UNIT-I

2/may/2019

Canonical form

Max

$\leq$

$x_i \geq 0$ ⇒ non -ve restriction

Standard form

Max z

=

$b_1, b_2 \ldots$ +ve   right side (+)ve

$x_i \geq 0$

$\leq$ Con

$x_i \geq 0$

1) Max $z = 4x_1 + 3x_2$

$x_1 + 4x_2 \leq 2$ less than or = means add one variable s_1

$x_2 + 3x_1 = 1$

$x_1 - 3x_2 \geq 1$ greater than or = means less one variable $-S_2$

Soln:

Max $z = 4x_1 + 3x_2$

$x_1 + 4x_2 + S_1 = 2$ → slack

$x_2 + 3x_1 = 1$

$x_1 - 3x_2 - S_2 = 1$

→ surplus.

Scanned by CamScanner
Scanned by CamScanner

simplex method

Max $z = 4x_1 + 10x_2$

$2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$ & $x_1, x_2 \ge 0$

S_1, S_2, S_3 are slack variable

Max $z = 4x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3$

subject to constraints

$2x_1 + x_2 + S_1 = 50$

$2x_1 + 5x_2 + S_2 = 100$

$2x_1 + 3x_2 + S_3 = 90$

CB → cost for the basis

YB → variable

		C_j	4	10	0	0	0
CB	YB	XB	x_1	x_2	S_1	S_2	S_3
0	S_1	50	2	1	1	0	0
0	S_2	100	2	5	0	1	0
0	S_3	90	2	3	0	0	1
	z_j	0	0	0	0	0	0
$z_j - c_j$		0	-4	-10	0	0	0

To find z_j Multiply CB × XB + next next Value.

Select most value

$$\text{Min}\left(\frac{X_{Bi}}{a_{ij}}\right)$$

$\frac{50}{1} = 50$

$\frac{100}{5} = 20$ *

$\frac{90}{3} = 30$

Find minimum value

20 is min

5 is pivotal element

To make pivotal element as 1 and others as 0.

x_2 enters the basis

S_2 leaves the basis

FORMULA

old eqn − (column coefficient) × New pivot eqn.

CB	YB	XB	X_1	X_2	S_1	S_2	S_3
	C_j		4	10	0	0	0
0	S_1	30	8/5	0	1	−1/5	0
10	x_2	20	2/5	1	0	1/5	0
0	S_3	30	4/5	0	0	−3/5	1

$Z_j = CB \times X_j + CB \times C_j$

Z_i	200	4	10	0	2	0

Scanned by CamScanner
Scanned by CamScanner

$z_j - c_j$ 200 0 0 0 2 0

All $z_j - c_j \geq 0$

soln is optimal

Max $z = 200$

$x_1 = 0$ $x_2 = 20$

90	2	3	0	0	1
60	6/5	3	0	3/5	0
30	4/5	0	0	-3/5	1

Q) Max $z = 3x_1 + 2x_2 + 5x_3$

Sub to contraint

$x_1 + 4x_2 \leq 420$

$3x_1 + 2x_3 \leq 460$

$x_1 + 2x_2 + x_3 \leq 430$

Max $z = 3x_1 + 2x_2 + 5x_2 + 0S_1 + 0S_2 + 0S_3$

Scanned by CamScanner
Scanned by CamScanner

$x_1 + 4x_2 + S_1 = 420$

$3x_1 + 2x_3 + S_2 = 460$

$x_1 + 2x_2 + x_3 + S_3 = 430$

	C_j		3	2	5	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	420	1	4	0	1	0	0
0	S_2	460	3	0	(2)	0	1	0
0	S_3	430	1	2	1	0	0	1
	Z_j	0	0	0	0	0	0	0
	$Z_j - C_j$	0	-3	-2	-5	0	0	0

$C_j = \quad 3 \quad 2 \quad 5$

C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	420	1	4	0	1	0	0
5	x_3	230	3/2	0	1	0	1/2	0
0	S_3	200	-1/2	(2)	0	0	-1/2	1
	Z_j	1150	15/2	0 ↑	5	0	5/2	0
	$Z_j - C_j$	1150	9/2	-2	0	0	5/2	0

Scanned by CamScanner
Scanned by CamScanner

θ Min $\frac{x_{Bi}}{a_{ij}}$

$\frac{-1/2}{2/1}$ $-1/2 \times \frac{2}{1}$ $-1/2 \times 1/2 = -1/4$

$460/2 = 230$

$430/1 = 430$

x_2 enters S_3 leaves the basis

θ $420/4 = 105$

$200/2 = 100$

		C_j	3	2	5	0	0	0	
CB	YB	XB	x_1	x_2	x_3	S_1	S_2	S_3	θ
0	S_1	20	2	0	0	1	1	-2	
5	x_3	230	3/2	0	1	0	1/2	0	
2	x_2	100	-1/4	1	0	0	-1/4	1/2	

Z_j 1350 7 2 5 0 2 1

$Z_j - C_j$ 1350 4 0 0 0 2 1

O.E 420 1 4 0 1 0 0

N.E 400 -1 4 0 0 -1 2

20 2 0 0 1 1 -2.

All $z_j - c_j \geq 0$ The soln is optimal

$$\text{Max } z = 1350$$

$$x_1 = 0$$

$$x_2 = 100$$

$$x_3 = 230$$

---

③ solve the following lpp by simplex method.

$$\text{minimize} \quad z_1 = 8x_1 - 2x_2$$

$$-4x_1 + 2x_2 \leq 1$$

$$5x_1 - 4x_2 \leq 3 \quad \& \quad x_1, x_2 \geq 0$$

---

convert to max

$$\text{Min } z = -\text{Max}(-z) = -\text{max } z^*$$

$$\text{Max } z^* = -8x_1 + 2x_2$$

sub to constraint

$$-4x_1 + 2x_2 \leq 1$$

$$5x_1 - 4x_2 \leq 3$$

Scanned by CamScanner
Scanned by CamScanner

		c_j	-8	2	0	0	
CB	YB	XB	x_1	x_2	S_1	S_2	θ
0	S_1	1	-4	2	1	0	1/2
0	S_2	3	5	-4	0	1	—

z_j 0 0 0 0 0

$z_j - c_j$ 0 8 -2 0 0

c_j -8 2 0 0

			-8	2	0	0
CB	YB	XB	X_1	X_2	S_1	S_2
2	x_2	1/2	-2	1	1/2	0
0	S_2	5	-3	0	2	1
z_j		1	-4	2	1	0
$z_j - c_j$		1	4	0	1	0.

Max $z = 1$

$x_1 = 0 \quad x_2 = 1/2$

Min $z = -1$

$x_1 = 0 \quad x_2 = 1/2$

Scanned by CamScanner
Scanned by CamScanner

OE	3	5	-4	0	1
(4)NPE	2	-8	4	2	0
add	5	-3	0	2	1

(4) 10m.

(4) 5m

Primal - dual problem

	obj	constraints
1)	Max z	$\leq$
2)	Min z	$\geq$

Max means Min

Obje to constrant

$\leq$ to $\geq$

row to column

Column to row.

(1)

Scanned by CamScanner
Scanned by CamScanner

(1)

Max $F = x_1 + 2x_2 + x_3$

$2x_1 + x_2 - x_3 \leq 2$

$-2x_1 + x_2 - 5x_3 \geq -6$

$4x_1 + x_2 + x_3 \leq 6$, $x_1, x_2, x_3 \geq 0$

Max means $\leq$ but (2) contains greater than so change $\not\geq$ by -

Max $F = x_1 + 2x_2 + x_3$

$2x_1 + x_2 - x_3 \leq 2$

$2x_1 - x_2 + 5x_3 \leq 6$

$4x_1 + x_2 + x_2 \leq 6$

Min $W = 2y_1 + 6y_2 + 6y_3$

$2y_1 + 2y_2 + 4y_3 \geq 1$

$y_1 - y_2 + y_3 \geq 2$

$-y_1 + 5y_2 + y_3 \geq 1$

Scanned by CamScanner
Scanned by CamScanner

2) Max $z = 3x_1 - x_2 + x_3$

$4x_1 - x_2 \le 8$

Sub to $8x_1 + x_2 + x_3 \ge 12$

co.

$5x_1 - 6x_3 \le 13$

Max $z = 3x_1 - x_2 + x_3$

$4x_1 - x_2 + 0x_3 \le 8$

$-8x_1 - x_2 - x_3 \le -12$

$5x_1 + 0x_2 - 6x_3 \le 13$

Min $W = 8y_1 - 12y_2 + 13y_3$

Sub to constr

$4y_1 - 8y_2 + 5y_3 \ge 3$

$-y_1 - y_2 \ge -1$

$-y_2 - 6y_3 \ge 1$

Scanned by CamScanner
Scanned by CamScanner

DUAL SIMPLEX METHOD.

(1) Min $z = 2x_1 + x_2$

Sub to $\begin{cases} 3x_1 + x_2 \geq 3 \\ 4x_1 + 3x_2 \geq 6 \\ x_1 + 2x_2 \geq 3 \end{cases}$

Convert to max.

Min $z = -\text{Max}(-z)$

Max $z = -2x_1 - x_2$

$-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq -6$

$-x_1 - 2x_2 \leq -3$

		c_j	-2	-1	0	0	0
CB	YB	XB	x_1	x_2	S_1	S_2	S_3
0	S_1	-3	-3	-1	1	0	0
0	S_2	-6	-4	-3	0	1	0
0	S_3	-3	-1	-2	0	0	1
	z_j	0	0	0	0	0	0
	$c_j - z_j$	0	2	1	0	0	0

x_2 enters the basis
S_2 leaves the basis

$\max \left\{ \frac{z_j - c_j}{\alpha_{ik}}, \alpha_{ik} < 0 \right\}$

$\max \left\{ \frac{2}{-4}, \frac{1}{-3} \right\}$

$= \max \left\{ \frac{-1}{2}, \frac{-1}{3} \right\}$

$-0.5, -0.33$

~~max~~ value
max

	C_j	-2	-1	0	0	0
CB	YB	XB	x_1	x_2	S_1	S_2	S_3
0	S_1	-1	-5/3	0	1	-1/3	0
-1	x_2	2	4/3	1	0	-1/3	0
0	S_3	1	5/3	0	0	-2/3	1
	z_j	-2	-4/3	-1	0	1/3	0
	$z_j - c_j$	-2	2/3	0	0	1/3	0

$\max \left\{ \frac{z_j - c_j}{\alpha_{ik}} \right\} = \max \left\{ \frac{2/3}{-5/3}, \frac{1/3}{-1/3} \right\}$

$= \left\{ -\frac{2}{5}, -1 \right\}$

max value

Max value.

Scanned by CamScanner
Scanned by CamScanner

C_B	Y_B	c_j X_B	-2 x_1	-1 x_2	0 S_1	0 S_2	0 S_3
-2	x_1	$3/5$	1	0	$-3/5$	$-1/5$	0
-1	x_2	$6/5$	0	1	$4/5$	$-1/15$	0
0	S_3	0	0	0	1	$-1/13$	1
	z_j	$12/5$	-2	-1	$2/5$	$1/3$	0
	$z_j - c_j$	$12/5$	0	0	$2/5$	$1/3$	0

$2 \quad 4/3 \quad 1 \quad 0 \quad -1/3 \quad 0$

$4/5 \quad 4/3 \quad 0 \quad -4/5 \quad -4/15 \quad 0.$

$2-4/5 \quad 0 \quad 1 \quad 4/5 \quad -1/3+\frac{4}{15} \quad 0$

$1 \quad 5/3 \quad 0 \quad 0 \quad -2/3 \quad 1$

$1 \quad 5/3 \quad 0 \quad -1 \quad -1/3 \quad 0$

$5/3$ NPE

$0 \quad 0 \quad 1 \quad -1/3 \quad 1$

0

Max $z^* = \frac{12}{5}$

$x_1 = \frac{3}{5}$ $x_2 = \frac{6}{5}$

Min $z = \frac{-12}{5}$

$x_1 = 3/5$ $x_2 = 6/5$

In dual X_B must be positive

If $z_j - c_j$ comes - it fails can't procedure.

$\boxed{\text{Over unit I}}$

Scanned by CamScanner
Scanned by CamScanner

$$A \begin{pmatrix} 1 & 7 & 3 & 4 \\ 5 & 6 & 4 & 5 \\ 7 & 2 & 0 & 3 \end{pmatrix}$$

col II $\geq$ col III

col II is dominated by c - III

omit col II

$$\begin{array}{c} \begin{array}{ccc} I & III & IV \end{array} \\ \begin{pmatrix} 1 & 3 & 4 \\ 5 & 4 & 5 \\ 7 & 0 & 3 \end{pmatrix} \end{array}$$

I row $\leq$ II row

R I dominated by row II

omit Row I

col III $\leq$ by C IV

omit . IV

Scanned by CamScanner

Scanned by CamScanner

$$\begin{pmatrix} 5 & 4 \\ 7 & 0 \end{pmatrix}$$

$5x + 4(1-x) = 7x + 0$

$5x + 4 - 4x = 7x$

$x + 4 = 7x$

$6x = 4$

$x = 4/6 = x/3$

---

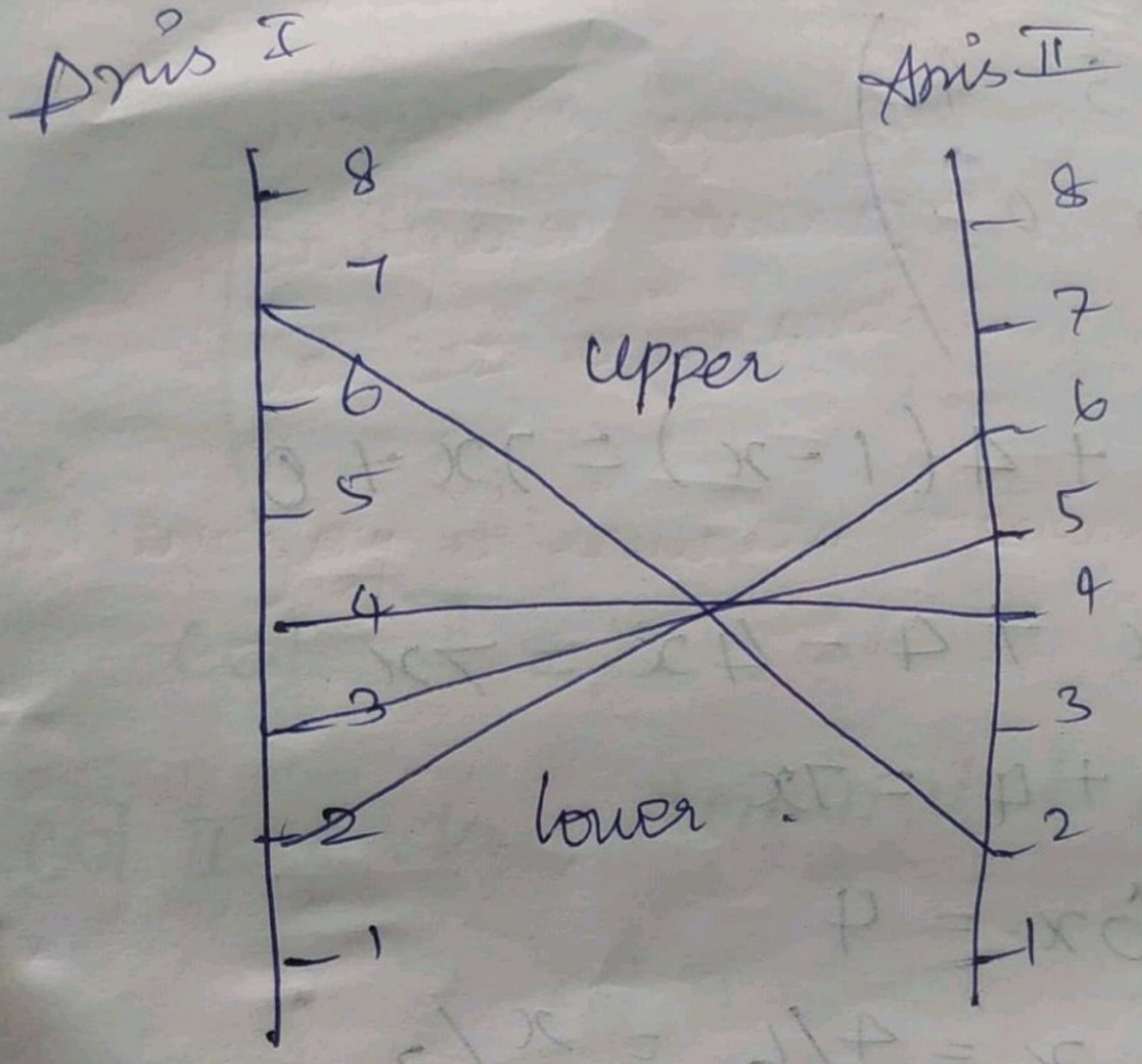
(2)
$$\begin{pmatrix} 18 & 4 & 6 & 4 \\ 6 & 2 & 13 & 7 \\ 11 & 5 & 17 & 3 \\ 7 & 6 & 12 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 4 \\ 2 & 7 \\ 5 & 3 \\ 6 & 2 \end{pmatrix}$$

1 to 2 axis
2 to 1 axis

Scanned by CamScanner
Scanned by CamScanner

Upper = min
Lower = Max



Scanned by CamScanner
Scanned by CamScanner

$2 \times 105 = 210 + 1150$

$$\begin{array}{r} 105 \\ \times\ 2 \\ \hline 210 \end{array}$$

$$\begin{array}{r} 1150 \\ 210 \\ \hline 1360 \end{array}$$

$$\begin{array}{r} 230 \\ 5 \\ \hline 1150 \end{array}$$

$2 \times 1/4 = \cancel{2}/\cancel{4}_2 \quad 1/2$

$5 \times 1/4 = \quad 5/4$

$\frac{1}{2} + \frac{5}{4}$

8

$\frac{1}{2} = \frac{4}{4} = \frac{4}{8} +$

$\frac{5}{4} \times \frac{2}{2} = \frac{10}{8}$

$7/4$

$\frac{\cancel{14}\ 7}{\cancel{8}\ 4}$

Scanned by CamScanner
Scanned by CamScanner

1) A project is defined as combination of Inter related activities all of which must be executed in a certain order to achieve a set goal.

A systematic scientific approach has become a necessity for such projects. So a number of methods applying network scheduling techniques has been developed programme Evaluation Review Technique [PERT] and critical Path method. are two of the main techniques using projects.

Main functions of project :-

planning,
Scheduling,
controlling.

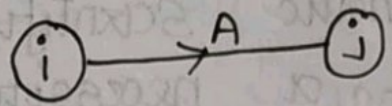
Basic terminologies :-

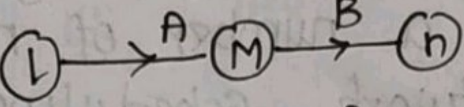
Activity :

Activity is a task or an item of work to be done in a project. An activity consumes resources like time, money, labour etc. An activity is

Scanned by CamScanner
Scanned by CamScanner

represented by an arrow with a node at the beginning and a node at the end indicating the start and termination of activity. Nodes are represented by circles





Here A is the immediate ~~predeess~~ predecessor of B and B is the immediate successor of A.

Notation:

A < B and

B > A

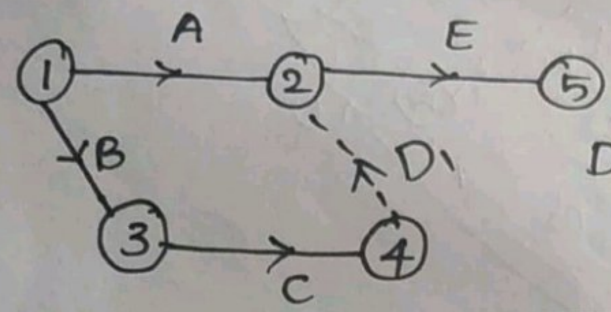
DUMMY ACTIVITY:

If the project contains two or more activities which have some of their immediate predecessors in common then there is a need for dummy activity. It is an imaginary activity which does not consume any resource which serves the

Scanned by CamScanner
Scanned by CamScanner

Purpose of indicating the predecessor or successor relationship clearly in any activity on arrow diagram

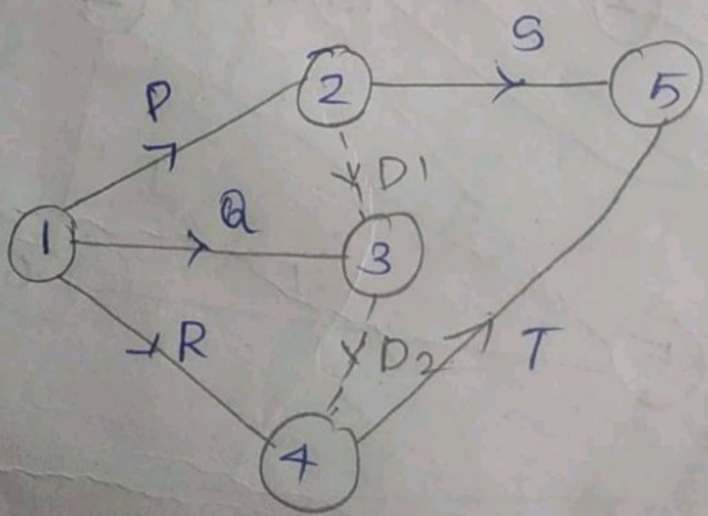
Eg:



D1 - Dummy activity

---

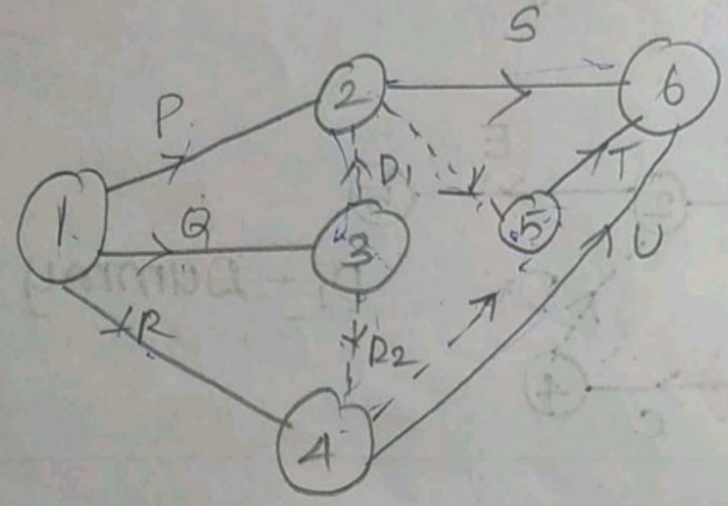
1) If there are 5 activities P, Q, R, S and T such that P, Q, R have no immediate predecessors but S and T have immediate predecessor of P, Q and Q, R respectively represent the situation by a network.



Scanned by CamScanner
Scanned by CamScanner

2) Activity : P Q R S T U

predecessor : – – – P, Q P, R Q, R

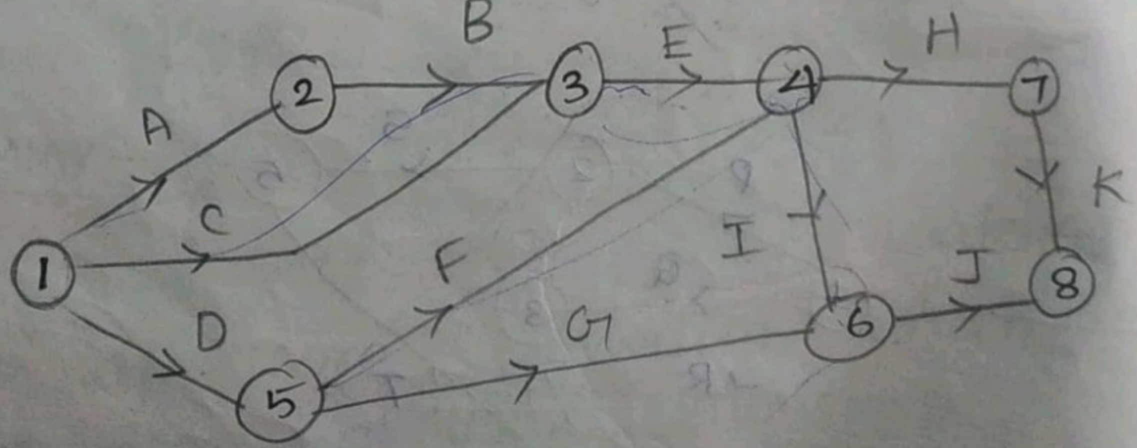


3) Draw the network for the following

A C D can start simultaneously

E > B, C ; F, G > D ; H, I > E, F ;

J > I, G ; K > H ; B > A

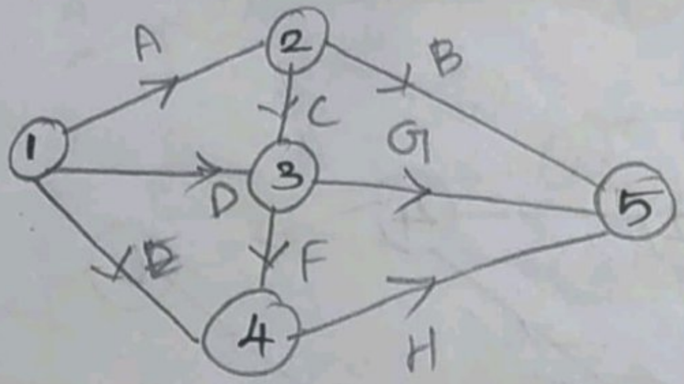


Scanned by CamScanner

Scanned by CamScanner

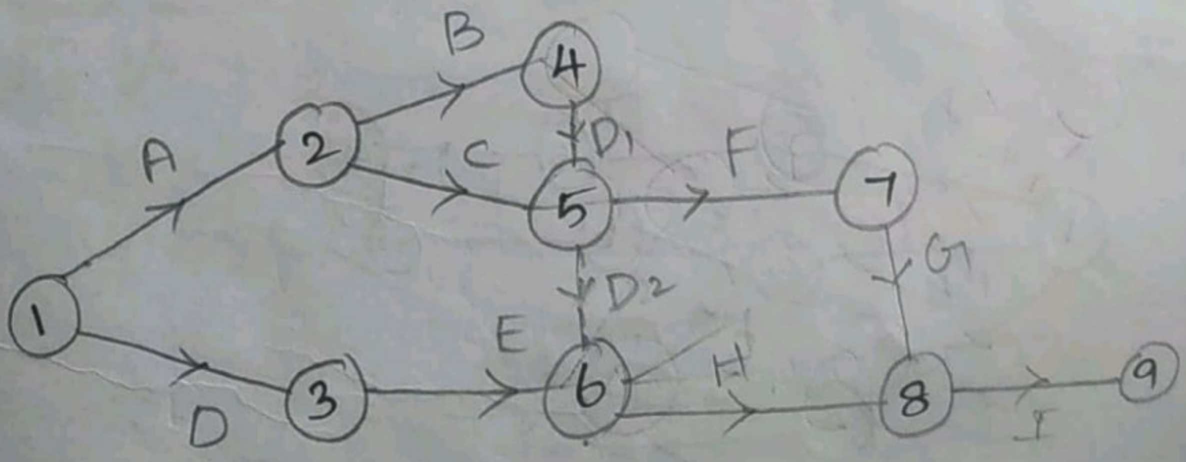
Scanned by CamScanner

4) Construct the network for following
A, D, E can Start simultaneously
B, C > A ; G, F > D, C ; H > E, F



5) Draw the network for the following

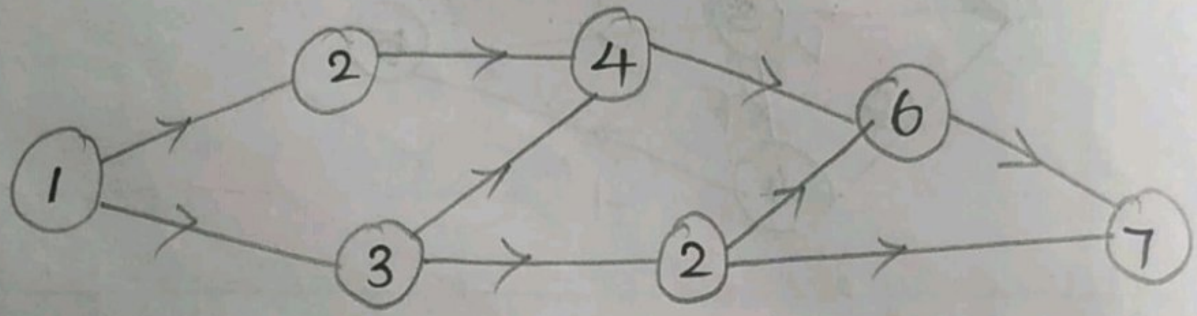
Activities:	A	B	C	D	E	F	G	H	I
predecessor:	–	A	A	–	D	B, C, E	F	E	G, H



Scanned by CamScanner
Scanned by CamScanner

6)

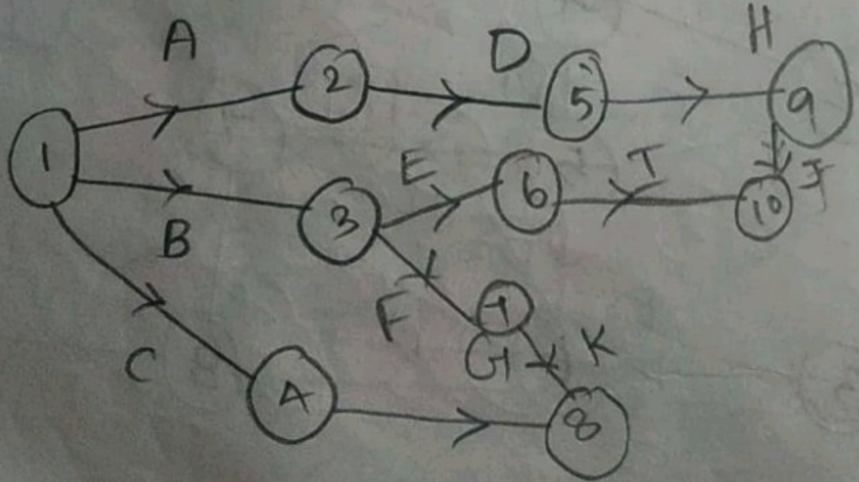
Event no :	1	2	3	4	5	6	7
Predecessor :	-	1	1	2,3	3	4,5	5,6



HW

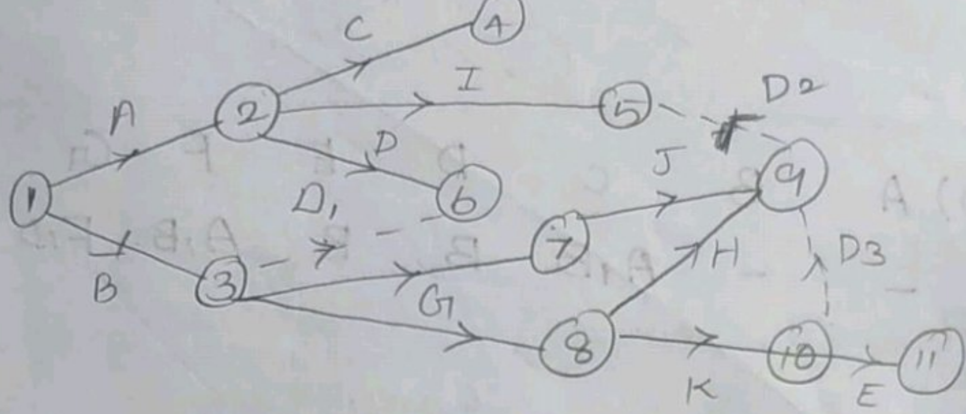
1.

Activity :	A	B	C	D	E	F	G	H	I	J	K
Predecessor :	-	-	-	A	B	B	C	D	E	H,I	F,G

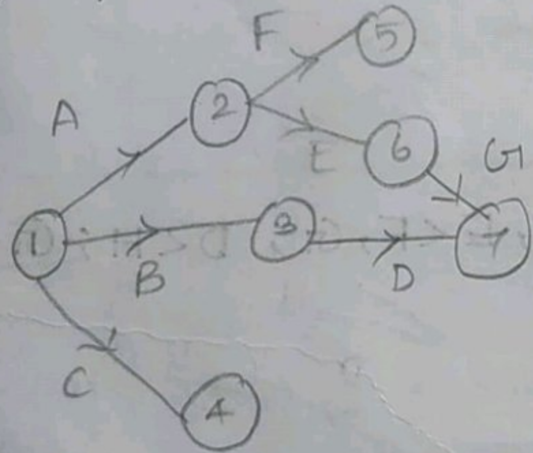


Scanned by CamScanner
Scanned by CamScanner

2) A<C, D, I ; B<G, F ; D<G, F ; F<H, K ;
G, H<J ; I, J, K <E

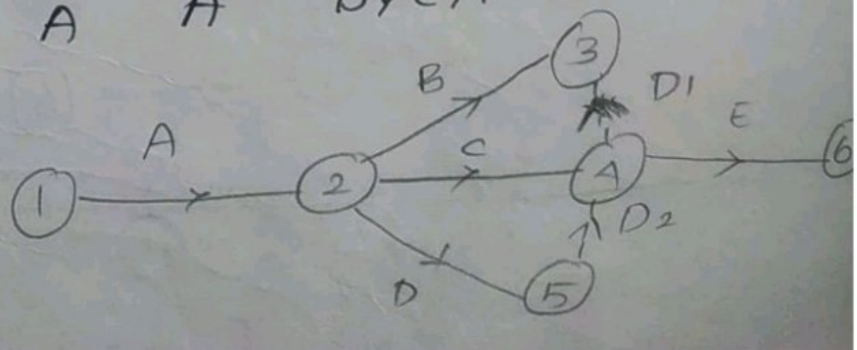


3) A, B, C can Start Simultanxously
A<F, E ; B<D ; C ; E, D<G.



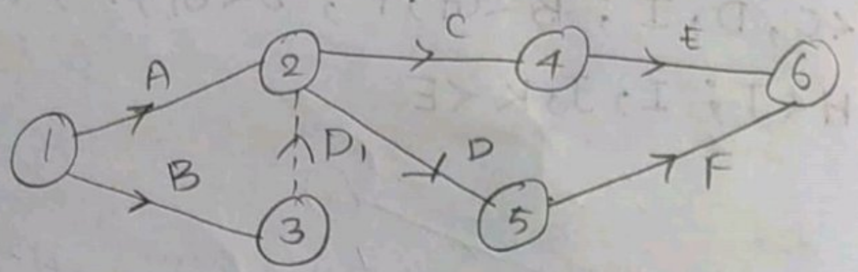
4)

A	B	C	D	E
–	A	A	A	B, C, D



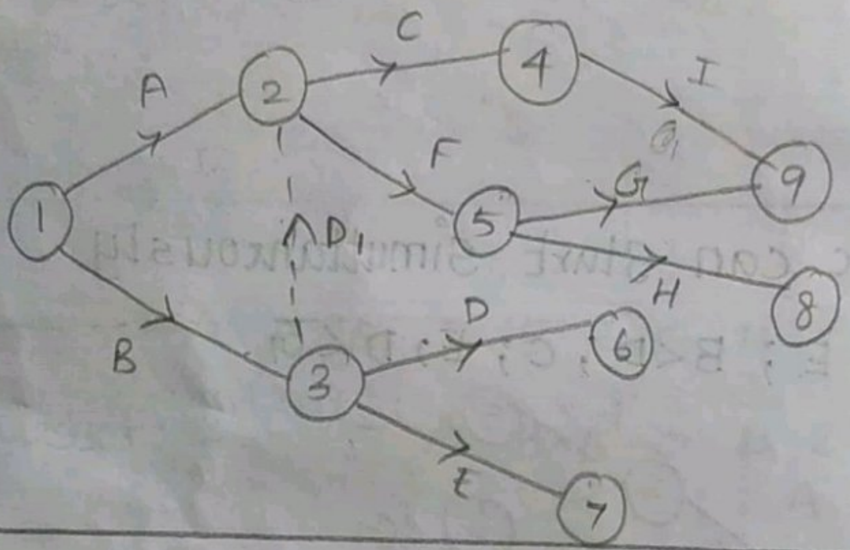
Scanned by CamScanner
Scanned by CamScanner

5) A < C, D ; B < C, D ; C < E ; D, E < F .



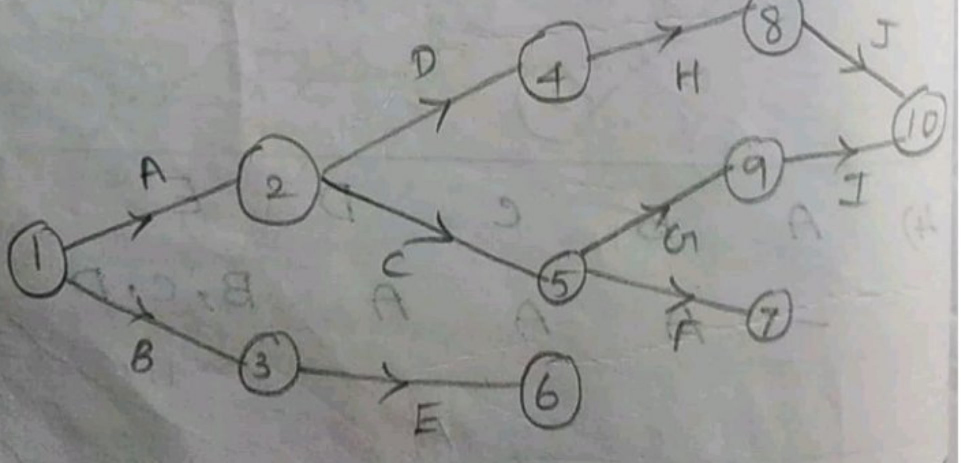
6)

A	B	C	D	E	F	G	H	I
–	–	A,B	B	B	A,B	F,D	F,D	C,G



7) A < C, D ; B < E ; C, E < F, G ; D < H ; G < I ;

H, I < J

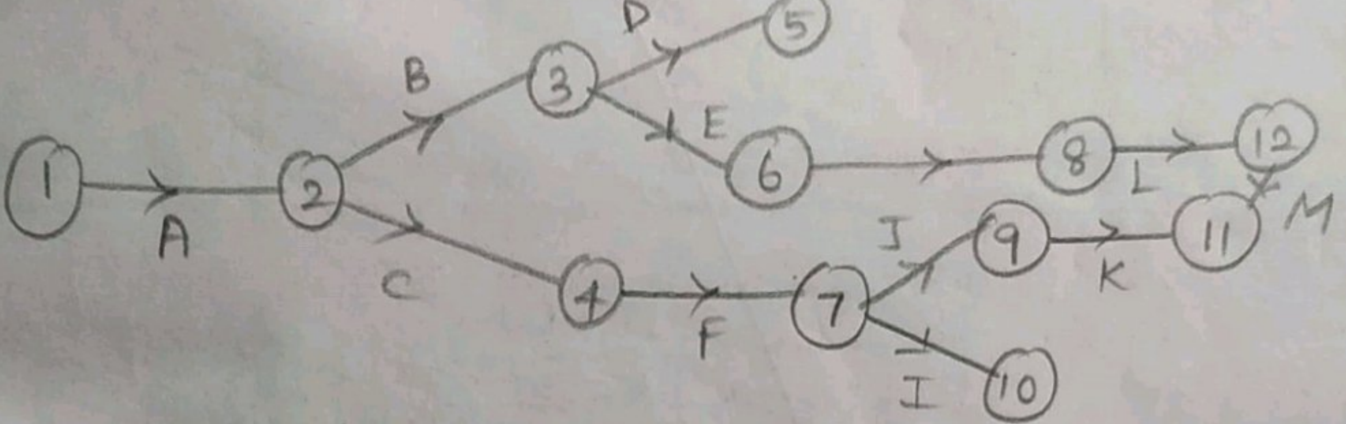


Scanned by CamScanner
Scanned by CamScanner

8) Draw the network data & number of the events

$A<C, B$; $B<D, E$; $C<F$; $E<G$;

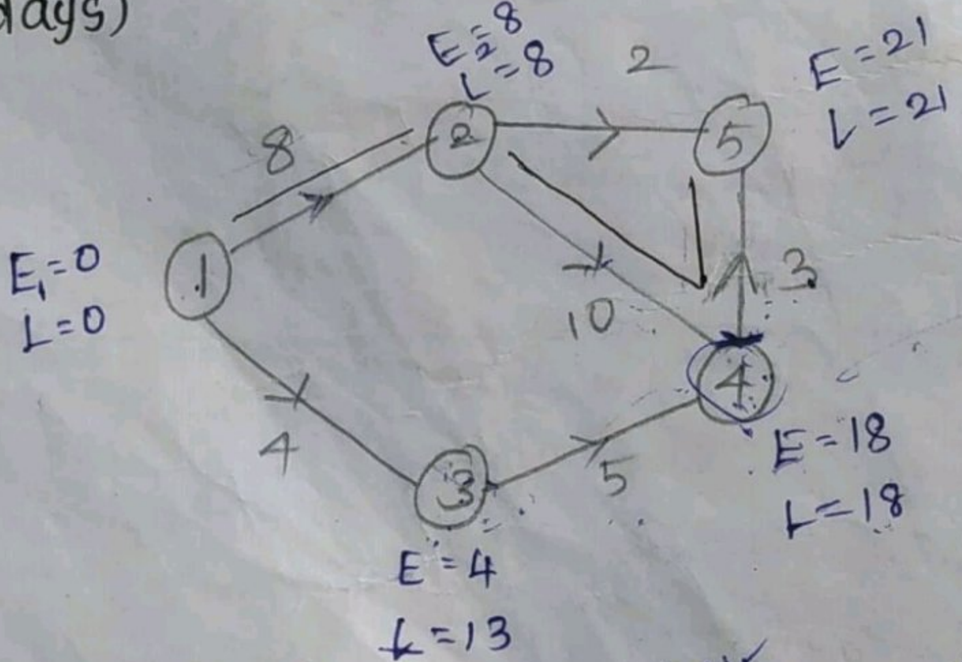
$F<I, J$; $J<K$; $G<L$; $K, L<M$



Scanned by CamScanner
Scanned by CamScanner

1) Compute Earliest start, Earliest finish, latest start and latest finish of each activity of the project given below

Activity :	1-2	1-3	2-4	2-5	3-4	4-5
Duration (in days) :	8	4	10	2	5	3



$E_1 = 0$

$E_2 = E_1 + t_{12}$

$0+8$

$E_2 = 8$

$E_3 = E_2 + t_{13}$

$8+4$

$=12$

$L_6 = L_7 - t_{45}$

$21 - 5$

$L_5 = L_7 - t_{34}$

$= 21 - 5$

Scanned by CamScanner
Scanned by CamScanner

Activity	duration	Earliest Start	Earliest finish
1-2	8	0	0+8=8
1-3	4	0	0+4=4
2-5	2	8	10
2-4	10	8	18
3-4	5	4	9
4-5	3	18	21

max value

Latest Start	Latest Finish
0	8
9	13
19	21
8	18
13	18
18	21

To find critical path.

- 1-2-5 = 10
- 1-3-4-5 = 12
- 1-2-4-5 = 21

Critical path

Path connecting the first initial node to the very last terminal node, of longest duration in any project network is called critical path.

All the activities in the

Scanned by CamScanner
Scanned by CamScanner

critical path are called critical activities. It is usually denoted by double lines. From the above problem critical path is

1-2-5

1-3-4-5

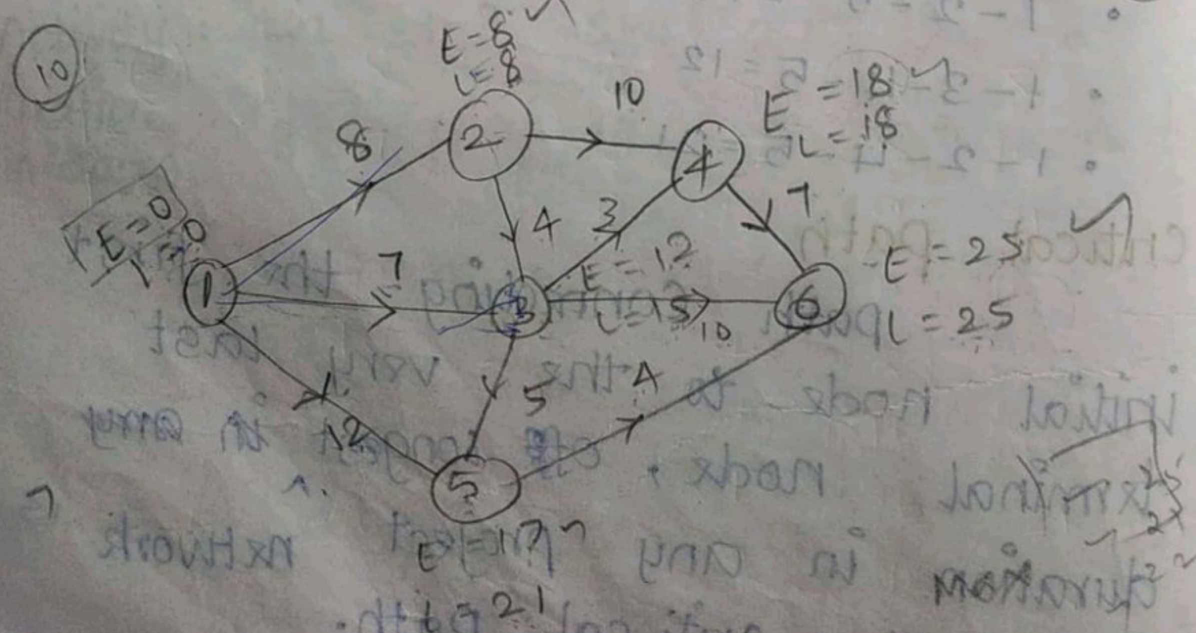
1-2-4-5

---

2) calculate E.S, E.F, L.S, L.F of each activity of the project given below and determine the critical path.

Activity	1-2	1-3	1-5	2-3	2-4	3-4
duration	8	7	12	4	10	3

3-5	3-6	4-6	5-6
5	10	7	4



Activity (Tail, head)	dur	ES (max)	EF (Add of)	LS	LF (min)
1-2	8	0	8	(8-8) 0	8
1-3	7	0	7	(15-7) 8	15
1-5	12	0	12	(21-12) 9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Floats:

Total Float = LF − EF (difference)

Free Float = (Total Float of i-j) − (Slack (L − E) of the head event j) pinnadi

Independent Float = Free float of i-j − Slack of Tail event tail → munnadi

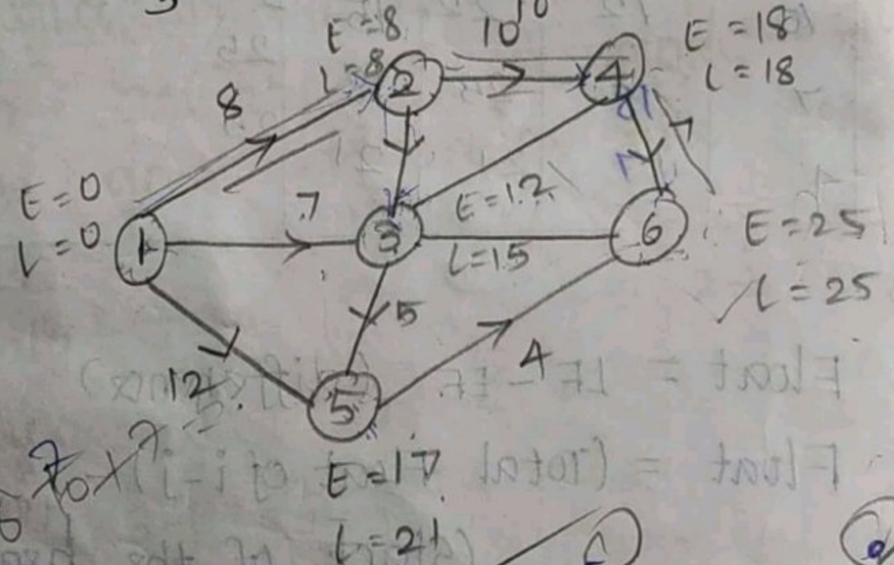
Interfering float = Total Float − Free Float.

Scanned by CamScanner
Scanned by CamScanner

1.) 8-2 calculate total float, free float, Independent float for the following activity.

Activity :	1-2	1-3	1-5	2-3	2-4
duration :	8	7	12	4	10

3-4	3-5	3-6	4-6	5-6
3	5	10	7	4



Activity	Duration	ES	EF	LS	LF
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Scanned by CamScanner
Scanned by CamScanner

Floats		
TF	FF	IF
0	0	0
8	(8-3) 5	5
9	(9-((21-17)) 5	5
3	0	0
0	0	0
3	3	0
4	0	-3
3	3	0
0	0	0
4	4	0

TF = LF - EF

LF - EF

0 - (0-0) = 0

(5 - 0) Total Float 0 zero means that is critical

5 - (0 - 5 - (0 path = 5

= 9 - (

Critical path :

3(0-3

0-(15-12) 1-2-4-6

0-(3) Critical path 1-2-4-6

Project time = 25 weeks

0 - 3

0 - (15 - (12) = 3 9 -

TF -

(2) Determine critical path, project duration, floats.

3 -

Activity :	0-1	1-2	1-3	2-4	2-5	3-4
duration : in weeks	3	8	12	6	3	3

3-6	4-7	5-7	6-7
8	5	3	8

9 - (21 - 17)

= 5

8 - (15-12) =

9 - 4

L - E

FF -

Slack

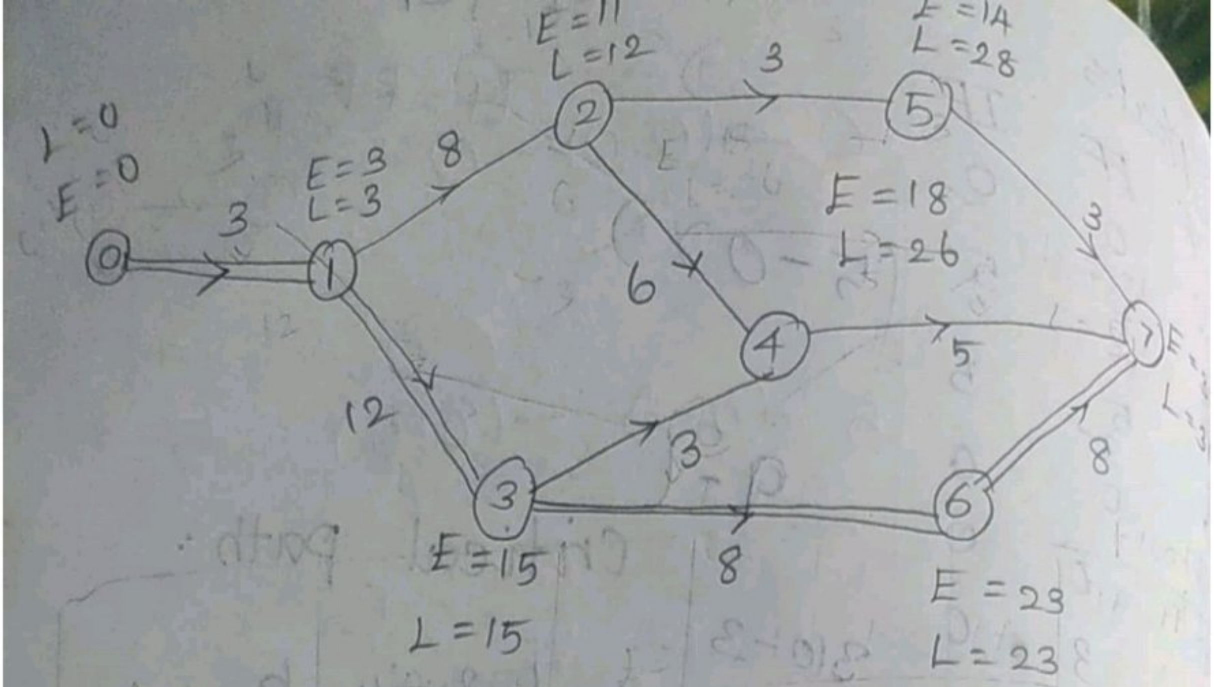
T H

1-2 = 0 8 - 3 TF of (i-j)

Scanned by CamScanner

Scanned by CamScanner

Scanned by CamScanner



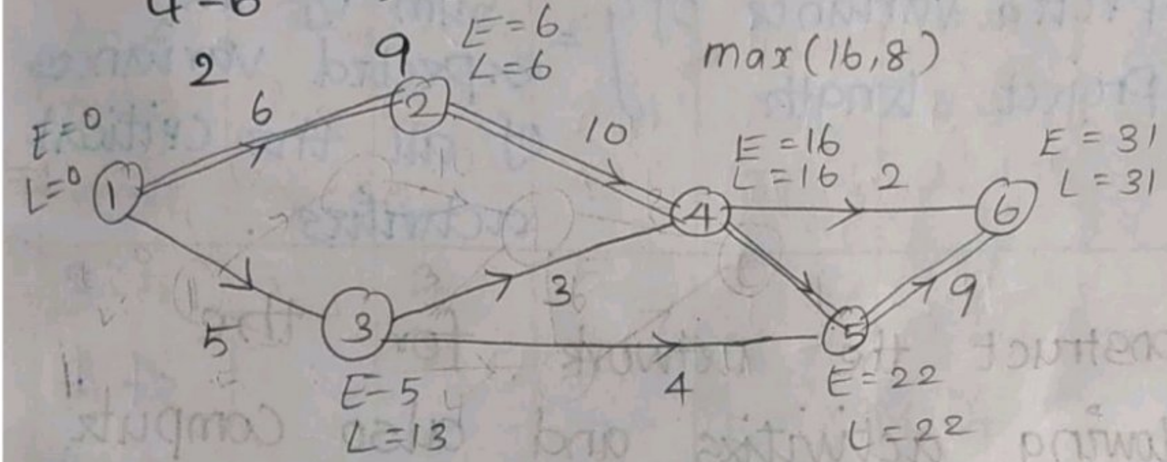
Activity	Duration	ES	EF	LS	LF	Floats TF	FF	
0-1	3	0	3	0	3	0	0	[illegible]
1-2	8	3	11	4	12	1	0	0
1-3	12	3	15	3	15	0	0	[illegible]
2-4	6	11	17	20	26	9	1	0
2-5	3	11	14	25	28	14	0	-1
3-4	3	15	18	23	26	8	10	10
3-6	8	15	23	15	23	0	0	0
4-7	5	18	23	26	31	8	8	0
5-7	3	14	17	28	31	14	14	0
6-7	8	23	31	23	31	0	0	0

Critical path : 0-1-3-6-7
project time : 31 weeks.

Scanned by CamScanner
Scanned by CamScanner

3) Draw the network determine critical free float and Independent float path of each activity for the given data

Jobs :	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-6
Duration :	6	5	10	3	4	6	2	9



Jobs	Duration	ES	EF	LS	LF	Floats TF	FF	IF
1-2	6	0	6	0	6	0	0	0
1-3	5	0	5	8	13	8	0	0
2-4	10	6	16	6	16	0	0	0
3-4	3	5	8	13	16	8	8	0
3-5	4	5	9	18	22	13	13	5
4-5	6	16	22	16	22	0	0	0
4-6	2	16	18	29	31	13	13	13
5-6	9	22	31	22	31	0	0	0

Critical path : 1-2-4-5-6

Scanned by CamScanner

Scanned by CamScanner

Scanned by CamScanner

PERT - Programme Evaluation Review Technique :

Expected duration $t_e = \dfrac{t_o + 4t_m + t_p}{6}$

Expected ~~value~~ variance $\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$

Expected variance of project length $\Big\}$ = Sum of the expected variances of all the critical activities.

---

1) Construct the network for the following activities and also compute expected duration, expected variance, Expected Variance of project length.

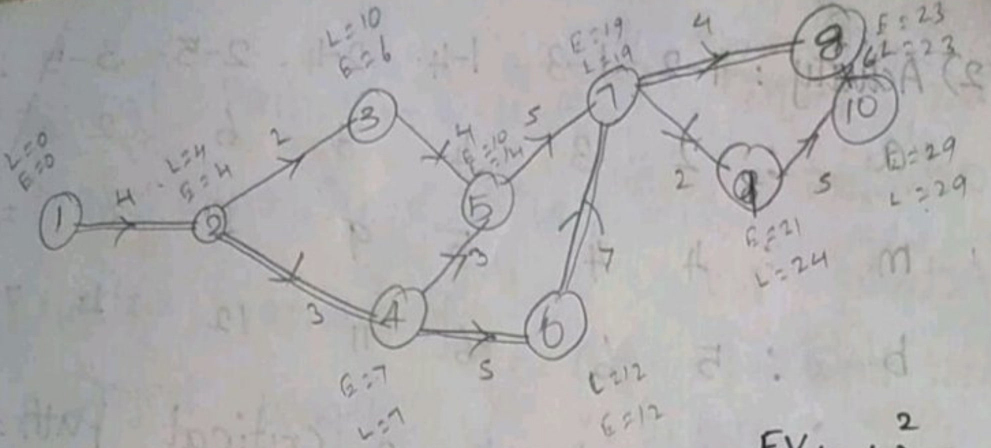
Activity	1-2	2-3	2-4	3-5	4-5	4-6	5-7	6-7	7-8	7-9	8-10	9-10
t_o	3	1	2	3	1	3	4	6	2	1	4	3
t_m	4	2	3	4	3	5	5	7	4	2	6	5
t_p	5	3	4	5	5	7	6	8	6	3	8	7

t_o - optimistic

t_m - pesimistic

t_p -

Scanned by CamScanner
Scanned by CamScanner



$$\sigma^2 = \left(\frac{tp - to}{6}\right)^2$$

Activity	to	tm	tp	Expected duration $te = to + 4tm + tp/6$	
1 - 2	3	4	5	4	0.1
2 - 3	1	2	3	2	0.1
2 - 4	2	3	4	3	0.1
3 - 5	3	4	5	4	0.1
4 - 5	1	3	5	3	0.4
4 - 6	3	5	7	5	0.4
5 - 7	4	5	6	5	0.1
6 - 7	6	7	8	7	0.1
7 - 8	2	4	6	4	0.4
7 - 9	1	2	3	2	0.1
8 - 10	4	6	8	6	0.4
9 - 10	3	5	7	5	0.4

$\frac{3+4(4)+5}{6}$

$3+16+5$

Critical path : 1-2-4-6-7-8-10

Project duration : 29

Variance of project length = 0.11 + 0.11 + 0.44 + 0.11 + 0.44 + 0.44

= 1.65

$\left(\frac{5-3}{6}\right)^2$

$4 + 3 + 5 + 7 + 4 + 6$

$3 + 2 + 3 + 6 + 2 + 4$

0.3 × 0.3 = 0.09

Scanned by CamScanner
Scanned by CamScanner

2) Activity : 1-2 1-3 1-4 2-4 2-5 3-5 4-5

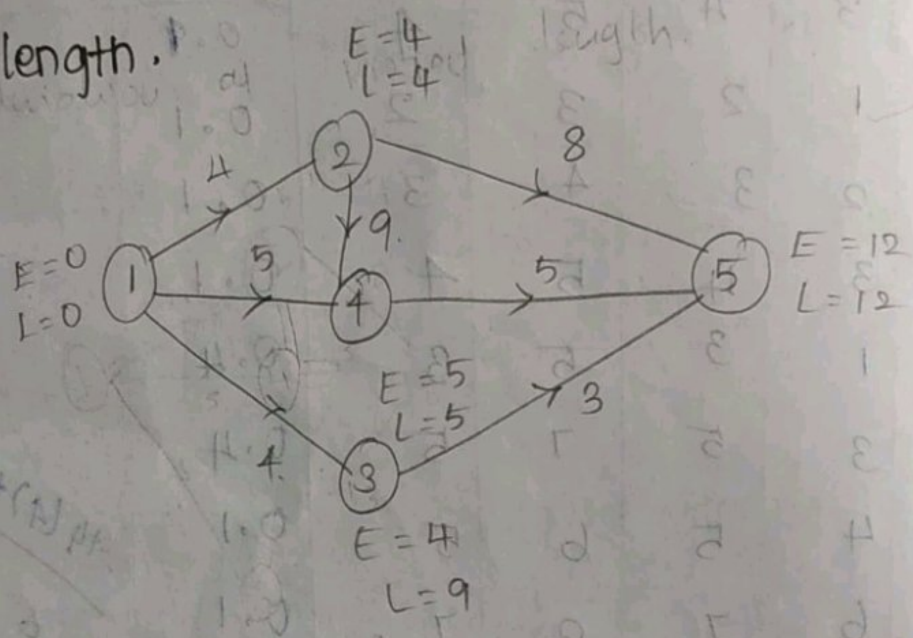
a : 2 3 4 8 6 2 2

m : 4 4 5 9 8 3 5

b : 5 6 6 11 12 4 7

Determine 1. Network 2. Critical Path.

3. Expected Standard deviation of project length.



Activity	to	tm	tp	te = to + 4tm + tp/6	$\sigma^2 = \left(\frac{tp-to}{6}\right)^2$
1-2	2	4	5	3.83	0.25
1-3	3	4	6	4.167	1
1-4	4	5	6	5	0.11
2-4	8	9	11	9.167	0.25
2-5	6	8	12	8.33	1
3-5	2	3	4	3	0.11
4-5	2	5	7	4.833	0.694

Scanned by CamScanner
Scanned by CamScanner

Critical Path : 1-2-4-5

project duration = 4+9+5 = 18

$$\left.\begin{array}{l}\text{Expected variance of} \\ \text{Project length}\end{array}\right\} = \begin{array}{l} 0.25+0.25+ \\ 0.694 \end{array}$$

$$= 1.194$$

S.D of Project length $= \sqrt{1.194}$

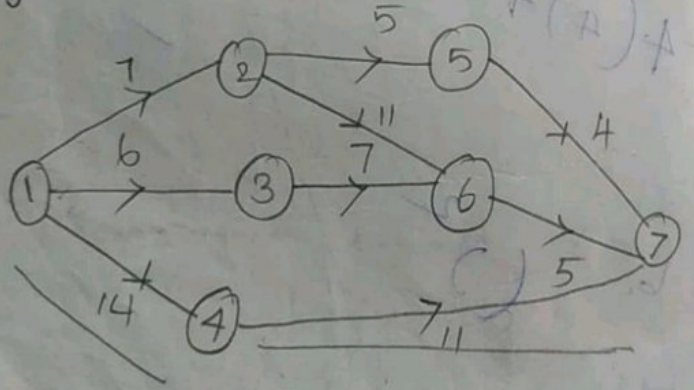
$$= 1.092$$

Scanned by CamScanner
Scanned by CamScanner

The project consist of the following activities and time estimate

Activity	1-2	1-3	1-4	2-5	2-6	3-6	4-7	5-7	6-7
least ~~type~~ time t_o (days)	3	2	6	2	5	3	3	1	2
greatest time t_p	15	14	13	8	17	15	27	7	8
Most likely time t_m	6	5	12	5	11	6	9	4	5

Find draw the network, standard deviation of project length.



Critical path : 1-4-7

Project duration : 14 + 11 = 25 days

Standard deviation: $\sigma = \sqrt{\sigma^2}$

Project variance σ^2 = 16 + 16 = 32

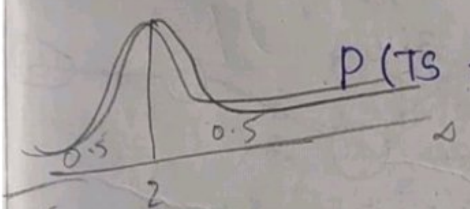
SD $\sigma = \sqrt{\text{variance}} = \sqrt{32} = 5.65$

Scanned by CamScanner
Scanned by CamScanner

Activity	to	tp	tm	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	3	15	6	7	2.25
1-3	2	14	5	6	4
1-4	6	13	12	14	2.25
2-5	2	8	5	5	1
2-6	5	17	11	11	4
3-6	3	15	6	7	4
4-7	3	27	9	11	16
5-7	1	7	4	4	1
6-7	2	8	5	5	1

(ii) What is the probability that the project will be completed in 27 days.

$$z = \frac{T_S - T_E}{\sigma} = \frac{27-25}{5.65} = \frac{2}{5.65} = 0.35$$



$$P(T_S \le 27) = P(z \le 0.35)$$
$$= 0.6368 \text{ (table)}$$
$$= 63.7\%$$

2) Three times estimates (in months) of all activities of a project are as given below.

Activity

	1-2	2-3	2-4	3-4	4-5	5-6
a	0.8	3.7	6.2	2.[illegible]	0.8	0.9
m	1.0	5.6	6.6	2.7	3.4	1.0
b	1.2	9.9	15.4	6.1	3.6	1.1

Scanned by CamScanner
Scanned by CamScanner

(i) Find the expected duration, standard deviation of each activity

(ii) Construct the project network.

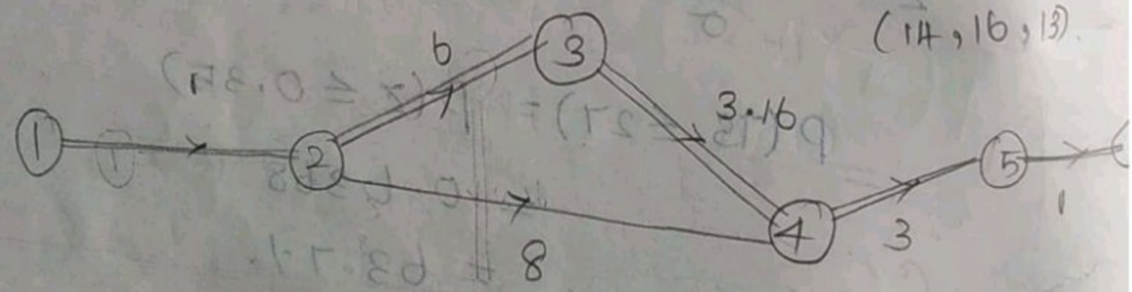
(iii) Determine the critical path expected project length, expected the variance of project length.

(iv) What is the probability that the project will be completed.

(i) 2 months later than expected

(ii) Not more than 3 months yearler than expected

(iii) What due date has above 90% has chance of being met.



Critical path : 1-2-3-4-5-6

Expected project duration : 1+6+3+3+1

$= \boxed{14}$ months.

$T_E = 14$

Scanned by CamScanner

Scanned by CamScanner

Activity	t_o	t_m	t_p	$t_e = \frac{t_o+4t_m+t_p}{6}$	$\sigma = \left(\frac{t_p-t_o}{6}\right)$ S.D
1-2	0.8	1.0	1.2	1	0.066
2-3	3.7	5.6	9.9	6	1.033
2-4	6.2	6.6	15.4	8	1.533
3-4	2.1	2.7	6.1	3.16	0.666
4-5	0.8	3.4	3.6	3	0.466
5-6	0.9	1.0	1.1	1	0.033

Variance of project length $\sigma = (0.066)^2 + (1.033)^2$

$$+ (0.666)^2 + (0.466)^2 + (0.033)^2$$

$$= 1.5374$$

$$\sigma = \sqrt{1.5374} = 1.2399$$

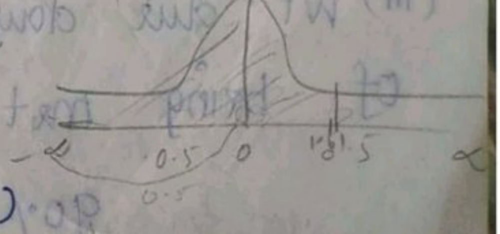
(i) Two months later than expected:

$T_S = (14+2)$ $\quad T_S = 16, \quad T_e = 14$

$T_e + 2$ $\qquad \sigma = 1.2399$

$$Z = \frac{T_S - t_e}{\sigma} = \frac{16-14}{1.2399}$$

$$= 1.61$$

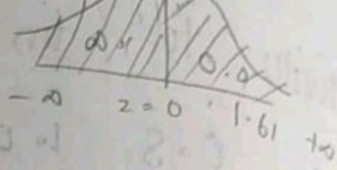


$$P(T_S \le 16) = P(Z \le 1.61)$$

$$= P(-\infty < Z < 0) + P(0 < Z < 1.61)$$

Scanned by CamScanner
Scanned by CamScanner

$= 0.5 + 0.4463$



$= 0.9463$

(or) 0.9463×100

$= 94.63\%$

ii) Not more than 3 months earlier than expected:

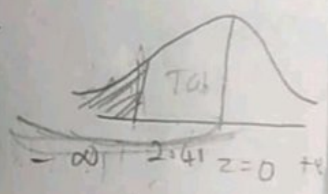
$T_S = 11$ $T_S = (14-3)$

$(T_e - 3)$

$t_e = 14$

$\sigma = 1.2399$

$$Z = \frac{T_S - T_e}{\sigma} = \frac{11-14}{1.2399}$$



$= -2.41$

$P(T_S \le 11) = P(Z \le -2.41)$

$= 0.5 - P(Z \le -2.41)$

$= 0.5 - 0.4920$

$= 0.008$ (or) 0.008×100

$= 0.80\%$

(iii) wt due day as above 90% chance of being met.

$90\% = 1.28$

$$z = \frac{T_S - T_E}{\sigma}$$

$$1.28 = \frac{T_S - 14}{1.2399}$$

$$(1.28)(1.2399) + 14 = T_S$$

$T_S = 15.59$ months nearly

INVENTORY MODEL

Introduction :-

Inventory may be defined as the stock of goods commodities (or) economic resources that are stored (or) reserved smooth & effecient running of business affairs. The Inventory may be kept in any one of the following firms.

(i) Raw material Inventory

Raw material which are kept in stock. for using in product of goods.

(ii) Work-in process Inventory.

Semi finished goods which are stored during production process

(iii) Finished goods inventory

Finished goods awaiting shipments from the factory.

Scanned by CamScanner
Scanned by CamScanner

Types of Inventory :

i) Fluctuation Inventories:

In real life problems, there are fluctuation in the demand & lead times that affect the production of the items. Such type safety stock are called fluctuation Inventories.

ii) Anticipated Inventories:

These are built up in advance for the season of large sales, a promotion programme (or) a plant shut down period. Anticipated Inventories keep men & machine hours for future participation.

iii) Lot size Inventory :

Generally rate of consum- is different from rate of production (or) purchasing. Therefore the items are produced in larger quantities, which result in lot-size Inventories.

Reason for maintaining Inventory:

Scanned by CamScanner
Scanned by CamScanner

1) Inventory helps in smooth & efficient running of business.

2) It provides service to the customer at short notice.

3) Because of long-~~tnine~~ uninterrupted runs, production cost is less.

4) It acts as a buffer stock if shop rejections are too many

5) It takes care of economic fluctuation ~~cost Involved~~

Cost Involved in inventory problems:

(i) Holding cost C_1 :-

The cost associated with carrying (or) holding the goods in stock is known as holding cost (or) carrying cost per unit of time. Holding cost is assumed to directly vary with the size of Inventory as well as the time the item is held in stock the following components :-

a) Interested capital cost :

This is the interest charge

Scanned by CamScanner
Scanned by CamScanner

over the capital Invested.

b) Record keeping & Administration costs.

c) Handling cost: These include costs associated with movement of stock, such as cost of labour, etc.

d) Storage costs

e) Depreciation costs

f) Taxes & Insurance costs

g) Purchase price (or) production cost.

Purchase price per unit item is affected by the quantity purchased due to quantity discount or price breaks. If P is the purchase price of an item and I is the stock Holding cost per unit time expressed as the fraction of stock value, holding cost

$$C_1 = IP$$

Shortage cost:

The penality cost that are incurred as a result of running out of stock are known as shortage cost

Scanned by CamScanner
Scanned by CamScanner

These are denoted by C_2. In case where the unfilled demand for the goods may be satisfied at a later day, these cost are assumed to vary directly with both their shortage quantity and the delaying time on the other hand if the unfilled demand is lost shortage cost become proportional to shortage quantity only.

Setup cost :

These cost are associated with obtaining goods ~~may be~~ through placing an order ~~or~~ purchasing or manufacturing or setting up a machinary before starting production. So they include cost of purchase requisition, follow-up receiving the goods, quantity control etc. These are called ordering cost or setup cost. It is usually denoted by C_3 per production run. They are assumed to be independent of the quantity ordered or produced.

Scanned by CamScanner
Scanned by CamScanner

Variables in Inventory Problem.

Quantity occure & time, stocked item completion — Controlled variables,

Uncontrolled variables → Holding, setup cost

lead time:

Elapsed time between the placement of the order and its Receipts in inventory is known as lead time.

Reordered level:

This is the time when we should place an order by taking into consideration the interval between placing the order and receiving the supply.

Economic Order quantity. (EOQ) 2m

Economic Order quantity is that size of order which minimizes total anual cost of Carring inventory and the cost of ordering under the assumed conditions of Certainity and that anual demands are known.

Scanned by CamScanner
Scanned by CamScanner

Deterministic Inventory models:

purchasing model with no shortages.

Manufacturing model with no shortages.

purchasing model with shortages.

Manufacturing model with shortages.

Model I: purchasing model with no shortages:

Average total cost : $\frac{1}{2} C_1 R t + \frac{C_3}{t}$

Time interval $t_0 = \sqrt{\frac{2C_3}{C_1 R}}$

$EOQ = q_0 = q = \sqrt{\frac{2C_3 R}{C_1}}$

Minimum average cost $= C_0(q) = \sqrt{2C_1C_3R}$

Model III : purchasing model with shortage

Case I :

This is the extension of Model I allowing shortage. The assumption are:

i) C_1 is the holding cost per quantity unit per unit time.

ii) C_2 is the shortage cost per quantity per unit time.

Scanned by CamScanner
Scanned by CamScanner

(iii) R is the demand rate

(iv) t_p is the scheduling time period which is constant. fixed lot size $q_p = Rt_p$

(v) q_p is the fixed lot size

(vi) z is the order level to which the Inventory is raised in the begining of each scheduling period. shortage if any, have to be made up. Here z is a variable

(vii) production rate is Infinite.

(viii) Lead time is zero.

Optimum period:

$$t^* = \sqrt{\frac{2C_3}{RC_1}\left[\frac{C_1+C_2}{C_2}\right]} \quad \text{(optimum period)}$$

optimum order quantity q is given by

$$q^* = Rt^* = R\sqrt{\frac{2C_3}{RC_1}\left[\frac{C_1+C_2}{C_2}\right]}$$

$$q^* = \sqrt{\frac{2C_3}{C_1}\,\frac{C_1+C_2}{C_2}}$$

$$C_{min} = C^* = \sqrt{2C_1C_3R}\sqrt{\frac{C_2}{C_1+C_2}}$$

Scanned by CamScanner

Scanned by CamScanner

Scanned by CamScanner

Model II : Manufactring model with no shortage uniform, production rate finite)

It is assumed that run sizes are constant & that a new run will be started whenever it Inventory is zero. let

R = number of items required per unit time.

k = number of items produced per unit time

C_1 = cost of holding per item per unit.

C_3 = cost of setting up a production run

q = number of items produced per run, $q = Rt$.

t = time Interval between runs.

∴ Total average cost per time. $\Big\} \; C(I_m, t) = \frac{1}{2} I_m C_1 + \frac{C_3}{t}$

EOQ ∴ optimum lot size $q_0 =$

$$\sqrt{\frac{k}{k-R}} \sqrt{\frac{2C_3 R}{C_1}}$$

∴ Optimum time Interval $t_0 = \frac{q_0}{R}$

Scanned by CamScanner
Scanned by CamScanner

The annual demand for item is 3200 units
The unit cost is ₹6 and inventory
carrying charges 25% per annum If
the cost of one procurement is
₹150

$$\begin{array}{cc|c} & B & \text{Row Minimum} \\ A & \begin{pmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{pmatrix} & \begin{array}{c} 1 \\ -4 \\ -1 \end{array} \end{array}$$

column max 1 5 1 ⟵ Minimum of column maximum

maximin = 1

minimax = 1

$\underline{V}$

maximum of Row minimum = $\underline{V}$ = Value of the game

$\overline{V}$

$\overline{V}$

$\overline{V} = \underline{V}$

1) Solve the transportation problem

	1	2	3	4	supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	43

$\Sigma a_i = \Sigma b_j = 43$

TP is balanced

21	16	25	$\boxed{11}$ 13	~~11~~	(3)
17	18	14	23	13	(3)
32	27	18	41	19	(9)
6	10	12	~~15~~ 4		
(4)	(2)	(4)	(10) ↑		

17	18	14	$\boxed{4}$ 23	~~13~~ 9	(3)
32	27	18	41	19	(9)
6	10	12	4		
(15)	(9)	(4)	(18) ↑		

$\boxed{6}$ 17	18	14	~~9~~ 3	(3)
32	27	18	19	(9)
↑ 6 (15)	10 (9)	12 (4)		

Scanned by CamScanner
Scanned by CamScanner

18	14	3	(4)
27	$\boxed{12}$ 18	~~19~~ 7	(9)
10	~~12~~		
(9)	(4)		

$\boxed{3}$ ~~18~~	~~3~~
27	7
~~10~~ 7	
(9)	

$\boxed{7}$ ~~27~~	~~7~~
~~7~~	

21	16	25	$\boxed{11}$ 13
$\boxed{6}$ 17	$\boxed{3}$ 18	14	$\boxed{4}$ 23
32	$\boxed{7}$ 27	$\boxed{12}$ 18	41

$m+n-1 = 3+4-1 = 6$

No of positive allocation $= 6$

$\therefore$ The soln is non degenarate.

Initial TP cost $= \text{Rs}((11\times13)+(6\times17)+(3\times18)$
$+(4\times23)+(7\times27)+(12\times18))$

$= \text{Rs } 796$/.

Scanned by CamScanner
Scanned by CamScanner

occupied Cells: (1,4) (2,1) (2,2) (2,4) (3,2) (3,3)

$C_{ij} = u_i + v_j$

let $u_2 = 0$

$C_{21} = u_2 + v_1$

$17 = 0 + v_1$

$v_1 = 17$

$C_{22} = u_2 + v_2$

$18 = 0 + v_2$

$v_2 = 18$

$C_{24} = u_2 + v_4$

$23 = 0 + v_4$

$v_4 = 23$

$C_{14} = u_1 + v_4$

$13 = u_1 + 23$

$u_1 = -10$

$C_{32} = u_3 + v_2$

$27 = u_3 + 18$

$u_3 = 9$

$C_{33} = u_3 + v_3$

$18 = 9 + v_3$

$v_3 = 9$

unoccupied Cells: (1,1) (1,2) (1,3) (2,3), (3,1) (3,4)

Scanned by CamScanner
Scanned by CamScanner

$d_{ij} = c_{ij} - u_i - v_j$

$d_{11} = c_{11} - u_1 - v_1 = 21 + 10 - 17 = 14 \geq 0$

$d_{12} = c_{12} - u_1 - v_2 = 16 + 10 - 18 = 8 \geq 0$

$d_{13} = c_{13} - u_1 - v_3 = 25 + 10 - 9 = 26 \geq 0$

$d_{23} = c_{23} - u_2 - v_3 = 14 - 0 - 9 = 5 \geq 0$

$d_{31} = c_{31} - u_3 - v_1 = 32 - 9 - 17 = 6 \geq 0$

$d_{34} = c_{34} - u_3 - v_4 = 41 - 9 - 23 = 9 \geq 0$

All $d_{ij} \geq 0$

The soln given below is optimal

Transportation model:

Vogel's approximation method:

Find the initial basic feasible solution for the following transportation problem by VAM.

Distribution Centre

		D_1	D_2	D_3	D_4	Availability
Origin	S_1	11	13	17	14	250
	S_2	16	18	14	10	300
	S_3	21	24	13	10	400
Requirements		200	225	275	250	

$\Sigma a_i = \Sigma b_j = 950$

TP is balanced.

Scanned by CamScanner
Scanned by CamScanner

$\boxed{200}$ 11	13	17	14	~~250~~ 50	(2)
16	18	14	10	300	(4)
21	24	13	10	400	(3)
200	225	275	250		
↑ (5)	(5)	(1)	(0)		

$\boxed{50}$ ~~13~~	~~17~~	~~14~~	50	(1)
18	14	10	300	(4)
24	13	10	400	(3)
175 ~~225~~	275	250		
↑ (5)	(1)	(0)		

$\boxed{175}$ 18	14	10	~~300~~ 125	(4)
24	13	10	400	(3)
175	275	250		
(6)	(1)	(0)		

~~14~~	$\boxed{125}$ ~~10~~	125	(4)
13	10	400	(3)
275	~~250~~ 125		
(1)	(0)		

Scanned by CamScanner
Scanned by CamScanner

13	[125] 10	~~400~~ 275	(3)
275	125		

[275] ~~13~~	~~275~~
~~275~~	

Initial basic feasible solution is given by.

[200] 11	[50] 13	17	14
16	[175] 18	14	[125] 10
21	24	[275] 13	[125] 10

The no of the allocation = 6

$m+n-1 = 3+4-1 = 6.$

The Solution is non degenerate

Transportation cost = RS ((200 × 11) + (50 × 13) + (18 × 175) + (10 × 125) + (13 × 275) + (10 × 125))

= RS 12075.

3) Find the initital basic feasibile Solution for the following Transportation problem by VAM.

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

Scanned by CamScanner

Scanned by CamScanner

$\Sigma a_i = \Sigma b_j = 30$

TP is balanced.

1	2	6	7	(1)
0	4	$\boxed{10}$ 2	~~12~~ 2	(2)
3	1	5	11	(2)
10	10	10		
(1)	(1)	(3) ↑		

1	2	7	(1)
$\boxed{2}$ 0	4	2	(4) ←
3	1	11	(2)
~~10~~ 8	10		
(1)	(1)		

1	2	7	(1)
3	$\boxed{10}$ 1	~~11~~ 1	(2)
8	10		
(2)	(1)		

$\boxed{7}$ 1	~~7~~
3	1
~~8~~ 1	
(2) ↑	

$\boxed{1}$ 3	~~1~~
~~1~~	

Scanned by CamScanner
Scanned by CamScanner

$\boxed{7}$ 1	2	6
$\boxed{2}$ 0	4	$\boxed{10}$ 2
$\boxed{1}$ 3	$\boxed{10}$ 1	5

$= (7\times1) + (2\times0) + (10\times2) + (1\times3) + (10\times1)$

$= 7+0+20+3+10$

$= 40.$

The no of allocation = 5

$m+n-1 = 3+3-1 = 6-1 = 5$

The Soln is non degenarate.

Transportation Cost = RS 40

4) obtain the optimum basic feasible Solution for the following Transportation problem

			Available
7	3	2	2
2	1	3	3
3	4	6	5
Demand 4	1	5	

$\Sigma a_i = \Sigma b_j = 10$

TP is balanced.

Scanned by CamScanner
Scanned by CamScanner

7	3	$\boxed{2}$ 2	$\cancel{2}$ (1) (5)
2	$\boxed{1}$ 1	$\boxed{2}$ 3	$\cancel{3}$ 2 (1) (1) (1)
$\boxed{4}$ 3	4	$\boxed{1}$ 6	5 (1) (3) (3)
4	$\cancel{1}$	$\cancel{5}$ $\cancel{3}$ 1	

(1)	(2)	(1)
(1)		(1)
(1)		(3)
(1)		(3)

7	3	$\boxed{2}$ 2
2	$\boxed{1}$ 1	$\boxed{2}$ 3
$\boxed{4}$ 3	4	$\boxed{1}$ 6

No of +ve allocation = 5

$m+n-1 = 3+3-1 = 5$

Non degenarate.

TP cost = $((2\times2)+(1\times1)+(3\times2)+(4\times3)+(1\times6))$

$= (4+1+6+12+6)$

$=$ ₹29

Scanned by CamScanner
Scanned by CamScanner

occupied cells: (1,3) (2,2) (2,3) (3,1) (3,3)

$C_{ij} = u_i + v_j$

let $v_3 = 0$

$C_{13} = u_i + v_3$

$2 = u_1 + 0$

$u_1 = 2$

$C_{23} = u_2 + v_3$

$3 = u_2$

$u_2 = 3$

$C_{33} = u_3 + v_3$

$u_3 = 6$

$C_{22} = u_2 + v_2$

$1 = 3 + v_2$

$v_2 = -2$

$C_{31} = u_3 + v_1$

$3 = 6 + v_1$

$v_1 = -3$

unnoccupied cells: (1,1) (1,2) (2,1) (3,2)

$d_{ij} = C_{ij} - u_i - v_j$

$d_{11} = C_{11} - u_1 - v_1 = 7 - 2 + 3 = 8 \geq 0$

$d_{12} = C_{12} - u_1 - v_2 = 3 - 2 + 2 = 3 \geq 0$

$d_{21} = C_{21} - u_2 - v_1 = 2 - 3 + 3 = 2 \geq 0$

$d_{32} = C_{32} - u_3 - v_2 = 4 - 6 + 2 = 0 \geq 0$

Scanned by CamScanner
Scanned by CamScanner

5) Find the non degenarate strating solution for the following Transportation problem:

10	20	5	7	10
13	9	12	8	20
4	5	7	9	30
14	7	1	0	40
3	12	5	19	50
60	60	20	10	

$\sum a_i = \sum b_j = 150$

Tp is balanced.

$\boxed{10}$ 10	20	5	7	~~10~~	(2) (5) (10)
13	$\boxed{20}$ 9	12	8	~~20~~	(1) (3) (4) (4)
4	$\boxed{30}$ 5	7	9	~~30~~	(1) (1) (1) (1) (9)
14	$\boxed{10}$ 7	$\boxed{20}$ 1	$\boxed{10}$ 0	~~40~~ ~~30~~ 10	(1) (6) (7) (7) (5) (5)
$\boxed{50}$ 3	$\boxed{\varepsilon}$ 12	5	19	~~50~~	(2) (2) (9) (9) (7) (7)
~~60~~ 50	~~60~~ ~~40~~ 10	~~20~~	~~10~~		
(1)	(2)	(4)	(7)		
(1)	(2)	(4)			
(1)	(1)				
(1)	(2)				
	(2)				

$m+n-1 = 5+4-1$

$= 9-1$

$= 8$

Initial TP cost = Rs $(0 \times 10) + (20 \times 1) + (10 \times 10)$
$+ (50 \times 3) + (20 \times 9) + (30 \times 5) +$
$(10 \times 7) + (\varepsilon \times 12)$

$= (10 + 20 + 100 + 150 + 180 + 150$
$+ 70 + 0)$

$= 670$

Transportation cost = Rs 670

$\boxed{10}$ 10	20	5	7
13	$\boxed{20}$ 9	12	8
4	$\boxed{30}$ 5	7	9
14	$\boxed{10}$ 7	$\boxed{20}$ 1	$\boxed{10}$ 0
$\boxed{50}$ 3	$\boxed{\varepsilon}$ 12	5	19

Scanned by CamScanner
Scanned by CamScanner

6) solve the transportation problem:

	To				Supply
From	1	2	3	4	6
	4	3	2	0	8
	0	2	2	1	10
Demand	4	6	8	6	

$\Sigma a_i = \Sigma b_j = 24$

TP is balanced

Scanned by CamScanner
Scanned by CamScanner

∴ The no of +ve allocations = 5

1	$\boxed{6}$ 2	3	4	6	(1) (1) (1) –
4	3	$\boxed{2}$ 2	$\boxed{6}$ 0	8	(2) (1) (1) (1) –
$\boxed{4}$ 0	$\boxed{\varepsilon}$ 2	$\boxed{6}$ 2	1	10	(1) (2) (0) (0) (0)
4	6	8	6		
(1)	(1)	(1)	(1)		
(1)	(1)	(1)	–		
–	(1)	(1)	–		
–	(1)	(0)	–		

The no of +ve allocations = 5

$m+n-1 = 3+4-1 = 6$

The soln is degenerate.

TP Cost = Rs $[(6\times2)+(2\times2)+(6\times0)+(4\times0)+(6\times2)+(2\times\varepsilon)]$

= Rs $[28 + 2\varepsilon]$ $\quad \varepsilon \to 0$

= Rs $(28+(2\times0))$

TP Cost = Rs 28

Scanned by CamScanner
Scanned by CamScanner

ASSIGNMENT PROBLEM :

1) consider the problem of assigning five jobs to five persons. The assignment cost are given below

Jobs

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline A & 8 & 4 & 2 & 6 & 1 \\ B & 0 & 9 & 5 & 5 & 4 \\ C & 3 & 8 & 9 & 2 & 6 \\ D & 4 & 3 & 1 & 0 & 3 \\ E & 9 & 5 & 8 & 9 & 5 \end{array}$$

Determine optimum assignment schedule solution: the cost matrix is given

Scanned by CamScanner
Scanned by CamScanner
Scanned by CamScanner

$$\begin{pmatrix} 8 & 4 & 2 & 6 & \underline{1} \\ \underline{0} & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & \underline{2} & 6 \\ 4 & 3 & \underline{1} & \underline{0} & 3 \\ 9 & 5 & 8 & 9 & \underline{5} \end{pmatrix}$$

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & \underline{1} & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 3 & \cancel{0} & 5 & (0) \\ (0) & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & (0) & 4 \\ 4 & 3 & (0) & \cancel{0} & 3 \\ 4 & (0) & 2 & 4 & \cancel{0} \end{pmatrix}$$

Optimum Assignment Schedule:

$A \to 5$, $B \to 1$, $C \to 4$, $D \to 3$,

$E \to 2$

Optimum Assignment cost $= 1 + 0 + 2 + 1 + 5$

$= 9$ units of cost.

Scanned by CamScanner
Scanned by CamScanner

2) Four different jobs can be done on four different machines. The setup and take down time cost are assumed to be prohibitadly high for change overs the matrix below gives the cost in Rs of processing job i on machine j

		M_1	M_2	M_3	M_4
		Machines			
Jobs	J_1	5	7	11	6
	J_2	8	5	9	6
	J_3	4	7	10	7
	J_4	10	4	8	3

Soln:

The cost matrix is given by

$$\begin{pmatrix} \underline{5} & 7 & 11 & 6 \\ 8 & \underline{5} & 9 & 6 \\ \underline{4} & 7 & 10 & 7 \\ 10 & 4 & 8 & \underline{3} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & \underline{4} & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{pmatrix}$$

Scanned by CamScanner
Scanned by CamScanner

$$\begin{pmatrix} (0) & 2 & 2 & 1 \\ 3 & \times & (0) & 1 \\ \times & 3 & 2 & 3 \\ 7 & 1 & 1 & (0) \end{pmatrix}$$

Tick the not assigned rows, and ✓ the assigned column and tick the assigned value row.

$$\begin{pmatrix} \times & 7 & 4 & \times \\ 4 & (0) & \times & 1 \\ (0) & 2 & 1 & 2 \\ 8 & 1 & 1 & (0) \end{pmatrix}$$

Unmarked rows
Marked columns

Add the value which intersect both row and coloumn.

$$\begin{pmatrix} (0) & \times & \times & \times \\ 5 & (0) & \times & 2 \\ \times & 1 & (0) & 2 \\ 8 & \times & \times & (0) \end{pmatrix}$$

Find the least value in remaining

Optimum assignment ~~Solution~~ Schedule: $J_1 \rightarrow M_4$, $J_2 \rightarrow M_2$, $J_3 \rightarrow M_1$, $J_4 \rightarrow M_3$

Optimum assignment Cost $= Rs\ (6+5+4+8)$
$= Rs\ 23$/-

Scanned by CamScanner
Scanned by CamScanner

unbalanced assignment problem:

1) A company has four machines to do three jobs each jobs can be assigned to one and only one machine the cost of each job on each machine is given in the following table

		Machines			
		1	2	3	4
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22
		0	0	0	0

The Dummy row.

$$\begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} (0) & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & (0) & 0 & 0 \end{pmatrix}$$

Scanned by CamScanner
Scanned by CamScanner

$$\begin{pmatrix} (0) & & 5 & 9 \\ \cancel{0} & (0) & 4 & 6 \\ \cancel{0} & \cancel{0} & 4 & 7 \\ 5 & \cancel{0} & (0) & \cancel{0} \end{pmatrix}$$

$$\begin{pmatrix} (0) & 1 & 1 & 5 \\ \cancel{0} & (0) & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & (0) & 3 \\ 9 & 4 & \cancel{0} & (0) \end{pmatrix}$$

$\left.\begin{array}{l}\text{Assignment} \\ \text{Schedule}\end{array}\right\}$: $A \rightarrow M_1$, $B \rightarrow M_2$, $C \rightarrow M_3$, $D \rightarrow M_4$

$\left.\begin{array}{l}\text{Assignment} \\ \text{cost}\end{array}\right\}$: $(18+13+19+0) = 50$

2) Maximization

Machines

Job		P	Q	R	S
	A	51	53	54	50
	B	47	50	48	50
	C	49	50	60	61
	D	63	64	60	60

Scanned by CamScanner
Scanned by CamScanner

$$\begin{pmatrix} 13 & 11 & 10 & 14 \\ 17 & 14 & 16 & 14 \\ 15 & 14 & 4 & 3 \\ 1 & 0 & 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 0 & 4 \\ 3 & 0 & 2 & \cancel{0} \\ 12 & 11 & 1 & 0 \\ 1 & \cancel{0} & 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & (0) & 4 \\ 2 & (0) & 2 & \cancel{0} \\ 11 & 11 & 1 & (0) \\ (0) & \cancel{0} & 4 & 4 \end{pmatrix}$$

$\left.\begin{array}{l}\text{Assignment} \\ \text{Schedule}\end{array}\right\} : \; A \to R,\; B \to Q,\; C \to S,\; D \to P$

$\left.\begin{array}{l}\text{Assignment} \\ \text{cost}\end{array}\right\} : \; (54 + 50 + 61 + 63)$

$= 228.$

Scanned by CamScanner
Scanned by CamScanner

Travelling Salesman problem :

1) Solve the following travelling Salesman Problem

		To			
		A	B	C	D
From	A	-	46	16	40
	B	41	-	50	40
	C	82	32	-	60
	D	40	40	36	-

Scanned by CamScanner
Scanned by CamScanner

The cost matrix is given by

$$\begin{pmatrix} \infty & 46 & 16 & 40 \\ 41 & \infty & 50 & 40 \\ 82 & 32 & \infty & 60 \\ 40 & 40 & 36 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & 30 & 0 & 24 \\ 1 & \infty & 10 & 0 \\ 50 & 0 & \infty & 28 \\ 4 & 4 & 0 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & 30 & (0) & 24 \\ (0) & \infty & 10 & \cancel{0} \\ 49 & (0) & \infty & 28 \\ 3 & 4 & \cancel{0} & \infty \end{pmatrix} \begin{matrix} \checkmark \\ \\ \\ \checkmark \end{matrix}$$

$$\begin{matrix} & & \checkmark & \end{matrix}$$

$$\begin{pmatrix} \infty & 27 & (0) & 21 \\ \cancel{0} & \infty & 13 & (0) \\ 49 & (0) & \infty & 28 \\ (0) & 1 & \cancel{0} & \infty \end{pmatrix}$$

Optimum Assignment Schedule $\left.\right\}$ $A \to C$, $B \to D$, $C \to B$, $D \to A$

$A \to C$, $C \to B$, $B \to D$, $D \to A$

$A \to \cancel{C} B \to D \to A$

Scanned by CamScanner
Scanned by CamScanner

$$\text{cost} = 16+32+40+40$$
$$= 128$$

2) solve the following travelling salesman Problem so as to minimize the cost per cycle

		To				
		A	B	C	D	E
From	A	-	3	6	2	3
	B	3	-	5	2	3
	C	6	5	-	6	4
	D	2	2	6	-	6
	E	3	3	4	6	-

The cost matrix is given by

$$\begin{pmatrix} \infty & 3 & 6 & 2 & 3 \\ 3 & \infty & 5 & 2 & 3 \\ 6 & 5 & \infty & 6 & 4 \\ 2 & 2 & 6 & \infty & 6 \\ 3 & 3 & 4 & 6 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & 1 & 4 & 0 & 1 \\ 1 & \infty & 3 & 0 & 1 \\ 2 & 1 & \infty & 2 & 0 \\ 0 & 0 & 1 & 3 & \infty \end{pmatrix}$$

Scanned by CamScanner
Scanned by CamScanner

$$\begin{pmatrix} \infty & 1 & 3 & (0) & 1 \\ 1 & \infty & 2 & \cancel{0} & 1 \\ 2 & 1 & \infty & 2 & (0) \\ (0) & \cancel{0} & 3 & \infty & 4 \\ \cancel{0} & (0) & \cancel{0} & 3 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & \cancel{0} & 2 & (0) & \cancel{0} \\ (0) & \infty & 1 & \cancel{0} & \cancel{0} \\ 2 & 1 & \infty & 3 & (0) \\ \cancel{0} & (0) & 3 & \infty & 4 \\ \cancel{0} & \cancel{0} & (0) & 4 & \infty \end{pmatrix}$$

$A \to D$, $B \to A$, $C \to E$, $D \to B$, $E \to C$

$A \to D$, $D \to B$, $B \to A$, $C \to E$, $E \to C$

$\therefore$ The route is not satisfied

The next minimum is 1.

$$\begin{pmatrix} \infty & \cancel{0} & 2 & (0) & \cancel{0} \\ \cancel{0} & \infty & (1) & \cancel{0} & \cancel{0} \\ 2 & 1 & \infty & 3 & (0) \\ \cancel{0} & (0) & 3 & \infty & 4 \\ (0) & \cancel{0} & \cancel{0} & 4 & \infty \end{pmatrix}$$

$A \to D$, $B \to C$, $C \to E$, $D \to B$, $E \to A$

$A \xrightarrow{2} D$, $D \xrightarrow{2} B$, $B \xrightarrow{5} C$, $C \xrightarrow{4} E$, $E \xrightarrow{3} A$

$A \to D \to B \to C \to E \to A$

Cost $= 2 + 5 + 4 + 2 + 3 = 16$

Scanned by CamScanner
Scanned by CamScanner

HW

1) Solve the assignment problem?

	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

The cost matrix is given by

$$\begin{pmatrix} \underline{1} & 4 & 6 & 3 \\ 9 & \underline{7} & 10 & 9 \\ \underline{4} & 5 & 11 & 7 \\ 8 & 7 & 8 & \underline{5} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 5 & 2 \\ 2 & 0 & 3 & 2 \\ 0 & 1 & 7 & 3 \\ 3 & 2 & 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} (0) & 3 & 2 & 2 \\ 2 & (0) & \cancel{0} & 2 \\ \cancel{0} & 1 & 4 & 3 \\ 3 & 2 & (0) & \cancel{0} \end{pmatrix}$$ ✓ ✓

Scanned by CamScanner
Scanned by CamScanner

3) Given the following matrix of Setup Costs show how to sequence production so as to minimize Setup cost per cycle.

		To				
		A	B	C	D	E
From	A	-	2	5	7	1
	B	6	-	3	8	2
	C	8	7	-	4	7
	D	12	4	6	-	5
	E	1	3	2	8	-

The cost matrix is given by

$$\begin{pmatrix} \infty & 2 & 5 & 7 & 1 \\ 6 & \infty & 3 & 8 & 2 \\ 8 & 7 & \infty & 4 & 7 \\ 12 & 4 & 6 & \infty & 5 \\ 1 & 3 & 2 & 8 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & 1 & 4 & 6 & 0 \\ 4 & \infty & 1 & 6 & 0 \\ 4 & 3 & \infty & 0 & 3 \\ 8 & 0 & 2 & \infty & 1 \\ 0 & 2 & 1 & 7 & \infty \end{pmatrix}$$

Scanned by CamScanner
Scanned by CamScanner

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 1 & 3 & 6 & (0) \\ B & 4 & \infty & (0) & 6 & \times \\ C & 4 & 3 & \infty & (0) & 3 \\ D & 8 & (0) & 1 & \infty & 1 \\ E & (0) & 2 & \times & 7 & \infty \end{array}$$

$A \to E,\ B \to C,\ C \to D,\ D \to B,\ E \to A$

$A \to E,\ E \to A,\ B \to C,\ C \to D,\ D \to B.$

The root is not satisfied.
The next minimum is 1

$$\begin{pmatrix} \infty & 1 & 3 & 6 & (0) \\ 4 & \infty & \times & 6 & \times \\ 4 & 3 & \infty & (0) & 3 \\ 8 & \times & (1) & \infty & 1 \\ (0) & 2 & \times & 7 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & (0) & 3 & 5 & \times \\ 3 & \infty & (0) & 5 & \times \\ 4 & 3 & \infty & (0) & 4 \\ 7 & \times & 1 & \infty & (1) \\ (0) & 2 & 1 & 7 & \infty \end{pmatrix}$$ $A \to B,\ B \to C,\ C \to D,$
$D \to E,\ E \to A.$

Optimum cost $= 2+3+4+5+1$
$= 15$

Optimum cost $= 15$

Scanned by CamScanner
Scanned by CamScanner

4) A salesman has to visit five cities A, B, C, D and E distances (in hundred km) b/w the five cities and follows:

		To				
		A	B	C	D	E
From	A	-	7	6	8	4
	B	7	-	8	5	6
	C	6	8	-	9	7
	D	8	5	9	-	8
	E	4	6	7	8	-

If the salesman starts from city A and has to come back to A, which route he should select so that the total distance travelled by him is minimized.

The cost matrix is given by

$$\begin{pmatrix} \infty & 7 & 6 & 8 & 4 \\ 7 & \infty & 8 & 5 & 6 \\ 6 & 8 & \infty & 9 & 7 \\ 8 & 5 & 9 & \infty & 8 \\ 4 & 6 & 7 & 8 & \infty \end{pmatrix}$$

Scanned by CamScanner
Scanned by CamScanner

$$\begin{pmatrix} \infty & 3 & 2 & 4 & 0 \\ 2 & \infty & 3 & 0 & 1 \\ 0 & 2 & \infty & 3 & 1 \\ 3 & 0 & 4 & \infty & 3 \\ 0 & 2 & 3 & 4 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & 3 & (0) & 4 & \cancel{0} \\ 2 & \infty & 1 & (0) & 1 \\ (0) & 2 & \infty & 3 & 1 \\ 3 & (0) & 2 & \infty & 3 \\ \cancel{0} & 2 & 1 & 4 & \infty \end{pmatrix} \begin{matrix} \\ \\ \checkmark \\ \\ \checkmark \end{matrix}$$

$\checkmark$

$$\begin{pmatrix} \infty & 3 & (0) & 4 & \cancel{0} \\ 3 & \infty & 1 & (0) & 1 \\ \cancel{0} & 1 & \infty & 2 & (0) \\ 4 & (0) & 2 & \infty & 3 \\ (0) & 1 & \cancel{0} & 3 & \infty \end{pmatrix}$$

$A \to C$, $B \to D$, $C \to E$, $D \to B$, $E \to A$

$A \to C$, $C \to E$, $E \to A$, $B \to D$, $D \to B$

The root is not Satisfied

The next minimum is

Scanned by CamScanner

Scanned by CamScanner

$$\begin{pmatrix} \infty & 3 & \cancel{0} & 4 & (0) \\ 3 & \infty & (1) & 0 & 1 \\ (0) & 1 & \infty & (2) & \cancel{0} \\ 4 & \cancel{0} & (2) & \infty & 3 \\ \cancel{0} & (1) & \cancel{0} & 3 & \infty \end{pmatrix}$$

$$\begin{pmatrix} \infty & 3 & \cancel{0} & 4 & \cancel{0} \\ (3) & \infty & 1 & \cancel{0} & 1 \\ \cancel{0} & 1 & \infty & 2 & \cancel{0} \\ 4 & \cancel{0} & 2 & \infty & 3 \\ \cancel{0} & 1 & (0) & 3 & \infty \end{pmatrix}$$

Scanned by CamScanner
Scanned by CamScanner

Complete Enumeration method:

Time in minutes

		Workers		
		A	B	C
Job	I	120	100	80
	II	80	90	110
	III	110	140	120

3×3

$n = 3$

$n! = 3! = 1 \cdot 2 \cdot 3 = 6$

$A_1 - B_2 - C_3 = 120 + 90 + 120 = 330$

$A_1 - B_3 - C_2 = 120 + 140 + 110 = 370$

$A_2 - B_1 - C_3 = 80 + 100 + 120 = 300$

$A_2 - B_3 - C_1 = 80 + 140 + 80 = 300$

$A_3 - B_1 - C_2 = 110 + 100 + 110 = 320$

$A_3 - B_2 - C_1 = 110 + 90 + 80 = 280$

Least that the minimum

Scanned by CamScanner
Scanned by CamScanner