ALLIED COURSE III - MATHEMATICS FOR MANAGEMENT- II

Objectives:

As the result of instructional activities, students will be able to:

1. Perform the operations of addition, subtraction, multiplication, and division on whole numbers, fractions, and decimals, *by hand*.
2. Evaluate numerical expressions involving whole number exponents and square roots.
3. Identify basic geometrical figures and find their perimeter and area.
4. Solve problems involving ratios and proportions.
5. Solve problems involving percents

UNIT - I

Introduction to Operations Research - Meaning - Scope – Models - Limitation. Linear Programming - Formulation – Application in Management decision making (**Graphical method only)**

UNIT – II

Transportation (Non- degenerate only) - Assignment problems - Simple Problems only

UNIT - III

Game Theory:- Queuing theory - Graphical Solution – mx2 and 2xn type. Solving game by Dominance property - fundamentals - Simple problems only. Replacement problem – Replacement of equipment that detoriates gradually (value of money does not change with time)

UNIT - IV

CPM - Principles - Construction of Network for projects – Types of Floats – Slack- crash programme.

UNIT -V

PERT - Time scale analysis - critical path - probability of completion of project - Advantages and Limitations.

Note: Theory and problem shall be distributed at 20% and 80% respectively.

REFERENCE BOOKS

1. Kanti Swarup, Gupta R.K. - Operations Research
2. P.R. Vittal - Operations Research
3. Gupta S.P. - Statistical Methods.

Course Out comes:

1. Compute a given integral using the most efficient method;
2. Use integrals to formulate and solve application problems in science and engineering;
3. Construct and plot parametric and polar curves;
4. Identify different types of series and determine whether a a particular series converges;
5. Find the interval of convergence of a power series;
6. Apply Taylor series to approximate functions and estimate the error of approximation

Unit - I

LINEAR PROGRAMMING

GRAPHICAL METHOD OF SOLVING a L.P.P [Graphical solution].

Linear programming problem involving only two variables can be effectively should by a graphical method which provides representation of the problem and its solution and varicim gives the basic complales used in solving general L.P.P

Working procedure for graphical method:-

Given a L.P.P optimize $z = f(x)$,

$$g(x) = \leq, =, \geq x_i \geq 0$$

$$i = 1, 2 \ldots$$

Step 1: Draw the straight line

Step 2:- Find the permissible region (feasible region (or) solution space (or) convex region)

Step 3:- Find the points of intersection

Step 4:- Find the value of z

Step 5:- For max z To choose vertex max

& For min z To choose vertex min

{A region of a set points is said to be convex) .

Problem :-

solve the following L.P.P by the graphical method

$$\max Z = 3x_1 + 2x_2$$

subject to $-2x_1 + x_2 \le 1$

$x_1 \le 2$

$x_1 + x_2 \le 3$

and $x_1, x_2 \geq 0$

sol :-

First consider the Inequality constraints as Equalities :-

$-2x_1 + x_2 = 1 \rightarrow$ ①

$x_1 = 2 \rightarrow$ ②

$x_1 + x_2 = 3 \rightarrow$ ③

and $x_1 = 0$

$x_2 = 0$

① $\Rightarrow -2x_1 + x_2 = 1$

put $x_1 = 0 \Rightarrow x_2 = 1 \Rightarrow [0,1]$

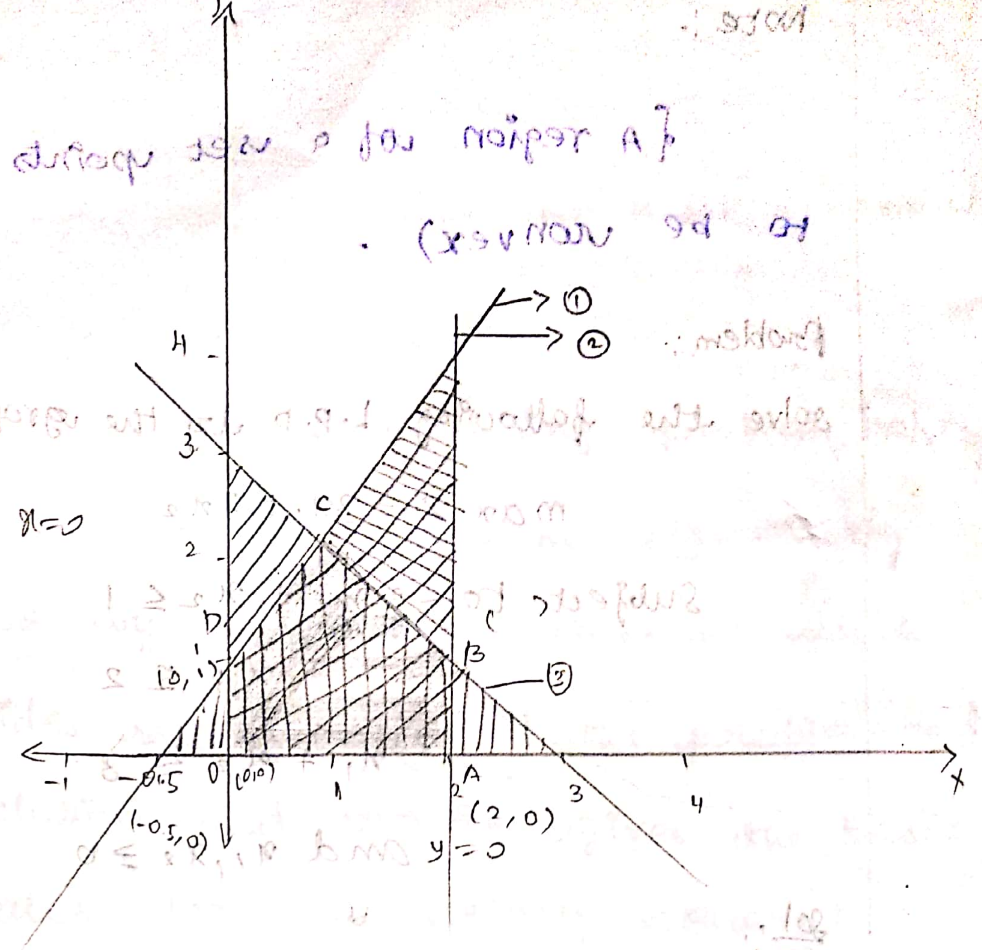
$x_2 = 0 \Rightarrow -2x_1 = 1$

$x_1 = -\frac{1}{2} \Rightarrow [\frac{1}{2}, 0]$

(or) $[0.5, 0]$

② $\Rightarrow x_1 = 2 \Rightarrow [2, 0]$

③ $\Rightarrow x_1 + x_2 = 3$



max $z = 3x_1 + 2x_2$

O(0,0)	0
A(2,0)	6
B(2,1)	8
C(2/3, 7/3)	20/3 = 6.3
D(0,1)	2

② ⇒ $x_1 = 2$

$x_1 + x_2 = 3$

$x_1 = 2 \Rightarrow x_2 = 1$

(2, 1)

C ⇒ $-2x_1 + x_2 = 1$

$x_1 + x_2 = 3$

$-3x_1 = -2$

$x_1 = 2/3$

$2/3 + x_2 = 3$

$x_2 = 3 - 2/3$

$x_2 = 7/3$

max $z = 8$; $x_1 = 2$; $x_2 = 1$

Solve the following L.P.P by the graphical method

minimize $Z = 3x_1 + 5x_2$

subject to $-3x_1 + 4x_2 \le 12$

$x_1 \leq 4$

$2x_1 - x_2 \geq -2$

$x_2 \geq 2$

$2x_1 + 3x_2 \geq 12$ and $x_1, x_2 \geq 0$.

Sol :-

$-3x_1 + 4x_2 = 12 \rightarrow (1)$

$x_1 = 4 \rightarrow (2)$

$2x_1 - x_2 = -2 \rightarrow (3)$

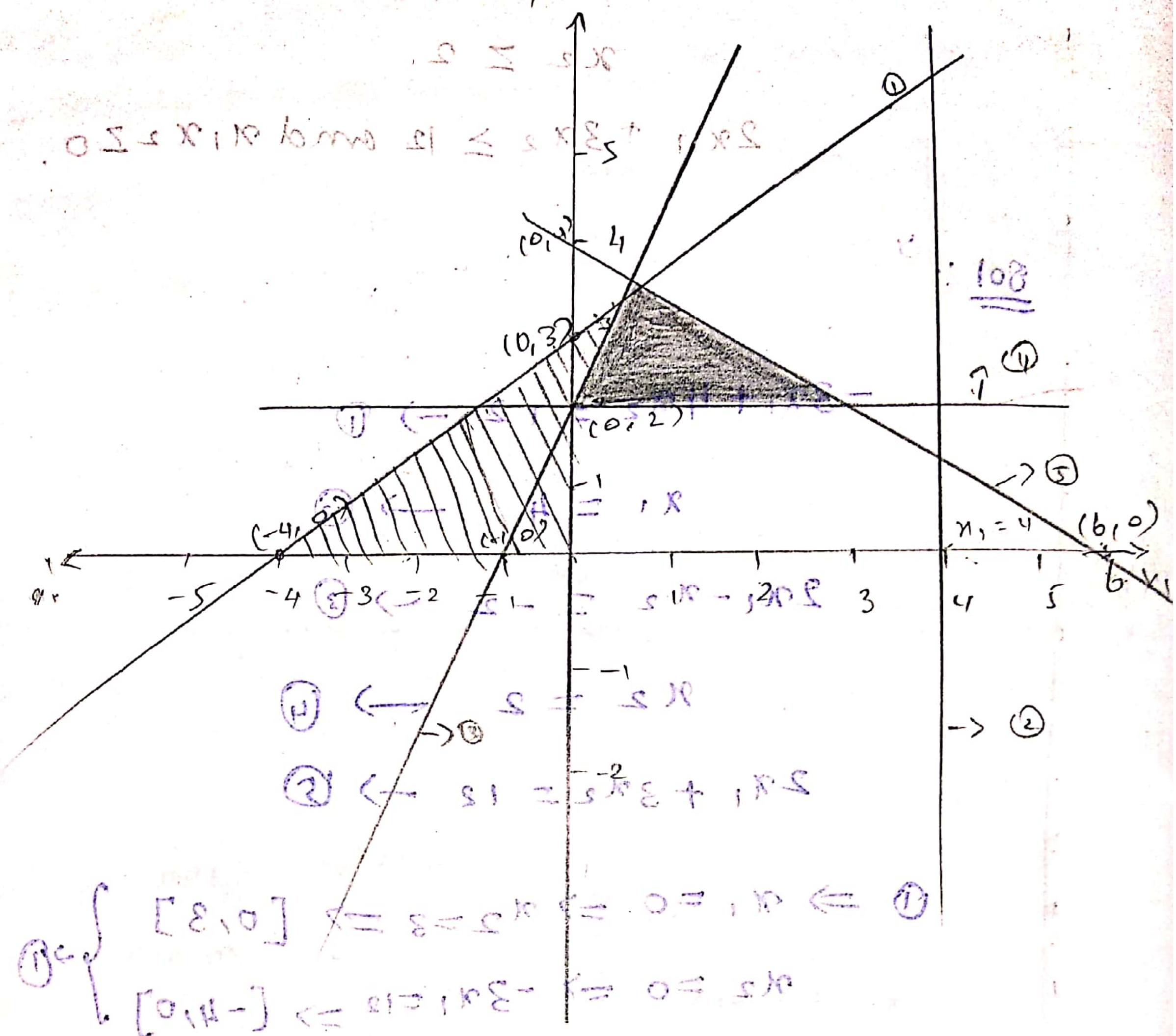
$x_2 = 2 \rightarrow (4)$

$2x_1 + 3x_2 = 12 \rightarrow (5)$

$$\left.\begin{array}{l} (1) \Rightarrow x_1 = 0 \Rightarrow x_2 = 3 \Rightarrow [0,3] \\ \quad x_2 = 0 \Rightarrow -3x_1 = 12 \Rightarrow [-4,0] \end{array}\right\} \rightarrow (1)$$

$$\left.\begin{array}{l} (3) \Rightarrow x_1 = 0 \Rightarrow -x_2 = -2 \Rightarrow [0,2] \\ \quad x_2 = 0 \Rightarrow 2x_1 = -2 \Rightarrow [-1,0] \end{array}\right\} \rightarrow (2)$$

$$\left.\begin{array}{l} (5) \Rightarrow x_1 = 0 \Rightarrow 3x_2 = 12 \Rightarrow [0,4] \\ \quad x_2 = 0 \Rightarrow 2x_1 = 12 \Rightarrow [6,0] \end{array}\right\} \rightarrow (3)$$



(A) → Equ (4) & (5)

$x_1 = 4$; $2x_1 + 3x_2 = 12$

$3x_2 = 12 - 8 = 4$

$x_2 = 4/3$

$A = 4, 4/3$

(B) → $x_1 = 4$, $x_2 = 2$

(C) → equ (1) & (2)

$x_1 = 4$, $-3x_1 + 4x_2 = 12$

$-3(4) + 4(x_2) = 12$

$4x_2 = 24$

$x_2 = 6$

$(4, 6)$

(1) → Eqn (1) & (5) $-3x_1 + 4x_2 = 12 \rightarrow A_1$

$2x_1 + 3x_2 = 12 \rightarrow A_2$

(5) ⇒

(2) $\times A_1 \Rightarrow -6x_1 + 8x_2 = 24$

(3) $\times A_2 \Rightarrow 6x_1 + 9x_2 = 36$

$17x_2 = 60$

$x_2 = 60/17$

$-6x_1 + 8(60/17) = 24$

$-6x_1 = 24 - \frac{480}{17}$

$x_1 = \cancel{0.0415}\ 0.7058$

$x_2 = 3.529$

$A(4, 4/3) = 3 \times 4 \times 5(4/3)$

$= 12 + \frac{20}{3} = \frac{56}{3} = 18.66$

$B(4, 2) = 12 + 10 = 22 = 22$

$C(4, 2) = 12 + 60 = 72 = 72$

$D(0.7058, 3.529) = (3 \times 0.7058) + 5(3.529)$

$= 19.762$

min $Z = 18.66$

$x_1 = 4$

$x_2 = 4/3$

A principle firm products two product canned pineapple and canned juice. the specific amounts of material, labour and equipment required to produce each product and the availability of each of these resources are show in the table given below.

	canned juice	canned pineapple	Available resources
Labour	3	2.0	12.0
equipment	1	2.3	6.9
material	1	1.4	4.9

Assuming one unit of canned juice and canned pineapple has profit margins Rs 2 and Rs 1 respectivels. formulate this as L.P.P. and solve this graphically also.

Sol:-

$$\max z = 2x_1 + x_2$$

$$3x_1 + 2x_2 \le 12$$

$$x_1 + 2.3x_2 \le 6.9$$

$$x_1 + 1.4x_2 \le 4.9$$

(1) => $3x_1 + 2x_2 = 12$ — (1)

$x_1 + 2.3x_2 = 6.9$ — (2)

$x_1 + 1.4x_2 = 4.9$ — (3)

(1) -> $x_1 = 0$ $2x_2 = 12$

$x_2 = 6$ => [0.6]

$x_2 = 0$ => $3x_1 = 12$ [4,0]

$x_1 = 4$

(2) ⇒ $x_1 = 0 \Rightarrow 2.3x_2 = 6.9$

$$\boxed{x_2 = 3} \quad [0, 3]$$

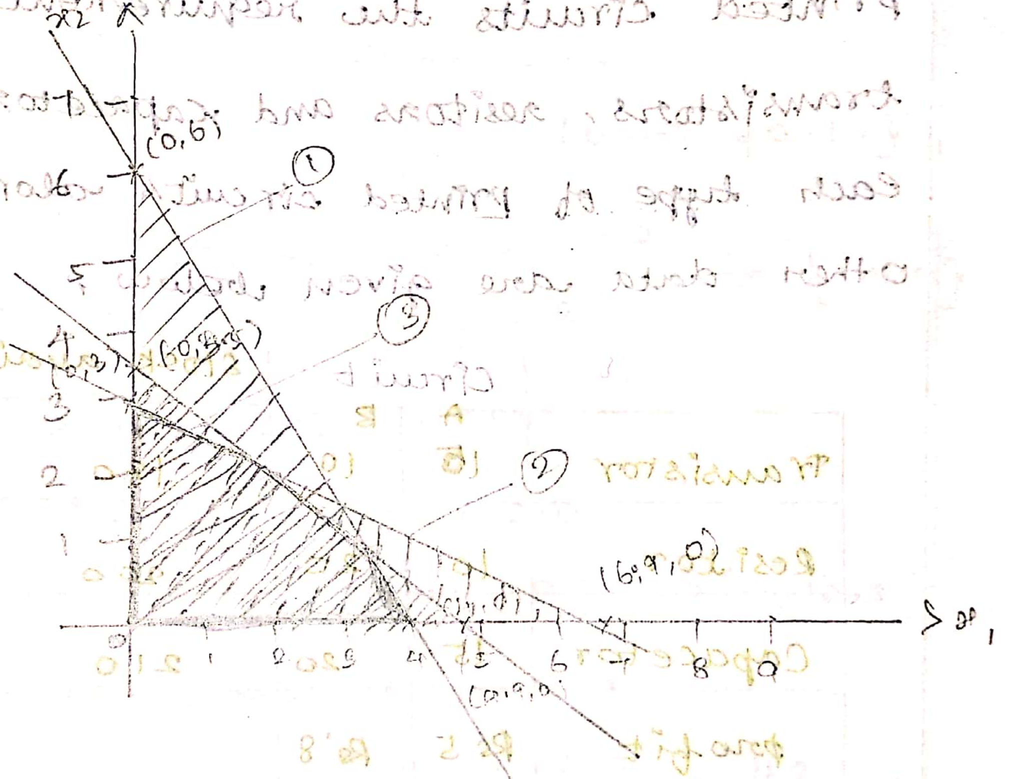
$x_2 = 0 \Rightarrow x_1 = 6.9 \quad [6.9, 0]$

(3) $x_1 = 0 \Rightarrow 1.4x_2 = 4.9$

$$x_2 = \frac{4.9}{1.4}$$

$$x_2 = 3.5 \quad [0, 3.5]$$

$x_2 = 0 \Rightarrow x_1 = 4.9 \quad [4.9, 0]$



$$x_1 + 2.3x_2 = 6.9 \quad \text{—— (2)}$$

$$x_1 + 1.4x_2 = 4.9 \quad \text{—— (3)}$$

$$(2.3 - 1.4)x_2 = 2$$

$$0.9\,x_2 = 2$$

$$x_2 = \frac{2}{0.9}$$

$$\boxed{x_2 = 2.2}$$

$$x_1 + 2.3(2.2) = 6.9$$

$$x_1 = 1.8$$

$O(0,0) = 0$

$A(0,3) = 3$

$B(1.8,1.2 \cdot (2)) = 5.8$

$C(4,0) = 8$.

maximum $Z = 8$

$x_1 = 4;\ x_2 = 0$.

A company manufactures 2 types of printed circuits the requirement of transistors, resitors and capacetors for each type of printed circuits along with other data are given below,

	Circuit A	Circuit B	stock available
Transistor	15	10	180
Resitor	10	20	200
Capacetor	15	20	210
profit	Rs 5	Rs 8	

How many circuits of each type should be company produce from the stock to earn maximum profit.

max $\leq$
min $\geq$

Sol:

$$\max : Z = 5x_1 + 8x_2$$

$$15x_1 + 10x_2 \leq 180$$

$$10x_1 + 20x_2 \leq 200$$

$$15x_1 + 20x_2 \leq 210$$

$$x_1, x_2 \geq 0.$$

$15x_1 + 10x_2 = 180 \rightarrow ①$

$10x_1 + 20x_2 = 200 \rightarrow ②$

$15x_1 + 20x_2 = 210 \rightarrow ③$

$① \Rightarrow x_1 = 0 \Rightarrow 10x_2 = 180$

$x_2 = 18 \rightarrow \{0, 18\}$

$x_2 = 0 \Rightarrow 15x_1 = 180$

$x_1 = 12 \quad \{12, 0\}$

$② \Rightarrow x_1 = 0 \Rightarrow 20x_2 = 200$

$x_2 = 10 \rightarrow \{0, 10\}$

$x_2 = 0 \Rightarrow 10x_1 = 200$

$x_1 = 20 \rightarrow \{20, 0\}$

$③ \Rightarrow 15x_1 + 20x_2 = 210$

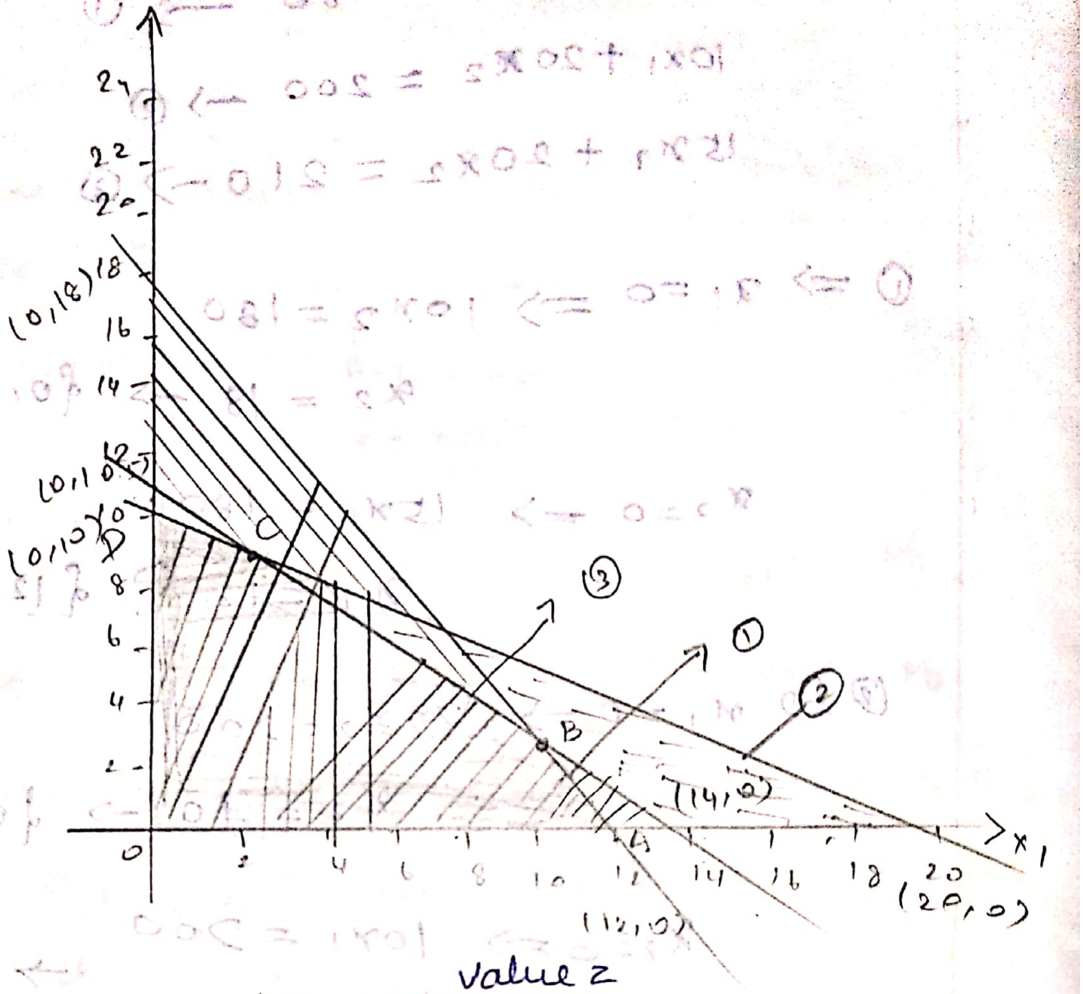
$x_1 = 0 \rightarrow 0 \quad 20x_2 = 210$

$x_2 = 10.5 \Rightarrow \{0, 10.5\}$

$x_2 = 0 \Rightarrow 15x_1 = 210$

$x_1 = \frac{210}{15} = \frac{42}{3} = 14$

$= \{14, 0\}.$



		value z
O	(0,0)	0
A	(12,0)	60
B	(10,3)	74
C	(2,9)	82
D	(0,10)	80

C: ⇒ 1 & 3 are intersect

D: ⇒ 2 & 3 are interse[ct]

Since the problem is of maximization type,

the optimal solution is

max $Z = 82$, $x_1 = 2$, $x_2 = 9$.

Here the maximum value of z occurs at two vertices A & B.

$\therefore$ thus are Infinite no. of solutions.

Solve the graphically the following L.P.P

maximize $Z = 4x_1 + 3x_2$

$x_1 - x_2 \leq -1 \Rightarrow$

$-x_1 + x_2 \leq 0$

and $x_1, x_2 \geq 0$

<u>Sol</u>:

$x_1 - x_2 \leq -1 \Rightarrow -x_1 + x_2 \geq 1 \rightarrow (1)$

$-x_1 + x_2 \leq 0 \rightarrow (2)$

$-x_1 + x_2 = 1 \rightarrow (1)$

$-x_1 + x_2 = 0 \rightarrow (2)$

$(1) \Rightarrow x_1 = 0 \Rightarrow -x_1 + x_2 = 1$

$x_2 = 1 \Rightarrow \{0, 1\}$

$x_2 = 0 \Rightarrow -x_1 + x_2 = 1$

$0 - x_1 = 1$

$x_1 = -1 \Rightarrow \{-1, 0\}$

$(2) \Rightarrow x_1 = 0 \Rightarrow -x_1 + x_2 = 0$

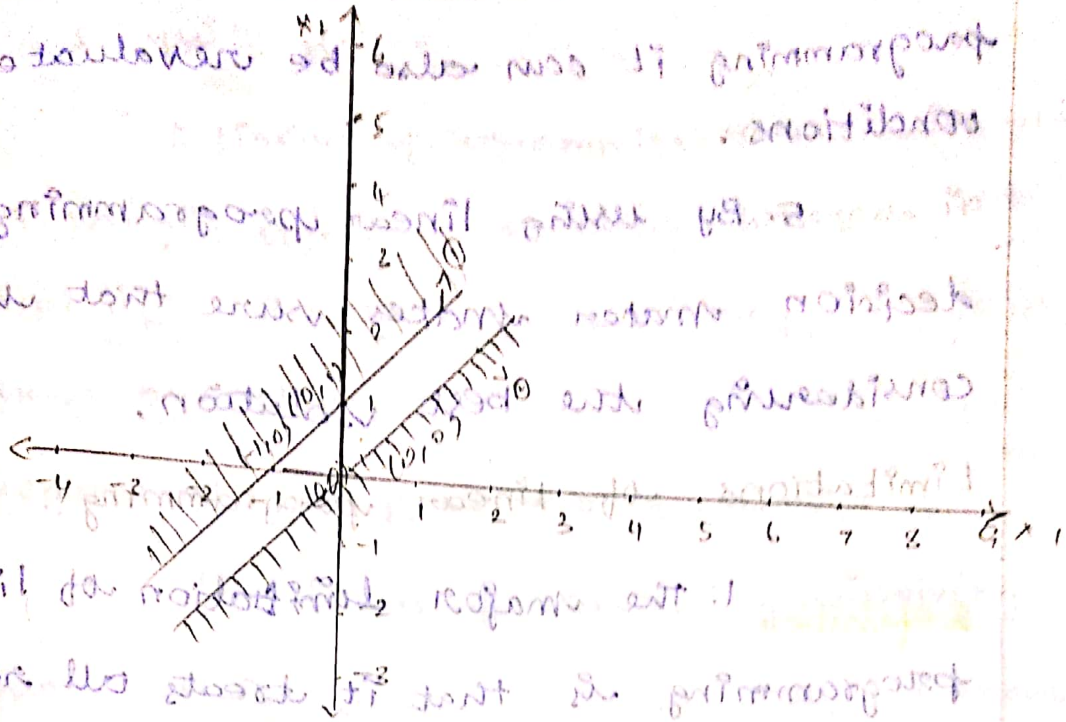
$x_2 = 0 \Rightarrow \{0, 0\}$

$x_2 = 0 \Rightarrow -x_1 + x_2 = 0$

$-x_1 = 0$

$x_1 = 0 \Rightarrow \{0, 0\}$

Scanned with CamScanner



Hence the problem is no feasible region.

Advantage of Linear Programming :-

1. It provides an in sight and perspective in to the problem environment. This generally results in clear picture of the true problem.

2. It makes a scientific and mathematical analysis of the problem situations

3. In gives an opportunity to the decision maker to formulate his strategies consistant with the constraints and the objectives

4. It deals with changing situations. Once a plan is arrived through the linear

programming it can also be revaluate conditions.

5. By using linear programming the decision maker makes sure that he is considering the best solution.

Limitations of Linear programming:

1. The major limitation of linear programming is that it treats all relations as linear. But it is not true in many real life situations.

2. The decision variables in some LPP would be meaningful only if they have integer values. But some time we get fractional values to the optimal solution, where only integer values are meaningful.

3. All the parameters in the linear programming model are assumed to be known contants. But it real life they may not be known completely or they may be probabilistic and they may be liable for changes from time to time.

4. The problem are complex if the number of variables and constraints

are quite large.

5. Linear programming deals with only a single objectives problems. whereas in real life situations there may be more than one objective.

General linear programming problem:

The linear programming involving more than two variables may be expressed as follows.

Maximize (or) Minimize $Z = c_1x_1 + c_2x_2 + \cdots c_nx_n$ subject to the constraints.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le \text{ or } = \text{ or } \ge b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le \text{ or } = \text{ or } \ge b_2$$

$$\cdots \quad \cdots \quad \cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le \text{ or } = \text{ or } \ge b_m$$

and the non negative restrictions.

$$x_1, x_2, x_3 \ldots x_n \ge 0.$$

Definition:

A set of values $x_1, x_2 \ldots x_n$ which satisfies the constraints of the LPP is called its solution.

Def:

Any solution to a LPP which satisfies the non-negativity restrictions of the LPP called its feasible solution.

Def :-

Any feasible solution which optimize (maximizes or minimize) the objective function of the LPP is called its optimum solution (or) optimal solution

Definition :-

If the constrains of a general LPP be

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad (i = 1, 2, 3 \ldots k) \rightarrow (1)$$

then the non-negative variables s_i which are introduced to convert the inequalities (1) to the equalities

$$\sum_{j=1}^{n} a_{ij} + s_i = b_i \quad (i = 1, 2, 3 \ldots k)$$ are

called slack variables. the value of these variables can be interpreted as the amount of unused resource.

Canonical and standard forms of LPP :

the general linear programming problem can always be expressed in the following form.

Maximize $Z = c_1 x_1 + c_2 x_2 + c_3 x_3 + \cdots + c_n x_n$ subject to the constrains.

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2$

- - - - - - - - - - -

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m$, and

the non-negativity restrictions. $x_1, x_2, \ldots x_n \ge 0$

This form of LPP is called the "conical form" of the LPP.

The standard form:

The general linear programming problem in the form, maximize $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ subject to the constraints.

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$

- - - - - - - - - -

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n$ and

$x_1, x_2 \ldots x_n \ge 0$ is known as standard form.

Characteristic of the cononical form:

i) The objective function is of maximization type

ii) All constraints are of $(\le)$ Type

iii) All variables x_j are non-negative.

Characteristic of the standard form:

i) The objective function is of maximization Type.

ii) All constrains are expressed as equations.

iii) Right hand side of each constant is non-negative.

iv) All variable are non-negative.

Note:

1. The minimization of a function $f(x)$ is equivalent to the maximization of the negative expression of this function

ie) $\min f(x) = -\max\{-f(x)\}$ ie) $\min z = -\max$

Eg: $\min z = c_1x_1 + c_2x_2$ is equivalent to $\max(-z) = -c_1x_1 - c_2x_2$.

Note: 2

2) An inequality in one direction can be converted into an inequality in the opposite direction by multiplying both sides by (-1).

eg: $ax_1 + bx_2 \geq c$

$-ax_1 - bx_2 \leq -c$

3) An equality constraint can be expressed as two inequality.

Eg: $ax_1 + bx_2 \leq c$

$ax_1 + bx_2 = c \Rightarrow ax_1 + bx_2 \geq c$ $\Big\} \Rightarrow$ $ax_1 + bx_2 \leq c$, $-ax_1 - bx_2 \leq -c$

4) An inequality constraint with its left hand side in the absolute form can

be expressed as two inequalities

e.g.) $|ax_1+bx_2| \le c \Rightarrow ax_1+bx_2 \le c$

$ax_1+bx_2 \ge -c$

5) If a variable is unconstrained or unrestricted (without specifying its sign) it can always be expressed as the different of two non-negative variables.

eg) If x_2 is unrestricted, then

$x_2 = x_2' - x_2''$ where $x_2', x_2'' \ge 0$

6) whenever slack/surplus variables are introduced in the constraints they should also appear in the objective function with zero co-efficients.

Example: 1

Express the following LPP in the canonical form maximize $Z = 2x_1+3x_2+x_3$

subject to the constraint $4x_1-3x_2+x_3 \le 6$

$x_1+6x_2-7x_3 \ge -4$

and $x_1, x_3 \ge 0$, x_2 is unrestricted.

Sol:

As x_2 is unrestricted $x_2 = x_2' - x_2''$

where

$x_2', x_2'' \ge 0$

$\therefore$ the given LPP becomes

maximize $z = 2x_1 + 3(x_2' - x_2'') + x_3$

subject to $4x_1 - 3x_2' + 3x_2'' + x_3 \le 6$

$x_1, x_2', x_2'', x_3 \ge 0$

Scanned with CamScanner

convert the second constraint $\leq$ type by multiplying both the sides by -1. Now the LPP becomes

maximize $Z = 2x_1 + 3x'_2 - 3x''_2 + x_3$

subject to $4x_1 - 3x'_2 + 3x''_2 + x_3 \leq 6$

$-x_1 - 5x'_2 + 5x''_2 + 7x_3 \leq 4$

and $x_1, x'_2, x''_2, x_3 \geq 0$

which in the canonical form.

Example: 2

Express the following LPP in standard form

Minimize $z = 5x_1 + 7x_2$

subject to the contraints

$x_1 + x_2 \leq 8$

$3x_1 + 4x_2 \geq 3$

$6x_1 + 7x_2 \geq 5$ and $x_1, x_2 \geq 0$.

Sol:

since $\min z = -\text{Max}(-z) = -\text{Max } z^*$

The given LPP becomes Minimize $z^* = -5x_1 - 7x_2$

subject to

$x_1 + x_2 \leq 8$

$3x_1 + 4x_2 \geq 3$

$6x_1 + 7x_2 \geq 5$ and $x_1, x_2 \geq 0$.

By introducing slack variable s_1 and surplus variables s_2, s_3 the standard form of the LPP is given by

Maximize $z^* = -5x_1 - 7x_2 + 0s_1 + 0s_2 + 0s_3$

subject to

$$x_1 + x_2 + s_1 = 8$$

$$3x_1 + 4x_2 - s_2 = 3$$

$$6x_1 + 7x_2 - s_3 = 5$$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$.

Example: 03

Express the following LPP in standard (Matrix) form Minimize $z = 4x_1 + 2x_2 + 6x_3$

subject to $2x_1 + 3x_2 + 2x_3 \geq 6$

$3x_1 + 4x_2 \geq 8$

$6x_1 - 4x_2 + x_3 \leq 10$ and $x_1, x_2, x_3 \geq 0$.

Sol:

By introducing the surplus variable s_1 and slack variable s_2, the standard form of the LPP becomes,

Minimize $z = 4x_1 + 2x_2 + 6x_3 + 0s_1 + 0s_2$

Subject to $2x_1 + 3x_2 + 2x_3 - s_1 + 0s_2 = 6$

$3x_1 + 4x_2 + 0x_3 + 0s_1 + 0s_2 = 8$

$6x_1 - 4x_2 + x_3 + 0s_1 + s_2 = 10$ and

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Thus the given problem in matrix form is

maximize $z = cx$.

Subject to $Ax = b$

$x \geq 0$

where $c = (4, 2, 6, 0, 0)$

$$A = \begin{pmatrix} 2 & 3 & 2 & -1 & 0 \\ 3 & 4 & 0 & 0 & 0 \\ 6 & -4 & 1 & 0 & 1 \end{pmatrix}, b \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}, x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{pmatrix}$$

Unit - II

Transportation problem:

Method 1 : North west corner rule

Method 2 : Least cost method (or) Matrix Minima method (or) Lowest cost entry method.

Method 3 : Vogels Approximation method (or) VAM method.

* The two set of constraints will be constant if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

(Total supply) (Total demand)

∴ problem satisfying condition are called Balanced Transportation problem ⇒ feasible solution.

* $\sum a_i \neq \sum b_j$ the transportation problem is unbalanced.

* A Basic feasible solution to a $(m \times n)$ transportation problem is said to be <u>non-degenerate Basic feasible solution</u>, if it contains exactly $m+n-1$

non-negative allocations.

* A Basic feasible solution that contains less than m+n-1 non-negative allocations is said to be degenerate Basic feasible solution.

problems:-

1. Determine Basic feasible solutions to the following transportation problem using North-west corner rule.

sink

	A	B	C	D	E	supply
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
Demand	3	3	4	5	6	

orgin

sol:-

Since $\Sigma a_i = \Sigma b_j$

$4+8+9 = 3+3+4+5+6$

$21 = 21$

∴ the given problem is Balanced.

∴ there exists a feasible solution to the transportation problem.

[3] 2	[1] 11	10	3	7	4 (1)
1	[2] 4	[4] 7	[2] 2	1	8 (6) (2)
3	9	4	[3] 8	[6] 12	9 (6)
3	3 (2)	4	5 (3)	6	

The initial Transportation cost $\Big\} = (3\times 2) + (1\times 11) + (2\times 4) + (4\times 7) + (2\times 2) + (3\times 8) + (6\times 12)$

$= 153$

$\therefore$ From this table allocations is equal to $m+n-1 = 7$.

$\therefore$ The solution is non-degenarate solution.

Find the initial Basic feasible solution for the following transportation problem by Least cost method.

					Supply
	2	1	4		30
	3	3	2	1	50
	4	2	5	9	20
Demand	20	40	30	10	

Scanned with CamScanner

Sol:

since $\sum a_i = \sum b_j$

$30+50+20 = 20+40+30+10$

$100 = 100$

∴ the given TPP is Balanced.

∴ There exists a feasible solution to the transportation problem.

$\boxed{20}$ 1	2	1	4	30 (10)
3	3	2	1	50
4	2	5	9	20
20	40	30	10	

2	$\boxed{10}$ 1	4	10
3	2	1	50
2	5	9	20
40	30 (20)	10	

3	2	$\boxed{10}$ 1	50 (40)
2	5	9	20
40	20	10	

3	(20) 2	40 (20)
2	1	20
40	20	

(20) 3	20
(20) 2	20
40 (20)	

∴ The transportation total cost

$= (20\times1) + (10\times1) + (10\times1) + (20\times2)$

$+ (20\times2) + (20\times3)$

$= 180$

∴ From this table allocations is equal to $m+n-1 = 6$

∴ The solution is non-degenarate solution.

3. To find LCM

	A	B	C	supply
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	

Sol:.

since $\Sigma a_i = \Sigma b_i$

$7+9+18 = 5+8+7+14$

$34 = 34$

∴ the given TPP is Balanced.

∴ The exists a feasible solution to the transportation problem.

2	7	4	5
3	3	1	8
5	4	7	7
1	6	2	14
7	9	18	

2	7	4	5
5	4	7	7
1 (11)	6	2	14 (4)
7	9	10	

7	4	5
4	7	7
6	2	7

9 10

7	[3] 4	5 (2)
4		

9 3

[2] 4	2
[7] 4	7
9	

∴ The transportation total cost

$= (8\times1) + (7\times1) + (7\times2) + (3\times4) + (7\times4)$

$+ (2\times7)$

$= 83.$

∴ From this table allocations

is equal to $m+n-1 = 6$.

∴ The solution is non-degenerate

solution.

4) To find LCM

50	30	220	1
90	45	170	3
250	200	50	4
4	2	2	

Since $\sum a_i = \sum b_j$

$4 + 2 + 2 = 1 + 3 + 4$

$8 = 8$

∴ The given TPP is Balanced.

∴ The exists a feasible solution to the transportation problem.

50	[1] 80	220	1
90	45	170	3
250	200	50	4
4	2	2	

(1)

90	[1] 45	170	3 (2)
250	200	50	4
4	~~1~~	2	

90	170	2
250	[2] 50	4 (2)
4	2	

[2] 90	120
[2] 250	2
4 (2)	

∴ The transportation total cost

$= (1\times30) + (1\times45) + (2\times50) + (90\times2) + (250\times2)$

$= 855.$

∴ From this table allocations is equal to $m+n-1 = 5$

∴ The solution is non-degeneral solution.

5. Find the initial basic feasible solution for the following transportation problem by VAM

	D_1	D_2	D_3	D_4	Availability
S_1	11	13	17	14	250
S_2	16	18	14	10	300
S_3	21	24	13	10	400
Requirement	200	225	275	250	

Sol:

$\Sigma a_i = \Sigma b_j$

$200 + 225 + 275 + 250 = 250 + 300 + 400$

$950 = 950$

∴ The given TPP is Balanced.

∴ There exists a feasible solution to the transportation problem.

$\boxed{200}$ 11	13	17	14	250 (50)	(2)
16	18	14	10	300	(4)
21	24	13	10	400	(3)
200	225	275	250		
(5)	(5)	1	0		

$\boxed{50}$ ~~13~~	~~17~~	~~14~~	50	(1)
18	14	10	300	(4)
24	13	10	400	(3)
225	275	250		
5	1	0		

$\boxed{175}$ 18	14	10	300 (125)	(4)
24	13	10	400	(3)
175	275	250		
6	1	0		

~~14~~	$\boxed{125}$ 10	125	(4)
13	10	400	(3)
275	250 (125)		
(1)	0		

$\boxed{275}$ 13	$\boxed{125}$ 10	400 (3) (125)
275	125	
(13)	10	

The transportation total cost

$= (200 \times 11) + (50 \times 13) + (175 \times 18) +$

$(125 \times 10) + (275 \times 13) + (125 \times 10)$

$= 12075$

$\therefore$ From this total allocations is equal to $m+n-1 = 6$

$\therefore$ the solution is non-degenarate solution.

5m 6. To Find i) NWCR (ii) LCM iii) VAM method.

iii) VAM method.

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

Sol:

since $\Sigma a_i = \Sigma b_i$

$10 + 10 + 10 = 7 + 12 + 11$

$30 = 30$

The given TPP is Balanced.

∴ There exists a feasible solution to the transportation problem.

1	2	6	7	1
0	14	$\boxed{10}$ 2	12 (2)	2
3	1	5	11	2
10	10	10		
(1)	(1)	3		

1	2	7	1
0	4	2	4
3	1	11	2
10	10	3	
(8)			

$\boxed{7}$ ~~1~~	~~2~~	~~7~~	1
3	1	11	2
8	10		
(1)			
2	1		

$\boxed{1}$ 3	$\boxed{10}$ 1	11	2
1	10		

is the transportation total cost

$= (10\times2) + (2\times0) + (7\times1) + (1\times3) + (0\times1)$

$= 40$

$\therefore$ From this total allocations is equal to $m+n-1 = 5$.

The solution is non-degenarate solution.

i) NWCR :-

Since $\Sigma a_i = \Sigma b_i$

$10+10+10 = 7+12+11$

$30 = 30$

$\boxed{7}$ 1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

$10-7=3$

$\boxed{3}$ 0	4	2	12	$12-3=9$
3	1	5	11	
3	10	10		

Scanned with CamScanner

$\boxed{9}$ 4	2	9
1	5	11
10	910	

$\boxed{1}$ 1	$\boxed{10}$ 5	11 (10)
1	10	

∴ The transportation total cost

$= (7\times1) + (3\times0) + (9\times4) + (1\times1) + (10\times5)$

$= 94$

∴ From this table allocation is equal to $m+n-1 = 5$

∴ The solution is non-degenerate solution.

ii) LCM:

1	2	6	7
$\boxed{10}$ 0	4	2	12 (2)
3	1	5	11
10	10	10	

2	6	7
4	2	2
$\boxed{10}$ 1	5	11 (1)
10	10	

6	7
$\boxed{2}$ 2	2
5	1

0 10 (8)

$\boxed{7}$ 6	7
$\boxed{1}$ 5	1

8 (7)

∴ The transportation total cost

$= (10\times0)+(10\times1)+(2\times2)+(1\times5)+(7\times6)$

$= 61$

∴ From this table allocation is equal to $m+n-1 = 6-1 = 5$

MODI METHOD :-

Solve the transportation problem

					Supply
	21	16	25	13	11
	17	18	14	23	13
	32	27	18	41	19
Demand	6	10	12	15	

<u>Sol</u> :-

Since $\Sigma a_i = \Sigma b_j$

$11+13+19 = 6+10+12+15$

$43 = 43$

∴ The given problem is balanced.

∴ There exists a feasible to the transportation problem:

21	16	25	$\boxed{11}$ 13	11	(3)
17	18	14	23	13	(3)
32	27	18	41	19	(9)
6	10	12	15		
			(4)		
(4)	(2)	(4)	(10)		

17	18	14	$\boxed{4}$ 23	13	(9)	13 − 4 = 9
32	27	18	41	19	(9)	
6	10	12	4			
(15)	(9)	(4)	(18)			

$\boxed{6}$ 17	18	14	9 (3)	(3)	
32	27	18	19	(9)	
6	10	12			
(15)	9	(4)			

$\boxed{3}$ 18	14	3	(4)
27	18	19	(9)
10 (7)	12		
(9)	(4)		

$\boxed{7}$ 27	$\boxed{12}$ 18	19 (12) (9)
7	12	

The transportation total cost

$= (11 \times 13) + (23 \times 4) + (17 \times 6) + (18 \times 3)$

$+ (27 \times 7) + (18 \times 12)$

$= 796.$

To find the optional solution.

Since $m+n-1 = 3+4-1$

$= 6$

We apply modi method.

21 7 14	16 8 8	25 -1 26	13 11	u_1
17 6	18 3	14 9 5	23 4	u_2
32 26 6	27 7	18 12	41 32 9	u_3
v_1	v_2	v_3	v_4	

$\because$ $u_2 + v_1 = 17$ $\quad$ $u_2 + v_2 = 18$ $\quad$ $u_2 + v_4 = 23$

$0 + v_1 = 17$ $\quad$ $v_2 = 18$ $\quad$ $v_4 = 23$

$u_1 + v_4 = 13$ $\quad$ $u_3 + v_2 = 27$ $\quad$ $u_3 + v_3 = 18$

$u_1 + 23 = 13$ $\quad$ $u_3 + 18 = 27$ $\quad$ $9 + v_3 = 18$

$u_3 = 9$ $\quad$ $v_3 = 9$

$u_1 = -10$

$d_{ij} \geq 0$

Total cost $= 796.$

Scanned with CamScanner

To find Modi method :-

7	3	2	2
2	1	3	3
3	4	6	5
4	1	5	

Sol :.

Since $\Sigma a_i = \Sigma b_j$

$5+3+2 = 4+1+5$

$10 = 10$

the given problem is Balanced

∴ there exists a feasible to the transportation .

7	3	2	2	(1)
2	1 [1]	3	3	(2) (1)
3	4	6	5	(1)
4	1	5		
(1)	(2)	(1)		

7 [2]	2 [2]	2	(5)
2	3	2	(1)
3	6	5 ~~3~~	(3)
4	5		
(1)	(3)		
	(1)		

2	[21] 3	2 (1)
3	6	5 (3) (3)
4	3	
	(1)	
(1)	(3)	

[4] 3	[1] 6	5 (4) (3)
4	1	
(3)	(6)	

the transportation total cost

$= (1 \times 1) + (2 \times 2) + (2 \times 3) + (1 \times 6) +$
(4×3)

$= 29$

To find the optional solution.

since $m + n - 1 = 6 - 1$
$= 5$

we apply modi method.

7 −1 8	3 0 3	2 2	$u_1 = 2$
2 0 2	1 1	3 2	$u_2 = 3$
3 4 4	4 4	9	$u_3 = 6$
$v_1 = 3$	$v_2 = -2$	$v_3 = 0$	

$u_3 + v_3 = 6$ $\quad u_3 + v_1 = 3$ $\quad u_2 + v_3 = 3$

$u_3 + 0 = 6$ $\quad 6 + v_1 = 3$ $\quad u_2 + 0 = 3$

$u_3 = 6$ $\quad v_1 = -3$ $\quad u_2 = 3$

$u_1 + v_3 = 2$ $\quad u_2 + v_2 = 1$

$u_1 + 0 = 2$ $\quad 3 + v_2 = 1$

$u_1 = 2$ $\quad v_2 = -2$

$\therefore d_{ij} \geq 0$

$\therefore$ The optimum solution is $= 29$.

---

1) Find the optimal transportation cost of the following matrix using least cost method for finding critical solutions.

	A	B	C	D	E	
P	4	1	2	6	9	100
Q	6	4	3	5	7	120
R	5	2	6	4	8	120
	40	50	70	90	90	

Sol:

since $\Sigma a_i = \Sigma b_j$

$40 + 50 + 70 + 90 + 90 = 100 + 120 + 120$

$340 = 340$

The given problem is Balanced.

There exists a feasible to the transportation.

4	$\overset{50}{1}$	2	6	9	100 (50)
6	4	3	5	7	120
5	2	6	4	8	120
40	50	70	90	90	

$\overset{50}{4}$	2	6	9	50
6	3	5	7	120
5	6	4	8	120
40	70 (20)	90	90	

6	$\overset{20}{3}$	5	7	120 100
5	6	4	8	120
40	20	90	90	

6	5	7	100
5	$\overset{90}{4}$	8	120 (30)
40	90	90	

6	7	100
~~5~~ 30	~~2~~	30
40 (10)	90	

10 6	90 7	100 (90)
10	90	

The transportation total cost

$= (50 \times 1) + (50 \times 2) + (20 \times 3) + (90 \times 4)$

$+ (30 \times 5) + (10 \times 6) + (90 \times 7)$

$= 1410.$

To find the optional solution.

since $m+n-1 = 7-1 = 6$

we apply modi method,

4 5 -1	1 (50)	2 (50)	6 4 2	9 6 3 2	$u_1 = -1$
6 (10)	4 2 2	3 (20)	5 5 0	7 (90)	$u_2 = 0$
5 (30)	2 1 1	6 2 4	4 (90)	8 6 2	$u_3 = -1$
$v_1 = 6$	$v_2 = 2$	$v_3 = 3$	$v_4 = 5$	$v_5 = 7$	

$v_5 + u_2 = 7$ $\qquad$ $v_1 + u_2 = 6$

$v_5 + 0 = 7$ $\qquad$ $v_1 + 0 = 6$

$v_5 = 7$ $\qquad$ $v_1 = 6$

$V_3 + u_2 = 3$

$V_3 + 0 = 3$

$V_3 = 3$

$V_1 + u_3 = 5$

$6 + u_3 = 5$

$u_3 = 5 - 6 = -1$

$V_4 + u_3 = 4$

$V_4 - 1 = 4$

$V_4 = 4 + 1$

$V_4 = 5$

$V_1 + u_3 = 5$

$V_1 + (-1) = 5$

$V_1 = 5 + 1$

$V_1 = 6$

$V_3 + u_1 = 2$

$3 + u_1 = 2$

$u_1 = 2 - 3$

$u_1 = -1$

$V_2 + u_1 = 1$

$V_2 + (-1) = 1$

$V_2 = 1 + 1$

$V_2 = 2$

$\theta = \min\{10, 50\} = 10$

4 5 (10)	1 (50)	2 (40)	6 3 3	9 6 3	$u_1 = 0$
6 5 1	4 2 2	3 (30)	5 4 1	7 (90)	$u_2 = 1$
5 (30)	2 2 0	6 3 3	4 (90)	8 7 1	u_3
$V_1 = 4$	$V_2 = 1$	$V_3 = 2$	$V_4 = 3$	$V_5 = 6$	

$v_1 + u_1 = 4$ $\quad v_2 + u_1 = 1$ $\quad v_3 + u_1 = 2$

$v_1 + 0 = 4$ $\quad v_2 + 0 = 1$ $\quad v_3 + 0 = 2$

$v_1 = 4$ $\quad v_2 = 1$ $\quad v_3 = 2$

$v_3 + u_2 = 3$ $\quad v_5 + u_2 = 7$

$2 + u_2 = 3$ $\quad v_5 + 1 = 7$

$u_2 = 3 - 2$ $\quad v_5 = 7 - 1$

$u_2 = 1$ $\quad v_5 = 6$

$v_1 + u_3 = 5$ $\quad v_4 + u_3 = 4$

$4 + u_3 = 5$ $\quad v_4 + 1 = 4$

$u_3 = 5 - 4$ $\quad v_4 = 4 - 1$

$u_3 = 1$ $\quad v_4 = 3$

$v_5 + u_2 = 7$

$v_5 + 1 = 7$

$v_5 = 7 - 1$

$v_5 = 6$

The total cost $= (4 \times 10) + (1 \times 50) + (2 \times 40) + (3 \times 30) + (7 \times 90) + (5 \times 30) + (4 \times 90) = 1400$

Assignment problem:

The assignment problem is a particular case of the transportation problem to equal no of columns & rows.

An assignment problem is always a degenerate from of a transportation problem.

Difference b/w the transportation problem and the Assignment problem.

Transportation problem	Assignment problem
a) suppose at any source may be any positive quantity a_i	a) supply at any source (machine) will be 1. $[a_i = 1]$
b) Demand at any destination may be any +ve quantity b_j	b) Demand at any destination (job) will be 1.
c) one or more source to any no of destinations.	one source to only one destination.

problem:-

1. Consider the problem of assigning five
15. jobs to five persons. The assignment
costs are given as follows.

$$\begin{array}{cc|c} & & \text{Job} \\ & & \begin{array}{ccccc} \; & 2 & 3 & 4 & 5 \end{array} \\ \text{person} & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix} \end{array}$$

Sol:

The cost of the given problem is

$$\begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix}$$

Row and column are equal.

Step : 1

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Step 2:

$$\begin{pmatrix} 7 & 3 & \cancel{0} & 5 & \boxed{0} \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & \cancel{0} & 3 \\ 4 & \boxed{0} & 2 & 4 & \cancel{0} \end{pmatrix}$$

$\therefore$ The optimum solution is

$$= (1+0+0+0+0+0+3+0+5+0)$$

$$= 9.$$

The Assignment cost of assigning any one operator to any one Mechnics is given by

$$\begin{pmatrix} 10 & 5 & 13 & 15 \\ 3 & 9 & 18 & 3 \\ 10 & 7 & 3 & 2 \\ 5 & 11 & 9 & 7 \end{pmatrix}$$

Sol:

The cost of the given problem is

$$\begin{pmatrix} 10 & 5 & 13 & 15 \\ 3 & 9 & 18 & 3 \\ 10 & 7 & 3 & 2 \\ 5 & 11 & 9 & 7 \end{pmatrix}$$

Scanned with CamScanner

Row and column are equal.

Step 1 :

$$\begin{bmatrix} 5 & 0 & 8 & 10 \\ 0 & 6 & 15 & 0 \\ 8 & 5 & 1 & 0 \\ 0 & 6 & 4 & 2 \end{bmatrix}$$

Step 2 :

$$\begin{bmatrix} 5 & \boxed{0} & 7 & 10 \\ \cancel{0} & 6 & 14 & \boxed{0} \\ 8 & 5 & \boxed{0} & \cancel{0} \\ \boxed{0} & 6 & 3 & 2 \end{bmatrix}$$

The optimum solution is

$= \cancel{[0+0+0+0+1+0}$

$= [5+0+0+3+9+0+5+0]$

$= 16.$

10m

3. The processing time in hours for the jobs when allocated to the different machines are indicated below. Assign the machines for the jobs so that the total processing time is minimum.

$$\begin{array}{c|ccccc} & M_1 & M_2 & M_3 & M_4 & M_5 \\ \hline J_1 & 9 & 22 & 58 & 11 & 19 \\ J_2 & 43 & 78 & 72 & 50 & 63 \\ J_3 & 41 & 28 & 91 & 37 & 45 \\ J_4 & 74 & 42 & 27 & 49 & 39 \\ J_5 & 36 & 11 & 57 & 22 & 25 \end{array}$$

Sol:.

The cost of the given problem wise

$$\begin{pmatrix} 9 & 22 & 58 & 11 & 19 \\ 43 & 78 & 72 & 50 & 63 \\ 41 & 28 & 91 & 37 & 45 \\ 74 & 42 & 27 & 49 & 39 \\ 36 & 11 & 57 & 22 & 25 \end{pmatrix}$$

Row and column are equal.

Step 1:.

$$\begin{pmatrix} 0 & 13 & 49 & 2 & 10 \\ 0 & 35 & 29 & 7 & 20 \\ 13 & 0 & 63 & 9 & 17 \\ 47 & 15 & 0 & 22 & 12 \\ 25 & 0 & 46 & 11 & 14 \end{pmatrix}$$

Step 2 :-

$$\begin{pmatrix} \cancel{0} & 13 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 35 & 29 & 5 & 10 \\ 13 & \boxed{0} & 63 & 7 & 7 \\ 47 & 15 & \boxed{0} & 20 & 2 \\ 25 & \cancel{0} & 46 & 9 & 4 \end{pmatrix}$$

$$\begin{bmatrix} \cancel{0} & 17 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 39 & 29 & 5 & 10 \\ 9 & \boxed{0} & 59 & 3 & 3 \\ 47 & 19 & \boxed{0} & 20 & 2 \\ 21 & \cancel{0} & 42 & 5 & \boxed{0} \end{bmatrix}$$

The optimum solution is

$= (11 + 43 + 28 + 27 + 25)$

$= 134.$

4. $$\begin{bmatrix} 5 & 7 & 11 & 6 \\ 8 & 5 & 9 & 6 \\ 4 & 7 & 10 & 7 \\ 10 & 4 & 8 & 3 \end{bmatrix}$$

Soln:-

The cost of the given problem is,

$$\begin{bmatrix} 5 & 7 & [illegible] & 6 \\ 8 & 5 & 9 & 6 \\ 4 & 7 & 10 & 7 \\ 10 & 4 & 8 & 3 \end{bmatrix} =$$

Row and column are equal:

Step:

$$\begin{bmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{bmatrix}$$

Step : 2

$$\begin{bmatrix} \boxed{0} & 2 & 2 & 1 \\ 3 & \boxed{0} & \cancel{0} & 1 \\ \cancel{0} & 3 & 2 & 3 \\ 7 & 1 & 1 & \boxed{0} \end{bmatrix}$$

$$\begin{bmatrix} \cancel{0} & 1 & 1 & \boxed{0} \\ 4 & \boxed{0} & \cancel{0} & 1 \\ \boxed{0} & 2 & 1 & 2 \\ 8 & 1 & 1 & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 5 & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & 1 & \boxed{0} & 2 \\ 8 & \cancel{0} & \cancel{0} & \boxed{0} \end{bmatrix}$$

The optimum solution is

$= 5 + 5 + 10 + 3$

$= 23.$

Unbalanced Assignment Models:

If the no. of rows is not equal to the no. of columns in the cost matrix of the given assignment problem, then the given assignment problem is said to be unbalanced.

The convert the unbalanced assignment problem is to a balanced one by dummy rows (or) columns with zero cost elements.

Problem:-

19. A company has four machines to do three of jobs. Each jobs can assigned to one and only one machine. The cost of each job on each is given in the following.

18	24	28	32
8	13	17	19
10	15	19	22

Scanned with CamScanner

Sol :

∴ The given problem is unbalanced

∵ Because the rows and columns are not equal.

To convert Balanced

$$\begin{array}{c} 18 \\ 8 \\ 10 \\ 0 \end{array}\begin{bmatrix} 24 & 28 & 32 \\ 13 & 17 & 19 \\ 15 & 19 & 22 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 1 :

$$\begin{bmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2 :

$$\begin{matrix} \boxed{0} & 6 & 10 & 14 \\ 0 & \boxed{5} & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & \boxed{0} & 0 & 0 \end{matrix}$$

$$\begin{bmatrix} \boxed{0} & \cancel{0} & 5 & 9 \\ \cancel{0} & \boxed{0} & 4 & 6 \\ \cancel{0} & \cancel{0} & 4 & 7 \\ 5 & \cancel{0} & \boxed{0} & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} \boxed{0} & 1 & 1 & 5 \\ \cancel{0} & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & \boxed{0} & 3 \\ 9 & 4 & \cancel{0} & \boxed{0} \end{bmatrix}$$

The optimum solution is

$= 18 + 13 + 19 + 0$

$= 50$

2. Assign four trucks 1, 2, 3 and 4 to vacant spaces, A, B, C, D, E and F so that the distance travelled is minimized. The matrix below show the distance.

20

4	1	3	7
8	2	5	5
4	9	6	9
7	5	4	8
6	3	5	4
6	8	7	3

Sol :-

$$\begin{bmatrix} 4 & 7 & 3 & 7 & 0 & 0 \\ 8 & 2 & 5 & 5 & 0 & 0 \\ 4 & 9 & 6 & 9 & 0 & 0 \\ 7 & 5 & 4 & 8 & 0 & 0 \\ 6 & 3 & 5 & 4 & 0 & 0 \\ 6 & 8 & 7 & 3 & 0 & 0 \end{bmatrix}$$

Step 1 :-

$$\begin{bmatrix} 4 & 7 & 3 & 7 & 0 & 0 \\ 8 & 2 & 5 & 5 & 0 & 0 \\ 4 & 9 & 6 & 9 & 0 & 0 \\ 7 & 5 & 4 & 8 & 0 & 0 \\ 6 & 3 & 5 & 4 & 0 & 0 \\ 6 & 8 & 7 & 3 & 0 & 0 \end{bmatrix}$$

Step 2 :-

$$\begin{bmatrix} \cancel{0} & 5 & \boxed{0} & 4 & \cancel{0} & \cancel{0} \\ 4 & \boxed{0} & 2 & 2 & \cancel{0} & \cancel{0} \\ \boxed{0} & 7 & 3 & 6 & \cancel{0} & \cancel{0} \\ 3 & 3 & 1 & 5 & \cancel{0} & \boxed{0} \\ 2 & 1 & 2 & 1 & \boxed{0} & \cancel{0} \\ 2 & 6 & 4 & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

Unit IV & Unit - V

PERT AND CPM

Introduction:

A project is defined as a combillation of interrelated activities all of which must be executed on a certain order to achieve a set goal.

Programme evaluation review technique (PERT) and critial path method (CPM) are two of the many network techniques which are widely used for planning, scheduling and the controlling large complex projects.

The three main Managerial function for any project are:

i) planning

ii) scheduling

iii) control

planning :-

The phase involves a visiting of tasks or jobs that must be performed complete a project under considerable.

scheduling :-

This phase involving the laying out of the actual activities of the project in a magical sequence of time in which they have to be performed.

Men and material requirements as well as the expected completion time of each activity at each stage of the projects are determined.

Control :-

This phase consists of remaining the progress of the project whether the actual performance is according to planned schedule and finding the reason for difference if any b/w the schedule and performance.

Basic termnologis

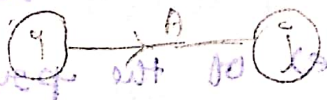
Activity : task or an items of work to be done is a project. An

Scanned with CamScanner

activity continuous resources like time, labours etc.

An activity is represented by an arrow with node (event) of the beginning & a node at the end, indicating the start and termination (finish) of the activity. Nodes are denoted by circles. Since this is a logical diagram length or shape of the arrow has no meaning, the direction indicates the progress of the activity.

For example if A is the activity where initial node is i and the terminal node is j, then it is denoted diagramatically by

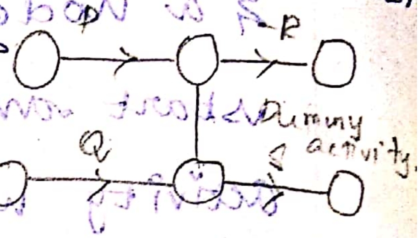


If an activity B can strict immediately order an activity A, then it is denoted by

A is called the immediate predecessor of B and B is called the immediate successor of A

Notation:

A is a producessor of B is denoted as $A < B$, B is a successor of A is denoted by $B > A$. [Let P and Q be the producessor of R and Q be the only Producessor



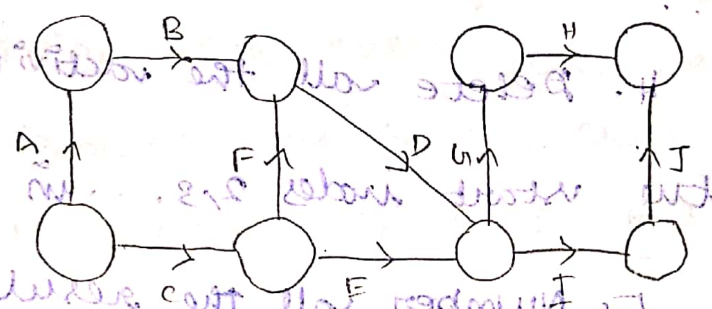
If the project contains two or more activities which have same of their immediate prodecessor is common there is a used for introducing that called dummy activity.

Activities which have no predecessor are called start activities can be made to have the same initial node. Activities which have no successors are called terminal activities of the project, these can be made to have the same terminal node (end node) of the project.

The diagram denoting all the activities of a project by arrows taking into account the technogical sequence of the activities is called the project nature represented by activity an arrow diagram or simply arrow diagram.

Rules for constructing a project network:-

1. There must be no loops. For example, the activities F, D, E obviously from a loop which is obviously not possible in any real project network.



only one activity should cannot any two nodes.

No dangling should appear in a project network. ie) no node of any activity except the terminal node of the project should be left without any activity emanating from it. such a node can be jointed to the terminal node of the project to avoid.

Nodes may be numbered using the rule given below:
(Ford and Fulkerson's Rule)

1. Number the start node which has no predecessor activity, as 1.

2. Delete all the activities emanating from this node 1.

3. Number all the resulting start nodes without any predecessor as 2, 3 . . . . .

4. Delete all the activities originating from the start nodes 2, 3 . . . in step 3.

5. Number all the resulting new start nodes without any predecessor next to the last number used in step 3.

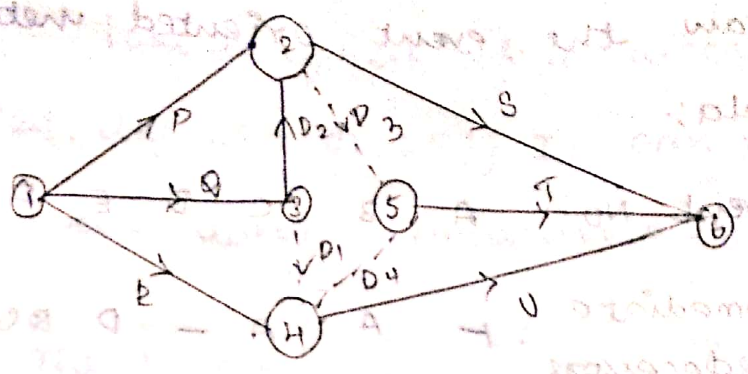
6. Repeat the process until the terminal node without any successor activity is reached and number this terminal node suitably.

1, Draw the network for the project whose activities and their precedence relationships are given below.

Activity:	P	Q	R	S	T	U
predecessor:	—	—	—	P, Q	P, R	Q, R

P, Q, R simultaneously can start

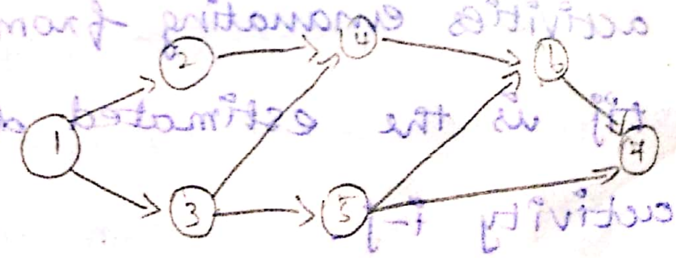
S > P; Q, T > P, R & U > Q, R.



Draw the network for the project whose activities with their predecessor relationships are given below A, C, D can start ; E > B, C ; F, G > D ; H, I > E, F ; J > I, G : K > H : B > A.

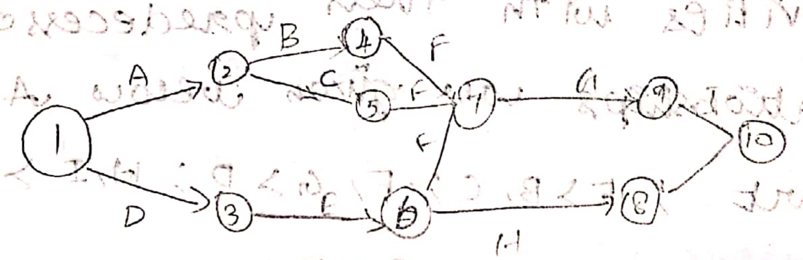
Draw the event oriented network for the following data:

Event No :	1	2	3	4	5	6	7
Immediate predecessors :	–	1	1	2, 3	3	4, 5	5, 6



4. Draw the event oriented network following data:

Event No :	A	B	C	D	E	F	G	H	I
Immediate Predecessors :	-	A	A	-	D	B,C,E	F	E	G,H



Critical path Method (CPM) :-

Earliest start of an activity i-j can be denoted as ES_i (or) ES_{ij}

It can also be called the earliest occurance of the event.

Earliest start of an activity (i-j) in a project network is given by,

$ES_i = \max\{ES_{ij} + E_{ij}\}$ where ES_i denoted the earliest start time of all the activities emanating from the node i and t_{ij} is the estimated duration of the activity i-j.

Latest finish of an activity can be denoted by LF_i or LF_{ij} It can also be called the latest occurance of the event j.

The Latest start time of all the activities emanating from the event i of the activity i-j. $LS = \min[LS_j - t_{ij}]$ for all defined i-j activities where. t_{ij} is the estimated duration of the activity i-j.

Critical path :

path connecting the first initial node to the very last terminal node of largest duration in any project network is called the critical path.

Critical path plays a very important role in project scheduling problems.

Floats :

Total floats of an activity (T.F) is as the difference b/w the latest finish and the finish of the activity (or) the difference b/w the latest start and the earliest start of the activity.

Total float of an activity $i-j = (LF)_{ij} - (EF)_{ij}$

$= (LS)_{ij} - (ES)_{ij}$

Note :

(L-E) of an event of $i-j$ is called the slack of the event j.

Free Float of an activity (F.F) is that portion of the total float which can be used for rescheduling that activity without affecting the succeeding activity.

Free float of an activity $i-j$ = Total float of $i-j$ – (L–E) of the event j

= Total float of $i-j$ – Slack of node event j

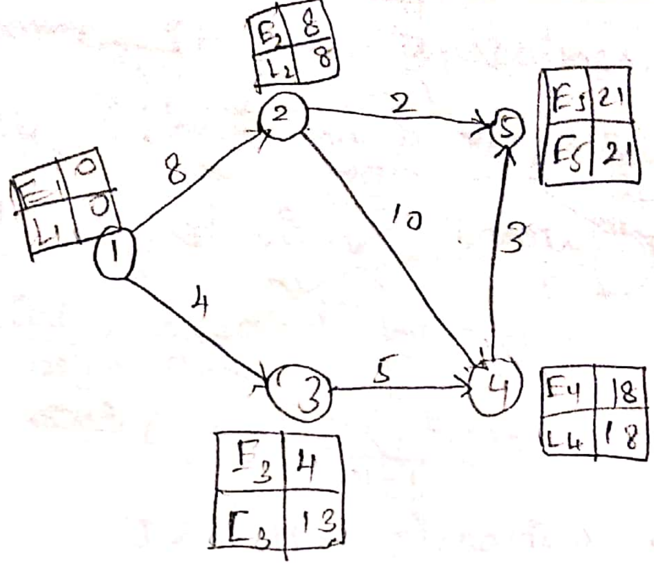
= Total float $I-J$ – Slack of node event j

where L = Latest occurance

E = Earliest occurance.

1) Compute the earliest start, earliest finish, latest start and latest finish of each activities of the project given below.

Activity :	1-2	2-3	2-4	2-5	3-4	4-5
Duration	8	4	10	2	5	3

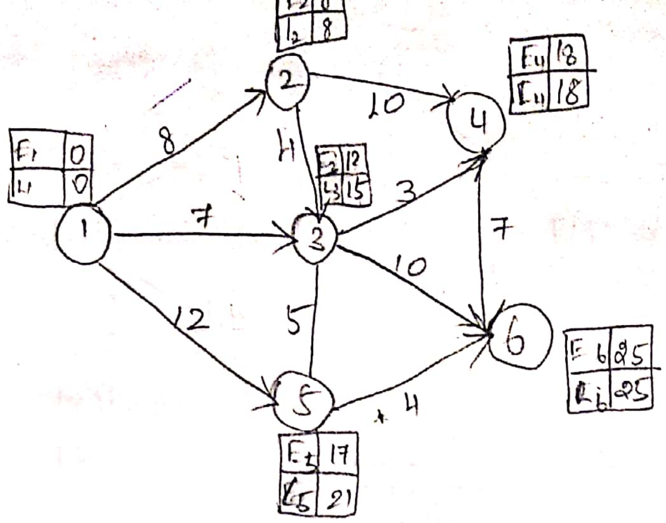


		Earliest		Latest	
Activity	Duration	Start ES	Finish EF $EF = ES + \text{duration}$	Start $LS = LF - \text{duration}$	Finish LF
1 - 2	8	0	8	0	8
1 - 3	4	0	4	9	13
2 - 4	10	8	18	8	18
2 - 5	2	8	10	19	21
3 - 4	5	4	9	13	18
4 - 5	3	18	21	18	21

2) Calculate the ES, EF, LS, LF of each activity of the project given below and determine the critical path of the project. 5m

Activity :	1-2	1-3	1-5	2-3	2-4	3-4
Duration :	8	7	12	4	10	3

3-5	3-6	4-6	5-6
5	10	7	4.



E3 = 18 - 3 = 15
8 - 4 = 4
25 - 10 = 15
17 - 5 = 12
0 - 7 = 7

		Earliest		Latest	
Activity	Duration	Start ES	Finish EF $EF = ES + t_{ij}$	Start $LS = LF - t_{ij}$	Finish LF
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

3. Calulate the total Float, Free float, Independent float for the project whose activities are given below:

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5
	8	7	12	4	10	3	5

3-6	4-6	5-6
10	7	4

and some time machines are working property money is a variable when ever headed there is no scarcity of raw material & headed etc....

pessimistic (greatest) time (tp or b)

Is the duration of any activity when almost every thing goes against our will and a lot of difficulties is forced while doing a project

Most likely time estimate (tm or m)

Is the duration of any activity when sometimes things go on very bad while doing the project.

To main assumptions made in PERT calculation are

i) The activity duration are independent ie) the time required to complete an activity will have no bearing on the completion times of any other activity of the project.

ii) The activity follow β-distribution. β distribution is a probability distribution with density function $k(t-a)^{\alpha}(b-t)^{\beta}$ with mean $t_3 = 1/3$ $[2t_m + 1/2(t_o - t_p)]$ and the standard derivation $\sigma_t = \frac{t_p - t_o}{6}$.

PERT Procedure :

1) Draw the project network.

2) Compute the expected duration each activity $t_e = \frac{t_o + 4t_m + t_p}{6}$

3) Compute the expected variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)$ of each activity

4) Compute the earliest start, earliest finish, latest start, latest finish and float of each activity.

5) Determine the critical path and identify critical activities

6) Compute the expected variance of the project length (also called the variance of the critical path, σ_c^2 which is the sum of the variance of all the critical activities.

7) compute the expected standard deviation of the project length σ_c and calculate the standard normal deviate $\frac{T_S - T_E}{\sigma_c}$ where,

T_S = specified or scheduled time to complete the project.

T_E = Normal expected project duration

σ_c = Expected standard deviation of the project length.

8) using (7) one can estimate the probability of completing the project within a specified line, using the normal curve (Area) tables.

Note :-

(2) (3) are valid because of assumption (ii). (6) is valid because of assumption (i).

PERT	CPM
1) PERT was developed in a brand R and D project it had to consider and deal with the uncertainties associated	CPM was developed for conventional projects which consists of well known routine tasks whose resource requirement and

with which projects. Thus the project duration is regarded as a random variable and therefore probabilities are calculate so as to characterise it.	duration were unknow with certainty.
	2) CPM is suited to establish a trade off for optimum balancing b/w schedule tim and cost of the project.
2) Emphasis is given to important of task rather than the activities required to be performed to reach a particular event or task in the analysis of network. i.e) PERT network is essentially an event-oriented network.	3) CPM is used for projects involving well known activities of repetitive in nature.
3) PERT is usually used for projects in which time estimate are uncertain Ex. R and D activities which are usually non-respetitive.	
4) PERT helps in identifying critical areas in a project so that suitable necessary adjustments may be made to meet the scheduled completion data of the project.	

Construct the network for the project whose activities and the three time estimate of these activities (in weeks) are given below. Complete.

a) Expected duration of each activity.

b) " Variance of each activity.

c) " Variance of the project length (or) critical length

Activity	t_o	t_m	t_p
1 – 2	3	4	5
2 – 3	1	2	3
2 – 4	2	3	4
3 – 5	3	4	5
4 – 5	1	3	5
4 – 6	3	5	7
5 – 7	4	5	6
6 – 7	6	7	8
7 – 8	2	4	6
7 – 9	1	2	3
8 – 10	4	6	8
9 – 10	3	5	7

Sol:-