$ul^2 + vm^2 + wn^2 = 0$

$u\left(\frac{(bm + cn)^2}{a^2}\right) + vm^2 + wn^2 = 0$

$u\left(\frac{b^2m^2 + c^2n^2 + 2bmcn}{a^2}\right) + vm^2 + wn^2 = 0$

$\frac{ub^2m^2 + uc^2n^2 + 2ubmcn + a^2vm^2 + a^2wn^2}{a^2} = 0$

$ub^2m^2 + uc^2n^2 + 2ubmcn + a^2vm^2 + a^2wn^2 = 0$

$(ub^2 + va^2)m^2 + (uc^2 + a^2w)n^2 + 2ubmcn = 0$ — (3)

$a^2b^2c^2 = (ub^2 + va^2)(uc^2 + a^2w)$

$\frac{m_1m_2}{n_1n_2} = \frac{ub^2 + va^2}{uc^2 + a^2w} \quad \frac{uc^2 + a^2w}{ub^2 + va^2}$

$$\cancel{\frac{m_1 m_2}{ub^2+va^2}} = \cancel{\frac{n_1 n_2}{uc^2+a^2w}} \quad \text{--- (4)} \qquad \frac{m_1 m_2}{vc^2+a^2w} = \frac{n_1 n_2}{ub^2+va^2} \quad \text{--- (7)}$$

||ly. $al+bm+cn=0$

$$cn = -al-bm$$

$$cn = -(al+bm)$$

$$n = \frac{-(al+bm)}{c}$$

$$n^2 = \frac{(al+bm)^2}{c^2}$$

$$ul^2+vm^2+wn^2=0$$

$$ul^2+vm^2+w\left[\frac{(al+bm)^2}{c^2}\right]=0$$

$$ul^2+vm^2+w\left[\frac{a^2l^2+b^2m^2+2albm}{c^2}\right]=0$$

$$ul^2c^2+c^2vm^2+wa^2l^2+wb^2m^2+2walbm=0$$

$$(uc^2+wa^2)\,l^2+(c^2v+wb^2)m^2+2walbm=0$$

$$a^2b^2c^2=(uc^2+wa^2)(c^2v+wb^2)$$

$$\frac{l_1l_2}{m_1m_2}=\frac{c^2v+wb^2}{uc^2+wa^2}$$

$$\frac{l_1l_2}{c^2\cancel{w}+wb^2}=\frac{m_1m_2}{uc^2+wa^2} \quad \text{--- (3)}$$

$$a^2+b^2+c^2=\frac{l_1l_2}{c^2v+wb^2}+\frac{m_1m_2}{uc^2+wa^2}+\frac{n_1n_2}{ub^2+va^2}$$

$$l_1l_2+m_1m_2+n_1n_2=K(c^2v+wb^2+uc^2+wa^2+ub^2+va^2)$$

⊥. $c^2v+wb^2+uc^2+wa^2+ub^2+va^2=0$

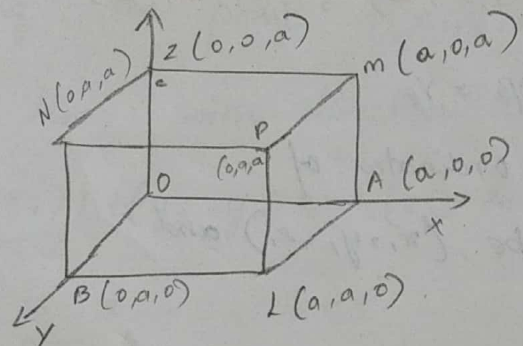
$$a(v+w)+b^2(u+w)+c^2(u+v)=0$$

$$\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0 \;//$$

3. A line make angle $(\alpha, \beta, \gamma, \delta)$ with the four diagonals of a cube, prove that.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3.$$

<u>Soln</u>. Let a be the length of each side of the cube taking three Co-terminous edges OA, OB, OC as axis the Co-ordinates of various Vertices will be $A(a,0,0)$, $L(a,a,0)$, $B(0,a,0)$, $P(a,0,a)$, $O(0,0,0)$, $N(0,a,a)$, $C(0,0,a)$, $M(a,0,a)$, $P(a,0,a)$, $O(0,0,0)$



direction Cosines of diagonals OP are.

$$\left[\frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}\right]$$

$$\Rightarrow \left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$$

Ily. Direction Cosines of AN are,

$$\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \text{ of BN } \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \text{and CL} \left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{5}}\right]$$

If l, m, n be the direction of a line which makes angle $\alpha, \beta, \gamma, \delta$ with these 4 diagnols of the Cube. then,

$$\cos\alpha = \frac{l+m+n}{\sqrt{3}}, \quad \cos\beta = \frac{-l+m+n}{\sqrt{3}}$$

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$\cos\gamma = \frac{1-m+n}{\sqrt{3}}$, $\cos\delta = \frac{l+m+n}{\sqrt{3}}$

Hence $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{1}{3}\left[(l+m+n)^2 + (-l+m+n)^2 + (l-m+n)^2 + (l+m-n)^2\right.$

$= \frac{4}{3}$

UNIT - II

(1). Find the equation of the plan through the points $P(2,2,-1)$, $Q(3,4,2)$, $R(7,0,6)$.

Soln. General equation of the plan.

$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$.

$A(x-2)+B(y-2)+C(z+1)=0$ — (1).

$Q(3,4,2)$.

$A(3-2)+B(4-2)+C(2+1)$.

$A(1)+B(2)+C(3)=0$

$A+2B+3C=0$ — (2)

$R(7,0,6)$.

$A(7-2)+B(0-2)+C(6+1)$

$5A-2B+7C=0$ (3)

(2) → $A+2B+3C=0$

(3) → $5A-2B+7=0$

$\frac{A}{14+6} = \frac{B}{7-15} = \frac{C}{-12-10}$.

$\frac{A}{20} = \frac{B}{-8} = \frac{C}{-12}$

$\frac{A}{5} = \frac{B}{-2} = \frac{C}{-3}$

$A(x-2)+B(y-2)+C(z+1)=0$

a	b	c
1	2	3
5	-2	7

$A(x-2)+B(y-2)+C(z+1)=0.$

$5(x-2)-2(y-2)-3(z+1)=0$

$5x-10-2y+4-3z+3=0$

$5x-2y-3z-9=0$

(2). Find the equation of the plan through the points $(2,2,1)$ and $(9,3,6)$ perpendicular to the plane $2x+6y+6z=0$.

General equation of the plan.

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

$$A(x-2)+B(y-2)+C(z-1)=0 \quad -(1)$$

$(9,3,6)$.

$$A(9-2)+B(3-2)+C(6-1)=0$$

$$A(7)+B(1)+C(5)=0$$

$$7A+B+5C=0 \quad -(2)$$

a	b	c
7	1	5
2	6	6

$(2) \rightarrow \quad 7A+B+5C=0$

$(3) \rightarrow \quad 2A+6B+6C=0$

$$\frac{A}{6-30}=\frac{B}{42-10}=\frac{C}{42-2}$$

$$\frac{A}{-24}=\frac{B}{32}=\frac{C}{40}$$

$$\frac{A}{-6} = \frac{B}{8} = \frac{C}{10}$$

$$A(x-2)+B(y-2)+C(z-1)=0$$

$$-6(x-2)+8(y-2)+10(z-1)=0.$$

$$-6x+12+8y-16+10z-10=0.$$

$$-6x+8y+10z$$

$$\frac{A}{-3} = \frac{B}{4} = \frac{C}{5}$$

$$A(x-2)+B(y-2)+C(z-1)=0.$$

$$-3(x-2)+4(y-2)+5(z-1)=0$$

$$-3x+6+4y-8+5z-5=0$$

$$-3x+4y+5z-7=0$$

$$3x-4y-5z+7=0.$$

(5). Find the equation of the plane passing through the point $(-1,1,1)$, $(1,-1,1)$ and perpendicular to the point $x+2y+2z=0$

General equation of the plan.

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

$$A(x+1)+B(y-1)+C(z-1)=0. \quad (1)$$

$(1,-1,1)$

$$A(1+1)+B(-1-1)+C(1-1)=0.$$

$$A(2)+B(-2)+C(0)=0$$

$$2A-2B=0 \quad (2)$$

a b
2 -2
1 2

(2) $\longrightarrow$ $2A \quad -2B \quad C(0) = 0$

(3) $\longrightarrow$ $A \quad 2B \quad 2C = 0$

$$\frac{A}{-4-0} = \frac{B}{4-0} = \frac{C}{4+2}$$

$$\frac{A}{-4} = \frac{B}{4} = \frac{C}{6}.$$

$$\frac{A}{-2} = \frac{B}{2} = \frac{C}{3}$$

$$A(x+1) + B(y-1) + C(z-1) = 0.$$

$$\cancel{A(x+1) + B(y-1) + C(z-1)} = 0$$

$$-2(x+1) + 2(y-1) + 3(z-1) = 0$$

$$-2x - 2 + 2y - 2 + 3z - 3 = 0$$

$$-2x + 2y + 3z - 7 = 0$$

$$2x - 2y - 3z + 7 = 0$$

Sm:

① The plane $lx + my = 0$ rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation of the plane in its new position is $lx + my \pm z\sqrt{l^2+m^2}\tan\alpha = 0$

Proof: $lx + my = 0$ & $z = 0$.

$$lx + my + \lambda z = 0 \quad \text{—①}$$

$$lx + my + \lambda z = 0$$

$$lx + my = 0.$$

α make angle

$$\cos\alpha = \frac{l.l + m.m + \lambda.0}{\sqrt{l^2+m^2+\lambda^2}\,\sqrt{l^2+m^2}}$$

$$= \frac{l^2+m^2}{\sqrt{l^2+m^2+\lambda^2}\,\sqrt{l^2+m^2}}$$

$\therefore$ ~~$2 = \sqrt{2}\sqrt{2}$~~. $\qquad 2 = \sqrt{2}\,\sqrt{2}$

$$l^2+m^2 = \sqrt{l^2+m^2}\,.\,\sqrt{l^2+m^2}$$

$$= \frac{\sqrt{l^2+m^2}\,.\,\sqrt{l^2+m^2}}{\sqrt{l^2+m^2+\lambda^2}\,\sqrt{l^2+m^2}}$$

$$\cos\alpha = \frac{\sqrt{l^2+m^2}}{\sqrt{l^2+m^2+\lambda^2}}$$

Square root of both side.

$$\cos^2\alpha = \frac{l^2+m^2}{l^2+m^2+\lambda^2}$$

$$\cos^2\alpha\,(l^2+m^2+\lambda^2) = l^2+m^2$$

$$\cos^2\alpha(l^2+m^2)+\lambda^2\cos^2\alpha = l^2+m^2$$

$$\lambda^2\cos^2\alpha = l^2+m^2-\cos^2\alpha(l^2+m^2)$$

$$= l^2+m^2(1-\cos^2\alpha)$$

$$\lambda^2\cos^2\alpha = l^2+m^2(\sin^2\alpha)$$

$$\lambda^2 = l^2+m^2\frac{\sin^2\alpha}{\cos^2\alpha}$$

$$\lambda^2 = l^2+m^2\tan^2\alpha$$

$$\lambda = \pm\sqrt{l^2+m^2}\tan\alpha.$$

Sub que ①.

$$lx+my+z(\sqrt{l^2+m^2}\tan\alpha = 0$$

Hence proved.

5m/10m:

② A point P moves on ~~afraio~~ a fixed plane. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, The plane through P perpendicular to OP meets the axes in A, B, C the plane through A, B, C parallel to Co-ordinate plane Intersect in Q. show that the Locus of Q is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}.$$

Proof.

Let the point $P(\alpha,\beta,\gamma)$. α

~~Equation~~ of . $\frac{\alpha}{a}+\frac{\beta}{b}+\frac{\gamma}{c}=1$

Equation of the perpendicular.

$$\alpha x+\beta y+\gamma z=d$$

$$\alpha(\alpha)+\beta(\beta)+\gamma(\gamma)=d$$

$$\alpha^2+\beta^2+\gamma^2=d.$$

$$\alpha x + \beta y + \gamma z = \alpha^2 + \beta^2 + \gamma^2$$

$$\alpha x = \alpha^2 + \beta^2 + \gamma^2 \quad \Big| \quad \beta y = \alpha^2 + \beta^2 + \gamma^2 \quad \Big| \quad \gamma z = \alpha^2 + \beta^2 + \gamma^2$$

$$x = \frac{\alpha^2 + \beta^2 + \gamma^2}{\alpha} \quad \Big| \quad y = \frac{\alpha^2 + \beta^2 + \gamma^2}{\beta} \quad \Big| \quad z = \frac{\alpha^2 + \beta^2 + \gamma^2}{\gamma}$$

$$\frac{1}{x} = \frac{\alpha}{\alpha^2 + \beta^2 + \gamma^2} \quad \Big| \quad \frac{1}{y} = \frac{\beta}{\alpha^2 + \beta^2 + \gamma^2} \quad \Big| \quad \frac{1}{z} = \frac{\gamma}{\alpha^2 + \beta^2 + \gamma^2}$$

$$\frac{1}{x^2} = \frac{\alpha^2}{(\alpha^2 + \beta^2 + \gamma^2)} \quad \Big| \quad \frac{1}{y^2} = \frac{\beta^2}{(\alpha^2 + \beta^2 + \gamma^2)^2} \quad \Big| \quad \frac{1}{z^2} = \frac{\gamma}{(\alpha^2 + \beta^2 + \gamma^2)^2}$$

$$\frac{1}{x} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{\alpha^2 + \beta^2 + \gamma^2}{(\alpha^2 + \beta^2 + \gamma^2)^2} = \frac{1}{\alpha^2 + \beta^2 + \gamma^2}$$

$$\frac{1}{x} = \frac{\alpha}{\alpha^2 + \beta^2 + \gamma^2} \quad \Big| \quad \frac{1}{y} = \frac{\beta}{\alpha^2 + \beta^2 + \gamma^2} \quad \Big| \quad \frac{1}{z} = \frac{\gamma}{\alpha^2 + \beta^2 + \gamma^2}$$

$$\frac{1}{ax} = \frac{\alpha}{a(\alpha^2 + \beta^2 + \gamma^2)} \quad \Big| \quad \frac{1}{by} = \frac{\beta}{b(\alpha^2 + \beta^2 + \gamma^2)} \quad \Big| \quad \frac{1}{cz} = \frac{\gamma}{c(\alpha^2 + \beta^2 + \gamma^2)}$$

$$\frac{1}{ax} + \frac{1}{by} + \frac{1}{cz} = \frac{\alpha/a + \beta/b + \gamma/c}{\alpha^2 + \beta^2 + \gamma^2}$$

$$= \frac{1}{\alpha^2 + \beta^2 + \gamma^2}$$

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}.$$

Hence proved.

(Q) 5m.