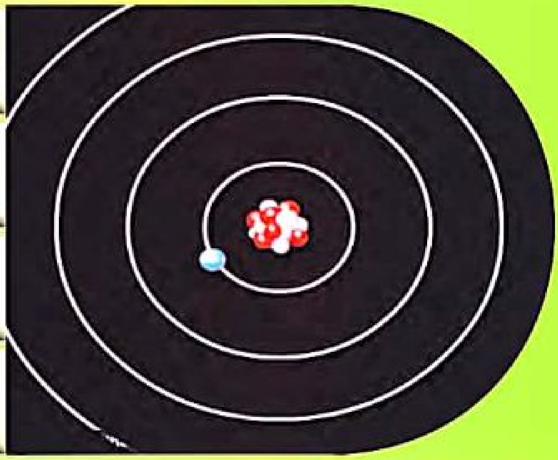


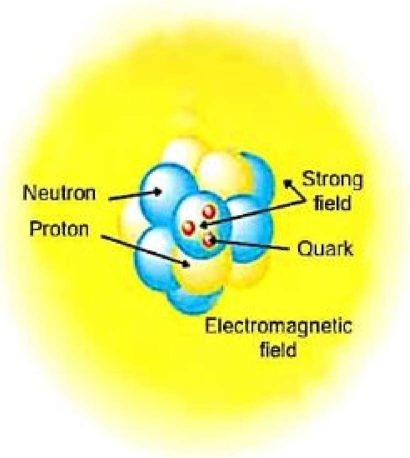
CHAPTER

27

INTRODUCTION TO THE NUCLEUS

27.1 Introduction

The atomic nucleus was discovered in 1911 by Rutherford. Rutherford's α-particle scattering experiments showed that the atom consists of a very small nucleus (≈ 10–14m in diameter) surrounded by orbiting electrons. It is the purpose of this book to explain all aspects of the nucleus, its structure and its behaviour under various conditions. In this chapter we introduce some of its most basic characteristics, its mass, size, shape, and other externally observable properties. We also consider some deeper questions, such as the force that holds the nucleus together.



The Nucleus.

Let us begin by reviewing a few fundamental facts that are probably already familiar. All atomic nuclei are made up of elementary particles called *protons* and *neutrons*. A proton has a positive charge of the same magnitude as that of an electron. A neutron is electrically neutral. The proton and the neutron are considered to be two different charge states of the same particle which is called a *nucleon*.

A species of nucleus, known as a *nuclide*, is represented schematically by $_ZX^A$ where Z, the *atomic number* indicates the number of protons, A, the mass number, indicates the total number of protons plus neutrons and X is the chemical symbol of the species.

$$N = \text{Number of neutrons} = A - Z$$

As an example, the chlorine nucleus $_{17}Cl^{35}$ has $Z = 17$ protons, $A = 35$ nucleons and $N = 35 - 17 = 18$ neutrons.

27.2 Classification of Nuclei

Atoms of different elements are classified as follows :

(*i*) *Isotopes* are nuclei with the same atomic number Z but different mass numbers A. The nuclei $_{14}Si^{28}$, $_{14}Si^{29}$, $_{14}Si^{30}$, and $_{14}Si^{32}$ are all isotopes of silicon. The isotopes of an element all contain the same number of protons but have different number of neutrons. Since the nuclear charge is what is ultimately responsible for the characteristic properties of an atom, all the isotopes of an element have identical chemical behaviour and differ physically only in mass.

(*ii*) Those nuclei, with the same mass number A, but different atomic number Z, are called *isobars*. The nuclei $_8O^{16}$ and $_7N^{16}$ are examples of isobars. The isobars are atoms of different elements and have different physical and chemical properties.

(*iii*) Nuclei, with an equal number of neutrons, that is, with the same N, are called *isotones*. Some isotones are $_6C^{14}$, $_7N^{15}$ and $_8O^{16}$ ($N = 8$ in each case).

(*iv*) There are atoms, which have the same Z and same A, but differ from one another in their nuclear energy states and exhibit differences in their internal structure. These nuclei are distinguished by their different life times. Such nuclei are called *isomeric nuclei* or *isomers*.

(*v*) Nuclei, having the same mass number A, but with the proton and neutron number interchanged (that is, the number of protons in one is equal to the number of neutrons in the other) are called *mirror nuclei*.

EXAMPLE. $_4Be^7$ ($Z = 4$ and $N = 3$) and $_3Li^7$ ($Z = 3$ and $N = 4$).

Note

Nuclei, consisting of an even (odd) number of protons and even (odd) number of neutrons, are said to be *even-even* (odd-odd). Nuclei consisting of an even (odd) number of protons and an odd (even) number of neutrons are said to be *even-odd* (odd-even).

27.3 General Properties of Nucleus

Nuclear size. Rutherford's work on the scattering of α-particles showed that the mean radius of an atomic nucleus is of the order of 10^{-14} to 10^{-15} m while that of the atom is about 10^{-10} m. Thus the nucleus is about 10000 times smaller in radius than the atom.

The empirical formula for the nuclear radius is

$$R = r_o A^{1/3}$$

where A is the mass number and $r_o = 1.3 \times 10^{-15}$ m = 1.3 *fm*. Nuclei are so small that the fermi (*fm*) is an appropriate unit of length. 1 *fm* = 10^{-15} m. From this formula we find that the radius of the $_6C^{12}$ nucleus is $R \approx (1.3)(12)^{1/3} = 3 fm$. Similarly, the radius of the $_{47}Ag^{107}$ nucleus is 6.2 *fm* and that of the $_{92}U^{238}$ nucleus is 8.1 *fm*.

The nuclear radius may be estimated from the scattering of neutrons and electrons by the nucleus, or by analysing the effect of the finite size of the nucleus on nuclear and atomic binding energies.

Fast neutrons of about 100 MeV energy, whose wavelength is small compared to the size of the nucleus, are scattered by nuclear targets. The fraction of neutrons scattered at various angles can be used to deduce the nuclear size. The results of these experiments indicate that the radius of a nucleus is given by $R \approx r_0 A^{1/3}$ where $r_0 \approx 1.3 - 1.4$ *fm*. The scattering can be done with proton beams as well. In this case, however, the effects due to Coulomb interaction have to be separated out. The observations are in agreement with the equation $R \approx r_0 A^{1/3}$ with $r_0 \approx 1.3 - 1.4$ *fm*.

The scattering of fast electrons of energy as high as 10^4 MeV, with a wavelength of about 0.1 *fm*, has the advantage that it can directly measure the charge density inside a nucleus. The results of the experiment are in agreement with the equation $R \approx r_0 A^{1/3}$ but with a somewhat smaller value of $r_0 \approx 1.2$ *fm*. The slight difference in the value of r_o may be ascribed to the fact that the electron scattering measures the charge density whereas the neutron and proton scattering experiments measure the region of large nuclear potential, which may be expected to be somewhat larger than the size of the nucleus.

EXAMPLE. *The radius of Ho^{165} is 7.731 fermi. Deduce the radius of He^4.*

SOL. Let R_1, A_1 and R_2, A_2 be the radius and mass number of Ho^{165} and He^4 respectively. Then $R_1 = r_0 A_1^{1/3}$ and $R_2 = r_0 A_2^{1/3}$.

$$\therefore \qquad \frac{R_1}{R_2} = \frac{A_1^{1/3}}{A_2^{1/3}} \text{ or}$$

$$R_2 = \frac{R_1 A_2^{1/3}}{A_1^{1/3}} = \frac{7.731 \times 4^{1/3}}{(165)^{1/3}} = 2.238 \ fm.$$

Nuclear mass. We know that the nucleus consists of protons and neutrons. Then the mass of the nucleus should be

assumed nuclear mass = $Zm_p + Nm_n$

where m_p and m_n are the respective proton and neutron masses and N is the neutron number. Nuclear masses are experimentally measured accurately by mass spectrometers. Measurements by mass spectrometer, however, show that

$$\text{real nuclear mass} < Zm_p + Nm_n.$$

The difference in masses

$$Zm_p + Nm_n - \text{real nuclear mass} = \Delta m$$

is called the *mass defect*.

Nuclear density. The nuclear density ρ_N can be calculated from $\rho_N = \frac{\text{Nuclear mass}}{\text{Nuclear volume}}$.

Nuclear mass = Am_N where A = mass number and m_N = mass of the nucleon = 1.67×10^{-27} kg.

Nuclear volume = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \frac{4}{3}\pi r_0^3 A$

$$\therefore \qquad \rho_N = \frac{Am_N}{\frac{4}{3}\pi r_0^3 A} = \frac{m_N}{\frac{4}{3}\pi r_0^3} = \frac{(1.67 \times 10^{-27})}{\frac{4}{3}\pi (1.3 \times 10^{-15})^3}$$

$$= 1.816 \times 10^{17} \text{ kg m}^{-3}.$$

Note the high value of the density of the nucleus. This shows that the nuclear matter is in an extremely compressed state. Certain stars (the "*white dwarfs*") are composed of atoms whose electron shells have collapsed owing to enormous pressure, and the densities of such stars approach that of pure nuclear matter.

Nuclear charge. The charge of the neucleus is due to the protons contained in it. Each proton has a positive charge of 1.6×10^{-19} *C*. The nuclear charge is *Ze* where *Z* is the atomic number of the nucleus. The value of *Z* is known from X-ray scattering experiments, from the nuclear scattering of α-particles, and from the X-ray spectrum.

Spin angular momentum. Both the proton and neutron, like the electron, have an intrinsic spin. *The spin angular momentum* is computed by $L_s = \sqrt{I(I+1)}\ h/2\pi$ where the quantum number *I*, commonly called the spin, is equal to 1/2. The spin angular momentum, then has a value $L_s = \frac{\sqrt{3}}{2}\frac{h}{2\pi}$.

Resultant angular momentum. In addition to the spin angular momentum, the protons and neutrons in the nucleus have an *orbital angular momentum*. The resultant angular momentum of the nucleus is obtained by adding the spin and orbital angular momenta of all the nucleons within the nucleus. The total angular momentum of a nucleus is given by $L_N = \sqrt{I_N(I_N+1)}\ \frac{h}{2\pi}$. This total angular momentum is called *nuclear spin*.

Nuclear magnetic dipole moments. We know that the spinning electron has an associated magnetic dipole moment of 1 Bohr magneton. *i.e.* $\mu_e = \frac{eh/2\pi}{2m_e}$. Proton has a positive elementary charge and due to its spin, it should have a magnetic dipole moment. According to Dirac's theory, $\mu_N = \frac{eh/2\pi}{2m_p}$ where m_p is the proton mass. μ_N is called a *nuclear magneton* and is the unit of nuclear magnetic moment. μ_N has a value of 5.050×10^{-27} J/T. Since $m_p = 1836\ m_e$, the nuclear magneton is only 1/1836 of a Bohr magneton. For nucleons, however, measurements give $\mu_p = 2.7925\ \mu_N$ and $\mu_n = -1.9128\ \mu_N$. Physicists have found that it is especially hard to understand how the neutron, which is a neutral particle, can have a magnetic moment.

Electric quadrupole moment. In addition to its magnetic moment, a nucleus may have an electric quadrupole moment. An electric dipole moment is zero for atoms and nuclei in stationary states. This is a consequence of the symmetry of nuclei about the centre of mass. However, this symmetry does not need to be spherical; there is nothing precluding the nucleus from assuming the shape of an ellipsoid of rotation, for instance. Indeed most nuclei do assume approximately such a shape, and the deviation from spherical symmetry is expressed by a quantity called the *electric quadrupole moment*. It is defined as

$$Q = \left(\frac{1}{e}\right) \int (3z^2 - r^2)\, \rho\, d\tau$$

where ρ is the charge density in the nucleus. *Q* is actually a measure of the eccentricity of the ellipsoidal nuclear surface. Evidently $Q = 0$ for a spherically symmetric charge distribution. A charge distribution stretched in the z-direction (prolate) will give a positive quadrupole moment, and an oblate distribution will give a negative quadrupole moment (Fig. 27.1). Since the expression is divided by the electronic charge, the dimension of the quadrupole moment is that of an area. In nuclear physics, area is measured in barns. (1 barn = 10^{-28} m^2).

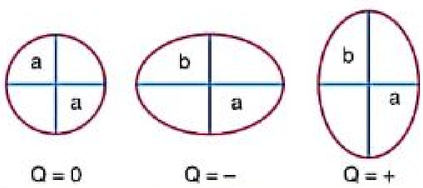
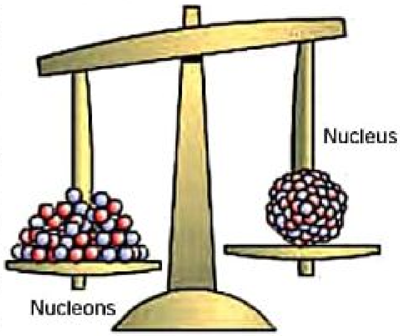


Fig. 27.1

27.4 Binding Energy

The theoretical explanation for the mass defect is based on Einstein's equation $E = mc^2$. When the Z protons and N neutrons combine to make a nucleus, some of the mass (Δm) disappears because it is converted into an amount of energy $\Delta E = (\Delta m) c^2$. This energy is called the *Binding energy* (*B.E*) of the nucleus. To disrupt a stable nucleus into its constituent protons and neutrons, the minimum energy required is the binding energy. The magnitude of the B.E. of a nucleus determines its *stability against disintegration*. If the B.E. is large, the nucleus is stable. A nucleus having the least possible energy, equal to the B.E., is said to be in the *ground state*. If the nucleus has an energy $E > E_{min}$, it is said to be in the *excited state*. The case $E = 0$ corresponds to dissociation of the nucleus into its constituent nucleons.



Binding Energy.

If M is the experimentally determined mass of a nuclide having Z protons and N neutrons,

$$\text{B.E.} = \{(Zm_p + Nm_n) - M\}c^2 .$$

If B.E. > 0, the nucleus is stable and energy must be supplied from outside to disrupt it into its constituents. If B.E.< 0, the nucleus is unstabe and it will disintegrate by itself.

Illustration: Let us illustrate the calculation of B.E. by taking the example of the deuteron. A deuteron is formed by a proton and a neutron.

Mass of proton = $1.007276u$.

Mass of neutron = $1.008665u$.

∴ Mass of proton + neutron in free state = $2.015941u$

Mass of deuteron nucleus = $2.013553u$

∴ Mass defect = $\Delta m = 0.002388u$

B.E. = 0.002388 × 931 = 2.23 MeV (∵ 1u = 931 MeV)

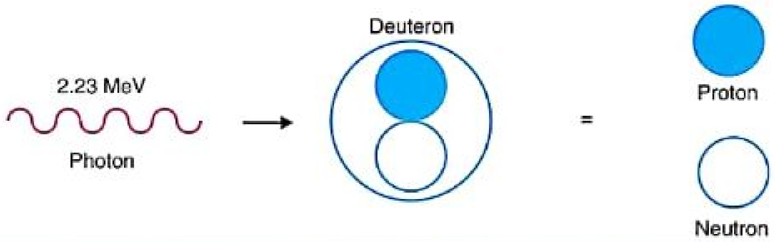


Fig. 27.2

Thus, when a deuteron is formed from a free proton and neutron, 2.23 MeV of energy is liberated. Conversely, 2.23 MeV must be supplied from an external source to break a deuteron up into a proton and a neutron. This is confirmed by experiments that show that a gamma-ray photon with a minimum energy of 2.23 MeV can split a deuteron into a free neutron and a free proton (Fig. 27.2).

Example 1. *Calculate the binding energy of an α-particle and express the result both in MeV and joules.*

Sol. Mass of 2 protons + 2 neutrons = $(2 \times 1.007276 + 2 \times 1.008665)u$

$= 4.031882u.$

Mass of the α-particle = $4.001506u$.

Mass defect $\Delta m = (4.031882 - 4.001506)\ u = 0.030376\ u$

∴ $\text{B.E.} = (0.030376 \times 931.3)\ \text{MeV} = 28.29\ \text{MeV}$.

$= 45.32 \times 10^{-13}$ J

Example 2. *Given the following isotope masses:*

$${}_3Li^7 = 7.016004,\ {}_3Li^6 = 6.015125 \text{ and } {}_0n^1 = 1.008665\ u$$

Calculate the B.E. of a neutron in the ${}_3Li^7$ *nucleus. Express the result in u, MeV and joules.*

Sol. B.E. of a neutron in $u = \text{M}({}_3Li^6 + {}_0n^1) - \text{M}({}_3Li^7)$

$= 6.015125 + 1.008665 - 7.016004$

$= 0.007786\ u$.

In MeV this becomes 0.007786×931 MeV $= 7.35$ MeV.

In joules, $7.35 \times 1.6 \times 10^{-13}$ J $= 1.18 \times 10^{-12}$ J.

Stability of Nucleus and Binding Energy

$$\text{B.E per nucleon} = \frac{\text{Total B.E. of a nucleus}}{\text{The number of nucleons it contains}}.$$

The Binding Energy per nucleon is plotted as a function of mass number A in Fig. 27.3. The curve rises steeply at first and then more gradually until it reaches a maximum of 8.79 *MeV* at $A = 56$, corresponding to the iron nucleus ${}_{26}Fe^{56}$. The curve then drops slowly to about 7.6 *MeV* at the highest mass numbers. Evidently, nuclei of intermediate mass are the most stable, since the greatest amount of energy must be supplied to liberate each of their nucleons. This fact suggests that a large amount of energy will be liberated if heavier nuclei can somehow be split into lighter ones or if light nuclei can somehow be joined to form heavier ones. The former process is known as *nuclear fission* and the latter as *nuclear fusion*. Both the processes indeed occur under proper circumstances and do evolve energy as predicted.

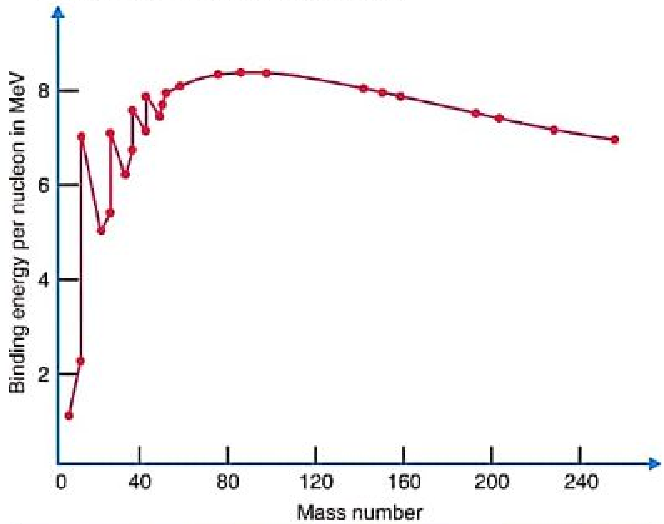


Fig. 27.3

Packing fraction. The ratio between the mass defect (Δm) and the mass number (A) is called the packing fraction (f);

$$f = \Delta m / A.$$

Packing fraction means the mass defect per nucleon. Since atomic masses are measured relative to C–12, the packing fraction for this isotope is zero. Packing fraction is a measure of the comparative stability of the atom.

Packing fraction is defined as

$$\text{Packing fraction} = \frac{\textit{Isotopic mass} - \textit{Mass number}}{\textit{Mass number}} \times 10^4.$$

Packing fraction may have a ***negative*** or a ***positive*** sign. If packing fraction is negative, the isotopic mass is less than the mass number. In such cases, some mass gets transformed into energy in the formation of that nucleus, in accordance with Einsein's equation $E = mc^2$. Such nuclei, therefore, are more stable. A positive packing fraction would imply a tendency towards instability. But this is not quite correct, especially for elements of low atomic masses.

A plot of packing fraction against the corresponding mass numbers of the various elements is shown in Fig. 27.4. It is seen that helium, carbon and oxygen atoms of mass numbers 4, 12 and 16 respectively, do not fall on this curve. Their paking fractions have small values. These elements are, therefore, stable. The transition elements, with mass numbers in the neighbourhood of 45, have lowest packing fractions with a negative sign, which indicates their high stability. The packing fraction beyond mass number 200 becomes positive and increases with increase in mass number. This indicates increasing instability of these elements. Elements with mass numbers beyond 230 are radioactive and undergo disintegration spontaneously.

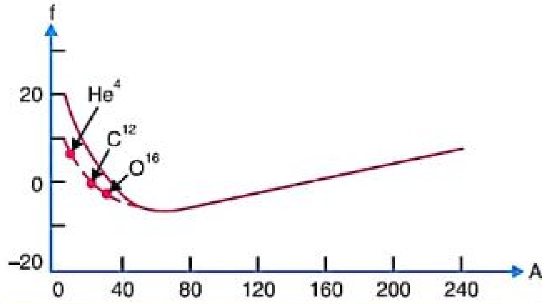


Fig. 27.4

27.5 Nuclear Stability

Table 27.1 shows how the 272 stable nuclei found in nature are classified according to even and odd numbers of protons and neutrons.

TABLE 27.1

Protons	*Neutrons*	*Stable Nuclides*
even	even	160
even	odd	56
odd	even	52
odd	odd	4
		272

The combination of an even number of protons and an even number of neutrons, composing the nucleus, is evidently preferred by nature for stable nuclides. The odd-odd combination of stable nuclides is found only in the light elements. The number of even-odd combinations is about the same. A plot of the number of neutrons versus the number of protons for the stable nuclides is shown in Fig. 27.5. Notice that for $Z < 20$, the stability line is a straight line with $Z = N$. For the heavier nuclides $Z > 20$, $N > 20$, the stability curve bends in the direction of $N > Z$. For example $_{20}Ca^{48}$ has $N = 28$, $Z = 20$; for larger values of Z, the tendency is more pronounced, as in the case of $_{91}Pa^{232}$ which has $N = 141$, $Z = 91$.

Evidently, for large values of Z, the coulomb electrostatic repulsion becomes important, and the number of neutrons must be greater to compensate this repulsive effect.

Thus the curve of Fig. 27.5 departs more and more from the $N = Z$ line as Z increases. For maximum stability, there is an optimum value of neutron/proton ratio. The number of neutrons N (= $A - Z$) required for maximum stability is plotted as a function of proton number Z in Fig. 27.6. All the stable nuclei fall within the shaded region. Nuclei above and below the shaded region are unstable. Artificial radioactive nuclei lie at the fringe of the region of stability. All nuclei with

Z > 83, and A > 209 spontaneously transform themselves into lighter ones through the emission of α and β particles. α and β decays enable an unstable nucleus to reach a stable configuration.

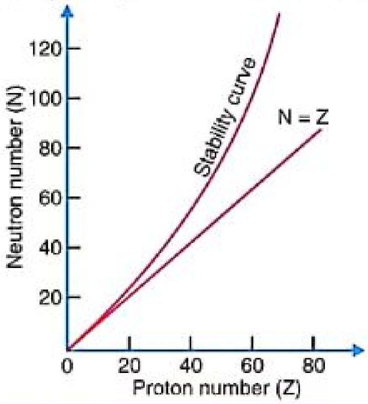


Fig. 27.5

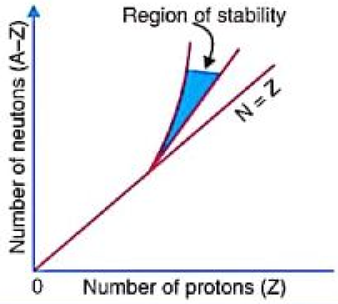


Fig. 27.6

27.6 Theories of Nuclear Composition

Several theories were proposed about the nuclear constitution. Prominent among these are proton–electron theory and proton–neutron theory.

Proton-electron hypothesis. Before the discovery of neutron by Chadwick (1932), it was assumed that the nucleus consists of protons and electrons. This concept arose because electrons were found to be emitted by radioactive nuclei in β-decay process. According to this hypothesis, a nucleus of mass number A and atomic number Z contains A protons and $A–Z$ electrons. The nucleus is surrounded by Z electrons, so that the atom is electrically neutral.

Example. *The nucleus of* $_{11}Na^{23}$ *should consist of 23 protons and 12 electrons. The total nuclear charge is 11 which is balanced by —11 units of electron charge in the extranuclear space.*

There was very strong theoretical evidence for the fact that electrons cannot exist in the nucleus. We discuss below some very strong arguments which forbid the existence of electrons inside the nucleus.

Why electrons cannot be present inside the nucleus?

(*i*) Nuclear size. Typical nuclei are less than 10^{-14} m in radius. If an electron exists inside a nucleus, the uncertainty in its position (Δx) may not exceed 10^{-14} m. According to Heisenberg's uncertainty principle, uncertainty in the electron's momentum is

$$\Delta p \geq \frac{h/2\pi}{\Delta x} \geq \frac{1.054 \times 10^{-34}}{10^{-14}} \geq 1.1 \times 10^{-20} \text{ kg ms}^{-1}.$$

If this is the uncertainty in the electron's momentum, the momentum itself must be at least comparable in magnitude.

∴ Approximate momentum of the electron = $p = 1.1 \times 10^{-20}$ kg ms^{-1}. An electron whose momentum is 1.1×10^{-20} kg ms^{-1} has a K.E. (T) many times greater than its rest energy $m_0 c^2$ *i.e.*, $T >> m_0 c^2$. Hence we can use the extreme relativistic formula $T = pc$ to find T.

$$\therefore \quad T = (1.1 \times 10^{-20})(3 \times 10^8) = 3.3 \times 10^{-12} J$$

$$= \frac{3.3 \times 10^{-12}}{1.6 \times 10^{-13}} MeV = 20.63 \ MeV.$$

This shows that if an electron exists in the nucleus, the K.E. of the electron must be more than 20 MeV. Electrons of such large energy are never found to be emitted during β-decay. The maximum energy of a β-particle emitted is only 2 to 3 *MeV*. Hence we conclude that electrons cannot be present within nuclei.

(*ii*) Nuclear spin. Electrons and protons have a spin of 1/2. Thus nuclei with an even number of protons and electrons should have integral spins, while those with an odd number of protons and electrons should have half-integral spins. Let us consider deuteron as an example. Deuteron nucleus has 3 particles (two protons and one electron). Hence the nuclear spin of deuteron should be 1/2 or 3/2. But experiment shows that the spin of the deuteron is 1. Thus the experimental result is in contradiction to the hypothesis.

(*iii*) Magnetic moment. Protons and electrons are endowed with magnetic properties. The magnetic moment of an electron is about one thousand times that of a proton. If electrons exist inside the nucleus, the magnetic moment of electrons will have a dominating influence and so nuclear magnetic moments ought to be of the same order of magnitude as that of the electron. However, the observed magnetic moments of nuclei are comparable with that of the proton. This experimental fact goes against the electrons existing inside the nucleus.

Due to these reasons, it is concluded that electrons cannot exist in the nucleus. Hence the proton-electron hypothesis regarding the constitution of the nucleus has been given up.

Proton-Neutron hypothesis. After the discovery of neutron by Chadwick in 1932, proton-neutron theory has been gaining support. According to this, a nucleus of atomic number Z and mass number A consists of Z protons and $A–Z$ neutrons. The number of electrons in the extra nuclear space is also Z, so that the atom as a whole is neutral.

Merits. (*i*) This assumption is able to explain the observed value of nuclear spin and nuclear magnetic moment.

(*ii*) There is no difficulty in explaining the existence of isotopes on this hypothesis. Isotopes of a given element differ only in the number of neutrons they contain.

(*iii*) The β-*ray emission of radioactive elements is explained as follows. The electron does not pre-exist in the nucleus. The electron is formed just at the instant of emission, caused by the transformation of a neutron into a proton. i.e.,* $n \rightarrow p + e^-$. *The positron emission is due to the converse process, viz., when a proton transforms itself into a neutron. i.e.,* $p \rightarrow n + e^+$.

(*iv*) The emission of α-particles from the nuclei of radioactive elements is due to the combination of 2 protons and 2 neutrons at the instant of emission. Thus, this hypothesis explains β-decay and α-decay.

27.7 Nuclear Forces

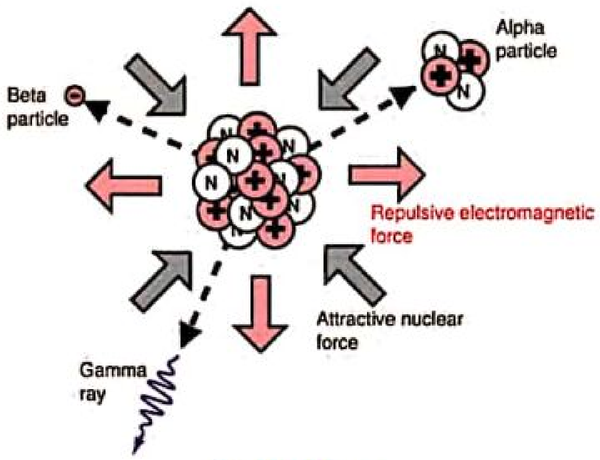
Since stable nuclei exist, it follows that there must be certain forces acting between their nucleons that bind them into the nucleus. These are called *nuclear forces. The nuclear force must be strongly attractive*, in order to overcome the electrostatic repulsion between protons. Only three kinds of attractive forces can be conceived in the nucleus, viz., neutron-neutron (n-n), neutron-proton (n-p) and proton-proton (p-p) interactions.

Yukawa attributed the following characteristics to the nuclear forces.

(1) Nuclear forces are effective only at short ranges. Nuclear forces are appreciable only when the distance between nucleons is of the order of 10^{-15} m or less. The force vanishes for all practical purposes at distances greater than a few times 10^{-15} m. These distances are called the *action radii* or *range* of the nuclear forces. In the up-to-date version of the exchange theory of nuclear forces, it is supposed that interaction between nucleons is accomplished by the exchange of π-mesons. The exchange version of nuclear forces explains their short range action. Let m be the rest mass of the π-meson. The rest energy of the π-meson = $\Delta E = mc^2$. According to Heisenberg's uncertainty principle, the time required for nucleons to exchange π-mesons cannot exceed Δt, for which $\Delta E \Delta t \geq h/2\pi$. The distance that a π-meson can travel away from a nucleon in the nucleus during the time Δt, even at a velocity ≈ c, is

$$R_0 \approx \frac{h/2\pi}{mc} \approx 1.2 \times 10^{-15} \ m$$

This approximately coincides with the value of the nuclear radius and is of the order of magnitude of the nuclear force range.



Nuclear Forces.

(2) Nuclear forces are charge independent. The nuclear forces acting between two protons, or between two neutrons, or between a proton and a neutron, are the same. It follows that nuclear forces are of a non-electric nature.

(3) Nuclear forces are the strongest known forces in nature.

(4) Nuclear forces have saturation property. Nuclear forces are limited in range. As a result, each nucleon interacts with only a limited number of nucleons nearest to it. This effect is referred to as the *saturation* of nuclear forces.

27.8 Meson Theory of Nuclear Forces

According to the meson theory of nuclear forces, all nucleons consist of identical cores surrounded by a "*cloud*" of one or more mesons. Mesons may be neutral or may have a positive or negative charge. The sole difference between neutrons and protons is supposed to lie in the composition of their respective meson clouds. Yukawa assumed that π meson is exchanged between the nucleons and that this exchange is responsible for the nuclear binding forces. The forces that act, between one neutron and another, and between one proton and another, are the result of the exchange of neutral mesons (π°) between them. The force between a neutron and a proton is the result of the exchange of charged mesons (π^+ and π^-) between them.

Thus a neutron emits a π^- meson and is converted into a proton:

$$n \rightarrow p + \pi^-$$

The absorption of the π^- meson by the proton (with which the neutron was interacting) converts it into a neutron:

$$p + \pi^- \rightarrow n.$$

In the reverse process, a proton emits a π^+ meson, becoming a neutron and the neutron, on receiving this π^+ meson, becomes a proton :

$$p \rightarrow \text{n} + \pi+$$

$$n + \pi^+ \rightarrow p.$$

Thus in the nucleus of an atom, attractive forces exist between (1) proton and proton (2) proton and neutron and (3) neutron and neutron. These forces of attraction are much larger than the electrostatic force of repulsion between the protons, thus giving a stability to the nucleus.

Just as a photon is a quantum of electromagnetic field, a meson is a quantum of nuclear field. Yukawa considered the equation for particle of mass m as,

$$\left(\nabla^2 - \frac{m^2c^2}{(h/2\pi)^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0 \quad ...(1)$$

This is a relativistic equation valid only for spinless particles.

Separating the time dependent part, the equation for the radial part is

$$(\nabla^2 - \mu^2)\,\phi\,(r) = 0 \quad ...(2)$$

where $\mu = mc/(h/2\pi)$.

The solution of Eq. (2) is $\phi\,(r) = -g\,\frac{e^{-\mu r}}{r}$ ...(3)

Here g is a constant, which plays the same role as the charge q in electromagnetic theory. In analogy with electromagnetism, the potential between two nucleons is then given by

$$V(r) = -g^2\,\frac{e^{-\mu r}}{r} \quad ...(4)$$

Here g^2 is called the '*Coupling constant*'. This argument made Yukawa predict the existence of *pion* as a quantum of nuclear force field.

The range of the pion field is $\frac{h/2\pi}{m_\pi\,c} \approx 1.4$ *fm*

The form of $V(r)$ given by Eq. (4) is known as the *one-pion-exchange potential* (*OPEP*).

On the basis of the range of nuclear force and the uncertainty principle, it is possible to estimate the mass of the meson. According to uncertainty principle $\Delta E \times \Delta t = h/2\pi$ where ΔE and Δt are the uncertainties in energy and time. The range of nuclear force is $R \approx 1.4 \times 10^{-15}$ *m*. Let us assume that the meson travels between nuclei at approximately the speed of light *c*. Let Δt be the time interval between the emission of meson from one nucleon and the absorption by the other nucleon.

Then $\Delta t = R/c$. $\therefore\ \Delta E = \frac{(h/2\pi)}{\Delta t}$

$\therefore$ The minimum meson mass is specified by $m \geq \frac{(h/2\pi)}{Rc}$.

In terms of the electronic mass m_e, the mass of the meson is

$$\frac{m}{m_e} = \frac{h/2\pi}{m_e\,Rc} = \frac{1.054 \times 10^{-34}}{(9.108 \times 10^{-31})\,(1.4 \times 10^{-15})\,(3 \times 10^{8})} = 275$$

i.e., mass of the meson $\approx$ **275 × mass of electron.**

In 1947, Powell discovered π meson of mass about 273 m_e. This particle showed strong interaction with nucleons and was recognized as the Yukawa particle.

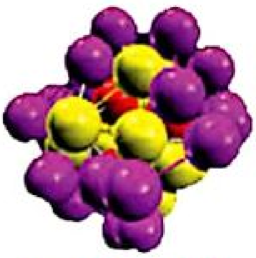
The discovery of the meson of mass of about 273 electron mass and the existence of positive, negative and neutral mesons, lends some support to this theory. The experimental values of the magnetic moments of a free proton and of a free neutron also lend some support to the "*Yukawa's meson field theory*" of nuclear forces. A free proton is, for a part of its life-time, a neutron with a closely bound meson. Hence the magnetic moment of a free proton can be the resultant of the true magnetic moment of the proton and the magnetic moment of the meson. Thus the net magnetic moment of a free proton will exceed that given by the simple theory. Similarly, a neutron is for a fraction of its life-time dissociated into a proton and a negative meson. This combination will have a negative magnetic moment. It follows that, though uncharged, a neutron will have a negative magnetic moment.

27.9 Models of Nuclear Structure

The precise nature of the forces acting in the nucleus is unknown. Hence, nuclear models are resorted to for investigation and theoretical prediction of its properties. Such models may be based on (*i*) the extrinsic analogy between the properties of atomic nuclei and those of a liquid drop (*ii*) the electron shell of an atom etc. The corresponding models are called the liquid-drop model, shell model, etc.

27.10 The Liquid Drop Model

In the liquid-drop model, the forces acting in the nucleus are assumed to be analogical to the molecular forces in a droplet of some liquid. This model was proposed by Neils Bohr who observed that there are certain marked similarities between an atomic nucleus and a liquid drop. The similarities between the nucleus and a liquid drop are the following:



Liquid Drop Model.

(*i*) The nucleus is supposed to be spherical in shape in the stable state, just as a liquid drop is spherical due to the symmetrical surface tension forces.

(*ii*) The force of surface tension acts on the surface of the liquid-drop. Similarly, there is a potential barrier at the surface of the nucleus.

(*iii*) The density of a liquid-drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.

(*iv*) The intermolecular forces in a liquid are short range forces. The molecules in a liquid drop interact only with their immediate neighbours. Similarly, the nuclear forces are short range forces. Nucleons in the nucleus also interact only with their immediate neighbours. This leads to the saturation in the nuclear forces and a constant binding energy per nucleon.

(*v*) The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiations almost immediately.

(*vi*) When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and the nucleus breaks up into two smaller nuclei.

Semi-empirical mass formula. The liquid-drop model can be used to obtain an expression for the binding energy of the nucleus. Weizacker proposed the semi-empirical nuclear binding energy formula for a nucleus of mass number A, containing Z protons and N neutrons. It is written as

$$\text{B.E.} = aA - bA^{2/3} - \frac{cZ(Z-1)}{A^{1/3}} - \frac{d(N-Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

where a, b, c, d and δ are constants.

Explanation of the terms. (1) The first term is called the *volume energy* of a nucleus ($E_v = aA$). The larger the total number of nucleons A, the more difficult it will be to remove the individual protons and neutrons from the nucleus. The B. E. is directly proportional to the total number of nucleons A.

(2) The nucleons, at the surface of the nucleus, are not completely surrounded by other nucleons. Hence energy of the nucleon on the surface is less than that in the interior. The number of surface nucleons depends upon the surface area of the nucleus. A nucleus of radius R has an area of $4\pi R^2 = 4\pi r_o^2 A^{2/3}$. Hence the *surface effect* reduces the B.E. by $E_s = b A^{2/3}$. The negative energy E_s

is called the *surface energy* of a nucleus. It is most significant for the lighter nuclei, since a greater fraction of their nucleons are on the surface.

(3) The *electrostatic repulsion* between each pair of protons in a nucleus also contributes towards decreasing its B.E. The Coulomb energy E_c of a nucleus is the work that must be done to bring together Z protons from infinity into a volume equal to that of the nucleus. Hence $E_c \propto Z(Z-1)/2$ (the number of proton pairs in a nucleus containing Z protons) and E_c is inversely proportional to the nuclear radius $R = r_o A^{1/3}$. E_c is negative because it arises from a force that opposes nuclear stability.

(4) The fourth term $E_a = \dfrac{d\,(N-Z)^2}{A}$ originates from the *lack of symmetry* between the number of protons (Z) and the number of neutrons (N) in the nucleus. The maximum stability of a nucleus occurs when $N = Z$. Any departure from this introduces an asymmetry N–Z, which results in a decrease in stability. The decrease in the B.E. arising from this is called the asymmetric energy (E_a). This is also negative.

(5) The final correction term δ allows for the fact that *even-even* nuclei are more stable than *odd-odd* nuclei. δ is positive for even-even nuclei, is negative for odd-odd nuclei and δ = 0 for an odd A.

The best values of the constants, expressed in MeV, are a = 15.760; b = 17.810, c = 0.711, d = 23.702, δ = 34.

The contributions of the various effects in Weizacker's empirical formula are represented schematically in the graph of Fig. 27.7.

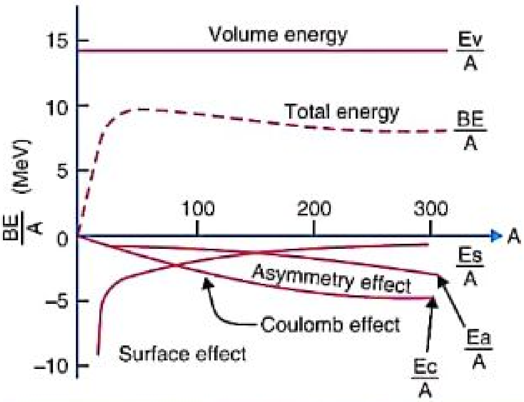


Fig. 27.7

Merits. (1) The liquid drop model accounts for many of the salient features of nuclear matter, such as the observed binding energies of nuclei and their stability against α and β disintegration as well as nuclear fission.

(2) The calculation of atomic masses and binding energies can be done with good accuracy with the liquid drop-model.

However, this model fails to explain other properties, in particular the magic numbers. It fails to explain the measured spins and magnetic moments of nuclei.

Example. *Calculate the atomic number of the most stable nucleus for a given mass number A.*

Sol. The most stable nucleus with a given mass number A is that which has the maximum value of the B.E. Thus we have to compute $\dfrac{\partial\,(B.E.)}{\partial Z}$ with A constant, and equate it to zero. In the formula for B.E., we can write $Z(Z-1) = Z^2$ and $N - Z = A - 2Z$. Then

$$B.E. = aA - bA^{2/3} - \frac{cZ^2}{A^{1/3}} - \frac{d\,(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

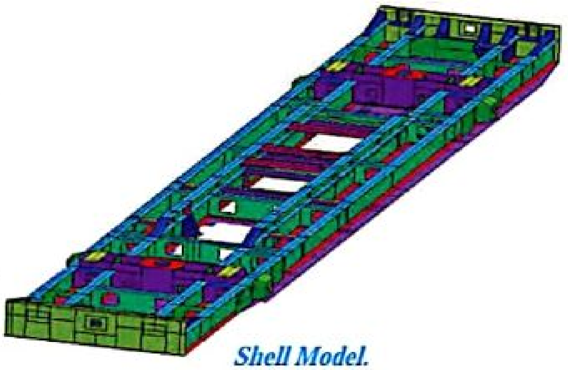
$$\frac{\delta E_b}{\delta Z} = -\,cZA^{-1/3} + 4d\,(A-2Z)\,A^{-1} = 0$$

or, introducing the numerical values of c and d, we have

$$Z = \frac{A}{2 + 0.0157\, A^{2/3}}.$$

For light nuclei, having small A, we can neglect the second term in the denominator. $\therefore\ Z \approx A/2$. This result is confirmed *experimentally.*

27.11 The Shell Model



Shell Model.

The shell model of the nucleus assumes that the energy structure (energy levels of the nucleons) of the nucleus is similar to that of an electron shell in an atom. According to this model, the protons and neutrons are grouped in shells in the nucleus, similar to extra-nuclear electrons in various shells outside the nucleus. The shells are regarded as "filled" when they contain a specific number of protons or neutrons or both. The number of nucleons in each shell is limited by the *Pauli exclusion principle.* The shell model is sometimes referred to as the *independent particle model* because it assumes that each nucleon moves independently of all the other nucleons and is acted on by an average nuclear field produced by the action of all the other nucleons.

Evidence for shell model. It is known that a nucleus is stable if it has a certain definite number of either protons or neutrons. These numbers are known as *magic numbers.* The magic numbers are 2, 8, 20, 50, 82 and 126. Thus nuclei containing 2, 8, 20, 50, 82 and 126 nucleons of the same kind form some sort of closed nuclear shell structures. The main points in favour of this inference are:

(*i*) The inert gases with closed electron shells exhibit a high degree of chemical stability. Similarly, nuclides whose nuclei contain a magic number of nucleons of the same kind exhibit more than average stability.

(*ii*) Isotopes of elements having an isotopic abundance greater than 60% belong to the magic number category.

(*iii*) Tin ($_{50}Sn$) has ten stable isotopes, while calcium ($_{20}Ca^{40}$) has six stable isotopes. So elements with Z = 50, 20 are more than usually stable.

(*iv*) The three main radioactive series (viz., the uranium series, actinium series and thorium series) decay to $_{82}Pb^{208}$ with $Z = 82$ and $N = 126$. Thus lead $_{82}Pb^{208}$ is the most stable isotope. This again shows that the numbers 82 and 126 indicate stability.

(*v*) It has been found that nuclei having a number of neutrons equal to the magic number, cannot capture a neutron because the shells are closed and they cannot contain an extra neutron.

(*vi*) It is found that some isotopes are spontaneous neutron emitters when excited above the nucleon binding energy by a preceding β-decay. These are $_8O^{17}$, $_{36}Kr^{87}$ and $_{54}Xe^{137}$ for which $N = 9$, 51 and 83 which can be written as 8 + 1, 50 +1, and 82 + 1. If we interpret this loosely bound neutron, as a valency neutron, the neutron numbers 8, 50, 82 represent greater stability than other neutron numbers.

It is apparent from the above conclusions that nuclear behaviour is often determined by the excess or deficiency of nucleons with respect to closed shells of nucleons corresponding to the magic numbers. It was, therefore, suggested that nucleons revolve inside the nucleus just as electrons revolve outside in specific permitted orbits. The protons and neutrons move in two separate systems of orbits round the centre of mass of all the nucleons. The extra-nuclear electrons revolve in the Coulomb field of a relatively distant heavy nucleus. But the nucleons move in orbits around a common centre of gravity of all the constituents of the nucleus. Each nucleon shell has a specific maximum capacity. When the shells are filled to capacity, they give rise to particular numbers (the magic numbers) characteristic of unusual stabiẊty.

The shell model is able to account for several nuclear phenomena in addition to magic numbers.

(*i*) It is observed that even-even nuclei are, in general, more stable than odd-odd nuclei. This is obvious from the shell model. According to Pauli's principle, a single energy sublevel can have a maximum of two nucleons (one with spin up and other with spin down). Therefore, in an even-even nucleus only *completed* sublevels are present which means greater stabiẊty. On the other hand, an odd-odd nucleus contains incomplete sublevels for both kinds of nucleon which means lesser stability.

(*ii*) The shell model is able to predict the total angular momenta of nuclei. In even-even nuclei, all the protons and neutrons should pair off so as to cancel out one another's spin and orbital angular momenta. Thus even-even nuclei ought to have zero nuclear angular momenta, as observed. In even-odd and odd-even nuclei, the half-integral spin of the single "extra" nucleon should be combined with the integral angular momentum of the rest of nucleus for a half-integral total angular momentum. Odd-odd nuclei each have an extra neutron and an extra proton whose half-integral spins should yield integral total angular momenta. Both these predictions are experimentally confirmed.

27.12 The Collective Model

The collective model was proposed by A. Bohr, B. R. Mottleson and James Rainwater. The model combines the best features of the liquid drop model and the shell model. In this model it is assumed that the particles within the nucleus exert a centrifugal pressure on the surface of the nucleus. That results in the permanent deformation to non-spherical shape. As a result, the surface may undergo periodic oscillations.

The particles within the nucleus move in a non-spherical potential. Thus the nuclear distortion reacts on the particles and modifies the independent particle aspect. Thus the nucleus is considered as a sheẊ structure capable of performing oscillations in shape and size. Thus the model can easily describe the drop like properties such as nuclear fission and at the same time it can retain the shell model characteristics. It is capable of explaining not only the large electric quadrupole moments but it can also predict the fine structure of nuclear level spectrum due to the energies associated with the vibrational and rotational motion of the core.

The total energy is expressed as

$$W = E_{rot} + E_{vib} + E_n$$

where E_{rot} is the energy due to rotational motion of the core, E_{vib} is the energy due to the vibrational coordinates and E_n is the energy due to the nucleonic coordinates. So the wave function is the product of three wave functions each containing the respective coordinates.

According to the collective model, all even-even nuclei such as $_2He^4$, $_8O^{16}$, $_{20}Ca^{40}$, $_{82}Pb^{208}$ wiẊ have spherical shapes and zero electric quadrupole moment, while the even-odd, odd-even or odd-odd nuclei will have non-spherical shapes and finite electric quardrupole moment. The nuclear energy levels predicted by the model agree closely with the ones given by γ–ray spectra of the nuclei.

27.13 Determination of Nuclear Radius–Mirror Nuclei Method

Two nuclei having same number of nucleons (protons + neutrons) but the number of protons in one of them being equal to the number of neutrons in the other are called the mirror-nuclei (Fig. 27.8.)

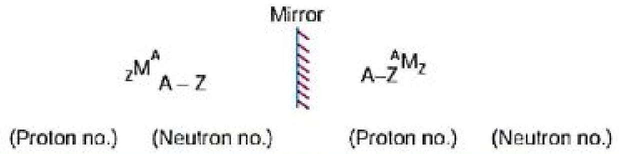


Fig. 27.8

Example. *$_6C^{11}$ and $_5B^{11}$ are mirror nuclei. $_6C^{11}$ contains 6 protons and 5 neutrons while $_5B^{11}$ contains 5 protons and 6 neutrons.*

Sol. The other examples of mirror nuclei are: ($_1^4\,H$, $_2^3\,He$); ($_3^7\,Li$, $_4^7\,Be$); ($_6^{13}\,C$, $_7^{13}\,N$); ($_{19}^{39}\,K$, $_{20}^{39}\,Ca$).

Bethe suggested that the difference of Coulomb energies between two neighbouring mirror nuclei can be used to calculate nuclear radii.

Consider a spherical nucleus of radius R and atomic number Z (Fig. 27.9). Each proton carries a charge $+\,e$. Thus the total charge is $+\,Ze$.

$\therefore$ Charge density,
$$\rho = \frac{\text{Charge on nucleus}}{\text{Volume of nucleus}} = \frac{Ze}{\frac{4}{3}\pi R^3} \qquad ...(1)$$

The electrostatic energy of nucleus is the work done against electrostatic forces in assembling such a sphere.

Suppose at any instant we have a spherical core of radius x.

Charge on this spherical core $= q_1 = \frac{4}{3}\pi x^3 \rho$

Now we bring a thin spherical shell of radius x and thickness dx from infinity in the process of assembling the charge.

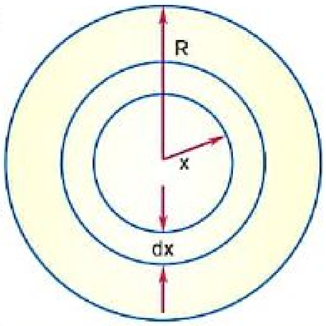


Fig. 27.9

The charge on this shell is

q_2 = volume of the shell × charge × density = $(4\pi x^2\, dx)\,\rho$.

The electrostatic potential energy of this spherical core and thin spherical shell

$$dE_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{x}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\left(\frac{4}{3}\pi x^3\rho\right)\left(4\pi x^2\, dx\,\rho\right)}{x} = \frac{1}{4\pi\epsilon_0}\left(\frac{16}{3}\pi^2\rho^2 x^4\, dx\right)$$

Total electrostatic potential energy of nucleus is obtained by integrating this expression between limits 0 and R.

$$E_c = \int_0^R \frac{1}{4\pi\epsilon_0}\left(\frac{16}{3}\pi^2\rho^2 x^4\, dx\right) = \frac{1}{4\pi\epsilon_0}\frac{16}{3}\pi^2\rho^2\left[\frac{R^5}{5}\right]$$

Substituting value of ρ from Eq. (1), we get

$$E_c = \frac{1}{4\pi\epsilon_0}\frac{16}{3}\pi^2 \cdot \left(\frac{Ze}{\frac{4}{3}\pi R^3}\right)^2 \cdot \frac{R^5}{5} = \frac{1}{4\pi\epsilon_0}\cdot\frac{3}{5}\frac{Z^2e^2}{R} \quad ...(2)$$

The difference in Coulomb energies between two neighbouring nuclei of charges (Ze) and $(Z+1)e$ having the same radius R is

$$\Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{(Z+1)^2e^2}{R} - \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Z^2e^2}{R} = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{e^2}{R}\left[(Z+1)^2 - Z^2\right]$$

$$\Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{e^2}{R}(2Z+1) \quad ...(3)$$

In the mirror nuclei $(2Z+1) = A$, where A is mass number. Eq. (3) becomes

$$\Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{e^2A}{R} \quad ...(4)$$

Taking experimental observations of mirror nuclei ($_{14}Si^{29}$, $_{15}P^{29}$), the difference in Coulomb-energies comes out to be 4.96 MeV. Substituting this value, $R = 4.96$ fermi.

From the relation $R = R_0 A^{1/3}$, we obtain the radial parameter, $R_0 \approx$ **1.5 fermi.**

27.14 Nuclear Fission

Example 1. *Obtain an expression for energy released in symmetric fission.*

(*Mumbai University, October 2010*)

Sol. In a *'symmetric fission'*, a nucleus (Z, A) breaks into two equal halves ($Z/2$, $A/2$). Semi-empirical mass formula gives a satisfactory account of fission energetics.

The energy released in a symmetric fission is:

$$Q = M(Z, A) - 2M(Z/2, A/2) \quad ...(1)$$

From the semi-empirical mass formula, binding energy for a nucleus (Z, A) is:

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_a\frac{(A-2Z)^2}{A} - a_p A^{-3/4} \quad ...(2)$$

Binding energy for two fission fragments ($Z/2$, $A/2$) is:

$$2B(Z/2, A/2) = a_v A - 2a_s\left(\frac{A}{2}\right)^{2/3} - 2a_c\frac{\frac{Z}{2}\left(\frac{Z}{2}-1\right)}{\left(\frac{A}{2}\right)^{1/3}} - 2a_a\frac{(A/2-Z)^2}{A/2} - 2a_p\left(\frac{A}{2}\right)^{-3/4} \quad ...(3)$$

$$Q = a_s A^{2/3}\left(1-2^{1/3}\right) + a_c\frac{Z(Z-1)}{A^{1/3}}\left(1-\frac{1}{2^{2/3}}\right) + \text{pairing energy difference.}$$

The difference in pairing energy is small and thus can be neglected as an approximation.

$$\therefore \quad Q = A^{2/3}\left(-0.260a_s + 0.370\frac{Z(Z-1)}{A}a_c\right) \quad ...(4)$$

Using the values of $a_s = 13$ MeV and $a_c = 0.60$ MeV, we get

$$Q \sim A^{2/3}\left(-3.38\text{ MeV} + 0.22\text{ MeV}\frac{Z(Z-1)}{A}\right) \quad ...(5)$$

From this we see that Q becomes > 0 for

$$\frac{Z^2}{A} \geq \frac{3.38}{0.22} \sim 15 \quad ...(6)$$

Thus for all nuclei having, $Z > 35$, $A > 80$, fission is possible and would release energy.

For the symmetric fission of $^{238}_{92}U$,

$$Q = (238)^{2/3}\left(-3.38 + 0.22\frac{92(92-1)}{238}\right)\text{MeV} \sim 170\text{ MeV} \quad ...(7)$$

EXAMPLE 2. *Explain mass yield in asymmetrical fission and discuss mass distribution of fission fragments for different fission chains of U^{235}.* (*Mumbai, 2010*)

SOL. Consider the following fission reaction:

$$^{235}_{92}\text{U} + ^{1}_{0}n \rightarrow \underset{\text{Compound nucleus}}{\left[^{236}_{92}\text{U}\right]^*} \rightarrow ^{143}_{56}\text{Ba} + ^{90}_{36}\text{Kr} + 3\left(^{1}_{0}n\right) + Q \quad ...(1)$$

The stable isotope of Barium with maximum A is ^{138}Ba. The stable isotope of Krypton with maximum A is ^{86}Kr. To get rid of excess neutrons, these fission fragments must exhibit much beta activity. Experimentally this is confirmed by the following beta decay observations:

$$^{143}_{86}\text{Ba} \longrightarrow ^{143}_{57}\text{La} + \beta^- + \bar{\nu}$$
$$\longrightarrow ^{143}_{58}\text{Ce} + \beta^- + \bar{\nu}$$
$$\longrightarrow ^{143}_{59}\text{Pr} + \beta^- + \bar{\nu}$$
$$\longrightarrow ^{143}_{60}\text{Nd} + \beta^- + \bar{\nu} \text{ (Stable)} \quad ...(2)$$

$$^{90}_{36}\text{Kr} \longrightarrow ^{90}_{37}\text{Rb} + \beta^- + \bar{\nu}$$
$$\longrightarrow ^{90}_{38}\text{Sr} + \beta^- + \bar{\nu}$$
$$\longrightarrow ^{90}_{39}\text{Y} + \beta^- + \bar{\nu}$$
$$\longrightarrow ^{90}_{40}\text{Zr} + \beta^- + \bar{\nu} \text{ (Stable)} \quad ...(3)$$

Eq. (1) is only one of the many ways in which $^{235}_{92}U$-nucleus undergoes fission.

There are more than 30 modes of fission, in each of which a different pair of nuclei (fission-fragments) is formed. Eqs. (2) and (3) are examples of a *fission chain*. There are about 60 such chains known experimentally.

Mass Distribution of Fission Fragments for Different Fission Chains of $^{235}_{92}$U

The fission yield $Y(A)$ is defined as the ratio of number of nuclei of mass number A formed in fission to the total number of fissions, *i.e.*,

$$Y(A) = \frac{\text{Number of nuclei of mass number } A \text{ formed in fission } (N_A)}{\text{Total number of fissions } (N_0)}$$

Percentage fission yield $Y(A)\% = \frac{N_A}{N_0} \times 100\%$

When this percentage of fission yield is plotted against mass number A for fission chains of ^{235}U, we get a mass distribution of fission fragments (Fig. 27.10).

- The maxima lie near mass numbers 95 and 139 implying that fission is highly asymmetric.
- The yield is minimum for A ~ 117, which corresponds to symmetrical fission (equal masses).
- Some of the fragments may have as low a mass number as 72 or as high as 158.
- The *asymmetric mass distribution* of fission fragments involves the *shell structure* of nuclei and the existence of *magic numbers*. Corresponding to the peak (A ~ 95 and 139) yields, the atomic numbers are Z~ 42 and 57. But the Z-value for minimum yield (A~ 117) is Z~ 50 that corresponds to a magic number. Hence its nuclear binding is strong, corresponding to a *closed nuclear shell*. Necessarily, therefore, the fission yield would be minimum. Thus, asymmetric fission is more probable for uranium.

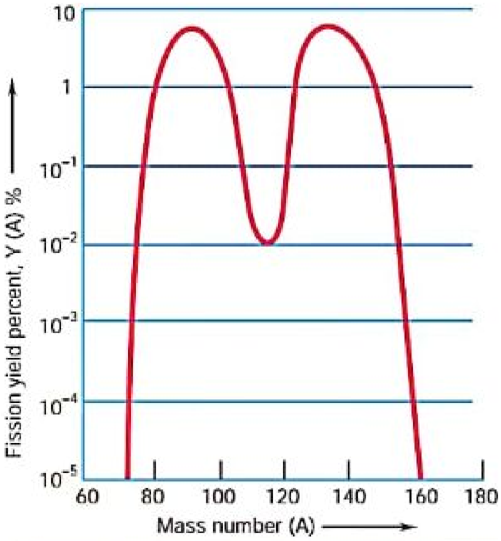


Fig. 27.10

27.15 Applications of Semi-Empirical Mass-Formula

1. Mass Parabolas: Prediction of stability of nuclei against β-decay.

The isobaric nuclei have the same mass number A but different atomic numbver Z.

The semi-empirical mass formula can be written as,

$$_ZM^A = Z(M_p - M_n) + A(M_n - a_v) + a_c\frac{Z^2}{A^{1/3}} + a_s A^{2/3} + a_a A - a_a 4Z + a_a\frac{4Z^2}{A} \pm \delta$$

$$_ZM^A = \alpha A + \beta Z + \gamma Z^2 \pm \delta \qquad ...(1)$$

Here, $\alpha = M_n - \left(a_v - a_a - \frac{a_s}{A^{1/3}}\right)$

$$\beta = -4a_a - (M_n - M_p)$$

$$\gamma = \left(\frac{4a_a}{A} + \frac{a_c}{A^{1/3}}\right)$$

δ = 0 for odd-Z, even N
= 0 for even-Z, odd-N
= + δ of odd-Z, odd-N
= − δ for even Z, even-N

$\pm\delta$ is the pairing energy and does not contain terms in Z.

When A is constant, Eq. (1) is the equation of a *parabola*. It is known as the *mass parabola*. Eq. (1) gives the dependence of nuclear mass on the nuclear charge with *constant A*. This dependence is *parabolic*.

The most stable nucleus has the *minimum* mass.

Differentiating Eq. (1) for constant A and equating it to zero, we have

$$\frac{\partial}{\partial Z}\left({}_Z M^A\right) = 0 = \beta + 2\gamma Z_0 \text{ at the point } Z = Z_0.$$

$$\therefore \quad Z_0 = \text{nuclear charge of the "most stable" isobar} = -\frac{\beta}{2\gamma} \quad \text{...(2)}$$

(i) Mass parabolas of odd A isobars

For Odd A nuclei, $\delta = 0$ and thus there is only one parabola, implying that there is only one stable nucleus.

The isobar $Z = Z_0 + 1$ lying on the right hand side of the parabola (Fig. 27.11), has greater mass. It will undergo β^+ decay, giving rise to a sable isobar with $Z = Z_0$.

Similarly, the isobar lying on the left of the curve will undergo β^- decay.

The situation is similar for two isobars $(A, Z_0 + 2)$ and $(A, Z_0 - 2)$ which can decay to $(A, Z_0 + 1)$ and $(A, Z_0 - 1)$, respectively; and so on.

$${}^{73}_{34}\text{Se} \xrightarrow{\beta^+, K} {}^{73}_{33}\text{As} \xrightarrow{K} {}^{73}_{32}\text{Ge} \text{ (stable)}$$

$${}^{73}_{30}\text{Zn} \xrightarrow{\beta^-} {}^{73}_{31}\text{Ga} \xrightarrow{\beta^-} {}^{73}_{32}\text{Ge} \text{ (stable)}$$

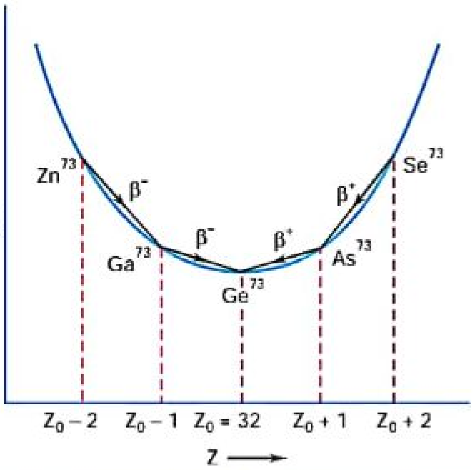


Fig. 27.11

Transition Energies

$${}_{Z_0}M^A = \alpha A + \beta Z_0 + \gamma Z_0^2 = \alpha A - \gamma Z_0^2 \quad \text{(using Eq. 2)}$$

$${}_Z M^A - {}_{Z_0}M^A = (\alpha A + \beta Z + \gamma Z^2) - (\alpha A - \gamma Z_0^2) = \gamma (Z - Z_0)^2 \quad \text{...(3)}$$

Energy released in $\{Z \to (Z + 1)\}$ transition is

$$Q_\beta^- = {}_Z M^A - {}_{Z+1}M^A = \gamma\,[(Z - Z_0)^2 - \{(Z + 1) - Z_0\}^2] = 2\gamma\,(Z_0 - Z - 1/2)$$

The Q value for $Z \to (Z - 1)$ transition is given by

$$Q_\beta^+ = {}_Z M^A - {}_{Z-1}M^A = \gamma\,\{(Z - Z_0)^2 - (Z - 1 - Z_0)^2\} = 2\gamma\,[Z - Z_0 - 1/2)$$

(ii) Mass parabolas of even A Isobars

Here the pairing term $\delta \neq 0$. Since both odd-odd and even-even nuclei are included, we have two parabolas. The lower parabola corresponds to more stable nuclei with even Z, while the upper parabola to less stable nuclei with odd Z. The vertical separation between two parabolas is 2δ due to the opposite sign of δ in two cases.

Fig. 27.12 shows the decay scheme. The reaction energy Q for transition $Z \to Z \pm 1$ for isobars of mass number A is

$$Q_\beta = 2\gamma\,\{\pm[Z_0 - Z] - 1/2\} \pm 2\delta \left| \begin{array}{l} +\,2\delta \text{ for odd } Z \\ -\,2\delta \text{ for even } Z \end{array} \right.$$

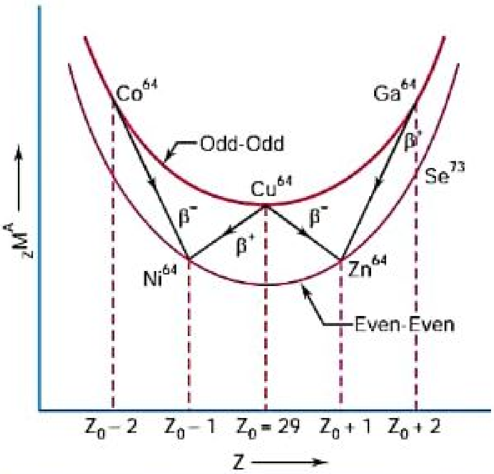


Fig. 27.12

2. Alpha Decay:

The α-decay process is conventionally written as

$$_{Z}X^{A} = {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q_{\alpha}$$

The Q_{α}-disintegration energy involved in α-decay process is written as

$$Q_{\alpha} = {}_{Z}M^{A} - {}_{Z-2}M^{A-4} - {}_{2}M^{4}\,({}_{2}He^{4})$$

In terms of B.E. of nuclei involved, this expression for large Z and A becomes

$$\begin{aligned}
Q_{\alpha}(A, Z) &= \text{B.E.}\,(A-4, Z-2) + \text{B.E.}\,({}_{2}He^{4}) - \text{B.E.}\,(A, Z) \\
&= a_v (A-4) - a_s (A-4)^{2/3} - a_c \frac{(Z-2)^2}{(A-4)^{1/3}} - a_a \frac{(A-2Z)^2}{(A-4)} \\
&\quad - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} + \text{B.E.}\left({}_{2}He^{4}\right) \\
&= \text{B.E.}\,({}_{2}He^{4}) - 4a_v + a_s \{A^{2/3} - (A-4)^{2/3}\} \\
&\quad + a_c \left\{ \frac{Z^2}{A^{1/3}} - \frac{(Z-2)^2}{(A-4)^{1/3}} \right\} + a_a (A-2Z)^2 \left(\frac{1}{A} - \frac{1}{(A-4)} \right) \\
&= 28.3 - 4a_v + \frac{8}{3} a_s A^{-1/3} + \frac{4a_c Z}{A^{1/3}} \left(1 - \frac{Z}{3A}\right) - \frac{4a_a (A-2Z)^2}{A(A-4)}
\end{aligned}$$

In the above formulation, pairing energy term has been neglected. If numerical values of a_v, a_s, a_c and a_a are employed, it is found that $Q_{\alpha} > 0$ for $A > 160$. Therefore, the nuclei $A > 160$ should be α-disintegrating. Practically α-disintegration is observed in nuclei with $A > 200$. For nuclei $A < 200$, the energy release is so small that barrier penetration becomes small.

EXAMPLE 3. *What are mass parabolas? Define mirror nuclei and obtain an expression for β disintegration energy of mirror nuclei.* *(Mumbai University, October 2010)*

Beta Disintegration Energy of Mirror Nuclei:

Mirror nuclei are pairs of isobaric nuclei in which the proton and neutron numbers are interchanged and differ by one unit. ${}_{6}C^{11}$ and ${}_{5}B^{11}$ are mirror nuclei. For such pairs $A = 2Z - 1$.

The members of the pairs with higher Z are usually found β^{+} emitters such as

$${}_{6}C^{11} \longrightarrow {}_{5}B^{11} + \beta^{+} + \nu$$

The maximum energy of the β^{+} emitted in this case is 0.96 MeV. No γ-ray energy is emitted in the transition.

We neglect pairing energy term δ (for odd A).

The semi-empirical mass formula is written as

$$\begin{aligned}
M(A, Z) &= ZM_p + NM_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \\
&= ZM_p + (Z-1)\,M_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + \frac{a_a}{A}
\end{aligned}$$

For the daughter term

$$M(A, Z-1) = (Z-1)\,M_p + (Z-2)\,M_n - a_v A + a_s A^{2/3} + a_c \frac{(Z-1)^2}{A^{1/3}} + \frac{a_a}{A}$$

$$M(A, Z) - M(A, Z-1) = M_p - M_n + \frac{a_c (2Z-1)}{A^{1/3}}$$

$$\therefore \quad Q_{\beta+} = M(A, Z) - M(A, Z-1) - 2m_e$$
$$= M_p - M_n + \frac{a_c(2Z-1)}{A^{1/3}} - 2m_e$$
$$= a_c A^{2/3} - (M_n - M_p + 2m_e)$$
$$Q_{\beta+} = a_c A^{2/3} + 1.804 \text{ MeV}$$

The equation shows that plot of disintegration energy $Q_{\beta+}$ against $A^{2/3}$ should be straight line with a slope $a_c = \dfrac{3e^2}{20\pi\varepsilon_0 r_0}$. The slope gives nuclear radius of the order 1.44 fermi.

EXERCISE

1. Find the energy release, if two $_1H^2$ nuclei can fuse together to form $_2He^4$ nucleus. The binding energy per nucleon of H^2 and He^4 is 1.1 MeV and 7.0 MeV respectively.

[**Hint.** B.E. for $_2He^4$ = 28 MeV; B.E. for $_1H^2$ nucleus = 2.2 MeV.

$$\Delta E = [\text{Mass of } _1H^2] - [\text{Mass of } _2He^4]$$
$$= 2[M_p + M_n - 2.2] - [2M_p + 2M_n - 28.0] = 23.6 \text{ MeV}]$$

2. Calculate the binding energy per nucleon in $_6C^{12}$. Masses of proton, neutron and electron are 1.007276, 1.008665 and 0.00055 u respectively. The mass of $_6C^{12}$ atom is 12.000000 u.

[**Hint.** mass of 6 protons + mass of 6 neutrons = (6 × 1.007276 + 6 × 1.008665) u = 12.095646 u.

Mass of carbon nucleus = mass of carbon atom – mass of 6 electrons

= (12.000000 – 6 × 0.00055) u = 11.996700 u

Δm = (12.095646 – 11.996700) u = 0.098946 u

B.E. = 0.098946 × 931.3 MeV = 92.148 MeV

B.E. per nucleon = 92.148/12 = 7.69 MeV]

3. The deviation of the charge distribution of a nucleus from spherical symmetry can be estimated by measuring its

(*a*) electric charge (*b*) electric dipole moment

(*c*) magnetic dipole moment (*d*) electric quadrupole moment.

[**Ans.** (*d*)]

4. *Give an account of the methods of separating isotopes.*

Ans. Since isotopes have the chemical properties, they cannot be separated from one another by chemical methods. Isotopes can be separated from one another only by making use of the differences in their physical properties. All the methods of separation are based on the difference in the masses of the isotopes.

(1) *Mass spectrograph method.* The photographic plate in the mass spectrograph of Aston's type is replaced by different vessels where the different isotopes can be separately collected. Complete separation is possible by this method, but the yield is extremely small. The main difficulty is that extremely high currents of ions or long intervals of time are required to separate any appreciable amount of an element into its isotopes.

(2) *Diffusion method.* The rate of diffusion of a gas through a porous partition is inversely proportional to the square root of the masses of the molecules. Hence different isotopes of a gas will diffuse with slightly different velocities.

Fig. 27.13 shows Hertz multiple-stage porous-tube apparatus for separating isotopes by diffusion.

The progress of the main volume of gas is from right to left, as it becomes enriched in the way heavy fractions. The lighter fractions diffuse out through the porous tubes (crosshatched) and are returned by the recirculating pumps P to the previous stage, eventually collecting in the reservoir L. The heavier fractions progress from state to stage, eventually collecting in the reservoir H. The over-all enrichment increases exponentially with the number of stages. Depending on the isotopes to be separated, some 10 to 50 or more stages may be used.

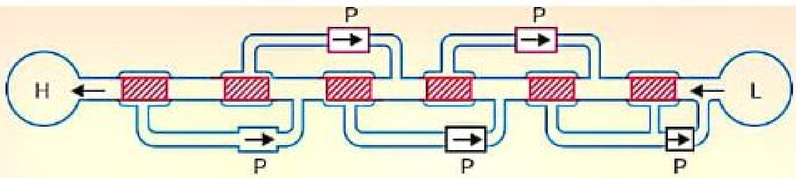


Fig. 27.13

(3) *Thermal diffusion method.* This is based on the fact that if a gaseous mixture of isotopes is placed in a vessel, one part of which is hotter than the other, the lighter molecules collect in the region of higher temperature.

Fig. 27.14 show the Clusius-Dickel thermal-diffusion isotope separator. A cooled vertical glass tube with an electrically heated wire along its axis is used. By thermal diffusion the heavier fraction tends to concentrate at the cool outer wall.

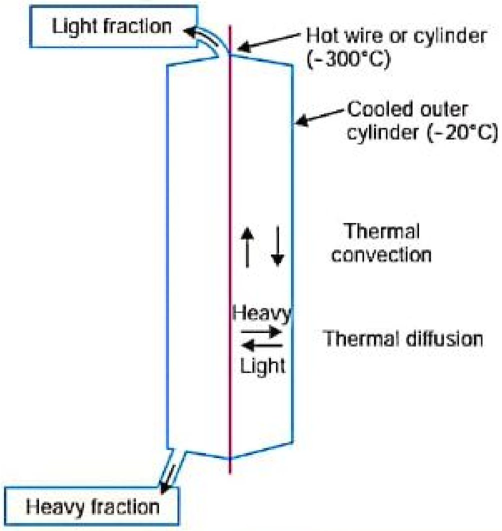


Fig. 27.14

The lighter fraction concentrates at the hot inner cylinder or wire. The action of gravity then causes thermal convection which provides an effective downward transport for the heavy fraction at the cool outer wall and an upward transport for the light fraction along the axis. The pressure is maintained at a low enough value to avoid turbulence in the thermal convective flow.

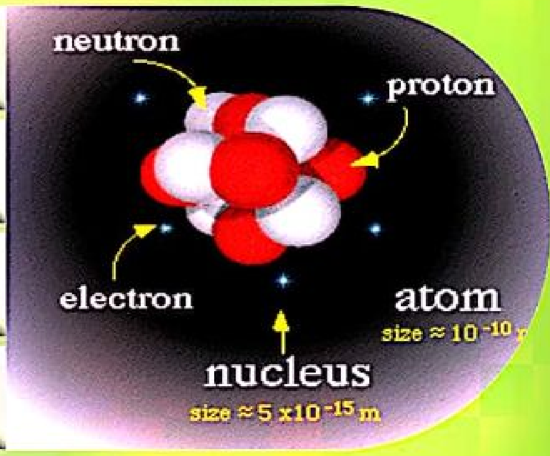
The process separation factor α, in terms of the equilibrium mole concentrations in the heavy and light reservoirs, is

$$\alpha = e^{lA(M_2 - M_1)/(M_2 + M_1)}$$

Here, M_1 and M_2 are the masses of the light and heavy isotopic molecules being separated, l is the length of the column.

A is a function of the viscosity, self-diffusion, and density of the gas, the temperatures and radii of the cylindrical walls, and the gravitational constant.

(4) *Pressure diffusion (or centrifugal) method.* This is based on the fact that if a gas or vapour flows into a rapidly rotating cylinder, the force acting on the molecules will result in an increased concentration of the heavier isotopes at the walls while the lighter ones collect near the axis of rotation. Isotopes of chlorine in carbon tetrachloride and of bromine in ethyl bromide have been separated by this method.



CHAPTER 28

NUCLEAR STRUCTURE

AT A GLANCE			
28.1	Measurement of Nuclear Radius	28.2	Parity
28.3	Statistics of Nuclei	28.4	Properties of Ground State of Deuterium Nucleus
28.5	Nuclear Models—The Shell Model	28.6	Fermi Gas Model of the Nucleus

28.1 Measurement of Nuclear Radius

Fig. 28.1 shows Hofstadter's experimental arrangement for nuclear radius measurement. This method consists of measurement of nuclear charge distribution and assumes that charge distribution and matter distribution within nuclei are essentially the same. The charge distribution in nuclei is obtained by analysing scattering of high-energy electrons from various nuclei. The elastic-scattering process is

$$e + X \rightarrow e + X. \quad ...(1)$$

Electron is chosen as the bombarding particle because its interaction with the nucleus, the familiar electro-magnetic interaction, is so well-known.

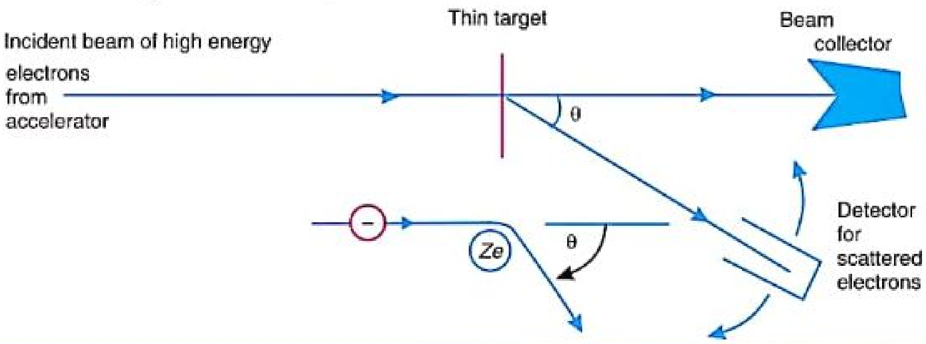


Fig. 28.1

The equipment includes an electron accelerator to prepare the high energy electron beam, a scattering target of species *X*, and a spectrometer to detect electrons scattered elastically in directions given by the indicated scattering angle θ.

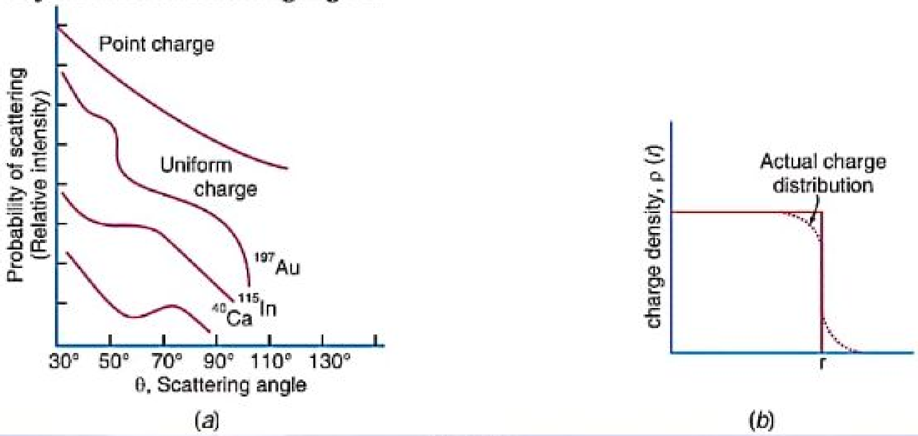


Fig. 28.2

The de Broglie wavelength of 200 MeV electrons is about 10^{-14} m which is the size of a nucleus. Thus to get information about nuclear charge distribution, the incident electron beam energy has to be of the order of 200 MeV. The assumption that nuclear charge is uniformly spread over a spherical volume and not a point charge leads one to expect diffraction effects. Portions of electron waves incident on different parts of the nucleus will be scattered in a particular direction, with phase differences resulting in constructive or destructive interference at some angle. Figure 28.2 (*a*) shows the angular distribution of elastically scattered 200 MeV electrons from nuclei assumed to have spherical uniform charge distribution (having uniform density up to radius *r*). The actual nuclear charge distribution is not exactly given by a step function, but is shown by the dotted line [Fig. 28.2 (*b*)].

Hofstadter assumed that the nuclear charge density has the form

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/b}} \qquad \text{...(2)}$$

Here, ρ_0 is density at the nuclear centre, *R* is the radius at which ρ falls to $\rho_0/2$ and *b* measures how rapidly ρ falls to zero at the nuclear surface.

Nuclear charge distribution found in some nuclei using high-energy electrons as probes is shown in Fig. 28.3. This analysis determines a nuclear radius *R* that grows with mass number *A* according to the formula

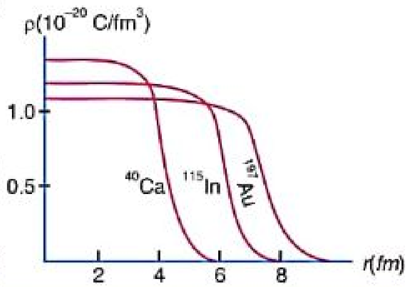


Fig. 28.3

$$R = R_0 A^{1/3} \qquad \text{...(3)}$$

28.2 Parity

The total spin of a nucleus consists of the sum of orbital angular momentum of nucleons and the sum of their spins. The orbital angular momentum $\sum_n l_n$ actually defines the parity of the nuclei.

The angular momentum eigen function can be expressed as a Spherical Harmonics Y_m which is an even function for even l_n and an odd function for an odd l_n. This shows that $\sum_n l_n$ is either even or odd for nuclei. According to this, the nuclear wave functions are said to have either an even parity or odd parity.

For positive or even parity, $\psi(x, y, z) = \psi(-x, -y, -z)$.

For negative or odd parity, $\psi(x, y, z) = -\psi(-x, -y, -z)$.

If $\sum_n l_n$ is even, the parity of nuclide is positive and if $\sum_n l_n$ is odd, the parity is negative.

As an example, the Deuteron nucleus contains a neutron and a proton in *S*-state with $l = 0$ and the parity of deuteron is positive.

Nuclei of various atoms in a ground state have a definite parity which is either positive or negative. When the nuclei are in an excited state, their parities are not always the same as in the ground state. Parity in nuclear transformations is conservable quantity but is not conserved in weak interactions like Beta decay. It is conserved in nuclear reactions and gamma decay.

28.3 Statistics of Nuclei

Since nuclear particles like protons are identical, the probability distribution $|\psi|^2$ remains unchanged by an exchange of particle co-ordinates. If x_1 are the general co-ordinates of one particle (which may include spin) and x_2 are the general co-ordinates for the other, then we have,

$$|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$$

$$\psi(x_1, x_2) = \pm\psi(x_2, x_1).$$

The sign we choose depends on the type of particle we are considering. For electrons, protons, etc., we must choose the negative sign, and the eigen function for the two electrons is antisymmetric for an exchange of coordinates. Such particles are said to satisfy *Fermi-Dirac statistics* and are called *fermions*. On the other hand, for photons, one must choose the positive sign so that the eigen function of two photons is symmetric for an exchange of coordinates. Photons are said to satisfy *Bose-Einstein statistics* and are called *bosons*.

In systems obeying Fermi-Dirac statistics, according to Pauli, not more than one particle can occupy the same quantum state. If spin also is taken as a coordinate in describing a particle, two similar particles with spins oriented in opposite way can occupy a quantum state. Neutrons and protons (nucleons) are spin $\frac{1}{2}$ particles and obey Fermi-Dirac statistics. The wave function of a nucleus with *A* particles will be odd or even according as $(-1)^A$ is positive or negative. Nuclei with odd *A* will obey Fermi-Dirac statistics and with even *A* Bose-Einstein statistics under this criterion.

28.4 Properties of Ground State of Deuterium Nucleus

(*i*) Binding Energy. Deuterium nucleus is the simplest among the atomic nuclei. It contains one proton and one neutron. These two particles are bound together with a binding energy of 2.225 MeV. There are essentially two direct methods of experimental measurement of the binding energy of deuteron. One is to measure the energy of gamma radiation emitted when a neutron and proton combine to form a nucleus of deuteron ($n - p$ capture reaction). The second method consists of measuring the energy of gamma radiation that will break the bond between the neutron and proton (photo disintegration of deuteron).

(*ii*) Angular Momentum and Parity. The angular momentum quantum number of the ground state of the deuteron is $\mathbf{I} = 1$. This is obtained as the resultant of the spin of proton (1/2) and of neutron (1/2). The orbital angular momentum for both the nucleons will be zero. The parity is taken as + ve, which is corroborated by experimental results.

***(iii)* Magnetic Dipole Moment.** The magnetic dipole moment of the deuteron is + 0.85739 μ_N as given by N.M.R. data. The magnetic dipole moment of a proton is + 2.79275 μ_N and that of a neutron is – 1.91315 μ_N. The magnetic dipole moment of deuteron is slightly smaller than the resultant obtained by the algebraic sum of the magnetic moments of proton and of neutron.

***(iv)* Electric Quadrupole Moment.** The electric quadrupole moment of the deuteron is Q = + 0.00282 barn. The quantity of the quadrupole moment is an indication that the charge distribution is not spherically symmetrical in deuteron. The + ve sign for Q indicates that the charge distribution is prolate rather than oblate.

***(v)* Radius of Deuteron.** The size of the deuteron is found by studying the scattering of high energy electrons. The average radius of the deuteron is found to be 4.2 fermi. [It is the root mean square distance between the neturon and proton].

***(vi)* The Force between a Neutron and Proton is Spin-dependent.** I, Angular momentum of deuteron = 1 in units of $\hbar$. *Since the spins of both, the neutron and the proton, is* $\frac{1}{2}$ *in the units of* $\hbar$*, the deuteron must be formed with the spin vectors of the neutron and the proton parallel. The* other possibility of the two spin vectors antiparallel giving a net spin = 0 *does not lead to a bound state for the deuteron.* Deuteron excited states do not exist and so the state to be studied is the *ground state.* For the ground state, $l = 0$. It is 1s state.

$$\therefore \qquad I = L + S = S = 1.$$

The 1s state (ground state) with $S = 0$ does not give a bound state for deuteron. The energy of the ground state of the deuteron is different if the spins of the neutron and proton are parallel than if they are anti-parallel. Otherwise why should the spin parallel system be stable and spin anti-parallel not stable? The important conclusion that can be drawn from this is that *the force between a neutron and a proton is spin-dependent. The* nuclear force keeping the two nucleons system bound, thus depends on whether the spins of the two nucleons are parallel or antiparallel *i.e.*, whether the total spin $S = 1$ or 0. This feature is quite different than, for example, the Coulomb force which goes as $1/r^2$ and so is just a function of r.

28.5 Nuclear Models—The Shell Model

Nuclear Energy Level Scheme and Explanation of Magic Numbers. To account for the observed magic numbers, Mayer and Jensen postulated a strong *nuclear spin-orbit interaction.* The magnitude of the spin-orbit interaction is such that the consequent splitting of energy levels into sublevels is many times larger than the analogous splitting of atomic energy levels. The nuclear spin-orbit splitting of a single-nucleon energy level is assumed to be *large* and also *inverted* (Fig. 28.4). We ascribe this behaviour to a nuclear interaction of the form

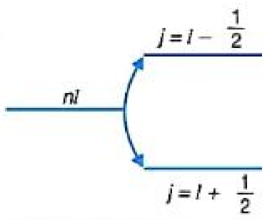


Fig. 28.4

$$V_{SL}\text{ (nucleus)} = -\frac{a_{SL}^2}{r}\frac{dV}{dr}\mathbf{S.L.}$$

The minus sign accomplishes the required inversion of the split levels.

The constant a_{SL}^2 produces the desired amount of energy splitting. The central-field function $V(r)$ appears along with the orbital and spin angular momenta of the nucleon. The exact form of the potential-energy function is not critical, provided that it more or less resembles a square well.

The shell theory assumes that LS coupling holds only for the very lightest nuclei, in which the l values are necessarily small in their normal configurations. In this scheme, the intrinsic spin angular momenta S_i of the particles concerned are coupled together into a total spin momentum **S**. The orbital angular momenta $\mathbf{L}_i$ are separately coupled together into a total orbital momentum **L**. Then **S** and **L** are coupled to form a total angular momentum **J** of magnitude $\sqrt{\mathbf{J}\,(\mathbf{J}+1)}\,\hbar$.

After a transition region in which an intermediate coupling scheme holds, the heavier nuclei exhibit **jj coupling**. In this case, the $\mathbf{S}_i$ and $\mathbf{L}_i$ of each particle are first coupled to form a $\mathbf{J}_i$ for that particle of magnitude $\sqrt{\mathbf{J}\,(\mathbf{J}+1)}\,\hbar$. The various $\mathbf{J}_i$ then couple together to form the total angular momentum **J**. The *jj* coupling scheme holds for the great majority of nuclei.

Fig. 28.5 shows the nucleon energy levels according to the shell model. The levels are designated by a prefix equal to the total quantum number *n*, a letter that indicates *l* for each particle in that level, and a subscript equal to *j*. The spin-orbit interaction splits each state of given *j* into $2j + 1$ substates. The accumulated population of nucleons corresponds to a magic number at every one of the larger energy gaps. Hence shells are filled when there are 2, 8, 20, 28, 50, 82 and 126 neutrons or protons in a nucleus.

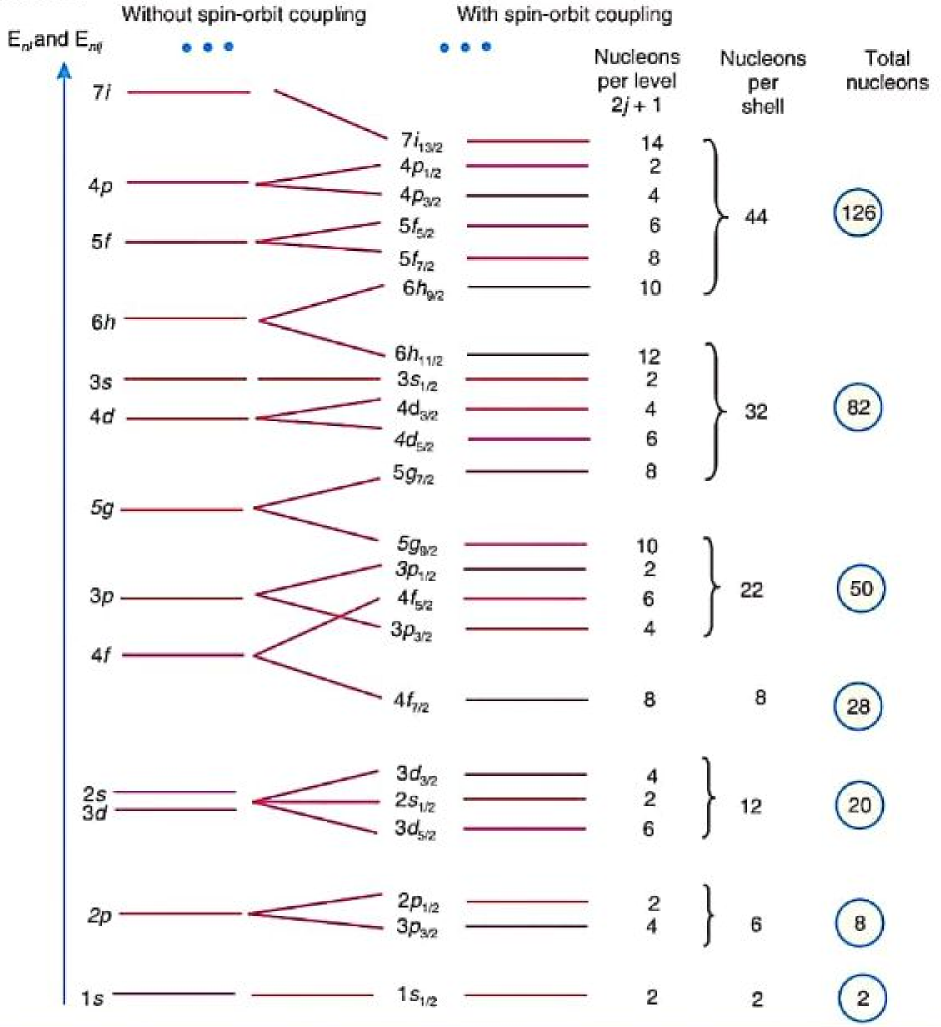


Fig. 28.5

28.6 Fermi Gas Model of the Nucleus

The degenerate gas model was suggested by Fermi and pictures the nucleus as a gas of neutrons and protons. In this model, the forces between pairs of nucleons and also the surface effects like capillarity are omitted. Protons and neutrons exist in a box of nuclear dimensions and fill the lowest available quantum states to the extent permitted by the exclusion principle. Since both protons and neutrons have spins of 1/2, they are fermions and obey Fermi-Dirac statistics.

It is assumed that the nucleons are confined to a small volume equal to $\frac{4}{3}\pi R_0^3 A$. Under these circumstances the nucleus is completely degenerate even in the first few excited states *i.e.*, unlike a classical gas, it occupies almost all the lowest energy states available. This is so because the spacing of energy levels in a gas confined to such a small volume as that of nuclear dimensions is of the order of several MeV. As the excitation energies of the first few excited states are not greater than this, the ground state and the first few excited states are therefore completely degenerate.

The de Broglie wave-length for a nucleus of radius R and A particles of mass M is, $\lambda = R/A^{1/3}$. The corresponding momentum

$$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda} = \frac{2\pi\hbar A^{1/3}}{R} = 2\pi\hbar (A^{1/3}/R).$$

The kinetic energy per particle will be proportional to $\frac{p^2}{2M}$.

$$\text{K.E} \propto \frac{A^{2/3}}{MR^2}.$$

For the whole nucleus, $\text{K.E} \propto \frac{A^{5/3}}{MR^2}$.

The potential energy is proportional to the number of interacting pairs *i.e.*, to $A(A-1)/2$.

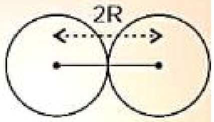
For A large enough, the potential energy is the main term. The nucleus would therefore collapse according to this model. The properties of neutron-proton interaction must be postulated to prevent this.

This model gives a good qualitative picture of the nucleus specially of heavy nuclei but the actual numerical results *e.g.*, energy levels are inaccurate.

EXERCISE

1. Mention various methods for determining the size of the nucleus and describe any one in detail. (*Meerut* 1980, 86)
2. Explain the terms, (*a*) Parity and (*b*) Statistics of nuclei.
3. Calculate the parity of deuteron, Li^7, B^{10} and N^{15} nuclei.
4. Explain the basic properties of deuteron nucleus.
5. Explain with arguments how the force between a proton and a neutron is spin dependent.
6. Explain how the shell model of a nucleus accounts for the existence of magic numbers.
7. Write a short-note on the Fermi gas model of the nucleus.

8. Calculate the electrostatic potential energy between two equal nuclei produced by the fission of $_{92}U^{235}$ at the moment of separation.



Sol. Before fission, $_{92}U^{235} + {}_0n^1$. *i.e.*, A = 235 + 1 = 236 and Z = 92.

Two equal nuclei produced will each have A = 118 and Z = 46.

The radius of the nucleus, $R = R_0 A^{1/3} = (1.3 \times 10^{-15})\ (118)^{1/3} = 6.376 \times 10^{-15}$ m

The electrostatic potential energy between the product nuclei is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(Ze)}{(2R)} = \frac{(46)^2(1.6\times10^{-19})^2}{4\pi\times(8.854\times10^{-12})\times2\times(6.376\times10^{-15})}$$

$$= 3.81952\times10^{-11}\,J = \frac{3.81952\times10^{-11}}{1.6\times10^{-13}}\,MeV = \mathbf{238.72\ MeV}$$

CHAPTER 29

DETECTORS OF NUCLEAR RADIATIONS

29.1 Introduction

Most of the nuclear reactions are accompanied by the emission of charged particles like α- particles, protons, electrons and radiations like γ-rays. In order to understand these particles and their interaction with atomic nuclei, precise information about their mass, momentum, energy, etc., are necessary. We shall describe in this chapter some of the common techniques employed for the detection of nuclear radiations and for analysing their energies.

Several nuclear radiation detectors depend for their operation on the *ionization* that is produced in them by the passage of charged particles. This group of detectors includes ionization chambers, proportional counters, G-M counters, semiconductor radiation detectors, cloud chambers and spark chambers. In other detectors, excitation and sometimes molecular dissociation also play important roles. These phenomena, in combination with ionization, bring about the luminescence in scintillation detectors and the latent images in photographic emulsions.

29.2 Interaction between Energetic Particles and Matter

(*a*) Heavy Charged Particles. A heavy charged particle (like a proton, α-particle or fission fragment) has a fairly definite *range* in a gas liquid, or solid. The particle loses energy primarily by the excitation and ionization of atoms in its path. The energy loss occurs in a large number of small increments. The primary particle has such a large momentum that its direction is usually not seriously changed during the slowing process. Eventually it loses all its energy and comes to rest. The distance traversed is called the *range* of the particle.

The energy loss per unit length ($-dE/dx$) is called the *stopping power*. The rate $-dE/dx$ at which a heavy particle of charge ze and speed v loses energy in an absorber of atomic number Z which contains N atoms per unit volume whose average ionization energy is I is given by

$$-\frac{dE}{dx} = \frac{z^2 e^4 NZ}{4\pi\varepsilon_0{}^2 m_0 v^2}\left[\text{In}\left(\frac{2m_0 v^2}{I}\right) - \text{In}\left(1-\frac{v^2}{c^2}\right) - \frac{v^2}{c^2}\right] \quad ...(1)$$

m_0 is the electron rest mass.

The range can be calculated by integrating Eq. (1) over the energies of the particle

$$R = \int_T^0 \left(-\frac{dE}{dx}\right)^{-1} dE \quad ...(2)$$

(*b*) Electrons. Electrons interact through coulomb scattering from atomic electrons, just like heavy charged particles. There are however, a number of important differences:

(1) Electrons travel at relativistic speeds.

(2) Electrons will suffer large deflections in collisions with other electrons, and therefore will follow erratic paths. The range will therefore be very different from the length of the path that the electron follows.

(3) Very energetic electrons ($E > 1$ MeV) lose an appreciable fraction of their energies by producing continuous X-rays (also called *Bremsstrahlung*). The cross section for this process increases with increasing E.

(*c*) The Absorption of γ-Rays. The interaction of γ-rays with matter is markedly different from that of charged particles such as α or β particles. γ-rays are extremely penetrating so that they are able to pass through considerable thicknesses of matter. γ-rays show the exponential absorption in matter. If radiation of intensity I is incident upon an absorbing layer of thickness dx, the amount of radiation absorbed dI is proportional both to dx and I. Hence,

$$dI = -\mu I\, dx \text{ or } I = I_0\, e^{-\mu x}.$$

Here, μ is a constant of proportionality which is a characteristic property of the medium, known as *linear absorption coefficient*. The *mass absorption coefficient* μ_m may be obtained by dividing μ by the density of the medium. $\mu_m = \mu/\rho$. The above relation gives the intensity (number of quanta per unit area per second) of the beam of initial intensity I_0, after traversing a thickness x of the homogeneous material. At low energies (0.1 MeV to 25 MeV) there are three important processes through which γ photons are absorbed by matter. [Fig. 29.1].

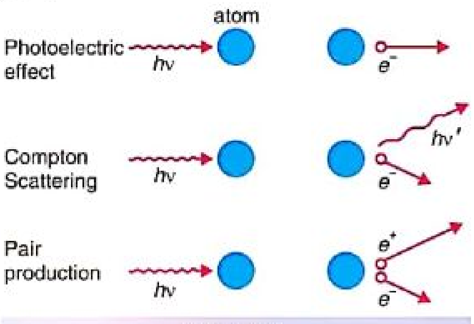


Fig. 29.1

(*i*) Photoelectric effect. In this process, the γ-rays knock out electrons from inside the atoms of the absorbing material. This results in the ionisation of the atoms and the emission of fluorescent radiations. Einstein's equation for those photo-electrons will be

$$\frac{1}{2}mv_k^2 = h\nu - W_k,\ \frac{1}{2}mv_L^2 = h\nu - W_L \text{ etc.}$$

where $h\nu$ is the photon energy, v_k, v_L represent the velocities of the photoelectrons arising in the *K*, *L*... shells, and W_k, W_L... are the binding energies of *K*, *L*... shells.

(*ii*) Pair Production. In this process, the photon disappears and is converted to an electron-positron pair. This process can take place only when the photon energy exceeds $2m_0c^2$. The pair production process cannot occur in free space and usually takes place in the presence of a nuclear field. The nucleus recoils in this process conserving momentum. But the K.E., carried away by the nucleus is negligibly small due to its large mass compared with that of the electron. Photon energy, if any, in excess of $2m_0c^2$ is shared as K.E. by the product particles.

(*iii*) Compton effect. It is an elastic scattering process in which the photon imparts energy to an electron. When a photon of energy $h\nu$ strikes the perfectly free electron (at rest), the photon with diminished energy $h\nu'$ is scattered at an angle θ with the direction of incident photon and the electron recoils at an angle ϕ. The energy absorbed by these Compton electrons is only a small fraction of the total energy of the incident γ-rays, unlike in the case of photoelectrons.

At low photon energies, the photoelectric effect is the chief mechanism of energy loss. The importance of the photoelectric effect decreases with increasing energy. In the lighter elements, Compton scattering becomes dominant at photon energies of a few tens of keV, whereas in the heavier ones this does not happen until photon energies of nearly 1 MeV are reached. Pair production becomes increasingly likely the more the photon energy exceeds the threshold of 1.02 MeV. Fig. 29.2 is a graph of the linear attenuation coefficient for photons in lead as a function of photon energy. The contributions to μ of the photoelectric effect, Compton scattering, and pair production are shown.

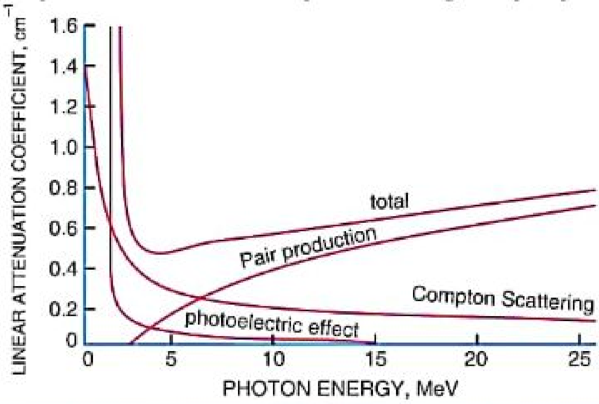


Fig. 29.2

Example. *The linear attenuation coefficient for 2-MeV gamma rays in water is about* $5m^{-1}$. (*a*) *Find the relative intensity of a beam of 2 MeV gamma rays after it has passed through* 0.1 *m of water.* (*b*) *How far must such a beam travel in water before its intensity is reduced to* 1 *per cent of its original value?*

Sol. We have, $I = I_0 e^{-\mu x}$.

(*a*) Here, $\mu = 5m^{-1}$; $x = 0.1\ m$, $I/I_0 = ?$

$$\frac{I}{I_0} = e^{-\mu x} = e^{-0.5} = 0.61.$$

(*b*) Here, $I_0/I = 100$, $\mu = 5m^{-1}$; $x = ?$

$$x = \frac{\log_e(I_0/I)}{\mu} = \frac{\log_e 100}{5} = 0.92\,m$$

29.3 Ionization Chamber

The principle employed here is that charged sub-atomic particles can ionise gases. The number of ion-pairs produced gives us information not only on the nature of the incident particles, but even on their energy. The ionisation chamber consists of a hollow metallic cylinder *C*, closed at both ends, with a window *W* at an end for the entry of the ionising particles or radiation [Fig. 29.3]. A metal rod *R*, well insulated from the cylinder, is mounted coaxially within the cylinder. *R* is connected to a quadrant electrometer *E*. A p.d. of several hundred volts is maintained between *C* and *R*. An earthed guard ring *G* prevents leakage of charge from the cylinder to the rod. The chamber contains some gas like sulphur dioxide or methyl bromide.

Ionization Chamber.

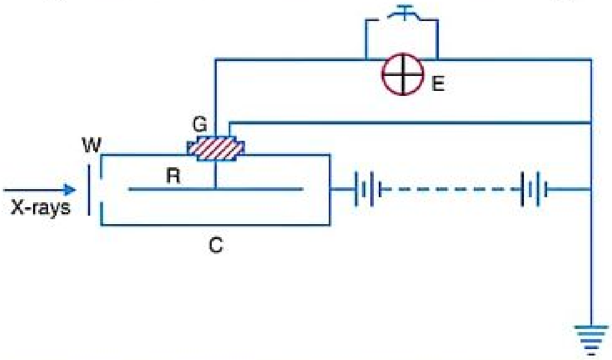


Fig. 29.3

When a charged particle enters the chamber, it produces a large number of ion pairs in the enclosed gas, along its path. Positive ions move towards *R* and negative ions towards *C*.

The quadrant electrometer *E* measures the rate of deposition of positive charges on *R*. The ionisation currents produced are quite small $\approx 10^{-12} - 10^{-15}$ amperes. Special electrometers and D.C. amplifying devices have to be employed to measure such small currents.

If individual particles are to be counted, then the pulses of current produced are fed to a pulse amplifier, which is joined to the ionisation chamber by a coupling capacitor [Fig. 29.4]. Ionisation chambers have been used to study α-particles, β-particles, protons, electrons and nuclei of lighter elements. Ionisation chambers were extensively used in the early studies of cosmic ray phenomena. Ionisation chambers can also be used for measurements on X-rays and γ-rays. For neutron detection, the chamber is filled with boron trifluoride vapour (where the boron is enriched with B^{10}) or the chamber walls are lined inside with a boron compound in the form of a paste.

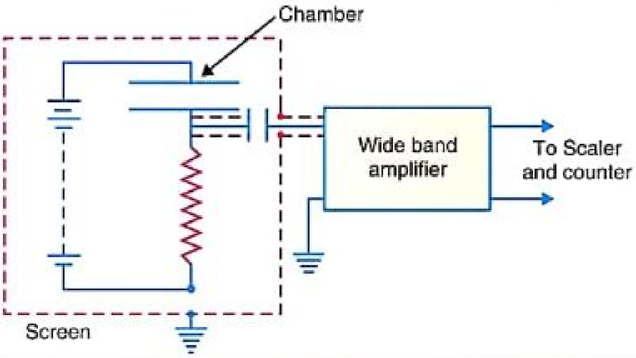


Fig. 29.4

An ionisation chamber is much less sensitive to β-particles (in comparison to α-particles) because β-particles produce fewer pairs of ions in their passage through the chamber.

For detecting γ-rays, an ionisation chamber of thick wall made of high atomic-number material (Pt, Bi) is employed. The γ-rays impinging on the *walls* of the chamber eject high-speed electrons which produce ionisation in the gas.

Example 1. *α-particles of energy 5 MeV pass through an ionisation chamber at the rate of 10 per second. Assuming all the energy is used in producing ion pairs, calculate the current produced. (35 eV is required for producing an ion pair and $e = 1.6 \times 10^{-19}$ C).*

Sol. Energy of α-particles = 5×10^6 *eV*.

Energy required for producing one ion pair = 35 *eV*

No. of ion pairs produced by one α-particle

$$= \frac{5\times10^6}{35} = 1.429\times10^5$$

Since 10 particles enter the chamber in one second,

No. of ion pairs produced per second

$$= 1.429 \times 10^5 \times 10 = 1.429 \times 10^6.$$

$$\text{Charge on each ion} = 1.6 \times 10^{-19}\ C.$$

$$\therefore \quad \text{Current} = (1.429 \times 10^6) \times (1.6 \times 10^{-19})\ C/s$$
$$= 2.287 \times 10^{-13}\ A.$$

Example 2. *An ionization chamber is connected to an electrometer of capacitance 0.5 pF and voltage sensitivity of 4 divisions per volt. A beam of α-particles causes a deflection of 0.8 divisions. Calculate the number of ion pairs required and the energy of the α-particles. Given that 1 ion pair requires energy of 35 eV and $e = 1.6 \times 10^{-19}$ coulomb.*

Sol. Voltage sensitivity of electrometer = 4 divisions/volt.

$$\therefore \quad \left.\begin{array}{r}\text{Voltage required to produce}\\ \text{a deflection of 0.8 divisions}\end{array}\right\} = \frac{0.8}{4}\text{ volt} = 0.2\text{ volt}$$

$$Q = CV = (0.5 \times 10^{-12}) \times 0.2 \quad [\text{since } C = 0.5\ pF = 0.5 \times 10^{-12}\ F]$$
$$= 10^{-13}\ C.$$

$$\therefore \quad \text{No. of ion pairs required} = \frac{10^{-13}}{1.6\times10^{-19}} \approx 6.25\times10^5$$

1 ion pair requires 35 *eV*.

$$\therefore \quad \text{Total energy required} = 35 \times (6.25 \times 10^5)\text{ eV}$$
$$= 21.88\text{ MeV}.$$

29.4 Solid-state Detectors

A *p-n* junction, used as a particle detector, is shown in Fig. 29.5. It consists of a *p-n* junction between *p*-type and *n*-type silicon. Contact is made with the *n*-type silicon layer by a thin evaporated film of gold. In order to minimise the current flowing in the detector, when no radiation is striking it, a reverse biased diode is always used. The positive (reverse) bias applied to the gold film will push all the positive charge carriers away from the junction and produce a *depletion layer*, indicated in the figure. The depletion layer contains almost no carriers of either sign. When an energetic charged particle travels through the depletion layer, its interaction with the electrons in the crystal, produces electron-hole

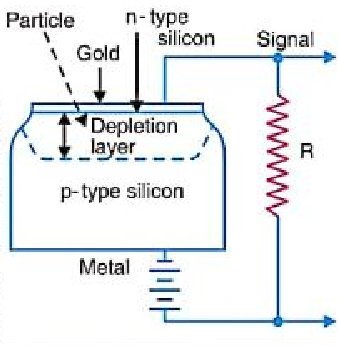


Fig. 29.5

pairs. There is an electron-hole pair for every 3.5 eV (in *Si*) of energy lost by the charged particle. The electrons and holes are swept away by the applied electric field and registered as a voltage pulse over the resistor R. The number of charge carrier pairs produced in a semiconductor material is approximately 10 times as large as the number of ion pairs produced in a gas ion chamber *i.e.*, the energy extended per pair is about 3.5 eV in silicon, compared to about 30 eV for gases. The voltage pulse will therefore be about 10 times larger. Hence this detector has much better *energy resolution* than other radiation detectors. In solid state detectors for charged particles, silicon has been used most because of its low intrinsic conductivity. This means that the detector can be operated at room temperature without excessive leakage current. For gamma ray work, germanium detectors are much better than silicon because of the larger density of germanium.

Solid-State Detector.

29.5 Proportional Counter

The proportional counter consists of a cylindrical gas filled tube with a very thin central wire which serves as the anode (Fig. 29.6). The outer cylinder serves as a cathode. In the case of the simple ionization chamber, the pulse height generated by an event is proportional to the intensity of the beam. But because of the comparatively low applied voltages, the current produced is always very small.

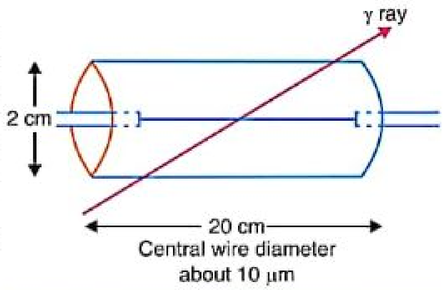


Fig. 29.6

If the voltage applied to an ionization chamber is increased past a certain value, the electrons acquire enough energy while moving toward the anode to create further ion pairs along the way. The resulting avalanche of secondary electrons that reaches the anode may represent a multiplication factor of as much as 1,000 with a correspondingly larger output pulse. In a certain range of applied voltages (See Fig. 29.7), the pulse size is proportional to the original number of ion pairs, and the device is called a proportional counter.

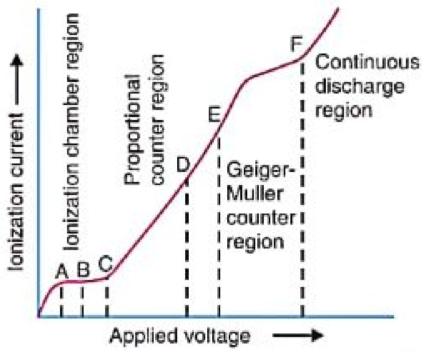


Fig. 29.7

Since the central wire is very thin and the p.d. fairly large, the electric field $E = dV/dr$ at a distance r from the centre is very high. If b is the radius of the cylinder and a the radius of the wire, the radial field E at a distance r from the centre is given by

$$E = \frac{V}{r \log_e (b/a)}$$

where V is the positive voltage of the central wire relative to the outer cylinder. Thus, in a proportional counter, the field strength near the wire is very great. Hence electrons travelling towards the wire are rapidly accelerated when near it, and produce additional electrons in that region due to the phenomena of ionization by collision. This process is called the *gas multiplication*.

The complete voltage pulse characteristics of this type of tube are shown in Fig. 29.7. The main regions used for measurements are : (1) The ionization chamber region AB (2) the proportional counter region CD (3) the Geiger-Muller region EF. After the point F, the tube becomes a simple discharge tube in which the current is produced even after the ionization event has ceased. Like the ionization chamber, the proportional counter gives single pulses of height proportional to the ionizing power of the radiation.

EXAMPLE. *It is required to operate a proportional counter with a maximum radial field of $10^7\ Vm^{-1}$. What is the applied voltage required if the radii of the wire and tube are 0.002 cm and 1 cm respectively?*

SOL. Radial field = $E = \dfrac{V}{r \log_e (b/a)}$

Radial field at the wire surface $E = \dfrac{V}{a \log_e (b/a)}$

$$\therefore \quad 10^7 = \frac{V}{(2\times10^{-5})2.302\log_{10}(10^{-2}/2\times10^{-5})}$$

or $\quad V = 1242$ volts.

29.6 Geiger-Muller Counter

It consists of a metal chamber *C* containing air or some other gas at a pressure of about 10 cm of Hg. A fine tungsten wire (*W*) is stretched along the axis of the tube and is insulated from it by ebonite plugs *EE* (Fig. 29.8). The wire is connected to the positive terminal of a high tension battery (about 1000 to 3000 volts) through a high resistance *R* (about 100 megohms) and the negative terminal is connected to the chamber *C*. The D.C. Voltage is kept slightly less than that which will cause a discharge between the electrodes.

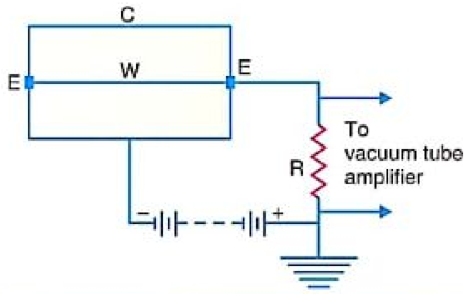


Fig. 29.8

Geiger-Muller Counter.

When an ionizing particle (say an α particle) enters the counter, ionisation takes place and a few ions are produced. If the applied P.D. is strong enough, these ions are multiplied by further collisions. An avalanche of electrons moves towards the central wire and this is equivalent to a smaX current impulse which flows through the resistance *R*. The critical potential is lowered momentarily, causing a sudden discharge through the resistance *R*. The p.d. thus developed across *R* is amplified by vacuum tube circuits and is made to operate a mechanical counter. In this way single particles can be registered. The sudden pulse of discharge sweeps away the ions from the chamber and the counter is ready to register the arrival of the next particle.

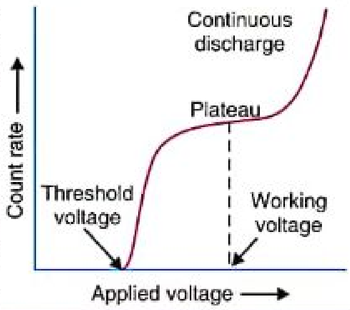


Fig. 29.9

The voltage characteristics of a Geiger-Muller counter are shown in Fig. 29.9. This is a plot of the counting rate against the counter potential with a radioactive source placed near the counter. It is seen that there is a threshold below which the tube does not work. This can be several hundred volts. As the applied potential is increased, the counting begins and rises rapidly to a flat portion of the curve called the *plateau*. This is the region of the counter operation where the counting rate is, more or less, independent of small changes in p.d. across the tube. Beyond the plateau the applied electric field is so high that a continous discharge takes place in the tube as shown in Fig. 29.9 and the count rate increases very rapidly. It does not require any ionizing event for this to happen so that the tube must not be used in this region.

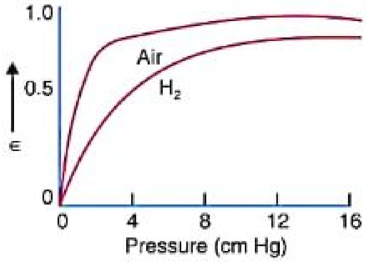


Fig. 29.10

The *efficiency of the counter* is defined as the ratio of the observed counts/sec. to the number of ionizing particles entering the counter per second. Counting efficiency is defined as the ability of its counting, if at least one ion pair is produced in it.

Counting efficiency = $\epsilon = 1 - e^{slp}$ where s = specific ionization at one atmosphere; p = pressure in atmospheres and l = path length of the ionization particle in the counter. The efficiency ε of a *GM* counter, as a function of pressure for air and hydrogen, is illustrated in Fig. 29.10.

The counter set-up is portable (with the transistorised electronics) and serves best for mineral prospecting, apart from its several other applications in cosmic ray work. A virtue of the Geiger counter is that the pulse height is constant over a range of applied voltages, as in Fig. 29.9. So the power supply does not have to be precisely regulated as it does for a proportional counter. Also, the pulses are several volts in height, which makes amplifiers unnecessary.

Disadvantages of the Geiger counter are: (*i*) it is insensitive for a period of 200 to 400 μs following each pulse, which prevents its use at very high counting rates. (*ii*) it cannot provide information about the particle or photon causing a pulse.

Example. *A self-quenched G-M counter operates at* 1000 *volts and has a wire diameter of* 0.2 *mm. The radius of the cathode is* 2 *cm and the tube has a guaranteed lifetime of* 10^9 *counts. What is the maximum radial field and how long will the counter last if it is used on an average for* 30 *hours per week at* 3000 *counts per minute? Consider 50 weeks to a year.*

Sol. The radial field at the central wire is

$$E_{max} = \frac{V}{r\log_e(b/a)} = \frac{1000}{0.0001 \times 2.3026 \log_{10}\left(\frac{2 \times 10^{-2}}{10^{-4}}\right)}$$

$$= 1.89 \times 10^6 \text{ volts/metre.}$$

If the lifetime of the tube is N years, the total number of counts recorded will be

$$N \times 50 \times 30 \times 60 \times 3000 = 2.7 \times 10^8 N$$

$$\therefore \quad 2.7 \times 10^8 \times N = 10^9$$

or

$$N = 3.7 \text{ years.}$$

29.7 The Wilson Cloud Chamber

Principle. If there is a sudden expansion of saturated vapour in a chamber, supercooling of the vapour occurs. Tiny droplets will be formed by condensation over the dust particles present in the chamber. If, therefore, we have completely dust-free and saturated air, and if it is suddenly allowed to expand and thereby cool, condensation will not take place. But if ions are available in the chamber during the expansion, they serve as nuclei for condensation. Hence, if an ionising particle passes through the chamber during an expansion, ions are produced along its path and droplets condense on these ions. Hence the "track" of the particle becomes visible.

Description. The apparatus consists of a large cylindrical chamber A, with walls and ceiling made of glass (Fig. 29.11). It contains dust-free air saturated with water vapour. P is a piston working inside the chamber. When the piston moves down rapidly, adiabatic expansion of the air inside the chamber takes place. The piston is connected to a large evacuated vessel *F* through a valve *V*. When the valve is opened, the air under the piston rushes into the evacuated vessel *F*, thereby causing the piston to drop suddenly. The wooden blocks *WW* reduce the air space inside the piston. Water at the bottom of the apparatus ensures saturation in the chamber. The expansion ratio can be adjusted by altering the height of the piston.

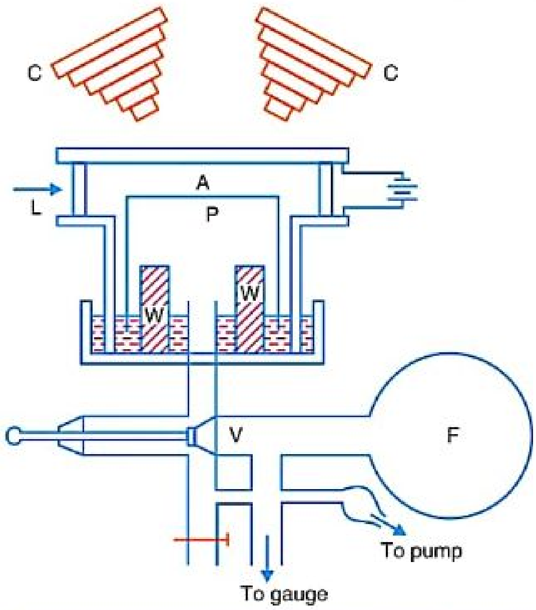


Fig. 29.11

As soon as the gas in the expansion chamber is subjected to sudden expansion, the ionizing particles are shot into the chamber through a side window. A large number of extremely fine droplets are formed on all the ions produced by the ionizing particles. These droplets form a track of the moving ionizing particles. At this stage, the expansion chamber is profusely illuminated by a powerful beam of light *L*. Two cameras *CC* are used to photograph the tracks. The process of expansion, shooting of the ionising particles into the expansion chamber, illuminating the chamber and clicking the camera must all be carried out in rapid succession in order to get satisfactory results.


Wilson Cloud Chamber

The ionising agent can be easily identified from its path in the cloud chamber. α-particles, being comparatively massive, go straight and their paths are *thick, straight* and *sharply defined*. β-particles being lighter, are easily deflected by collision and their paths are thin and *crooked*. The cloud chamber has led to the discovery of many elementary particles like positron, meson, etc.

Advantages. (1) Cloud chambers can be used to study the variation of specific ionisation along the track of a charged particle and the range of such particles.

(2) The sign of the electric charge and the momentum p of the particle can be determined if the chamber is placed in a strong magnetic field. Let a particle of mass m and charge q move with a velocity v perpendicular to the direction of the magnetic field of flux density B. The particle will be forced by the field to follow a circular path of radius R. The magnetic force Bqv is exactly balanced by the centrifugal force mv^2/R.

Thus $Bqv = Mv^2/R$ or $mv = p = BRq$.

The K.E. of the particle can be calculated, if the rest mass energy m_0c^2 of the particle is known, by the relation,

$$K.E. = E_k = \sqrt{\left[p^2c^2 + \left(m_0c^2\right)^2\right]} - m_0c^2$$

Limitations. (*i*) One is not always sure of the sense of track photographed.

(*ii*) The range of the particle may exceed the dimensions of the chamber so that the whole track is not photographed.

(*iii*) There remains a certain amount of uncertainty about the nature of the nuclei constituting the arms of the forked tracks.

29.8 Diffusion Cloud Chamber

The disadvantage of the cloud chamber lies in the fact that it needs a definite time to recover after an expansion. Hence it is not possible to have a continuous record of events taking place in the chamber. This difficulty was removed by the introduction of the diffusion cloud chamber.

The outline of the apparatus is shown in Fig. 29.12.

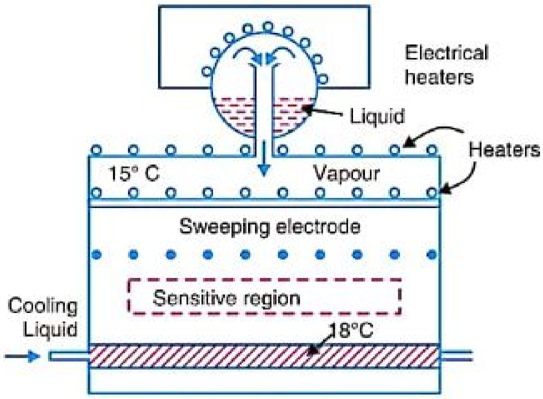


Fig. 29.12

It consists of a chamber containing a heavy gas which is kept warm at the top and cold at the bottom. Thermal gradient is maintained between the bottom and top of the chamber by external heating or cooling. The liquid (methyl alcohol) vaporises in the warm region, where the vapour pressure is high. The vapour diffuses downwards continuously where the vapour pressure is low and condensation takes place. In a region near the base, the supersaturation factor is high and condensation takes place around the available ions. The chamber remains continuously sensitive to ionizing particles until the supply of volatile liquid is exhausted. The system is illuminated by a strong source of light and the track of the particle is photographed by camera.

29.9 Bubble Chamber


Bubble Chamber.

Principle. We know that normally the liquid boils with the evolution of bubbles of vapour at the boiling point. If the liquid is heated under a high pressure to a temperature well above its normal boiling point, a sudden release of pressure will leave the liquid in a *superheated state*. If an ionising particle passes through the liquid within a few milliseconds after the pressure is released, the ions left in the track of a particle act as condensation centres for the formation of vapour bubbles. The vapour bubbles grow at a rapid rate and attain a visible size in a time of the order of 10 to 100μs. Thus in a bubble chamber, a vapour bubble forms in a superheated liquid, whereas in a cloud chamber, a liquid drop forms in a supersaturated vapour. Thus an ionising particle passing through the superheated liquid leaves in its wake a trail of bubbles which can be photographed.

A schematic diagram of a liquid hydrogen bubble chamber, operating at a temperature of 27 *K* is shown in Fig. 29.13. A box of thick glass walls is filled with liquid hydrogen and connected to the expansion pressure system. To maintain the chamber at constant temperature, it is surrounded by liquid nitrogen and liquid hydrogen shields. High energy particles are allowed to enter the chamber from the side window *W*. A sudden release of pressure from the expansion valve is followed by light flash and camera takes the stereoscopic view of the chamber.

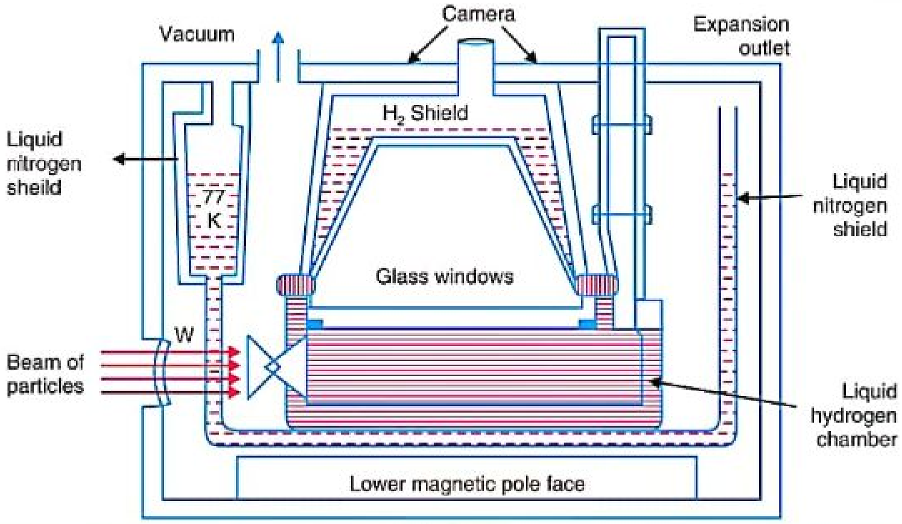


Fig. 29.13

The incoming beam triggers the chamber. The charge of the tracks can be identified by the direction of their curvature in the magnetic field applied over the bubble chamber. From the curvature and length of the track, the momentum and energy of the particle can be found. The bubble chamber is used to *study particle interaction* and to *detect very high energy particles.*

Advantages. (1) The density of a liquid is very large when compared to that of a gas of even high pressure. Hence the chances of collision of a high energy particle with a molecule of the liquid are very much greater. Consequently there is a greater chance of their track being recorded. So the chances of recording events like cosmic ray phenomena are improved when compared with cloud chambers.

(2) The bubbles grow rapidly and as a result the tracks are not likely to get distorted due to convection currents in the liquid.

(3) The bubble chamber is sensitive even to particles of low ionising power.

29.10 Spark Chamber

A spark chamber consists of a set of conducting plates alternately connected to a source of high DC voltage (Fig. 29.14)

The chamber is filled with an inert gas. Sudden application of very high voltages to alternate plates, while the others are left at ground potential, results in very high electrical fields across the gaps. Electrical breakdown then occurs along the trails of ions. So the trajectory of a given particle through the system is marked by a series of sparks. The spark trails are photographed stereoscopically. If the chamber is located in a magnetic field, the charge and momentum of the particle can be determined from the curvature of the track. Vidicons are frequently used in place of photography.

The spark chamber has an important advantage over the bubble chamber in that it can be rendered sensitive for only a very short time. Suppose, for example, that in an intense beam from a high-energy particle accelerator a rather rare particle is produced. Scintillation or Cerenkov counters can signal the production of this kind of particle. The pulse voltage is then applied to the spark chamber. The track of this particular particle can thus be determined. There is little likelihood that one of the much more numerous *background* particles will cause another spark track in the short time that the chamber is sensitive. Thus rare events and processes can be studied.

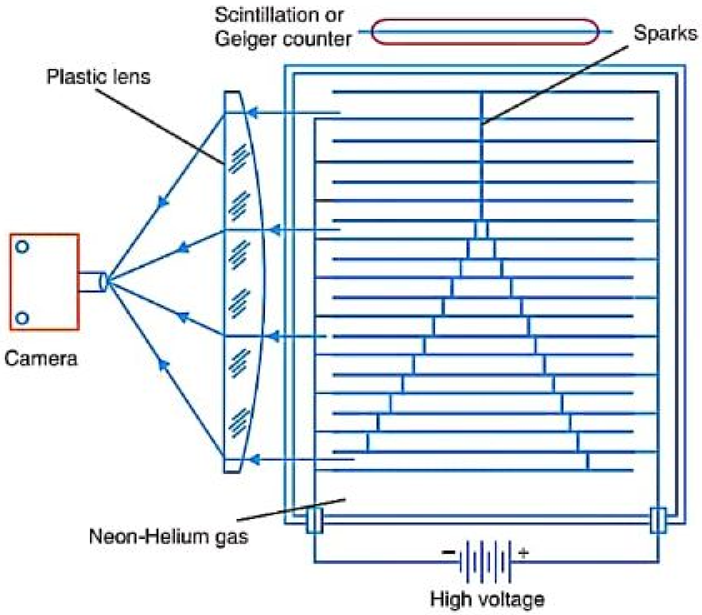


Fig. 29.14

29.11 Nuclear Emulsions

Nuclear Emulsion is the Detector with the best Spatial Resolution.

It has been known for a long time that charged particles affect a photographic plate. A heavy charged particle traversing a photographic emulsion produces a latent image of its track. The track is revealed when the plate is developed. Ordinary optical photographic emulsions are not suitable for quantitative studies with nuclear radiations. The sensitivity of such emulsions is low. Further, the tracks due to charged particles have non-clear range because the developed crystal grains are large and widely spaced. The composition of the emulsion was changed so as to make it more suitable for the study of various ionising particles, such as α-particles, protons, mesons and even electrons. The nuclear emulsions differ from the optical emulsions in that they have considereably higher silver halide content and smaller grain size. In nuclear emulsion, the thickness is greater than that of optical emulsion (Table 29.1). The smaller the grain size, the more sensitive is the emulsion to ionising particles. Thus different commercially available emulsions, differing chiefly in grain size, can be used to discriminate between different particles.

TABLE 29.1 **Nuclear and Optical Emulsions**

Property	*Optical Emulsion*	*Nuclear Emulsion*
AgBr : Gelatin (mass)	47 : 53	80 : 20
AgBr : Gelatin (volume)	15 : 85	45 : 55
Grain size (micron)	1–3.5	0.1–0.6
Thickness (,,)	2–3	25–2000
Sensitivity to light	Very high	Poor
Response to α particle	Dense blackening	Individual tracks
" " β "	Moderate	Faint fog
" " γ "	Faint	Almost none

Advantages. (1) The emulsion is relatively light and cheap. Because of their lightness, they can be sent in balloons, spaceships etc., for high altitude cosmic ray experiments. The cosmic ray events once recorded can be studied by developing the exposed plates conveniently in the laboratory. Emulsions were widely employed in cosmic ray studies and led to the discovery of the π and K mesons.

(2) The high density of the emulsion gives it a stopping power about a thousand times that of standard air. Unstable high energy particles are brought to rest in the emulsions and their decay schemes can thus be studied.

(3) The emulsion is continuously sensitive and is consequently always available to record an event. But the cloud chamber is sensitive only for a fraction of a second after an expansion and remains ineffective for several seconds between successive expansions.

Limitations. (1) The main drawback of nuclear emulsions is that their sensitivity and thickness are affected by temperature, humidity, age of the emulsions before development and the conditions under which they are developed. The scanning of the plates and the analysis of the tracks obtained are also laborious, when done manually.

(2) As compared with cloud chamber, photographic emulsions have the following drawback. It is very difficult to determine the sign of the charge and the momentum of a particle from observations of the curvature of the path in a strong magnetic field. The actual path-length in the emulsion is small compared to that in a cloud chamber.

29.12 The Scintillation Counters

One of the earliest methods of radiation detection was the spinthariscope (Fig. 29.15). It consists of a small wire, the tip of which is dipped in Radium bromide (R) or any other radioactive salt. It is placed in front of a zinc sulphide screen S and viewed through a microscope. When an α or β-particle falls on the zinc sulphide screen, they produce light flashes which can be seen by a microscope (M) in a dark room. The visible luminescence excited in zinc sulphide by α-particles was used by Rutherford for counting the particles. The process of counting these scintillations through

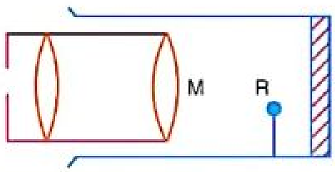


Fig. 29.15


Scintillation Counter.

a low power microscope is a tedious one and the limitations of observation with the eye restrict the counting rate to about 100 per minute. This process, whereby the energy of the particle is converted to light, is the basis of scintillation counter.

Fig. 29.16

The main parts of a scintillation counter are shown in Fig. 29.16. The atoms of the phosphor are excited or ionised by the energy loss of an impinging α, β or γ ray. When the atoms return to their ground states, photons are emitted, in the blue and ultraviolet regions of the optical spectrum. The phosphor is optically coupled to the envelope of a photomultiplier tube. The photons strike the photocathode, causing the ejection of photo-electrons (Fig. 29.17). As these photo-electrons leave the photocathode, they are directed by a focusing electrode to the first multiplier electrode or *dynode*. This electrode has the property of emitting three, four or five electrons for every single

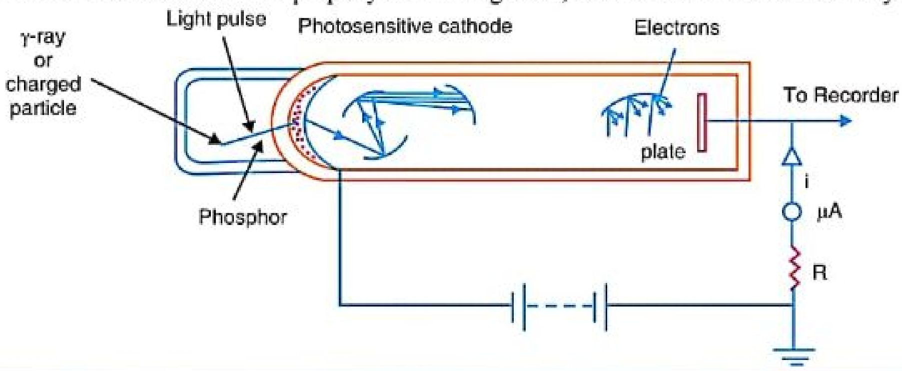


Fig. 29.17

electron which strikes its surface. There may be from 10 to 14 such multiplier stages in a given tube. Hence, from the emission of one single electron from the cathode, a burst of one million electrons may impinge on the final stage in the tube (the anode). The output pulse from the photomultiplier is fed to a pulse amplifier followed by a scaler circuit.

29.13 Cerenkov Counter

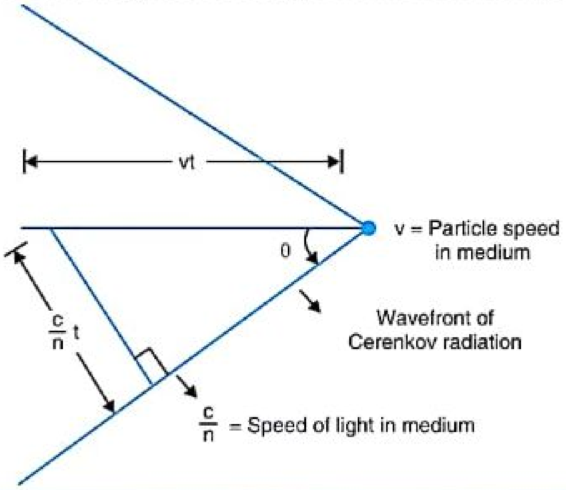
When a charged particle moves through a transparent dielectric medium with a velocity greater than the velocity of light in that medium, a cone of light waves is emitted. These light waves are known as *Cerenkov radiation*. In a dielectric medium of refractive index n, photons move with a velocity c/n. Consider a charged particle moving with a velocity v through the dielectric medium. Fig. 29.18 shows the interactions when $v > c/n$. The envelope of the radiation is a cone of half angle θ with the particle at its apex, where

Cerenkov Counter.

$$\sin\theta = \frac{(c/n)t}{vt} = \frac{c}{nv}.$$

The angle θ of the cone of radiation depends on the speed v of the particle. When a beam of fast charged particles moves through a medium such as glass or transparent plastic, the radiation can be detected and the angle θ measured. Thus the speed of the particle can be determined.

Fig. 29.19 shows a Cerenkov detector designed by Marshall. A collimated beam of pions is allowed to pass along the axis of a large hemispherical perspex lens. The Cerenkov radiation radiated in this lens is reflected by the cylindrical mirror M_1 on to one of the plane mirrors M_2, M_3 and then on to the cathodes of the photomultiplier tubes (*IP*28). The two photomultipliers operate in coincidence. The position of the radiator lens is adjusted so that the Cerenkov light cone is correctly focused on to the phototubes to give maximum coincidence counting rate.



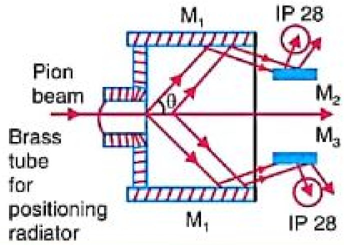


Fig. 29.18 **Fig. 29.19**

The Cerenkov detector is used for the detection of high energy particles. It has a high efficiency and a high counting rate.

EXERCISE

1. Describe the construction and working of an ionisation chamber.
2. Describe a G.M. counter and explain its working as a particle detector.
3. Explain fully the working of a cloud chamber. How is it used to determine the energy of a particle passing through it?
4. What are the constructional features of the bubble chamber? How does the instrument work? Enumerate its advantages over the cloud chamber.
5. Give a brief account of the nuclear emulsion technique in the study of ionising radiations.
6. Give an account of the mode of operation of a scintillation counter and describe how it may be utilised in the study of nuclear reactions.
7. Write a brief essay on the various techniques which are used for the detection of elementary particles indicating the advantages and the limitations pertaining to each method.
8. A G.M. counter wire collects 10^8 electrons per discharge. When the counting rate is 500 counts/min, what will be the average current in the circuit? **[Ans.** 1.33×10^{-10} *A*]
9. A Geiger-Muller counter collects 10^7 electrons per discharge. The average current in the circuit is 1.333×10^{-11} amp. Find the counting rate per minute. $e = 1.6 \times 10^{-19}$ coulomb. **[Ans.** 500]
10. A high energy singly ionised particle leaves a track in a cloud chamber placed in a magnetic field of flux density 2.5 Wb/m^2 and the radius of curvature of the track is 200 cm. Calculate the energy of the particle.

    [Hint. Energy $= E = pc = BRqc = 2.5 \times 2 \times 1.602 \times 10^{-19} \times (3 \times 10^8)\ J = 240\ MeV$]
11. Calculate the ionisation current produced by 3 *MeV* deuterons passing through a gas at 1000 per sec. assuming that 25 *eV* is required to produce an ion pair. **[Ans.** 1.923×10^{-11} *A*]

12. An ionisation chamber exposed to a beam of α-particles registers a current of 3.85×10^{-13} *A*. On the average, 20 α-particles enter the chamber per second. Assuming that the production of an ion pair in the chamber involves the expenditure of 35 *eV* of energy, calculate the energy of the α-particles. **[Ans.** 4.21 *MeV*]

13. An organic-quenched G.M. tube operates at 1000 volts and has a wire diameter of 0.2 mm. The radius of the cathode is 2 cm, and the tube has a guaranteed life-time of 10^9 counts. What is the maximum radial field and how long will the counter last if it is used on the average for 60 hours per week at 2000 counts per minute? **[Ans.** $E_{max} = 1.89 \times 10^6\ Vm^{-1}$; $N = 2.77$ years]

14. Measurements on the track of a particle carrying a charge *e* in a detector give the value of 330 *MeV/c* for its momentum and 215 *MeV* for its kinetic energy. Find the rest mass of the particle. If the detector is kept in a magnetic field of 1 *T* and the particle moves normal to the field, what is the radius of curvature of its track in the detector? **[Ans.** 145.8 *MeV*; 1.1 *m*]

15. What is the meaning and significance of quenching in a *GM* counter?

Ans. The process of removing all the ions from the chamber due to continuous discharge and making it ready for fresh event is called *quenching*. This is achieved by using quenching agents like alcohol or halogens.

Quenching Process. When a G.M. tube operates in Geiger region, the secondary electrons increase the current pulse by further ionisation of gas molecules. The object of counter is to produce a single pulse due to entry of a single particle. The tube should not then give any succeeding spurious pulses but should recover as quickly as possible to be in the state to record the next entering particle. Therefore, it is desirable to prohibit the production of spurious pulses following the main required pulse due to a single particle entry. The process of prohibiting the undesirable secondary pulses is called the *quenching*.

Self-quenching of GM-counter. A typical GM-counter contains 90% Ar and 10% ethyl alcohol by weight. The ionisation potential of alcohol (11.3 eV) is lower than that of argon (15.7 volt). As a result, the inert gas ions get neutralized by transferring their charge to alcohol ions during their long path towards cathode. The alcohol ions produced, capture electrons from the cathode and are neutralized. Hence there is no multiple pulsing and the discharge is quenched soon (fraction of a milli-second) after the initial ionisation.

16. What is meant by the *resolving time* of a GM-counter?

Ans. Fig. 29.20 gives the three time-intervals—*dead time, recovery time* and *resolving time*—of operation of a *GM*-counter.

(*i*) **Dead time.** The time, during which the counter is incapable of responding to a second ionizing event, is known as *dead time* (~ 200 μ sec). It is called *dead time* because during this period the counter is insensitive (dead) to further pulses. After the initial ionisation, the cloud of slow moving massive positive ions formed over the anode wire effectively lowers its potential relative to cathode. Thus the G.M counter is dead for about 200 μs after each ionising event.

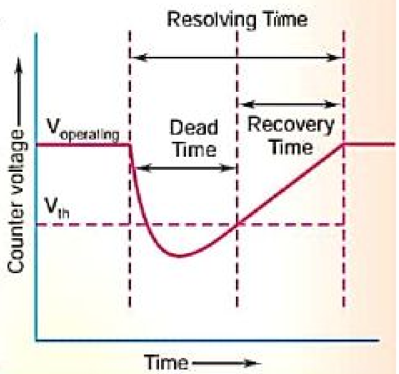


Fig. 29.20

(*ii*) **Recovery time.** As the positive ion-sheath moves further toward the cathode, the counter sensitivity gradually returns to its original value and the second pulse can now be recorded. This time, during which pulses of reduced size are poduced, is known as *recovery time*.

(*iii*) **Resolving time.** The dead time plus recovery time corresponds to the time which positive ion sheath takes to reach the cathode, and is known as the *resolving time* of the counter. This is the time between just recorded pulses.

Expression for resolving time. Assume that a counter with a resolving time τ responds at a rate *n* counts per unit time when exposed to *N* particles per unit time. In unit time, the total insensitive time is $n\tau$. The number of counts missed is $Nn\tau$.

The number of counts missed is the error in counting.

$$Nn\tau = N - n \qquad \text{...(1)}$$

$\therefore \qquad N = n/(1 - n\tau)$ ...(2)

From Eq. (2), we can find the *true* counting rate N from the *observed* counting rate n, if τ is known.

Determination of τ. By measuring separately the counting rates n_1' and n_2' with two radioactive sources 1 and 2 of nearly equal strength, τ can be determined. Let n_t' denote the total counting rate when both sources are placed in the same position as before and n_b be the background rate (*i.e.*, with no source).

$\therefore \qquad n_1 = n_1' - n_b; \quad n_2 = n_2' - n_b; \quad n_t = n_t' - n_b$ ...(3)

Here, n_1, n_2, n_t are the above counting rates corrected for background.

If N_1, N_2 and N_t be the true rates of arrival of particles in the counter in the above three situations, we have, using Eq. (2),

$$N_1 = \frac{n_1}{1 - n_1\tau}; \quad N_2 = \frac{n_2}{1 - n_2\tau}; \quad N_t = \frac{n_t}{1 - n_t\tau} \qquad ...(4)$$

Also, $\qquad N_t = N_1 + N_2$ ...(5)

Assuming the values of terms involving τ^2 to be negligible, we have, solving for τ

$$\tau = \frac{n_1 + n_2 - n_t}{2n_1 n_2} \qquad ...(6)$$

17. In a scintillation counter, the scintillator generally used is

(*a*) Anthracene (*b*) Argon (*c*) Camphor (*d*) Neon (*Barkatullah University, 2011*)

[Ans. (*a*)]

18. The wire in a GM counter collects 10^{10} electrons per discharge. If the count rate is 1000/minute, calculate the average current in the circuit. (charge on electron is 1.6×10^{-19} C). (*Andhra University, 2010*).

Ans. Counting rate = 1000 counts/min.

The wire collects 10^{10} electrons per discharge.

$\therefore$ Total number of electrons collected in one min. $= n = 1000 \times 10^{10} = 10^{13}$

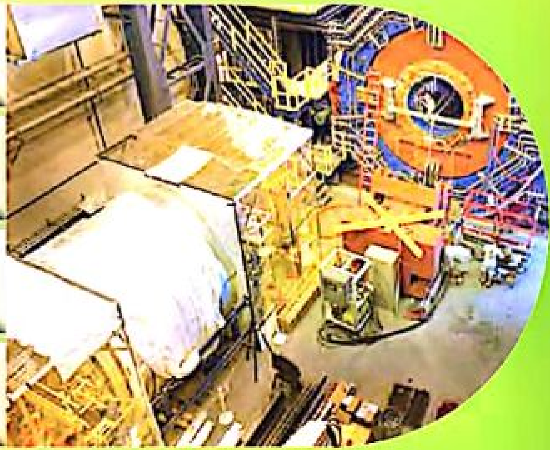
$$\text{Charge/min} = n\,e = 10^{13} \times (1.6 \times 10^{-19}) = 1.6 \times 10^{-6} \text{ coul/min.}$$

$$\text{Charge/second} = \frac{1.6 \times 10^{-6}}{60} = 2.66 \times 10^{-8} \text{ Amp.}$$

$\therefore \qquad$ Average current = charge/second = 2.66×10^{-8} Amp.

CHAPTER

30



PARTICLE ACCELERATORS

AT A GLANCE

30.1 Introduction

A particle accelerator is a device for increasing the K.E. of electrically charged particles. Methods of acceleration can be classed into three groups : *direct field*, *inductive*, and *resonance*. According to the shape of the path of the particles, accelerators are classified as *linear* and *cyclic*. In linear accelerators, the paths of particles are approximately straight lines; in cyclic accelerators, they are circles or spirals. (*i*) In a direct field linear accelerator, a particle passes only once through an electric field with a high p.d. set up by electrostatic generators. (*ii*) The only accelerator of the inductive type is the betatron. (*iii*) In *magnetic resonance* accelerators, the particle being accelerated repeatedly passes through an alternating electric field along a closed path, its energy being increased each time. A strong magnetic field is used to control motion of particles and to return them periodically to the region of the accelerating electric field. The particles pass definite points of the alternating electric field approximately when the field is in the same phase ("in resonance"). The simplest resonance accelerator is the cyclotron.

The Cockcroft-Walton tension multiplier

The principle of this instrument is that of cascade voltage rectification or voltage multiplication by suitable arrangement of high tension rectifier tubes and charging capacitors, as shown in

Fig. 30.1. It consists of a high voltage transformer T and two banks of identical type series capacitors alternately connected by identical type high voltage rectifiers. The high voltage step-up transformer T supplies the A.C. voltage to be rectified by H.T. valves V_1, V_2, V_3, V_4 etc. If in the first half-cycle the valve V_1 rectifies the A.C., it charges the capacitor C_1 to the maximum A.C. voltage V_o. In the next half-cycle, V_1 will not conduct, but V_2 will rectify and charge C_2 to a voltage $2V_o$. In this manner, the capacitors C_3, C_4 etc., will go on getting charged in series to higher and higher voltages. If a large number of such steps are used, a very high voltage D.C. supply will be available. The output of this supply will be approximately equal to nV_o where n is the number of H.T. valves used and V is the maximum A.C. voltage amplitude. This output voltage can be used to accelerate ions.

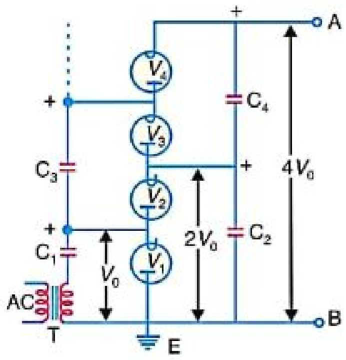
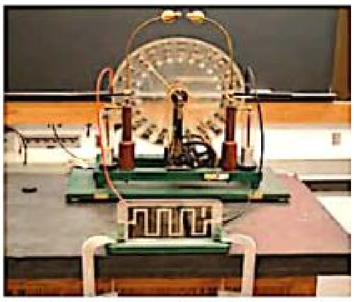


Fig. 30.1

30.2 Van de Graaff Generator

In this electrostatic accelerator, electric charge is carried continuously to an insulated hollow spherical conductor (S) and delivered to it. The potential of the hollow sphere can thus be gradually raised. When the potential has attained a sufficiently high value, it can be used to accelerate positive ions in the accelerator tube.



Van De Graaff Generator.

A well insulated endless belt B is rapidly rotated between two pulleys P_1 and P_2 by means of a motor [Fig. 30.2 (a)]. The upper pulley P_2 is placed inside a hollow spherical conductor S. Near the lower pulley P_1, there is a metallic comb C connected to a source of steady voltage of 50 kV. Positive charge is sprayed over the moving belt by the metallic comb C. The belt moving upward carries this charge into the hollow sphere. This positive charge, on entering the sphere S, induces an equivalent negative charge on its inside and a corresponding positive charge on its outside. The negative charge is sprayed by the comb C' on to the belt, thus neutralising the positive charge on it. The net result is that the positive charges are in effect transferred from the moving belt to the outside of the sphere. The belt, returning to the bottom, is charged again and delivers up the charge, when inside the sphere. Charges are thus accumulated and distributed on the surface of the sphere. As the charge on the sphere accumulates, its potential rises to a very high value.

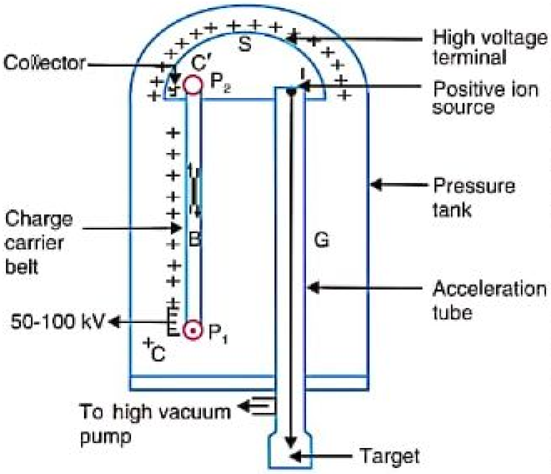


Fig. 30.2(a)

The charged particles which are to be accelerated are produced by a gas discharge ion source I placed inside the hollow sphere S over the accelerating tube G. The accelerating tube is evacuated and contains a number of insulated metal cylinders (C) arranged with short gaps between them [Fig. 30.2]. The H.T. is applied to the topmost cylinder near the ion source and the bottom-most cylinder is earthed. A potential gradient exists from the top to the bottom. The ions are accelerated at the gaps between the cylinders, where intense electric field exists. The highly accelerated ions are deviated by a suitable and adjustable magnetic field *M* and made to strike the target (*T*) at the bottom of the tube. By reversing the potential of the spray voltage, the Van de Graaff generator can also be used to accelerate electrons.

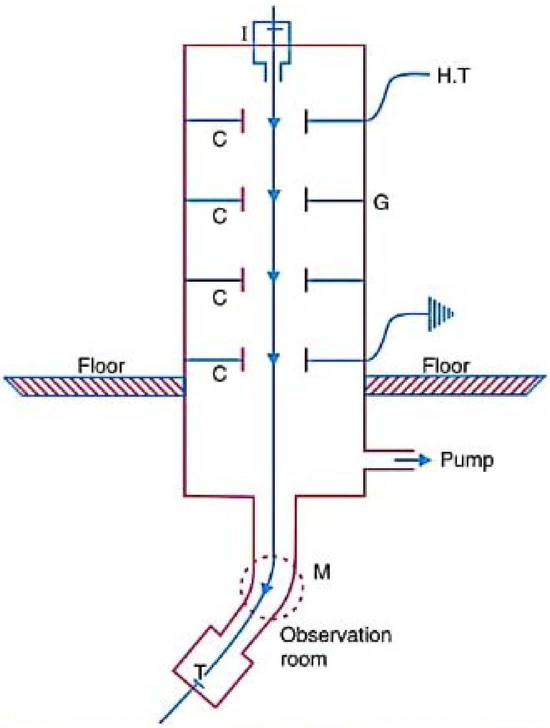


Fig. 30.2 (b)

30.3 The Linear Accelerator

Direct acceleration of particles by potentials above 10 million volts is a difficult problem due to insulation difficulties. For such high energies, acceleration of the particles is achieved in small successive steps. In such machines, the P.D. between different parts of the machine and between the machine and earth, is maintained low, compared with the P.D. corresponding to the ultimate energy acquired by the particles. One machine employing this method is the linear accelerator. In this machine, high energy particles are produced without employing high P.D.'s, by using the principle of *synchronous acceleration*.

Linear accelerator.

Fig. 30.3 shows the schematic diagram of a linear accelerator. It consists of a series of coaxial hollow metal cylinders or drift tubes 1, 2, 3, 4 etc. They are arranged linearly in a glass vacuum chamber. The alternate cylinders are connected together, the odd numbered cylinders being joined to one terminal and the even numbered ones to the second terminal of a H.F. oscillator. Thus in one-half cycle, if tubes 1 and 3 are positive, 2 and 4 will be negative. After half a cycle the polarities are reversed *i.e.*, 1 and 3 will be negative and 2 and 4 positive. The ions are accelerated only in the gap between the tubes where they are acted upon by the electric field present in the gaps. The ions travel with constant velocity in the *field-free space* inside the drift tubes.

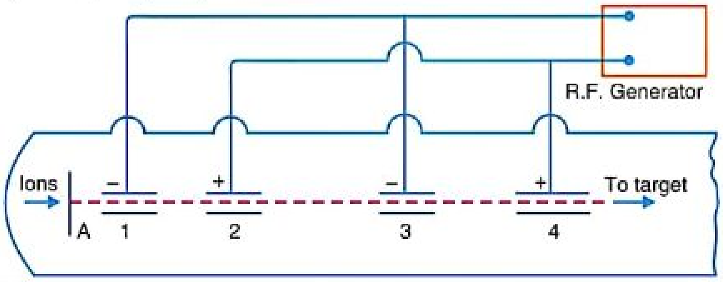


Fig. 30.3

Positive ions enter along the axis of the accelerator from an ion source through an aperture *A*. Suppose a positive ion leaves *A* and is accelerated during the half cycle, when the drift tube 1 is negative with respect to *A*. Let *e* be the charge and *m* the mass of the ion and *V* potential of drift tube 1 with respect to *A*. Then velocity v_1 of the ion on reaching the drift tube is given by

$$\frac{1}{2}mv_1^2 = Ve \text{ or } v_1 = \sqrt{\frac{2\,Ve}{m}}.$$

The length of the tube 1 is so adjusted that as the positive ions come out of it, the tube has a positive potential and the next tube (tube No. 2) has a negative potential, *i.e.*, the potentials change sign. The positive ion is again accelerated in the space between the tubes 1 and 2. On reaching the tube 2, the velocity v_2 of the positive ion is given by

$$\frac{1}{2}mv_2^2 = 2\,Ve \text{ or } v_2 = \sqrt{2}\sqrt{\frac{2\,Ve}{m}} = \sqrt{2}\,v_1.$$

This shows that v_2 is $\sqrt{2}$ times v_1. In order that this ion, on coming out of tube 2, may find tube 3 just negative and the tube 2 positive, it must take the same time to travel through the tube 2. Since $v_2 = \sqrt{2}\,v_1$, the length of tube 2 must be $\sqrt{2}$ times the length of tube 1. For successive accelerations in successive gaps the tubes 1, 2, 3, etc., must have lengths proportional to 1, $\sqrt{2}$, $\sqrt{3}$ etc. *i.e.*, $l_1 : l_2 : l_3$: etc. = 1 : $\sqrt{2}$, $\sqrt{3}$: etc.

Energy of the ion. If n = the number of gaps that the ion travels in the accelerator and v_n = the final velocity acquired by the ion, then

$$\left.\begin{array}{l}\text{Velocity of the ion, as it}\\ \text{emerges out of the } n^{th} \text{ tube}\end{array}\right\} = \sqrt{n}\sqrt{\frac{2\,Ve}{m}}.$$

$\therefore$ K.E. acquired by the ion = $\frac{1}{2}mv_n^2 = nVe.$

Thus the final energy of the ions depends upon (*i*) the total number of gaps and (*ii*) the energy gained in each gap.

The limitations of this accelerator are : (*i*) The length of the accelerator becomes inconveniently large and it is difficult to maintain vacuum in a large chamber. (*ii*) The ion current available is in the form of short interval impulses because the ions are injected at an appropriate moment.

EXAMPLE. *In a linear accelerator, proton accelerated thrice by a potential of* 40 *kV leaves a tube and enters an accelerating space of length* 30 *cm before entering the next tube. Calculate the frequency of the r. f. voltage and the length of the tube entered by the proton.*

SOL. Let v_1 and v_2 be the velocities of the proton on entering and leaving the accelerating space. Let *e* and *m* be the mass and charge of the proton respectively. Then

$$\frac{1}{2}mv_1^2 = 3 \times e \times 40000$$

$$\therefore \quad v_1 = [(2 \times 3 \times 40000 \times (e/m)]^{1/2}$$
$$= [2 \times 3 \times 40000 \times (9.578 \times 10^7)]^{1/2}$$
$$= 4.794 \times 10^6 \; ms^{-1}.$$

Similarly, $$v_2 = \left[2\times4\times40000\times(9.578\times10^7)\right]^{1/2}$$
$$= 5.536 \times 10^6 \; ms^{-1}.$$

Mean velocity while travelling the 0.3 *m* distance

$$= 5.165 \times 10^6 \ ms^{-1}.$$

The time taken to travel 0.3 *m* is half the period (*T*/2) of the *r.f.* voltage.

$$\therefore \quad \frac{T}{2} = \frac{0.3}{5.165 \times 10^6} \text{ or } T = \frac{2 \times 0.3}{5.165 \times 10^6}$$

$$\therefore \quad \left.\begin{array}{l}\text{frequency of the} \\ r.f.\text{ voltage}\end{array}\right\} = \frac{5.165 \times 10^6}{2 \times 0.3}$$

$$= 8.608 \times 10^6 \ Hz = 8.608 \ MHz.$$

The protons travel through the next tube for half a period with a velocity of $5.536 \times 10^6 \ ms^{-1}$.

∴ length of the tube entered by the protons

$$L = 5.536 \times 10^6 \times \frac{1}{2 \times 8.608 \times 10^6} = 0.3216 \ m$$

30.4 The Cyclotron

Construction. The cyclotron (Fig. 30.4) consists of two hollow semicircular metal boxes, D_1, D_2 called "dees". A source of ions is located near the mid-point of the gap between the "dees". The

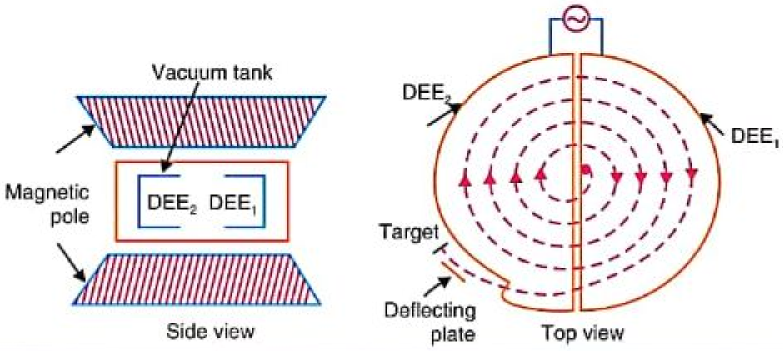


Fig. 30.4

"dees" are insulated from each other and are enclosed in another vacuum chamber. The "dees" are connected to a powerful radio-frequency oscillator. The whole apparatus is placed between the pole-pieces of a strong electromagnet. The magnetic field is perpendicular to the plane of the "dees".

Theory. Suppose a positive ion leaves the ion source at the centre of the chamber at the instant when the "dees" D_1 and D_2 are at the maximum negative and positive A.C. potentials respectively. The positive ion will be accelerated towards the negative dee D_1 before entering it. The ions enter the space inside the dee with a velocity *v* given by $Ve = \frac{1}{2}mv^2$, where *V* is the applied voltage and *e* and *m* are the charge and mass of the ion respectively. When the ion is inside the "dee" it is not accelerated since this space is field free. Inside the dee, under the action of the applied magnetic field, the ions travel in a circular path of radius *r* given by

Cyclotron.

$$Bev = mv^2/r \qquad ...(1)$$

where B = the flux density of the magnetic field.

or $$r = mv/Be \quad ...(2)$$

The angular velocity of the ion in its circular path = $\omega = \frac{v}{r} = \frac{Be}{m}$ ...(3)

The time taken by the ion to travel the semicircular path = $t = \frac{\pi}{\omega} = \frac{\pi m}{Be}$ ...(4)

Suppose the strength of the field (B) or the frequency of the oscillator (f) are so adjusted that by the time the ion has described a semicircular path and just enters the space between D_1 and D_2, D_2 has become negative with respect to D_1. The ion is then accelerated towards D_2 and enters the space inside it with a greater velocity. Since the ion is now moving with greater velocity, it will describe a semicircle of greater radius in the second "dee". But from the equation $t = \pi m/Be$ it is clear that the time taken by the ion to describe a semicircle is independent of both the radius of the path (r) and the velocity of the ion (v). Hence the ion describes all semicircles, whatever be their radii, in exactly the same time. This process continues until the ion reaches the periphery of the dees. The ion thus spirals round in circles of increasing radius and acquires high energy. The ion will finally come out of the dees in the direction indicated, through the window.

Energy of an ion. Let r_{max} be the radius of the outermost orbit described by the ion and v_{max} the maximum velocity gained by the ion in its final orbit. Then the equation for the motion of the ion in a magnetic field is

$$Bev_{max} = \frac{mv_{max}^2}{r_{max}}$$

or $$v_{max} = B\frac{e}{m}r_{max} \quad ...(5)$$

∴ The energy of the ion

∴ $$E = \frac{1}{2}mv_{max}^2 = \frac{B^2 r_{max}^2}{2}\left[\frac{e^2}{m}\right] \quad ...(6)$$

The condition for acceleration of the ion in the inter-dee gap is that

The time taken by the ion to travel the semicircular path } = Half the time period of oscillation of the applied high frequency voltage

i.e., $$\frac{\pi m}{Be} = \frac{T}{2} \text{ or } T = \frac{2\pi m}{Be}$$

∴ Frequency of the oscillator

$$f = \frac{Be}{2\pi m} \quad ...(7)$$

Hence the energy of the ion is given by

$$E = 2\pi^2 r_{max}^2 f^2 m \quad ...(8)$$

The particles are ejected out of the cyclotron not continuously but as pulsed streams.

Limitations of the Cyclotron. The energies to which particles can be accelerated in a cyclotron are limited by the relativistic increase of mass with velocity. The mass of a particle, when moving with a velocity v is given by $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ where m_0 is the rest mass and c the velocity of light.

According to equation (4),

The time taken by the ion to travel the semicircular path $\} = t = \frac{\pi m}{Be} = \frac{T}{2}$

$\therefore$ Frequency of the ion$\} = n = \frac{1}{T} = \frac{Be}{2\pi m}$ or $n = \frac{Be\sqrt{1-v^2/c^2}}{2\pi m_0}$.

Therefore, the frequency of rotation of the ion decreases with increase in velocity. The ions take longer time to describe their semicircular paths than the fixed period of the oscillating electric field. Thus, the ions lag behind the applied potential and finally they are not accelerated further. Due to this reason, the energy of the ions produced by the cyclotron is limited. This limitation can be overcome in the following two ways.

Now, the frequency of the ion = $Be\frac{\sqrt{1-v^2/c^2}}{2\pi m_0}$.

(*i*) Field variation. The frequency of the ion can be kept constant by increasing the magnetic field (B) at such a rate that the product $B\sqrt{1-v^2/c^2}$ remains constant. For this purpose, the value of the magnetic field B should increase, as velocity of the ion increases, so that the product $B\sqrt{1-v^2/c^2}$ remains unchanged. This type of machine in which the frequency of electric field is kept constant and magnetic field is varied is called *synchrotron.*

(*ii*) Frequency modulation. In another form of apparatus, the frequency of the applied A.C. is varied so that it is always equal to the frequency of rotation of the ion. This type of machine in which magnetic field is kept constant and the frequency of the applied electric field is varied is called a *frequency modulated cyclotron* or *synchro-cyclotron.*

EXAMPLE 1. *A cyclotron in which the flux density is 1.4 weber/m² is employed to accelerate protons. How rapidly should the electric field between the dees be reversed? Mass of the proton* $= 1.67 \times 10^{-27}$ *kg and charge* $= 1.6 \times 10^{-19}$ *C.*

SOL. Here, $B = 1.4$ weber/m²; $m = 1.67 \times 10^{-27}$ kg; $e = 1.6 \times 10^{-19}$ C.

$$\therefore \quad t = \frac{\pi m}{Be} = \frac{\pi(1.67\times10^{-27})}{1.4\times(1.6\times10^{-19})} = 2.342\times10^{-8}\ s.$$

EXAMPLE 2. *Deuterons in a cyclotron describe a circle of radius 0.32 m just before emerging from the dees. The frequency of the applied e.m.f. is 10 MHz. Find the flux density of the magnetic field and the velocity of deuterons emerging out of the cyclotron. Mass of deuterium* $= 3.32 \times 10^{-27}$ *kg;* $e = 1.6 \times 10^{-19}$ *C.*

SOL. We have, $f = \frac{Be}{2\pi m} \quad \therefore \ B = \frac{2\pi m f}{e}$

Here, $m = 3.32 \times 10^{-27}$ kg; $f = 10$ MHz $= 10^7$ Hz, $e = 1.6 \times 10^{-19}$ C

$$\therefore \quad B = \frac{2\pi(3.32\times10^{-27})10^7}{1.6\times10^{-19}} = 1.303\ \text{weber/m}^2$$

We have $\frac{mv^2}{r_{max}} = Bev$ or $v = \frac{Ber_{max}}{m}$

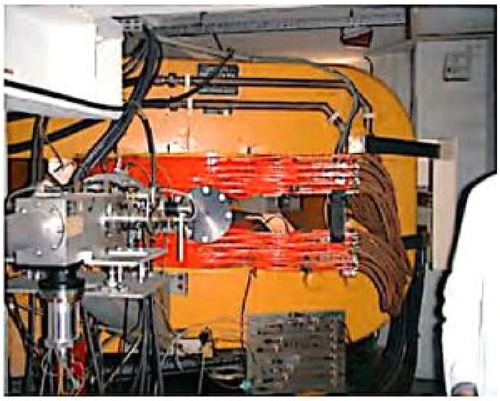
Here, $B = 1.303$ weber/m²; $e = 1.6 \times 10^{-19}$ C; $r_{max} = 0.32$ m and $m = 3.32 \times 10^{-27}$ kg.

$$\therefore \qquad v = \frac{Be\,r_{max}}{m} = \frac{1.303\,(1.6\times10^{-19})\,0.32}{3.32\times10^{-27}}$$

$$= 2.009 \times 10^7\ \text{ms}^{-1}.$$

30.5 The Synchrocyclotron

Synchrocyclotron is a modified form of the Lawrence cyclotron. This consists of only one dee placed in a vacuum chamber between the poles of an electromagnet (Fig. 30.5). Instead of the second dee, opposite the opening of the dee, there is a metal sheet connected to the earth. The alternating P.D. is applied between the dee and the metal plate (Fig. 30.6). The alternating potential applied to the "dee" is made to rise and fall periodically, instead of remaining constant. The frequency is changed at such a rate that as the ion lags a little due to the increase in mass caused by increase in velocity, the electric field frequency also automatically lags in variation. Hence the particle always enters the dee at the correct moment, when it can experience maximum acceleration. An advantage of using one dee is that it leaves sufficient space in the vacuum chamber for the ion source and the target. The pole-pieces of the magnet are of suitable shape such that the field decreases outwards from the centre. This ensures good focusing of the accelerated ions.



Synchrocyclotron

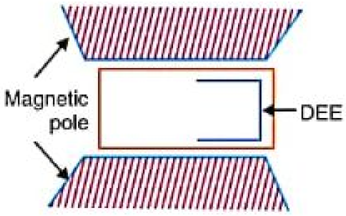


Fig. 30.5

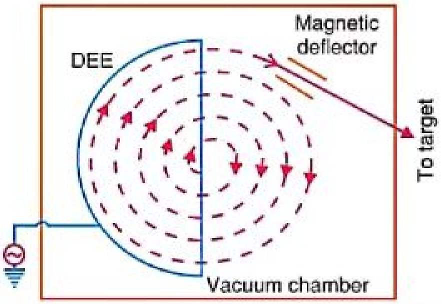
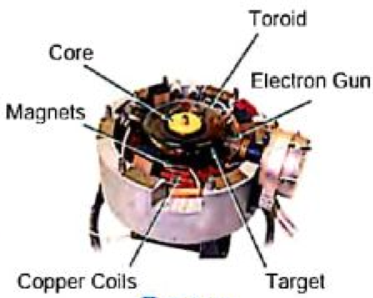


Fig. 30.6

30.6 The Betatron

Betatron is a device to accelerate electrons (beta particles) to very high energies. It was constructed in 1941 by D.W. Kerst. The action of this device depends on the principle of a transformer.



Betatron

Construction. It consists of a doughnut—shaped vacuum chamber placed between the pole-pieces of an electromagnet. The electromagnet is energised by an alternating current. The magnet produces a strong magnetic field in the doughnut. The electrons are produced by the electron gun (*FG*) and are allowed to move in a circular orbit of constant radius in the vacuum chamber

(Fig. 30.8). The magnetic field varies very slowly compared with the frequency of revolution of the electrons in the equilibrium orbit.

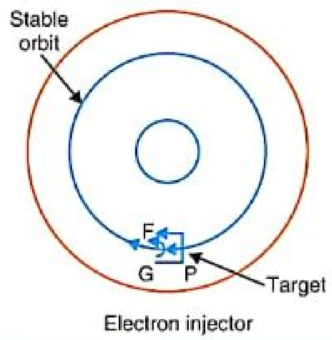


Fig. 30.7

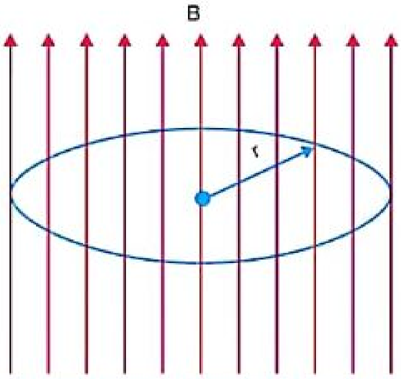


Fig. 30.8

The varying magnetic field, acting parallel to the axis of the vacuum tube, produces two effects on the electrons viz., (*i*) The changing flux due to the electromagnet produces the induced e.m.f. which is responsible for the acceleration of the electrons. (*ii*) The field of the magnet serves at the same time to bend the electrons in a circular path in the chamber and confine them to the region of the changing flux.

Theory. Consider the electron moving in an orbit of radius r (Fig. 30.9). Let ϕ be the flux linked with the orbit. The flux increases at the rate $d\phi/dt$ and the induced e.m.f. in the orbit is given by

$$E = -\frac{d\phi}{dt} \quad ...(1)$$

The work done on an electron of charge e in one revolution $= Ee = -e\frac{d\phi}{dt}$ ...(2)

Let F be the tangential electric force acting on the orbiting electron.

For one revolution, the path length is $2\pi r$. Then

the work done on the electron in one revolution $= F \times 2\pi r$

$$\therefore \quad F \times 2\pi r = -e\frac{d\phi}{dt}$$

or

$$F = -\frac{e}{2\pi r}\frac{d\phi}{dt} \quad ...(3)$$

When the velocity of the electron increases due to the above force, it will try to move into an orbit of larger radius. Because of the presence of the magnetic flux perpendicular to the plane of the electron orbit, the electron will experience a radial force inward given by

$$Bev = mv^2/r \quad ...(4)$$

Here B is the value of the magnetic field intensity at the electron orbit of constant radius r, v = velocity of the electron and m = mass of the electron. From (4),

the momentum of the electron $= mv = Ber$ ...(5)

From Newton's second law of motion,

$$F = \frac{d}{dt}(mv) = er\frac{dB}{dt} \quad ...(6)$$

To maintain the constant radius of the orbit, the values of F given in equations (3) and (6) must be numerically equal.

$$\therefore \qquad \frac{e}{2\pi r}\frac{d\phi}{dt} = er\frac{dB}{dt} \text{ or } d\phi = 2\pi r^2\, dB$$

Integrating, $\int_0^{\phi} d\phi = \int_0^{B} 2\pi r^2\, dB$

or $$\phi = 2\pi r^2 B \qquad ...(7)$$

If the uniform magnetic field B acts over an area πr^2, the magnetic flux $\phi' = \pi r^2 B$. Therefore the flux through the orbit is twice the flux enclosed by the orbit, if the magnetic field were to be uniform over the area. Equation (7) represents the condition under which a betatron works and is called *betatron condition*. This distribution of magnetic flux is obtained by the special pole-pieces where the magnetic field is greater at the centre of the orbit than at its circumference.

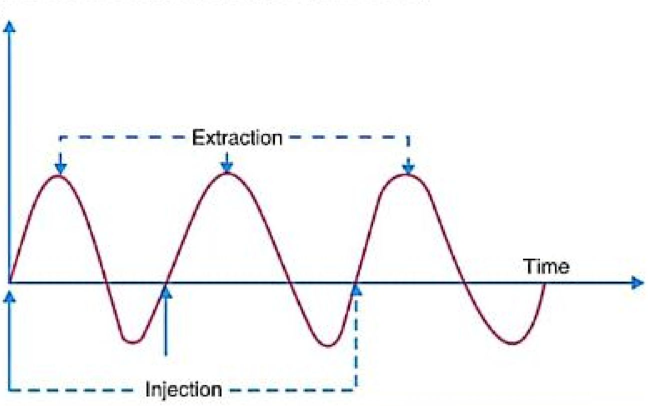


Fig. 30.9

Fig. 30.9 shows the variation of magnetic field with time. Electrons are injected into the chamber when magnetic field just begins to rise. The electrons are then accelerated by the increasing magnetic flux linked with the electron orbit. During the time the magnetic field reaches its peak value, the electrons make several thousand revolutions and get accelerated. If they are allowed to revolve any more, the decreasing magnetic field would retard the electrons. Hence, the electrons are extracted at this stage by using an auxiliary magnetic field to deflect them from their normal course. The high energy electron beam can be made to strike the target, generating X-rays. Alternately the electrons can be made to emerge out of the apparatus and used for transmutation work.

EXAMPLE. *In a certain betatron the maximum magnetic field at orbit was 0.4 Wb/m², operating at 50 Hz with a stable orbit diameter of 1.524 m. Calculate the average energy gained per revolution and the final energy of the electrons.*

SOL. In the betatron, the electron velocities are nearly c.

$\therefore$ the total distance travelled in the acceleration time (*i.e*, one quarter cycle) $= c\times\frac{T}{4} = c\times\frac{\pi}{2\omega}$

Total number of revolutions

$$= N = \frac{c\pi/2\omega}{2\pi r} = \frac{c}{4\omega r}.$$

Here, frequency $= f = 50$ Hz. $\therefore\ \omega = 2\pi f = 2\pi \times 50 = 100\,\pi$; $r = 0.762$ m, and $c = 3\times10^8\ ms^{-1}$.

$$\therefore \qquad N = \frac{3\times10^8}{4(100\pi)\,0.762} = 3.132\times10^5$$

Let E be the final energy acquired by the electrons. Since the electrons must be treated relativistically,

momentum of the electron $= mv = E/c$

But $mv^2/r = Bev$ or $mv = Ber$ or $E = Berc$.

$$\therefore \qquad E = \frac{0.4\,(1.6\times10^{-19})\,(0.762)\,(3\times10^{8})}{1.6\times10^{-13}}\,MeV$$

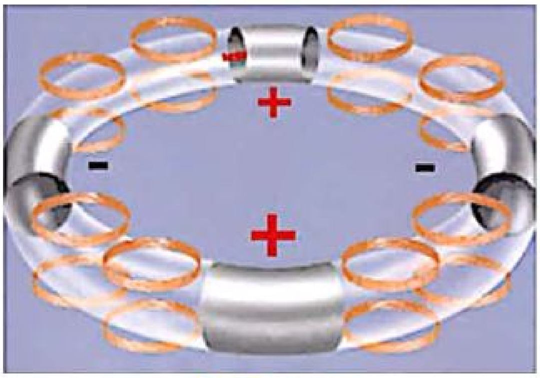
$$= 91.45\ MeV.$$

$$\text{Average energy gained per revolution} = \frac{91.45\times10^{6}}{3.132\times10^{5}} = 291.9\,eV\cdot$$

30.7 The Synchrotrons

There are two types of synchrotrons : (*a*) electron synchrotron and (*b*) proton synchrotron.

Electron synchrotron. The electron synchrotron is based on the principle of the combined working of betatron and cyclotron. Electrons are injected into an orbit of fixed radius at an initial energy of about 50 to 80 keV. The main accelerating tube, the torus, is made of glass or some plastic with a circular "dee" (*D*) made of a metal. An alternating potential is applied to the "dee" as shown in Fig. 30.10. A varying magnetic field is applied perpendicular to the torus. The radius of the orbit is kept constant by increasing the magnetic field as in a betatron. The increments of energy are given, as in a cyclotron, at the beginning and ending of the *D*. The electrons, after acceleration, are made to strike the required target. Using tungsten as target, very hard X-rays of energy about 300 MeV have been produced.



Synchrotrons.

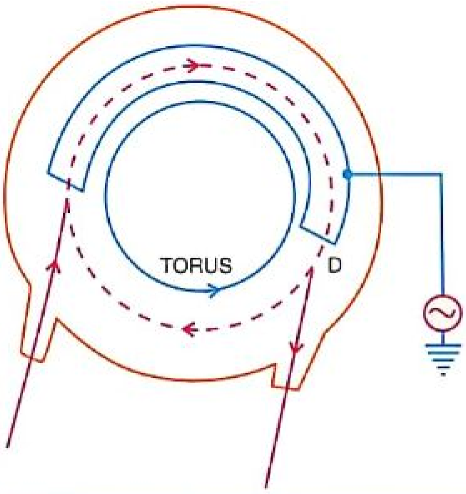


Fig. 30.10

30.8 The Proton Synchrotron (Bevatron, Cosmotron)

The proton synchrotron has been designed to provide protons of very high energy of the order of billion electron volts. The bevatron magnet consists of four segments connected by four equal straight sections (Fig. 30.11). The segments are quadrants of a circle of radius 30 feet and the connecting straight sections are 20 feet each. The protons are first accelerated by a linear accelerator and then injected into the synchrotron at one of the straight sections. The straight sections have an arrangement for applying the high frequency alternating potential. The magnetic field acts vertically upward. As the protons pass through one of the straight sections, they are accelerated and tend to have a bigger orbital radius. The magnetic field also increases at the same rate to constrain the proton beam to move in an orbit of constant radius. The output beam consists of a series of pulses of protons. By a suitable arrangement, the high energy particle may be made to strike a target.

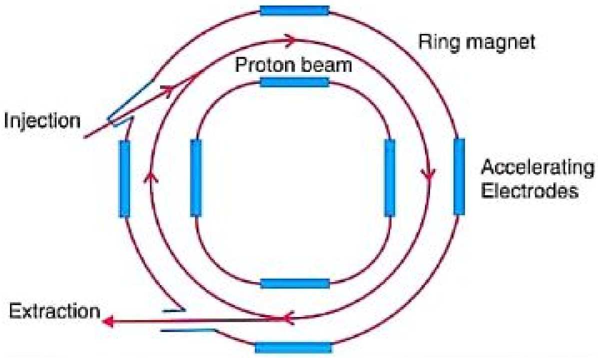


Fig. 30.11

Proton Synchrotron.

Theory. The frequency of revolution of the ion in a circular orbit is given by $n = Be/2\pi m$. In this derivation, we have neglected the relativistic variation in the mass of the accelerated ion. To take the relativistic effects into consideration, let us write

$$n = \frac{1}{2\pi}\frac{Bec^2}{mc^2} = \frac{1}{2\pi}\frac{Bec^2}{(m_0 c^2 + T)} \qquad ...(1)$$

where m_0c^2 is the rest energy and T is the K.E. of the ion. Eqn. (1) gives the frequency of revolution of an ion in the absence of the straight sections. In the presence of four straight sections, each of length L, the frequency n' of circulation of protons is given by

$$n' = \frac{Bec^2}{2\pi(m_0c^2 + T)}\frac{2\pi r_0}{(2\pi r_0 + 4L)} \qquad ...(2)$$

The proton is always guided by the magnetic field B to travel in the equilibrium orbit. Hence the following relation between B and the proton momentum p should hold at all times.

$$p = Ber_0 \qquad ...(3)$$

If T is the K.E. of a proton of momentum p then

$$T + m_0c^2 = [(pc)^2 + (m_0c^2)^2]^{1/2} = [(Becr_0)^2 + (m_0c^2)^2]^{1/2}$$

or $$T^2 + 2T m_0c^2 + m_0^2c^4 = B^2 e^2 c^2 r_0^2 + m_0^2 c^4$$

$\therefore$ $$B = \frac{[T(T + 2m_0c^2)]^{1/2}}{ecr_0} \qquad ...(4)$$

Eqn. (4) describes the way in which the magnetic field B at the equilibrium orbit has to increase with the increase in the energy T of the proton as it circulates in the orbit. Substituting for B from Eqn. (4) in Eqn. (2), we get

$$n' = \frac{c[T(T + 2m_0c^2)]^{1/2}}{(m_0c^2 + T)(2\pi r_0 + 4L)} \qquad ...(5)$$

In the proton synchroton, the accelerating voltage has to be in phase with the circulation frequency of the proton in the equilibrium orbit. Hence the radio-frequency of the accelerating voltage should always match n' and vary in the manner in which n' varies with the proton energy T.

Example. *What radius is needed in proton synchrotron to attain particle energies of 10 GeV, assuming that a guide field of 1.8 Wb/m² is available?*

Sol. Equivalent mass of proton = 10 GeV + rest mass

$= (10 + 0.938)\ GeV = 11.75\ u = 11.75 \times (1.66 \times 10^{-27})$ kg

$$\text{Radius of the orbit} = \frac{mv}{Be} = \frac{(11.75 \times 1.66 \times 10^{-27})(3 \times 10^8)}{1.8 \times (1.6 \times 10^{-19})} = 20.31\,m$$

EXERCISE

1. Describe with diagram the construction and action of Van de Graaff generator. Mention its uses.
2. Can a Van de Graaff generator be used to accelerate electrons and protons?
3. Give the construction and working of a linear accelerator.
4. Describe the construction and action of a cyclotron. Discuss its limitations.
5. What is the difference between a cyclotron and a synchrocyclotron? Explain the theory and principle of working of a synchrocyclotron.
6. Describe giving necessary theory the working of a betatron. Show that to keep the electron in a constant radial motion, the magnetic induction in the area enclosed by the path has to be maintained at any instant equal to double the induction over the circular orbits.
7. Give an account of the theory, construction and working of a modern synchrotron. Give its uses.
8. A linear accelerator for the acceleration of protons to 45.3 MeV is designed so that, between any pair of accelerating gaps, the protons spend one complete radio-frequency cycle inside a drift tube. The r.f. frequency used is 200 Mc/sec.; (*i*) What is the length of the final drift tube? (*ii*) If the first drift tube is 5.35 cm long, at what K.E. are the protons injected into the linac? (*iii*) If the peak accelerating potential is 1.49×10^6 volts, calculate the total length of the accelerator.

   [**Ans.** (*i*) 0.4654 m (*ii*) 0.60 MeV (*iii*) 9.4 m]
9. Assume that in the 70 MeV betatron, the radius of the stable electron orbit is 28 cm. Calculate (*i*) the angular velocity of the electrons (*ii*) the frequency of the applied electric field and (*iii*) the value of the magnetic field intensity at the orbit, for this energy.

   [**Ans.** (*i*) 1.07×10^9 radian /sec. (*ii*) 1.7×10^8 Hz (*iii*) 0.83 Wb m^{-2}]
10. Show that the radius of curvature R of the path of a particle inside the dees of a cyclotron is proportional to the $N^{1/2}$, where N is the number of times the particle has been accelerated across the space between the "dees".
11. Deuterons are accelerated in a fixed frequency cyclotron to a maximum dee orbit radius of 88 cm. The magnetic field is 1.4 weber/m^2. Calculate the energy of the emerging deuteron beam and the frequency of the dee voltage. What change in magnetic flux density is necessary if doubly charged helium ions are accelerated? Given atomic masses : H^2 = 2.014102 u, He^4 = 4.002603 u.

   [**Ans.** 36.3 MeV ; 10.67 MHz; 0.01 Wbm^{-2}]
12. A fixed frequency cyclotron has an oscillator frequency of 12 MHz and dee radius of 0.55 m. It is used to accelerate deuterons. Calculate : (*a*) the magnetic flux density needed, and (*b*) energy to which deuterons are accelerated. [**Ans.** (*a*) 1.574 Wb/m^2 (*b*) 17.95 MeV]

    (c) If the above cyclotron is required to accelerate the doubly charged Helium ions, find how and by what amount magnet flux density will have to be changed at a fixed dee voltage.

    Given : mass of helium ion = 6.642397×10^{-27} kg.

    [**Hint.** Let B_1 and B_2 be the values of the magnetic flux densities when the cyclotron accelerates two ions having charges Z_1e and Z_2e and masses m_1 and m_2 respectively. Then,

    $B_1 = 2\pi m_1 f/Z_1 e$ and $B_2 = 2\pi m_2 f/Z_2 e$

$$\therefore \quad B_1 - B_2 = \frac{2\pi f}{e}\left(\frac{m_1}{Z_1} - \frac{m_2}{Z_2}\right)$$

$$= \frac{2\pi(12\times10^6)}{1.6\times10^{-19}}\left(\frac{3.34245\times10^{-27}}{1} - \frac{6.642397\times10^{-27}}{2}\right)$$

$$= 0.01008 \text{ Wb m}^{-2} \text{ (to be decreased)}].$$

13. A betatron has the following parameters : Magnet current supply frequency = 50 *Hz*. Peak magnetic flux density at the orbit = 0.4 Wb m^{-2}. Electron orbit radius = 0.75 m. Calculate (*a*) the final energy of the electrons (*b*) energy gained per revolution and (*c*) total time of flight of the electrons.

[**Ans.** (*a*) 90.01 *MeV* (*b*) 282.7 *eV* (*c*) 4.999×10^{-3} seconds]

14. 0.5 *MeV* protons are injected into a 50 *MeV* linear accelerator powered by a 200 *MHz* radio-frequency supply. Find the approximate lengths of the first and the last drift tubes.

[**Ans.** 0.02446 m; 0.2354 m]

15. Calculate the average energy per revolution and final energy gained by electrons in a betatron to which is applied a maximum magnetic field of 0.5 tesla operating at 60 Hz in a stable orbit of diameter 2 metres. [**Ans.** 750 eV, 150 MeV].

16. What total accelerator voltage will impart protons a velocity of 95% of the speed of light? (Rest mass of proton = $m_0 = 1.672 \times 10^{-27}$ kg).

[**Hint.** $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.672\times10^{-27}}{\sqrt{1-\left(\frac{95}{100}\right)^2}} = 5.356\times10^{-27}\text{ kg}$$

$\therefore$ $\Delta m = (m - m_0) = (5.356 - 1.672) \times 10^{-27}$ kg

$\therefore$ $KE = \Delta m.c^2 = (3.684 \times 10^{-27}) \times (3 \times 10^8)^2 \text{ J} = 2.07 \times 10^9$ eV

Hence accelerating voltage = 2.07 BeV].

17. In a synchrocyclotron, the magnetic flux density decreases from 1.5 Wb m^{-2} at the centre of the magnet to 1.43 Wb m^{-2} at the limiting radius of 2.06 m. Calculate the range of frequency modulation required for deuteron acceleration and the maximum kinetic energy of the deuterons. The rest mass of the deuteron is 3.34×10^{-27} kg; the electronic charge is 1.6×10^{-19} *C*.

[**Hint.** $$\frac{m_0 v}{\sqrt{1-(v^2/c^2)}} = Ber \quad ...(1)$$

where B is the magnetic induction at the limiting radius r and v is the final velocity attained by the deuteron.

Here, $m_0 = 3.34 \times 10^{-27}\ kg$; $B = 1.43\ Wb\ m^{-2}$;

$e = 1.6 \times 10^{-19}\ C$; $r = 2.06\ m$, $c = 3 \times 10^8\ ms^{-1}$; $\therefore$ $v = 1.277 \times 10^8\ ms^{-1}$

$$\text{Frequency of revolution} = \frac{v}{2\pi r} = \frac{1.277\times10^8}{2\pi\times2.06} Hz = 9.87\ MHz.$$

The initial frequency of the oscillator must equal the simple cyclotron frequency

$$f = \frac{Be}{2\pi m} = \frac{1.5\times(1.6\times10^{-19})}{2\pi\times3.34\times10^{-27}} = 11.44\ MHz.$$

Therefore the range of frequency modulation is from 11.44 *MHz* to 9.87 *MHz*.

$$E = m_0 c^2\left[\frac{1}{(1-v^2/c^2)^{12}} - 1\right] = 197.2\ MeV].$$