CORE COURSE VI
QUANTITATIVE TECHINIQUES FOR BUSINESS DECISIONS

Objective : To acquaint the students with the Statistical tools and techniques for managerial decisions.

UNIT I:

Meaning of Quantitative Techniques – Role of Quantitative Techniques – Advantages and Limitations of Quantitative Techniques – Correlation Analysis – Simple – Partial and Multiple –Regression Analysis – Time Series.

UNIT II:

Probability – Problems applying Additional and Multiplication Theorem – Mathematical Expectations – Theoretical Distributions – Binomial – Poisson – Normal Distribution.

UNIT III:

Significance Tests in Small Samples (t test) – Testing the significance of the mean of a random sample – Testing difference between means of two samples (Independent and Dependent Samples) – Chi-square test- Analysis of Variance (One way and two way classification).

UNIT IV:

Linear Programming – Graphical Method – Simplex Method – Transportation Problems – Initial Basic Feasible Solution - Modi Method – Assignment Problems.

UNIT V:

Interpolation and Extrapolation – Methods of Interpolation – Binomial Expansion Method – Newton's Method – Lagrange's Method – Parabolic Curve Method – Extrapolation – Vitai Statistics – Life Tables.

Note: Theory 25 Marks : Problems 50 Marks

EQUAL IMPORTANCE TO BE GIVEN TO ALL UNITS

Text and Reference Books (Latest revised edition only)

1. S.P. Gupta, Statistical Methods - Sultan Chand & Sons, New Delhi – 600 002.
2. S. Gurusamy,Operations Research, Vijay Nicole Imprints Pvt. Ltd, Chennai.
3. D. Joseph Anbarasu,Business Statistics –Vijay Nicole Imprints Pvt. Ltd., Chennai.
4. C.R.Kothari, Quantitative Techniques –Vikas Publishing House, New Delhi.
5. Levin, Richard I. and David S Rubin: Statistics for Management, Prentice Hall, Delhi.
6. Hooda, R.P: Statistics for Business and Economics, Macmilla 3rd edition, New Delhi.
7. Hein, L.W: Quantitative Approach to Managerial Decisions, Prentice Hall, Delhi

Unit - V

Interpolation and Extrapolation

Meaning of Interpolation :-

Interpolation consists in reading a value which lies between two extreme points.

Meaning of Extrapolation :-

Extrapolation is reading a value that lies outside the two extreme points.

Method of Interpolation :-

I. Graphic Method

II. Algebraic Method

(i) Binomial expansion method

(ii) Newton's method

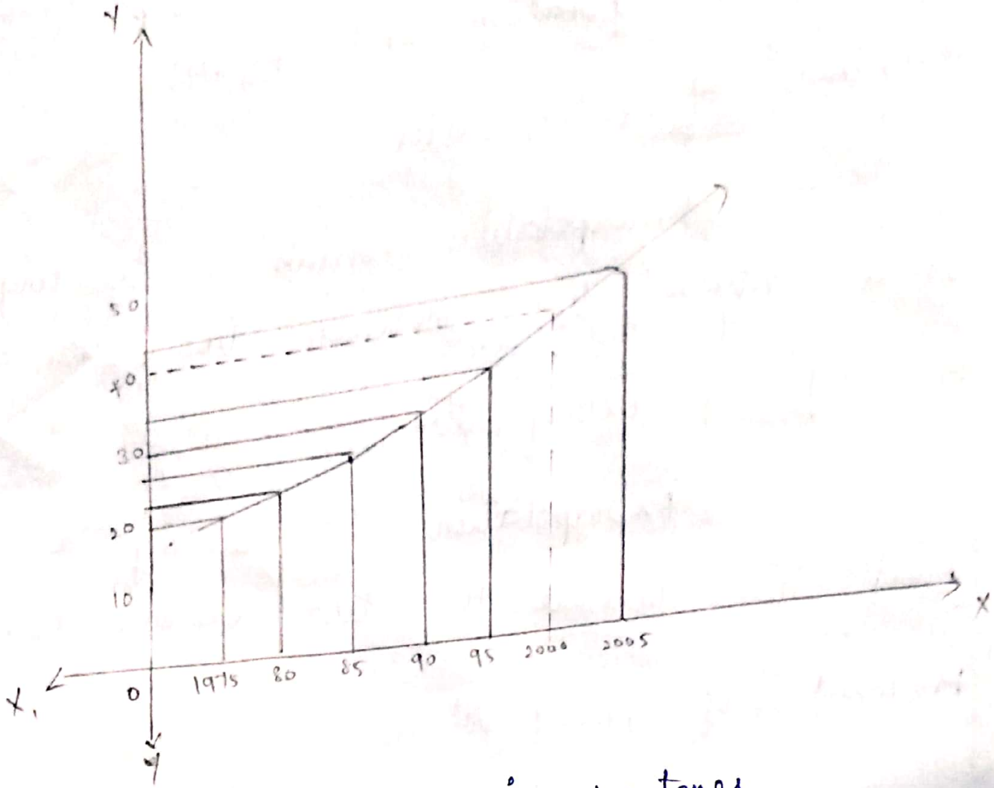
(iii) Langrange's Method

(iv) Parabolic curve method.

Graphic Method :-

1) Estimate the Production for the year 2000 with the help of the following :-

Year	1975	1980	1985	1990	1995	2000	2005
Production (in tones)	20	22	26	30	35	?	43
	y_0	y_1	y_2	y_3	y_4	y_5	y_6



Production of 2000 is = 40 tones.

Algebraic method :-

Binomial expansion method :-

No. of known values	Equation of determining the unknown values
2.	$y_2 - 2y_1 + y_0 = 0$
3	$y_3 - 3y_2 + 3y_1 - y_0 = 0$
4	$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$
5	$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$
6	$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$
7	$y_7 - 7y_6 + 21y_5 - 35y_4 + 35y_3 - 21y_2 + 7y_1 - y_0 = 0$
8	$y_8 - 8y_7 + 28y_6 - 56y_5 + 70y_4 - 56y_3 + 28y_2 - 8y_1 + y_0 = 0$
9	$y_9 - 9y_8 + 36y_7 - 84y_6 + 126y_5 - 126y_4 + 84y_3 - 36y_2 + 9y_1 - y_0 = 0$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

$$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$$

$$43 - 6(y_5) + 15(35) - 20(30) + 15(26) - 6(22) + 20 = 0$$

$$43 - 6(y_5) + 525 - 600 + 390 - 135 + 20 = 0$$

$$\cancel{43} - \cancel{978} \quad -6y_5 + 978 - 732 = 0$$

$$-6y_5 + 246 = 0$$

$$-6y_5 = -246$$

$$y_5 = \frac{-246}{-6}$$

$$y_5 = 41$$

In the year 2000 Production is 41 Tonnes.

2.) Find out algebraic interpolation of the index number for 2013 from the following table of index number of Production of certain article in India.

year	2011	2012	2014	2015
index no.	100	107	157	212

year	2011	2012	2013	2014	2015
index no	100	107	?	157	212
	y_0	y_1	y_2	y_3	y_4

Sol:

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$212 - 4(157) + 6(y_2) - 4(107) + 100 = 0$$

$$212 - 628 + 6y_2 - 428 + 100 = 0$$

$$6y_2 + 312 - 1056 = 0$$

$$6y_2 - 744 = 0 \quad (1)$$

$$6y_2 = 744$$

$$y_2 = 744/6$$

$$y_2 = 124$$

Extrapolate the profit for the year 2016 from the following data.

year	2011	2012	2013	2014	2015	2016
Profit	31	42	51	65	80	?
	y_0	y_1	y_2	y_3	y_4	y_5

Sol:

$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$

$$y_5 - 5(80) + 10(65) - 10(51) + 5(42) - 31 = 0$$

$$y_5 - 400 + 650 - 510 + 210 - 31 = 0$$

$$y_5 + 860 - 941 = 0$$

$$y_5 - 81 = 0$$

$$\boxed{y_5 = 81}$$

The annual sales of a concern are given below assuming the condition of market to be the same estimate for 2015.

year	2010	2011	2012	2013	2014	2015
sales (Rs)	125	163	204	238	282	?
	y_0	y_1	y_2	y_3	y_4	y_5

$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$

$$y_5 - 5(282) + 10(238) - 10(204) + 5(163) - 125 = 0$$

2

$$y_5 - 1410 + 2380 - 2040 + 815 - 125 = 0$$

$$y_5 - 3195 - 3575 = 0$$

$$y_5 - 380 = 0$$

$$\boxed{y_5 = 380}$$

Estimate the missing term in the following table

x	1	2	3	4	5
y	2	4	8	?	37
	y_0	y_1	y_2	y_3	y_4

$$y_4 - 4y_2 + 6y_2 - 4y_1 + y_0 = 0$$

$$37 - 4(y_3) + 6(8) - 4(4) + 2 = 0$$

$$37 - 4y_3 + 48 - 16 + 2 = 0$$

$$-4y_3 + 87 - 16 = 0$$

$$-4y_3 + 71 = 0$$

$$-4y_3 = -71$$

$$y_3 = \frac{-71}{-4}$$

$$\boxed{y_3 = 17.75}$$

Lagrange's Method :-

$$y_x = y_0\left[\frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)}\right]$$
$$+ y_1\left[\frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)}\right]$$
$$+ y_2\left[\frac{(x-x_0)(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)}\right]$$
$$+ y_n\left[\frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})}\right]$$

The following table given the normal weight of a baby during the first six month of life.

Age (month)	0	2	3	5	6
weight (lbs)	5	7	8	10	12

Estimate the weight of a baby at age of 4 month

$$y_4 = y_0\left[\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\right]$$
$$+ y_1\left[\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}\right]$$
$$+ y_2\left[\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}\right]$$
$$+ y_3\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}\right]$$
$$+ y_4\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}\right]$$

$$y_4 = 5\left[\frac{(4-2)(4-3)(4-5)(4-6)}{(0-2)(0-3)(0-5)(0-6)}\right] + 7\left[\frac{(4-0)(4-3)(4-5)(4-6)}{(2-0)(2-3)(2-5)(2-6)}\right]$$

$$+ 8\left[\frac{(4-0)(4-2)(4-5)(4-6)}{(3-0)(3-2)(3-5)(3-6)}\right] + 10\left[\frac{(4-0)(4-2)(4-3)(4-6)}{(5-0)(5-2)(5-3)(5-6)}\right]$$

$$+ 12\left[\frac{(4-0)(4-2)(4-3)(4-5)}{(6-0)(6-2)(6-3)(6-5)}\right]$$

$$y_4 = 5\left[\frac{2\times1\times(-1)\times(-2)}{-2\times(-3)\times(-5)\times(-6)}\right] + 7\left[\frac{4\times1\times(-1)\times(-2)}{2\times(-1)\times(-3)\times(-6)}\right]$$

$$+8\left[\frac{4\times2\times(-1)\times(-2)}{3\times1\times(-2)\times(-3)}\right] + 10\left[\frac{4\times2\times1\times(-2)}{5\times3\times2\times(-1)}\right] + 12\left[\frac{4\times2\times1\times(-1)}{6\times4\times3\times1}\right]$$

$$y_4 = 5\left[\frac{4}{180}\right] + 7\left[\frac{8}{-24}\right] + 8\left[\frac{16}{18}\right] + 10\left[\frac{-16}{-36}\right] + 12\left[\frac{-8}{72}\right]$$

$$= \frac{1}{9} - \frac{7}{3} + \frac{64}{9} + \frac{16}{3} - \frac{4}{3}$$

$$= 0.11 - 2.33 + 7.11 + 5.33 - 1.33$$

4

$$= 8.89$$

$$\boxed{y_4 = 9}$$

The value of x and y are given below.

x	5	6	9	11
y	12	10	14	16.

Find the value of y when $x=10$ by using Langrange method.

$$y_{10} = 12\left[\frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}\right] + 10\left[\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}\right] +$$

$$14\left[\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}\right] + 16\left[\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}\right]$$

year	2011	2012	2013	2014	2015
index no	100 y_0	107 y_1	? y_2	157 y_3	212 y_4

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

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l:

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$$+ y_1\left[\frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)}\right]$$

$$+ y_2\left[\frac{(x-x_0)(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)}\right]$$

$$+ y_n\left[\frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})}\right]$$

1. The following table given the normal weight of a baby during the first six month of life.

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$$+ y_4\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}\right]$$

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$$+ 12\left[\frac{(4-0)(4-2)(4-3)(4-5)}{(6-0)(6-2)(6-3)(6-5)}\right]$$

$$y_4 = 5\left[\frac{2\times1\times(-1)\times(-2)}{-2\times(-3)\times(-5)\times(-6)}\right] + 7\left[\frac{4\times1\times(-1)\times(-2)}{2\times(-1)\times(-3)\times(-6)}\right]$$

$$+ 8\left[\frac{4\times2\times(-1)\times(-2)}{3\times1\times(-2)\times(-3)}\right] + 10\left[\frac{4\times2\times1\times(-2)}{5\times3\times2\times(-1)}\right] + 12\left[\frac{4\times2\times1\times(-1)}{6\times4\times3\times1}\right]$$

$$y_4 = 5\left[\frac{4}{180}\right] + 7\left[\frac{8}{-24}\right] + 8\left[\frac{16}{18}\right] + 10\left[\frac{-16}{-36}\right] + 12\left[\frac{-8}{72}\right]$$

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The value of x and Y are given below.

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y	12	10	14	16

Find the value of y when $x = 10$ by using Langrange method.

Sol:

$$y_{10} = 12\left[\frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}\right] + 10\left[\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}\right] +$$

$$14\left[\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}\right] + 16\left[\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}\right]$$

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$$= 12\left[\frac{4\times1\times(-1)}{(-1)(-4)(-6)}\right] + 10\left[\frac{5\times1\times(-1)}{1\times(-3)(-5)}\right] + 14\left[\frac{5\times4(-1)}{4\times3(-2)}\right] + 16\left[\frac{5\times4\times1}{6\times5\times2}\right]$$

$$= 12\left[\frac{-4}{-24}\right] + 10\left[\frac{-5}{15}\right] + 14\left[\frac{-20}{-24}\right] + 16\left[\frac{20}{60}\right]$$

$$= 12\left[\frac{1}{6}\right] + 10\left[\frac{-1}{3}\right] + 14\left[\frac{10}{12}\right] + 16\left[\frac{1}{3}\right]$$

$$= \frac{12}{6} - \frac{10}{3} + \frac{140}{12} + \frac{16}{3}$$

$$= 2 - 3.33 + 11.67 + 5.33$$

$$= 19 - 3.33$$

$$\boxed{y_{10} = 15.67}$$

(3)

The values of x and y are given below

x	5	9	11	12
y	121	73	25	16

$$y_8 \Rightarrow 121\left[\frac{(8-9)(8-11)(8-12)}{(5-9)(5-11)(5-12)}\right] + 73\left[\frac{(8-5)(8-11)(8-12)}{(9-5)(9-11)(9-12)}\right]$$

$$+ 25\left[\frac{(8-5)(8-9)(8-12)}{(11-5)(11-9)(11-12)}\right] + 16\left[\frac{(8-5)(8-9)(8-11)}{(12-5)(12-9)(12-11)}\right]$$

$$\Rightarrow 121\left[\frac{(-1)(-3)(-4)}{(-4)(-6)(-7)}\right] + 73\left[\frac{(3)(-3)(-4)}{(4)(-2)(-3)}\right] +$$

$$25\left[\frac{(3)(-1)(-4)}{(6)(2)(-1)}\right] + 16\left[\frac{(3)(-1)(-3)}{(7)(3)(1)}\right]$$

$$\Rightarrow 121\left[\frac{-12}{-168}\right] + 73\left[\frac{36}{24}\right] + 25\left[\frac{12}{12}\right] + 16\left[\frac{9}{\ }\right]$$

$$= \frac{121}{14} + \frac{219}{2} - 25 + \frac{48}{7}$$

$$= 8.6 + 109.5 - 25 + 6.8$$

$$= 99.9$$

$$\boxed{y_8 = 100}$$

The values of x and y are given below.

x	14	17	31	35
y	68.7	64.0	44.0	39.1

6

$x = 27$, $y = ?$

$$y_{27} \Rightarrow 68.7\left[\frac{(27-17)(27-31)(27-35)}{(14-17)(14-31)(14-35)}\right] + 64.0\left[\frac{(27-14)(27-31)(27-35)}{(17-14)(17-31)(17-35)}\right]$$

$$+ 44\left[\frac{(27-14)(27-17)(27-35)}{(31-14)(31-17)(31-35)}\right] + 39.1\left[\frac{(27-14)(27-17)(27-31)}{(35-14)(35-17)(35-31)}\right]$$

$$\Rightarrow 68.7\left[\frac{10(-4)(-8)}{(-3)(-17)(-21)}\right] + 64\left[\frac{13(-4)(-8)}{3(-14)(-18)}\right] + 44\left[\frac{13\times10\times(-8)}{17\times14(-4)}\right] +$$

$$39.1\left[\frac{13\times10\times(-4)}{21\times18\times4}\right]$$

$$\Rightarrow 68.7\left(\frac{320}{-1071}\right) + 64\left(\frac{416}{756}\right) + 44\left(\frac{-1040}{-952}\right) + 39.1\left[\frac{-520}{1512}\right]$$

$$\Rightarrow -20.52 + 35.21 + 48.06 - 13.44$$

$$\Rightarrow 83.27 - 33.96$$

$$\boxed{y_{27} \Rightarrow 49.31}$$

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(7)

Newton's method :-

1. Newton's Advancing Difference Method :-

$$y_x = y_0 + x\Delta_0^1 + \frac{x(x-1)}{2!}\Delta_0^2 + \frac{x(x-1)(x-2)}{3!}\Delta_0^3 + \frac{x(x-1)(x-2)(x-3)}{4!}\Delta_0^4 + \dots$$

Table showing Finite or advancing Differences

x	y	Differences							
		First differences Δ^1		second differences Δ^2		Third differences Δ^3		Fourth differences Δ^4	
x_0	y_0								
		$y_1 - y_0$	Δ^1_0						
x_1	y_1			$\Delta^1_1 - \Delta^1_0$	Δ^2_0				
		$y_2 - y_1$	Δ^1_1			$\Delta^2_1 - \Delta^2_0$	Δ^3_0		
x_2	y_2			$\Delta^1_2 - \Delta^1_0$	Δ^2_1			$\Delta^3_1 - \Delta^3_0$	Δ^4_9
		$y_3 - y_2$	Δ^1_2			$\Delta^2_2 - \Delta^2_1$	Δ^3_1		
x_3	y_3			$\Delta^1_3 - \Delta^1_2$	$\Delta^3_{\cdot 2}$				
		$y_4 - y_3$	Δ^1_3						
x_4	y_4								

Given the following pairs of corresponding values of x and y

x	20	25	30	35	40
y	73	198	573	1,198	1450

Estimate the value of Y for $x = 22$

x		y		Differences: First Δ^1		Second Δ^2		Third Δ^3		Fourth Δ^4	
20	x_0	73	y_0								
				125	Δ^1_0						
25	x_1	198	y_1			250	Δ^2_0				
				375	Δ^1_1			0	Δ^3_0		
30	x_2	573	y_2			250	Δ^2_1			-623	Δ^4_0
				625	Δ^1_2			-623	Δ^3_1		
35	x_3	1198	y_3			-373	Δ^2_2				
				252	Δ^1_3						
40	x_4	1450	y_4								

$$y_{x} = y_0 + x\Delta^1_0 + \frac{x(x-1)}{2!}\Delta^2_0 + \frac{x(x-1)(x-2)}{3!}\Delta^3_0 + \frac{x(x-1)(x-2)(x-3)}{4!} + \Delta^4_0 + \dots$$

$$\therefore x = \frac{22-20}{5} = 0.4$$

8

$$y_{22} = 73 + (0.4 \times 125) + \left[\frac{0.4(0.4-1)}{1\times 2} \times 250\right] + \left[\frac{0.4(0.4-1)(0.4-2)}{1\times 2\times 3} \times 0\right] + \left[\frac{0.4(0.4-1)(0.4-2)(0.4-3)}{1\times 2\times 3\times 4} \times -623\right]$$

$$= 73 + 50 + \left[\frac{0.4\times(-0.6)}{2} \times 250\right] + 0 + \left[\frac{0.4(-0.6)(-1.6)(-2.6)}{24} \times -623\right]$$

$$= 73 + 50 - 30 + 25.9$$

$$= 148.9 - 30$$

$$\boxed{y_{22} = 118.9}$$

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From the following data interpolate the value of y when $x = 1.5$

x	1	1.2	1.4	1.6	1.8	2.0
y	5	6	7.5	9.5	12.0	16.0

Sol:

x		y		I Δ		II Δ		III Δ		IV Δ	
1.0	x_0	5	y_0								
				1.0	Δ'_0						
1.2	x_1	6	y_1								
				1.5	Δ'_1	0.5	Δ^2_0				
1.4	x_2	7.5	y_2					0.0	Δ^3_0		
				2.0	Δ'_2	0.5	Δ^2_1			0	Δ^4_0
1.6	x_3	9.5	y_3					0.0	Δ^3_1		
				2.5	Δ'_3	0.5	Δ^2_2			1	Δ^4_1
1.8	x_4	12.0	y_4			1.5	Δ^2_3	1.0	Δ^3_2		
				4.0	Δ'_4						
2.0	x_5	16.0	y_5								

$$y_x = y_0 + \left[x\Delta'_0\right] + \left[\frac{x(x-1)}{2!}\Delta^2_0\right] + \left[\frac{x(x-1)(x-2)}{3!}\Delta^3_0\right] + \left[\frac{x(x-1)(x-2)(x-3)}{4!}\Delta^4_0\right] + \left[\frac{x(x-1)(x-2)(x-3)(x-4)}{5!}\Delta^5_0\right]$$

$$\therefore x = \frac{1.5-1}{0.2} = \frac{0.5}{0.2} = 2.5$$

$$\boxed{x = 2.5}$$

$$y_{1.5} = 5 + (2.5 \times 1.0) + \left[\frac{2.5(2.5-1)}{1\times2} \times 0.5\right] + \left[\frac{2.5(2.5-1)(2.5-2)}{1\times2\times3} \times 0\right] +$$

$$\left[\frac{2.5(2.5-1)(2.5-2)(2.5-3)}{1\times\ldots} \times 0\right] + \left[\frac{2.5(2.5-1)(2.5-2)(2.5-3)(2.5-\ldots)}{\ldots}\right.$$

$= 5 + 2.5 + 0.938 + 0 + 0 + 0.012$

$$\boxed{y_{1.5} = 8.45}$$

Interpolate the figure of Population for the year 2001 from the following data.

year	1985	1995	2005	2015
Population of a Town	25494	29,003	32,528	36,070

$$x = \frac{2001 - 1985}{10} = \frac{16}{10} = 1.6 \Rightarrow \boxed{x = 1.6}$$

sol:-

x		y		first Δ		second Δ		Third Δ	
1985	x_0	y_0	25494						
				3509	Δ'_0				
1995	x_1	y_1	29,003			~~3016~~ 16	Δ^2_0		
				3525	Δ'_1			~~8999~~ 1	Δ^3_0
2005	x_2	y_2	32,528			~~+5983~~ 17	Δ^2_1		
				3542	Δ'_2				
2015	x_3	y_3	36,070						

$$y_x = y_0 + x\Delta'_0 + \left[\frac{x(x-1)}{2!}\Delta^2_0\right] + \left[\frac{x(x-1)(x-2)}{3!}\Delta^3_0\right]$$

$$y_{2001} = 25494 + [1.6 \times 3509] + \left[\frac{1.6(1.6-1)}{1 \times 2} \times 16\right] + \left[\frac{1.6(1.6-1)(1.6-2)}{1 \times 2 \times 3} \times 1\right]$$

$$= 25,494 + 5614.4 + \frac{15.36}{2} + \left(\frac{-0.384}{6}\right)$$

$$= 25494 + 5614.4 + 7.68 + \left(\frac{-0.384}{6}\right)$$

$$= 25494 + 5614.4 + 7.68 - 0.064 \Rightarrow 31,115.68 - 0.064$$

31,116

Binomial Expansion Method :-

	x_0	x_1	x_2	x_3
year	1985	1995	2005	2015
Population	25494	29003	32528	36070
	y_0	y_1	y_2	y_3

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$36070 - 4(32528) + 6y_2 - 4(29003) + 25494 = 0$$

$$36070 - 130112 + 6y_2 - 116012 + 25494 = 0$$

$$6y_2 + 61564 - 246124 = 0$$

$$6y_2 + 184560 = 0$$

$$6y_2 = 184560$$

$$y_2 = \frac{184560}{6}$$

$$\boxed{y_2 = 30760}$$

Lagrange's Method :-

$$y_{2001} \Rightarrow \overset{25494}{y_0}\left[\frac{(2001-1995)(2001-2005)(2001-2015)}{(1985-1995)(1985-2005)(1985-2015)}\right]$$

$$+ 29003\left[\frac{(2001-1985)(2001-2005)(2001-2015)}{(1995)-1985)(1995-2005)(1995-2015)}\right]$$

$$+ 32528\left[\frac{(2001-1985)(2001-1995)(2001-2015)}{(2005-1985)(2005-1995)(2005-2015)}\right]$$

$$+ 36070\left[\frac{(2001-1985)(2001-1995)(2001-2005)}{(2015-1985)(2015-1995)(2015-2005)}\right]$$

$$\Rightarrow 25494\left[\frac{6\times(-4)(-14)}{(-10)(-20)(-30)}\right] + 29003\left[\frac{16\,(-4)(-14)}{10\times(-10)(-20)}\right]$$

$$+\ 32528\left[\frac{16\,(6)(-14)}{20\times(10)(-10)}\right] + 36070\left[\frac{16\,(6)(-4)}{30\times 20\times 10}\right]$$

$$\Rightarrow 25494\left[\frac{336}{-6000}\right] + 29003\left[\frac{896}{2000}\right] + 32528\left[\frac{1344}{2000}\right] +$$

$$36070\left[\frac{-384}{6000}\right]$$

$$\Rightarrow -25494\,(0.056) + 29003\left[0.448\right] + 32528\left[0.672\right) +$$

$$36070\,(-0.064)$$

$$\Rightarrow -1427.66 + 12993.34 + 21858.81 - 2308.48$$

$$\Rightarrow 34852.15 - 3736.14$$

$y_{2001} \Rightarrow 31,116.$

Using the following data find the newton's interpolating Polynomial and also find the value of y at $x = 24$

	x_0	x_1	x_2	x_3	x_4
x	20	35	50	65	80
y	3	11	24	50	98
	y_0	y_1	y_2	y_3	y_4

Binomial Expansion method :

$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$

$$98 - 5(50) + 10(24) - 10(11) + 5y_1 - 3 = 0$$

$$98 - 250 + 240 - 110 + 5y_1 - 3 = 0$$

$$5y_1 + 338 - 363 = 0$$

$$5y_1 - 25 = 0$$

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$5y_1 = 25$

$y_1 = 25/5$

$\boxed{y_1 = 5}$

Newton's Advancing difference method :-

x		y		differences							
				First Δ		second Δ		Third Δ		Fourth Δ	
20	x_0	3	y_0								
				8	Δ^1_0						
35	x_1	11	y_1			5	Δ^2_0				
				13	Δ^1_1			8	Δ^3_0		
50	x_2	24	y_2			13	Δ^2_1			1	Δ^4_0
				26	Δ^1_2			9	Δ^3_1		
65	x_3	50	y_3			22	Δ^2_2				
				48	Δ^1_3						
80	x_4	98	y_4								

$$y_x = y_0 + x\Delta^1_0 + \left[\frac{x(x-1)}{2!}\Delta^2_0\right] + \left[\frac{x(x-1)(x-2)}{3!}\Delta^3_0\right] + \left[\frac{x(x-1)(x-2)(x-3)}{4!}\Delta^4_0\right]$$

$$x = \frac{24-20}{15} = \frac{4}{15} = 0.26$$

$$y_{24} = 3 + [0.26 \times 8] + \left[\frac{0.26(0.26-1)}{1\times2} \times 5\right] + \left[\frac{0.26(0.26-1)(0.26-2)}{1\times2\times3} \times 8\right] + \left[\frac{0.26(0.26-1)(0.26-2)(0.26-3)}{1\times2\times3\times4} \times 1\right]$$

$$= 3 + 2.08 + (-0.48) + 0.44 + (-0.03)$$

$$= 3 + 2.08 - 0.48 + 0.44 - 0.03$$

$$= \cancel{4.93}\ 5.07$$

$\boxed{y_{24} = 5}$

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2. Estimate the value of f when $x = 5$ from the following use lagrange's method of interpolation.

x	3	7	9	10
y	168	120	73	63

$$y_5 \Rightarrow 168\left[\frac{(5-7)(5-9)(5-10)}{(3-7)(3-9)(3-10)}\right] + 120\left[\frac{(5-3)(5-9)(5-10)}{(7-3)(7-9)(7-10)}\right]$$

$$+ 73\left[\frac{(5-3)(5-7)(5-10)}{(9-3)(9-7)(9-10)}\right] + 63\left[\frac{(5-3)(5-7)(5-9)}{(10-3)(10-7)(10-9)}\right]$$

$$\Rightarrow 168\left[\frac{-2(-4)(-5)}{-4(-6)(-7)}\right] + 120\left[\frac{2(-4)(-5)}{4(-2)(-3)}\right] + 73\left[\frac{2(-2)(-5)}{6(2)(-1)}\right] + 63\left[\frac{2(-2)(-4)}{7\times3\times1}\right]$$

$$\Rightarrow 168\left[\frac{-40}{-168}\right] + 120\left[\frac{40}{24}\right] + 73\left[\frac{20}{-12}\right] + 63\left[\frac{16}{21}\right]$$

$$\Rightarrow 40 + 200 - \frac{365}{3} + 48$$

$$\Rightarrow 40 + 200 - 121.6 + 48$$

$$\boxed{y_5 \Rightarrow 166.4}$$

3. From the following data interpolate the index no. for 2010 under binomial expansion method.

x	2007	2008	2009	2010	2011	2012
y	173	149	145	?	131	141
	y_0	y_1	y_2	y_3	y_4	y_5

Sol:

$$\cancel{y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0}$$

$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$

$$141 - 5(131) + 10y_3 - 10(145) + 5(149) - 173 = 0$$

$$141 - 655 + 10y_3 - 1450 + 743 - 173 = 0$$

$$10y_3 + 886 - 2278 = 0$$

$$10y_3 - 1392 = 0$$

$$10y_3 = 1392$$

$$y_3 = \frac{1392}{10}$$

$$\boxed{y_3 = 139.2}$$

V.good

Unit - IV

Assignment Problem

Meaning :-

In an assignment Problem, the number of Jobs available equals of Machines. Each Job is assigned to only one Machine and each Machine is assigned only one job such that the total cost is a Minimum that is the assignment is on a one to one basis.

Find the optimal solution for the assignment Problems with the following cost Matrix.

Sales man	Area: W	X	Y	Z
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12

sol : step : 1 : choose the least element in each Row and subtract it from all the elements of the Row

area

salesman	w	x	y	z
A	3	9	0	8
B	3	1	6	0
C	1	4	3	0
D	4	0	2	1

step : 2

choose the least elements in each columns and subract it from all the elements of the each columns from step 1.

area

sales man	W	x	y	z
A	2	8	0	10
B	2	0	4	0
C	0	~~9~~ 3	7	6
D	3	3	4	5

area

salesman	W	x	y	Z
A	2	9	0	8
B	2	1	6	0
C	0	4	3	0
D	3	0	2	1

step : 3

Optimal assignment

salesman	area	cost Rs.
A	Y	8
B	Z	6
C	W	13
D	X	10
Total		37

2) A company has 5 jobs to be done on 5 Machine any job can be done on any Machine. The cost of doing the jobs on different machines are given below. Assign the jobs for different machine so as to minimize the total cost

Jobs	Machines A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

step : 1

choose the least elements in each rows and subtract from all the element of that row.

$$\begin{pmatrix} 5 & 0 & 8 & 10 & 11 \\ 0 & 6 & 15 & 0 & 3 \\ 8 & 5 & 0 & 0 & 0 \\ 0 & 6 & 4 & 2 & 7 \\ 3 & 5 & 6 & 0 & 8 \end{pmatrix}$$

objective function
constraints
Non-negativity
linearity
finiteness

step : 2

choose the least element in each columns and subtract from all the elements from the step : 1.

$$\begin{pmatrix} 5 & 0 & 8 & 10 & 11 \\ 0 & 6 & 15 & 0 & 3 \\ 8 & 5 & 0 & 0 & 0 \\ 0 & 6 & 4 & 2 & 7 \\ 3 & 5 & 6 & 0 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 5 & \boxed{0} & 8 & 10 & 11 \\ 0 & 6 & 15 & 0 & 3 \\ 8 & 5 & \boxed{0} & 0 & 0 \\ \boxed{0} & 6 & 4 & 2 & 7 \\ 3 & 5 & 6 & \boxed{0} & 8 \end{pmatrix}$$

step : 3

Assigned zero in the Rows and columns only one zero, the Remaining zero cancelled and not assigned row tick it.

row is mark line
column is unmark line

$$\begin{pmatrix} 8 & \boxed{0} & 8 & 13 & 11 \\ 0 & 3 & 12 & \cancel{0} & \boxed{0} \\ 11 & 5 & \boxed{0} & \cancel{0} & 0 \\ \boxed{0} & 3 & 1 & 2 & 4 \\ 3 & 2 & 3 & \boxed{0} & 5 \end{pmatrix}$$

Optional minimize cost

Job	Machine	cost
1	B	8
2	E	12
3	C	4
4	A	6
5	D	12
	Total	42

Transportation Problem

Meaning :-

The distribution of a product from several ~~places~~ sources to numerous Locality.

Methods of Transportation :-

1. North west corner rules
2. Matrix minima (or) Least cost method (MNM)
3. Vogels Approximation method (VAM)

Find the initial basic feasible solution to the following transportation problem using north-west corner rule.

From | to | availability

	E	F	G	H	
A	4	8	10	16	100 a_1
B	7	2	3	1	200 a_2
C	5	9	11	2	300 a_3
Demand	160	240	105	95	
	b_1	b_2	b_3	b_4	

sol:

$\Sigma a = a_1 + a_2 + a_3 = 100 + 200 + 300 = 600$

$\Sigma b = b_1 + b_2 + b_3 + b_4 = 160 + 240 + 105 + 95 = 600$

step :-

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step :-

4	8	10	16	100
7	2	3	1	200
5	9	11	2	300
160	240	105	95	

7	2	3	1	140 200
5	9	11	2	300
60	240	105	95	

140 2	3	1	140
9	11	2	300
240 100	105	95	

100 9	11	2	200 300
100	105	95	

105 11	2	95 200
105	95	

2	95
95	

Transport cost $= (100 \times 4) + (60 \times 7) + (140 \times 2) + (100 \times 9) + (105 \times 11) + (2 \times 95)$

$= 400 + 420 + 280 + 900 + 1155 + 190$

Transport cost $= 3345$

Find an initial basic feasible solution to the following problem by north - west corner rule.

Distribution Supply

sources	I	II	III		
S_1	7	3	4	12	a_1
S_2	2	1	3	13	a_2
S_3	3	4	6	15	a_3
	14	11	15		
	b_1	b_2	b_3		

Sol :

step :-

$\Sigma a = a_1 + a_2 + a_3 = 12 + 13 + 15 = 40$

$\Sigma b = b_1 + b_2 + b_3 = 14 + 11 + 15 = 40$

^{12} 7	3	4	~~12~~
2	1	3	13
3	4	6	15
~~14~~	11	15	
2			

^{2} 2	1	3	13 ^{11}
3	4	6	15
2	11	15	

^{11} 3		~~11~~
4	6	15
~~11~~	15	

^{15} 6	15
15	

Transport cost =>

$= (12 \times 7) + (2 \times 2) + (1 \times 11) + (6 \times 15)$

$= 84 + 4 + 11 + 90$

$= 189$

Find the optional cost solution for the assignment Problem with the following cost matrix.

Job	workers			
1	41	72	39	52
2	22	29	49	65
3	27	39	60	51
4	45	50	48	52

sol:

step : 1

$$\begin{pmatrix} 2 & 33 & 0 & 13 \\ 0 & 7 & 27 & 43 \\ 0 & 12 & 33 & 24 \\ 0 & 5 & 3 & 7 \end{pmatrix}$$

step : 2 :

$$\begin{pmatrix} 2 & 28 & 0 & 6 \\ 0 & 2 & 27 & 36 \\ 0 & 7 & 33 & 17 \\ 0 & 0 & 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 28 & \boxed{0} & 6 \\ \boxed{0} & 2 & 27 & 36 \\ \cancel{0} & 7 & 33 & 17 \\ \cancel{0} & \boxed{0} & 3 & \cancel{0} \end{pmatrix}$$

$$\begin{pmatrix} 4 & 28 & \boxed{0} & 6 \\ 0 & \boxed{0} & 25 & 34 \\ \boxed{0} & 5 & 31 & 15 \\ 0 & \cancel{0} & 3 & \boxed{0} \end{pmatrix}$$

optimal minimize cost

Job	workers	cost
1	C	39
2	B	29
3	A	27
4	B	53
	Total	147

Find the assignment of job to workers that will minimize the total assignment cost.

$$\begin{array}{cc|cccc} & & & \text{Job} & & \\ & & W & X & Y & Z \\ \hline & A & 7 & 6 & 8 & 4 \\ \text{workers} & B & 8 & 9 & 2 & 5 \\ & C & 11 & 1 & 6 & 7 \\ & D & 5 & 4 & 9 & 6 \end{array}$$

step : 1

Row minimum

$$\begin{pmatrix} 3 & 2 & 4 & 0 \\ 6 & 7 & 0 & 3 \\ 10 & 0 & 5 & 6 \\ 1 & 0 & 5 & 2 \end{pmatrix}$$

step : 2

columns minimum with step : 1

$$\begin{pmatrix} 2 & 2 & 4 & 0 \\ 5 & 7 & 0 & 3 \\ 9 & 0 & 5 & 6 \\ 0 & 0 & 5 & 2 \end{pmatrix}$$

step : 3

assigned zero, Rows & columns only one zero remaining zero cancelled.

$$\begin{pmatrix} 2 & 2 & 4 & \boxed{0} \\ 5 & 7 & \boxed{0} & 3 \\ 9 & \boxed{0} & 5 & 6 \\ \boxed{0} & 0 & 5 & 2 \end{pmatrix}$$

optimal minimize cost :-

worker	job	cost
A	z	4
B	y	2
C	x	1
D	w	5
Total		12

Matrix minima method (or) Least cost method (mmm)

① obtain and initial basic feasible solution to the following T.P using least cost method.

From	To				Availability
	D	E	F	G	
A	11	3	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
demand	200	225	275	250	

sol:

$\Sigma a = a_1 + a_2 + a_3 = 250 + 300 + 400 = 950$

$\Sigma b = b_1 + b_2 + b_3 + b_4 = 200 + 225 + 275 + 250 = 950$

$\Sigma a = \Sigma b$

step:

11	13	17	14	250
16	18	14	10	300
21	24	13	10 (250)	400 150
250	225	275	~~250~~	

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200 11	13	17	50 250
16	18	14	300
21	24	13	150
200	225	275	

13	17	50
18	14	300
~~24~~	150 ~~13~~	~~150~~
225	~~275~~ 125	

50 ~~13~~	17	~~50~~
18	14	300
~~225~~ 175	125	

18	125 14	~~300~~ 175
175	~~125~~	

18	175
175	

T.P cost $= (10 \times 250) + (11 \times 200) + (13 \times 150) + (13 \times 50) + (14 \times 125) + (18 \times 175)$

$= 2500 + 2200 + 1950 + 650 + 1750 + 3150$

T.P cost $= 12,200$

Find an initial basic feasible solution for the following Problem using least cost method.

	A	B	C	D	supply
I	1	5	3	3	34
II	3	3	1	2	15
III	0	2	2	3	12
IV	2	7	2	4	19
Demand	21	25	17	17	

sol:

$\Sigma a = a_1 + a_2 + a_3 + a_4 = 34 + 15 + 12 + 19 \Rightarrow 80$

$\Sigma b = b_1 + b_2 + b_3 + b_4 = 21 + 25 + 17 + 17 \Rightarrow 80$

$\Sigma a = \Sigma b$

step:

1	5	3	3	34
3	3	1	2	15
12 0	2	2	3	~~12~~
2	7	2	4	19
~~21~~ 9	25	17	17	

9 1	5	3	3	~~34~~ 25
3	3	1	2	15
2	7	2	4	19
~~9~~	25	17	17	

~~3~~ ~~3~~ ~~1~~ ~~2~~ 15

5	3	3	25
3	15 1	2	~~15~~
7	2	4	19
25	17	~~17~~	

2

5	3	3	25
7	2 2	4	~~19~~ 17
25	~~2~~	17	

5	17 ~~3~~	~~25~~ 8
7	4	17
25	~~17~~	

8 ~~5~~	~~8~~
7	17
~~25~~ 17	

17 7	~~17~~
~~17~~	

T.P cost = $(0 \times 12) + (9 \times 1) + (1 \times 15) + (2 \times 2) + (17 \times 3) + (8 \times 5) + (17 \times 7)$

$= 0 + 9 + 15 + 4 + 51 + 40 + 119$

T.P cost = 238

Linear Programming Problem (LPP)

Meaning:

Linear programming means planning. All the relationship between the variables considered in these problems are linear.

General form of LPP:

Maximize (or) minimize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ → (a)

Subject to the constraints,

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le \text{ or } = \text{ or } \ge b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le \text{ or } = \text{ or } \ge b_2$$
$$\vdots$$
$$a_{m_1}x_1 + a_{m_2}x_2 + \dots + a_{mn}x_n \le \text{ or } = \text{ or } \ge b_m$$
→ (b)

and $x_1, x_2, \dots x_n \ge 0$ → (c)

A company makes, Three Products X, Y and Z which pass through three departments Drill, Lathe and assembly. The hours available in each departments hours required by each Product is each department and Profit contribution of each Product are given below.

Time required in hours

Product	Drill	Lathe	Assembly	Profit Per unit Rs.
x	3	3	8	9
y	6	5	10	15
z	7	4	12	20
hours available	210	240	260	

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x_1 be the number of units of production of x

x_2 be the number of units of production of y

x_3 be the number of units of production of z

Profits of x, y, and z $\Rightarrow$ c_1, c_2, c_3.

Maximize $z = 9x_1 + 15x_2 + 20x_3 \longrightarrow$ (a)

Subject to the constraints,

$$\left.\begin{aligned} 3x_1 + 6x_2 + 7x_3 &\le 210 \\ 3x_1 + 5x_2 + 4x_3 &\le 240 \\ 8x_1 + 10x_2 + 12x_3 &\le 260 \end{aligned}\right] \longrightarrow \text{(b)}$$

and x_1, x_2 and $x_3 \geq 0 \longrightarrow$ (c)

---

(2) Find the minimum cost solution for the following transportation problem which has cost structure as,

From	To			Availabilities
	16	19	12	14
	22	13	19	16
	14	28	8	12
Requirement	10	15	17	

Ans :

$\Sigma a = a_1 + a_2 + a_3 = 14 + 16 + 12 = 42$

$\Sigma b = b_1 + b_2 + b_3 = 10 + 15 + 17 = 42$

$\Sigma a = \Sigma b$

16	19	12	14
22	13	19	16
14	28	12 8	~~12~~
10	15	~~17~~ 5	

16	19	$\cancel{12}$ 5	$\cancel{14}$ 9
22	13	19	16
10	15	$\cancel{5}$	

16	19	9
22	13 (15)	16
10	$\cancel{15}$	

16 (9)	$\cancel{9}$
22	1
$\cancel{10}$	

1

22 (1)	$\cancel{1}$
$\cancel{1}$	

T.P cost $= (8\times12) + (12\times5) + (13\times15) + (16\times9) + (22\times1)$

$= 96 + 60 + 195 + 144 + 22$

T.P cost $= 517$ //.

Determine basic feasible solution to the following transportation Problems using north west corner Rule.

Sin K

Origin	A	B	C	D	E	supply
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
demand	3	3	4	5	6	

$\Sigma a = a_1 + a_2 + a_3 = 4+8+9 = 21$

$\Sigma b = b_1 + b_2 + b_3 + b_4 + b_5 = 3+3+4+5+6 = 21$

$\Sigma a = \Sigma b$

^{3} 2	11	10	3	7	$\cancel{4}$ 1
1	4	7	2	1	8
3	9	4	8	12	9
$\cancel{3}$	3	4	5	6	

^{1} $\cancel{11}$	10	3	7	$\cancel{1}$
4	7	2	1	8
9	4	8	12	9
$\cancel{3}$ 2	4	5	6	

^{2} $\cancel{4}$	7	2	1	$\cancel{8}$ 6
9	4	8	12	9
$\cancel{2}$	4	5	6	

^{4} $\cancel{7}$	$\cancel{2}$	1	$\cancel{6}$ 2
4	8	12	9
$\cancel{4}$	5	6	

^{2} $\cancel{2}$	1	$\cancel{2}$
8	12	9
$\cancel{5}$	6	

8	12

$\cancel{9}$

$\cancel{3}$ 6

12

$\cancel{6}$

$\cancel{6}$

T.P cost $= (2\times3)+(1\times11)+(4\times2)+(7\times4)+(2\times2)+(8\times3)+(12\times6)$

$= 6+11+8+28+4+24+72$

T.P cost $= 153.$

VAM

1. Find an initial basic feasible solution for the following transportation problem using VAM.

	A	B	C	D	supply
I	1	2	1	4	30
II	3	3	2	1	50
III	4	2	5	9	20
Demand	20	40	30	10	

Ans:

$\Sigma a = a_1+a_2+a_3 = 30+50+20 = 100$

$\Sigma b = b_1+b_2+b_3+b_4 = 20+40+30+10 = 100$

$\Sigma a = \Sigma b$

1	2	1	10 4	$\cancel{30}$ 20	(1)
3	3	2	1	50	(1)
4	2	5	9	20	(2)
20	40	30	$\cancel{10}$		
(2)	(1)	(1)	(3)		

1^{20}	2	1	$\cancel{20}$	(1)
3	3	2	50	(1)
4	2	5	20	(2)
$\cancel{20}$	40	30		
(2)	(1)	(1)		

3	2^{30}	$\cancel{50}$ 20	(1)
2	5	20	(3)
40	$\cancel{30}$		
(1)	(3)		

3^{20}	$\cancel{20}$
2	20
$\cancel{40}$ 20	

2^{20}	$\cancel{20}$
$\cancel{20}$	

VAN $= (10 \times 4) + (1 \times 20) + (2 \times 30) + (3 \times 20) + (2 \times 20)$

$= 40 + 20 + 60 + 60 + 40$

VAM $= 220$

Linear Programming Problem.

A Firm Manufactures Three Products A, B and C. The Profit on each unit of three Products are Rs. 3, Rs. 2 and Rs. 4. respectively. The firm has two Machines D & E Below are given the Processing times in minutes of each product on each machine.

		Product		
		A	B	C
Machines	D	4	3	2
	E	2	2	4

Machines D and E have 2000 and 2500 Machine - minutes Respectively. The firm must Manufacture atleast 100 units of the Product A, 200 units of B and 50 units of C but not more than 150 units of A. set up an LPP to maximize the Profit.

	Machine		profits	unit
product	D	E	Rs	
A	4	2	3	100
B	3	2	2	200
C	2	4	4	50
	2000	2500		

Maximize $z = 3x_1 + 2x_2 + 4x_3 \longrightarrow$ (a)

Subject to constraints,

$$4x_1 + 3x_2 + 2x_3 \leq 2000$$
$$2x_1 + 2x_2 + 4x_3 \leq 2500 \longrightarrow \text{(b)}$$

$$\left.\begin{array}{r} 100 \le x_1 \le 150 \\ x_2 \le 200 \\ x_3 \le 50 \end{array}\right] \longrightarrow (c)$$

Graphic Method - L.P.P

Illus: ① solve the equation under graphic method

Maximize $Z = 3x_1 + 4x_2$

subject to constraints, $4x_1 + 2x_2 \le 80$

$2x_1 + 5x_2 \le 180$

$x_1, x_2 \ge 0$

Sol:

$4x_1 + 2x_2 \le 80$

Put $x_1 = 0$, $4(0) + 2x_2 = 80$

$2x_2 = 80$

$x_2 = 80/2$

$\boxed{x_2 = 40}$

Point $(0, 40)$

Put $x_2 = 0$, $4x_1 + 2(0) = 80$

$4x_1 = 80$

$x_1 = 80/4$

$\boxed{x_1 = 20}$

Point $(20, 0)$

$2x_1 + 5x_2 \le 180$

Put $x_1 = 0$, $2(0) + 5x_2 = 180$

$5x_2 = 180$

$x_2 = 180/5$

$\boxed{x_2 = 36}$

Point $(0, 36)$

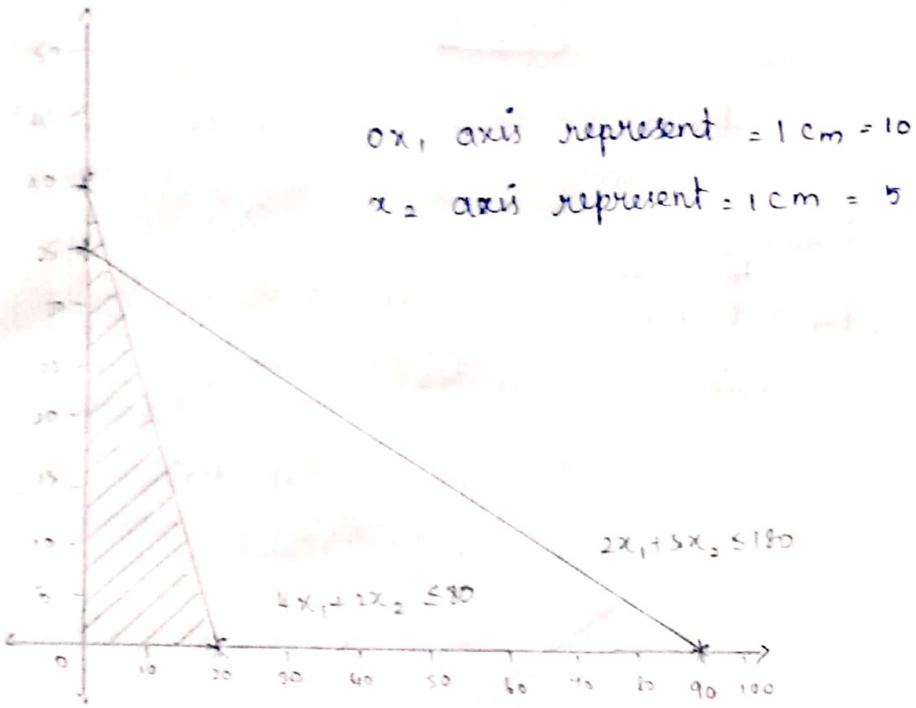
Put $x_2 = 0$, $2x_1 + 5(0) = 180$

$2x_1 = 180$

$x_1 = 180/2$

$\boxed{x_1 = 90}$

Point (90,0)



2) solve the following L.P.P graphically.

maximize $Z = 3x_1+2x_2$

subject to contraints, $-2x_1+x_2 \le 1$

$x_1+x_2 \le 3$

$x_1 \le 2$

$x_1, x_2 \ge 0$

sol:

$-2x_1+x_2 \le 1$

Put $x_1 = 0$, $2(0)+x_2 = 1$

$\boxed{x_2 = 1}$

Point (0,1)

Put $x_2=0$

$-2x_1+0 = 1$

$-2x_1 = 1$

$x_1 = -1/2$

$\boxed{x_1 = -0.5}$

Point (-0.5,0)

(2) $x_1+x_2 \le 3$

Put $x_1 = 0$, $0+x_2 = 3$

$\boxed{x_2 = 3}$ Point (0,3)

Put $x_2 = 0$, $x_1 + 0 = 3$

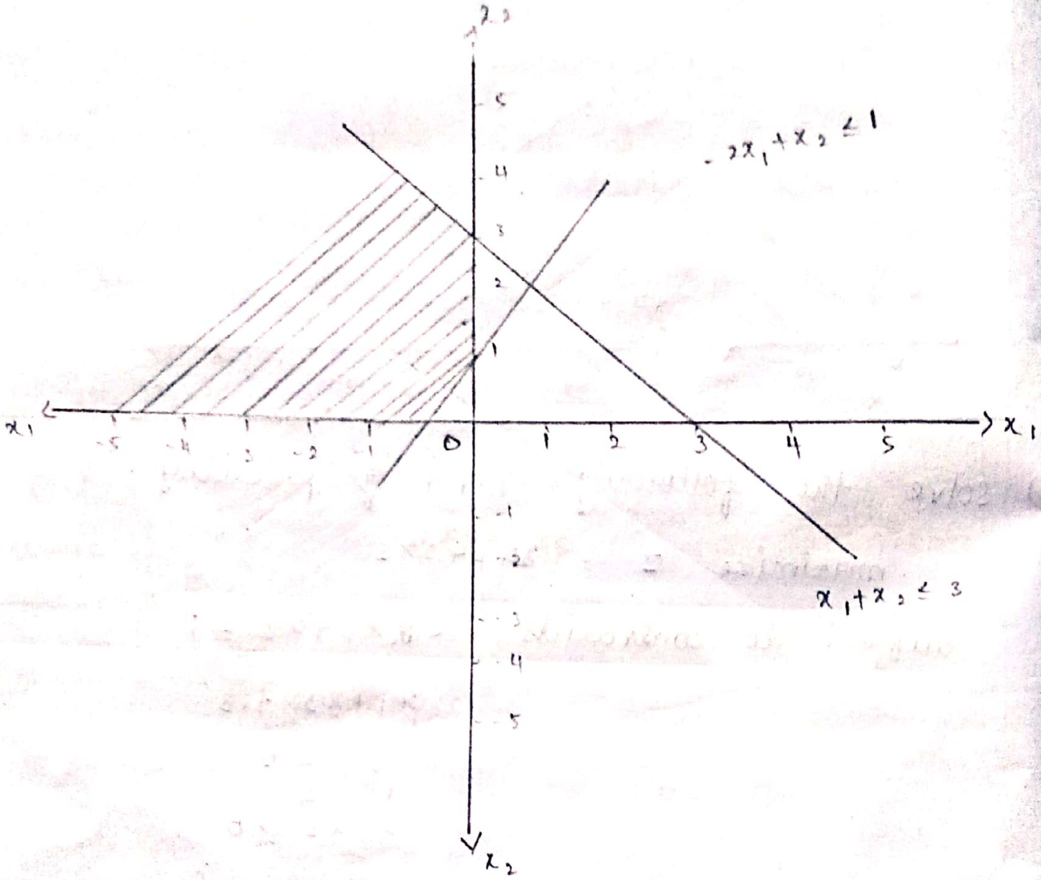
$\boxed{x_1 = 3}$

Point $(3, 0)$

$x_1 \leq 2$

$\boxed{x_1 = 2}$

Point $(2, 0)$



3) Maximize $z = 4x + 7y$

subject to constraints, $x + y \leq 60$

$x \leq 40$

$y \leq 40$

$x, y \geq 0$

$x + y \leq 60$

$x + y = 60$

Put $x = 0$, $0 + y = 60$

$\boxed{y = 60}$

Point $(0, 60)$

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Put $y = 0$, $x + 0 = 60$

$\boxed{x = 60}$

Point $(60, 0)$

x axis represent 1cm = 10

y axis represent 1cm = 10

$x \leq 40$

$x = 40$

$(40, 0)$

$y \leq 40$

$(0, 40)$

4. Maximize $z = -3x_1 + 4x_2$

subject to constraints, $x_1 + x_2 \leq 4$

$2x_1 + 3x_2 \geq 18$

$x_1, x_2 \geq 0$

sol:

1. $x_1 + x_2 \leq 4$

$x_1 + x_2 = 4$

Put $x_1 = 0$, $0 + x_2 = 4$

$\boxed{x_2 = 4}$

Point $(0, 4)$

Put $x_2 = 0$, $x_1 + 0 = 4$

$\boxed{x_1 = 4}$

Point $(4, 0)$

2. $2x_1 + 3x_2 \geq 18$

Put $x_1 = 0$, $2(0) + 3x_2 = 18$

$3x_2 = 18$

Point $(0, 6)$ $x_2 = 18/3$ $\boxed{x_2 = 6}$

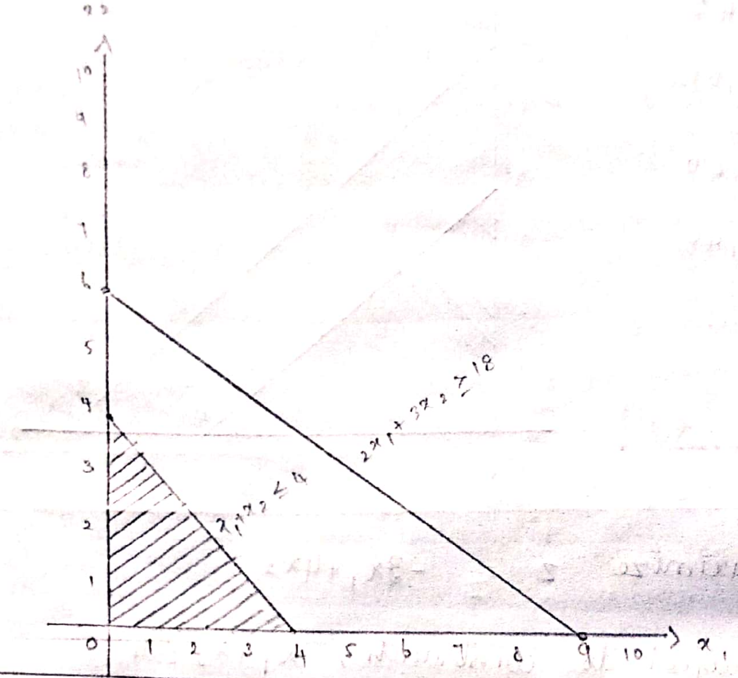
Put $x_2 = 0$, $\quad 2x_1 + 3(0) \geq 18$

$$2x_1 \geq 18$$

$$x_1 = 18/2$$

$$\boxed{x_1 = 9}$$

Point $(9, 0)$



Unit - III

CHI - SQUARE TEST (χ^2 - TEST)

The value of chi-square describes the magnitude of difference between observed frequencies and expected frequencies under certain assumptions. χ^2 value (χ^2 quantity) ranges from zero to infinity. It is zero when the expected frequencies and observed frequencies completely coinside. so greater the value of χ^2, greater is the discrepancy between observed and expected frequencies.

χ^2 test is a statistical test which tests the significance of difference between observed frequencies and corresponding theoretical frequencies of a distribution without any assumption about the distribution of the population. This is one of the simplest and most widely used non-parametric test in statistical work. This test was developed by Prof. Karl Pearson in 1990.

uses of x^2 - Test :-

* useful for the test of goodness of fit.
* useful for the test of independence of attributes
* useful for the test of homogeneity
* useful for the testing given population varience.

P. no 119 ① Test whether the accidents occur uniformity over week days on the basis of the following information

Days of the week	Sun	Mon	Tue	Wed	Thu	Fri	Sat
no. of Accidents	11	13	14	13	15	14	18

H_0: There is goodness of fit between observed and expected frequencies. ie. accidents occur uniformity over week days.

H_1: There is no goodness of fit between observed and expected frequencies. ie. accidents do not ~~occure~~ occur uniformity over week days.

computation of x^2 value

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
11	14	-3	9	$\frac{9}{14} = 0.6429$
13	14	-1	1	$\frac{1}{14} = 0.0714$
14	14	0	0	$\frac{0}{14} = 0.0000$
13	14	-1	1	$\frac{1}{14} = 0.0714$
15	14	1	1	$\frac{1}{14} = 0.0714$
14	14	0	0	$\frac{0}{14} = 0.0000$
18	14	4	16	$\frac{16}{14} = 1.1429$
98				
		$\Sigma \frac{(O-E)^2}{E}$		$= 2.0000$

The value of x^2 at 5% level of significance and $n-r-1 = 7-0-1 = 6$ d.o.f $= 12.592$

calculated value is less than table value, we accept the null hypothesis. Ho is accepted.

2) The following table gives data regarding election to an office.

Attitude towards election	Economic status Rich	Poor	Total
Favourable	50	155	205
non Favourable	90	110	200
Total	140	265	405

is attitude towards election influenced by economic status of workers?

H_0 => The two attributes, election and economic status are independent.

H_1 : The attributes, election and economic status are dependent.

computation of expected Frequency

Economic status → / attributes towards election ↓	Rich	Poor	Total
Favourable	$\frac{140 \times 205}{405} = 71$	$\frac{265 \times 205}{405} = 134$	205
not Favourable	$\frac{140 \times 200}{405} = 69$	$\frac{265 \times 200}{405} = 131$	200
Total	140	265	405

computation of x^2 value

O	E	O - E	$(O-E)^2$	$\frac{(OE)^2}{E}$
50	71	-21	441	$\frac{441}{71} = 6.21$
90	69	21	441	$\frac{441}{69} = 6.39$
155	134	21	441	$\frac{441}{134} = 3.29$
110	131	-21	441	$\frac{441}{131} = 3.37$
				19.26

calculation of Table value 5% significance

Degree of freedom $= (r-1)(c-1) \Rightarrow (2-1)(2-1)$

$\Rightarrow 1 \times 1 \Rightarrow 1$ $\therefore$ table $= 3.841$

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calculated value is greater than the table value.

∴ we reject the H_0. ∴ Election and economic status are not independent. accept the H_1.

P.g no 122

H_0 : The two attributes between sex and Tea habit are independent.

H_1 : The two attributes between sex and Tea habit are dependent.

Town - 'A'

2x2 contingency table of observed frequencies

Tea habits \ Sex	Male	Female	Total
Tea drinkers	19	12	31
not - Tea drinkers	32	37	69
Total	51	49	100

calculation of expected frequency

Tea habits \ Sex	Male	Female	Total
Tea drinks	$\frac{51 \times 31}{100} = 16$	$\frac{49 \times 31}{100} = 15$	31
not tea drinks	$\frac{51 \times 69}{100} = 35$	$\frac{49 \times 69}{100} = 34$	69
Total	51	49	100

calculation of x^2 value

O	E	O-E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
19	16	3	9	$\frac{9}{16} = 0.5625$
32	35	-3	9	$\frac{9}{35} = 0.2571$
12	15	-3	9	$\frac{9}{15} = 0.6000$
37	34	3	9	$\frac{9}{34} = 0.2647$
			$\Sigma \frac{(O-E)^2}{E}$	$= 1.6843$

calculation of Table :

$$\text{Degree of freedom} = (r-1)(c-1)$$
$$= (2-1)(2-1)$$
$$= 1 \times 1$$
$$\text{DOF} = 1$$

Test significance @ 5% , level is the Degree of freedom = 3.84

comparision :

calculated value		Table value
1.6843	<	3.84

H_0 : The two attributes between sex and Tea habit are independent in Town A . The null hypothesis is accepted .

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Town - 'B'.

2×2 contingency Table of observed frequency :

Sex / Tea drinkers	Male	female	Total
Tea drinkers	17	9	26
not tea drinkers	29	45	74
Total	46	54	100

calculation of Expected frequency

Sex / Tea drinkers	Male	Female	Total
Tea drinkers	$\frac{46 \times 26}{100} = 12$	$\frac{54 \times 26}{100} = 14$	26
not tea drinkers	$\frac{46 \times 74}{100} = 34$	$\frac{54 \times 74}{100} = 40$	74
Total	46	54	100

calculation x^2 value

O	E	O - E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
17	12	5	25	$\frac{25}{12} = 2.0833$
29	34	-5	25	$\frac{25}{34} = 0.7352$
9	14	-5	25	$\frac{25}{14} = 1.7857$
45	40	5	25	$\frac{25}{40} = 0.6250$
			$\Sigma \frac{(O-E)^2}{E}$	5.2292

calculation of Table :

Degree of freedom $= (r-1)(c-1)$

$= (2-1)(2-1)$

$= 1 \times 1$

DOF $= 1$

Test of significance @ 5% level is the degree of freedom = 3.84

comparison :

calculated value		Table value
5.2292	>	3.84

H_1 : The two attributes between sex and Tea habits are dependent.

Out of 8000 graduates in a town, 800 are female and out of 1600 graduates employees 120 are females. use x^2 to determine of any discrimination is made an appointment on the basis of sex.

contingency table observed frequency

Sex / Graduates	Male	Female	Total
employed	1480	120	1600
unemployed	5720	680	6400
Total	7200	800	8000

calculation of expected frequency

sex / graduate	Male	Female	Total
employed	$\frac{7200 \times 1600}{8000} = 1440$	$\frac{800 \times 1600}{8000} = 160$	1600
unemployed	$\frac{7200 \times 6400}{8000} = 5760$	$\frac{800 \times 6400}{8000} = 640$	6400
Total	7200	800	8000

H_0 :- The attributes sex and graduates are independent

H_1 :- The attributes sex and graduates are dependent calculation of x^2 value

O	E	O - E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1480	1440	40	1600	$\frac{1600}{1440}$ - 1.1111
5720	5760	-40	1600	$\frac{1600}{5760}$ = 0.2777
120	160	-40	1600	$\frac{1600}{160}$ = 10.000
680	640	40	1600	$\frac{1600}{640}$ = 2.5000
			$\Sigma \frac{(O-E)^2}{E}$	13.8888

Degree of freedom = (r-1) (c-1)

= (2-1) (2-1)

= 1 × 1

DOF = 1

Test of significance at 5% level is the degree of freedom = 3.84

comparison :-

x^2 value calculated value		Table value
13.8888	>	3.84

H_1 :- The attributes sex and graduates are dependent is accepted.

Analysis of varience :-

1. Set up ANOVA table for the following per hectare yield for three varities of wheats. each grown in four plots.

Per hectare yield (in hundred kgs)

Plots in land	variety of wheat		
	A_1	A_2	A_3
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	7

Also work out F-Ratio and test whether there is significance differences among the Average yields in the 3 varieties of wheat.

A :-

H_0 : There is no significance among the means of three varities.

H_1 : There is significant among the means of three varities.

$A_1=x_1$	$A_2=x_2$	$A_3=x_3$	x_1^2	x_2^2	x_3^2
6	5	5	36	25	25
7	5	4	49	25	16
3	3	3	9	9	9
8	7	4	64	49	16
24	20	16	158	108	66
Σx_1	Σx_2	Σx_3	Σx_1^2	Σx_2^2	Σx_3^2

No. of observations $= 4 \times 3 = 12$

SST (sum of square Total) = sum of square all observations $- \frac{T^2}{\cancel{N^2}\, N_2}$

$$= \left(\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2\right) - \frac{(\Sigma x_1 + \Sigma x_2 + \Sigma x_3)}{N}$$

$$= (158 + 108 + 66) - \frac{(24 + 20 + 16)^2}{12}$$

$$= 332 - \frac{(60)^2}{12}$$

$$= 332 - \frac{3600}{12}$$

$$= 332 - 300$$

$$\boxed{SST = 32}$$

SSC = sum of squares between samples

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_2} + \frac{(\Sigma x_3)^2}{N_3} - \frac{T^2}{N} \quad (\Sigma x_1 + \Sigma x_2 + \Sigma x_3)^2$$

$$= \frac{(24)^2}{4} + \frac{(20)^2}{4} + \frac{(16)^2}{4} - \frac{(24+20+16)^2}{12}$$

$$= \frac{574}{4} + \frac{400}{4} + \frac{256}{4} - \frac{(60)^2}{12}$$

$$= (144 + 100 + 64) - \frac{3600}{12}$$

$$= 308 - 300$$

$$\boxed{SSC = 8}$$

ONE way ANOVA Table

	Sources of variation	sum of square	Degree of freedom	mean square	F. Ratio
SSC	Between sample	8	(C-1) (3-1) 2	$MSC = \frac{SSC}{d.f} = \frac{8}{2} = 4$	$F = \frac{MSC}{MSE} = \frac{4}{2.6}$
SSE	within sample	24	9	$MSE = \frac{SSE}{d.F} = \frac{24}{9} = 2.6$	$F = 1.53$
SST	Total	32	11		

Table value @ 5% level is 4.26

∴ The H_0 is accepted. The calculated value is less than table value.

Two way Analysis

2. Apply the technique of Analysis of Variance of the following data relating to yields of 4 varieties of wheat in 3 blocks.

Varieties	Blocks		
	X	Y	Z
A	10	9	8
B	7	7	6
C	8	5	4
D	5	4	4

carry two-way analysis of variance.

Ans: H_0 : There is no significant difference between blocks.

There is no significant difference between varieties.

H_1 : There is significant difference between blocks.

There is significant difference between varieties.

Variety	$x(x_1)$	$y(x_2)$	$z(x_3)$	Total	x_1^2	x_2^2	x_3^2	Total
A (y_1)	10	9	8	(27) Σy_1	100	81	64	245 (Σy_1^2)
B (y_2)	7	7	6	20 (Σy_2)	49	49	36	134 $(\Sigma y_2)^2$
C (y_3)	8	5	4	17 (Σy_3)	64	25	16	105 $(\Sigma y_3)^2$
D (y_4)	5	4	4	13 (Σy_4)	25	16	16	57 $(\Sigma y_4)^2$
Total	30	25	22	77	238	171	132	54
	Σx_1	Σx_2	Σx_3		Σx_1^2	Σx_2^2	Σx_3^2	

between columns

SSC = sum of square of all item - $\frac{(\text{all observation})^2}{N}$

$$= \left[\frac{(\Sigma x_1)^2}{N} + \frac{(\Sigma x_2)^2}{N} + \frac{(\Sigma x_3)^2}{N}\right] - \frac{(\text{All observations})^2}{N}$$

$$= \left[\frac{30^2}{4} + \frac{25^2}{4} + \frac{\cancel{484}\ 22^2}{4}\right] - \frac{(77)^2}{12}$$

$$= \left[\frac{900}{4} + \frac{625}{4} + \frac{484}{4}\right] - \frac{5929}{12}$$

$$= (225 + 156.25 + 121) - 494.083$$

$$= 502.25 - 494.083$$

$$\boxed{SSC = 8.167}$$

SST = sum of square of all item - $\frac{(\text{All observations})^2}{N^2}$

$$= (\Sigma x_1^2) + (\Sigma x_2^2) + (\Sigma x_3^2) - \left(\frac{T^2}{N^*}\right)$$

$$= (238 + 171 + 132) - \frac{77^2}{12}$$

$$= (238 + 171 + 132) - \frac{5929}{12}$$

$$= 541 - 494.083$$

$$\boxed{SST = 46.917}$$

SSR = sum of squares between Rows

$$= \frac{(\Sigma y_1)^2}{N} + \frac{(\Sigma y_2)^2}{N} + \frac{(\Sigma y_3)^2}{N} + \frac{(\Sigma y_4)^2}{N} - \frac{(\text{all observations})^2}{N}$$

$$= \frac{(27)^2}{3} + \frac{(20)^2}{3} + \frac{(17)^2}{3} + \frac{(13)^2}{3} - \frac{(77)^2}{N}$$

$$= \frac{729}{3} + \frac{400}{3} + \frac{289}{3} + \frac{169}{3} - \frac{5929}{12}$$

$$= 243 + 133.33 + 96.33 + 56.33 - 494.08$$

$$= 528.99 - 494.08$$

$$\boxed{SSR = 34.91}$$

Two way ANOVA Table

Sources of variation	Sum of square	Degree of freedom	Mean square	F. Ratio
Between columns	$SSC = 8.167$	$c-1 = 3-1 = 2$	$MSC = \frac{SSC}{c-1} = \frac{8.167}{2} = 4.084$	$F_C = \frac{MSC}{MSE} =$ [illegible] $= 6.39$
Between Rows	$SSR = 34.91$	$R-1 = 4-1 = 3$	$MSR = \frac{SSR}{R-1} = \frac{34.91}{3} = 11.639$	$F_R = \frac{MSR}{MSE} =$ [illegible] $= 18.21$
Residual	$SSE = 3.84$	$(c-1)(R-1) = 6$ $(3-1)(4-1)$ 2×3	$MSE = \frac{SSE}{(C-1)(R-1)} = \frac{3.84}{6} = 0.639$	
Total	$SST = 46.917$	$N-1 = 12-1 = 11$		

Calculation of H_0 accept / Reject

Sources of variation	Calculated value: Degree of freedom	Calculated value: F. Ratio	5% level table value	Comparison	Accepted / Rejected
SSC	$(2, 6)$	6.39	5.1433	$6.39 > 5.1433$	Rejected
SSR	$(3, 6)$	18.21	4.7371	$18.21 > 4.7371$	Rejected

1. From the following data test if the difference between the variences is significant at 5% level of significance.

sum of squares of deviations from the mean	84.4	102.6
size	8	10
sample	A	B

2. In a sample of 8 observations, the sum of the squared deviations of items from the mean was 94.5. In another sample of 10 observations, the value was found to be 101.7. Test whether the difference is the variences is significant at 5% level.

sol

1) $H_0 \rightarrow$ There is no significance between varience of A & B

$H_1 \rightarrow$ There is significance between varience of A & B

sample A and B, Put $A \rightarrow S_1$, $B \rightarrow S_2$

sample size A, $N_1 \rightarrow 8$ B $N_2 \rightarrow 10$

varience of the sample $A \rightarrow S_1^2 = \dfrac{\Sigma(x-\bar{x})^2}{N_1} = \dfrac{84.4}{8} = 10.55$

varience of the sample $B \rightarrow S_2^2 = \dfrac{\Sigma(x-\bar{y})^2}{N_2} = \dfrac{102.6}{10} = 10.26$

The estimated varience of the Population from which the sample A and B are drawn are given by,

$$S_1^2 = \frac{N_1 \, \Sigma(x-\bar{x})^2}{N_1 - 1} = \frac{8 \times 10.55}{7} = 12.06$$

$$S_2^2 = \frac{N_2 \, \Sigma(x-\bar{y})^2}{N_2 - 1} = \frac{10}{9} \times 10.26 = 11.4$$

Here $S_1 > S_2$

$$F.\ Ratio = \frac{S_1}{S_2} = \frac{12.06}{11.4} = 1.06$$

calulation of Ho accept / rejected

sources of varience	calculated value		5% level Table value	comparison	Accepted Rejected
	Degree of freedom	F Ratio			
A & B	$(N_1-1)\ (N_2-1)$ (8-1) (10-1) (7, 9)	1.06	3.29	1.06 < 3.29	Accepted

T – Test

Sample items (N) = 900

use z – Test (sample size is greater than 30)

$$\text{standard Error} = \frac{S}{\sqrt{N}}$$

$H_0 : \mu = 26.8$

$H_1 : \mu \neq 26.8$

$\bar{x} = 26.8$ (Population mean)

$\mu = 25$ (sample mean)

$\sigma = 15$

$$SE = \frac{\sigma}{\sqrt{N}} = \frac{15}{\sqrt{900}} = \frac{15}{30} = 0.5$$

$$z = \frac{\bar{x} - \mu}{SE} = \frac{26.8 - 25}{0.5} = \frac{1.8}{0.5} = 3.6$$

The Table value (1.96) is less than calculated value (3.6). Ho is rejected. H_1 is accepted.

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(2) H_0: There is no significance between varience of A & B
H_1: There is significance between varience of A & B

sample A & B , let $A \to S_1$, $B \to S_2$

sample A, $N_1 \to 8$, B, $N_2 \to 10$

varience of the sample $A \to S_1^2 = \frac{\Sigma(X-\bar{X})^2}{N_1 - 1} = \frac{94.5}{8-1} = \frac{94.5}{7}$

$S_1^2 = 13.5$

varience of the sample $B \to S_2^2 = \frac{\Sigma(X-\bar{X})^2}{N_2-1} = \frac{101.7}{10-1} = \frac{101.7}{9}$

$S_2^2 = 11.3$

Here, $S_1 > S_2$

F - Ratio $= \frac{S_1}{S_2} = \frac{13.5}{11.3} = 1.19$

calculation of Ho accept / Rejected

sources of varience	calculated value		5% level Table value	comparison	acept/ Reject
	Degree of freedom	F-Ratio			
A & B	$(N_1-1)(N_2-1)$ $(8-1)(10-1)$ $(7,9)$	1.19	3.29	1.19 > 3.29	Accepted

Pg. no 101. $H_0 \Rightarrow$ There is no significance of Population mean.

$H_1 \Rightarrow$ There is significance of Population mean.

Population $(\bar{X})$ = 1600 hours

Sample mean (M) = 1570

Sample size (N) = 100

standard deviation (σ) = 120 hours

standard Error $= \frac{\sigma}{\sqrt{N}} = \frac{120}{\sqrt{100}} = \frac{120}{10} = 12$

standard Error = 12

$$Z = \frac{\bar{x} - \mu}{SE}$$

$$= \frac{1600 - 1570}{12}$$

$$= \frac{30}{12}$$

$$Z = 2.5$$

table value at 5% level is 1.96

comparison :-

The calculated value is greater than the table value. so we reject the H_0 and accept the H_1.

Pg. no 102

H_0 => There is no significance of Population mean.

H_1 => There is significance of Population mean.

Population mean ($\bar{x}$) = ? 67.5

sample mean (μ) = 65

sample size (N) = 10

standard deviation (σ) = ? 2.65

X	x^2
66	4356
65	4225
69	4761
70	4900
69	4761
71	5041
70	4900
63	3969
64	4096
68	4624
675	45633
Σx	Σx^2

$$\bar{x} = \frac{\Sigma x}{N} = \frac{675}{10} = 67.5$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \frac{(\Sigma x)^2}{N}}$$

$$= \sqrt{\frac{45633}{10} - \frac{(675)^2}{10}}$$

$$= \sqrt{45633 - (67.5)^2}$$

$$= \sqrt{4563.3 - 4556.25}$$

$$= \sqrt{7.05}$$

$$\sigma = 2.65$$

$$\text{standard Error} = \frac{\sigma}{\sqrt{n-1}} = \cancel{\frac{67.5-65}{\sqrt{10-1}}} = \frac{2.65}{\sqrt{10-1}}$$

$$= \frac{2.65}{\sqrt{9}}$$

$$= \frac{2.65}{3}$$

$$S.E = 0.88$$

$$T = \frac{\bar{x}-\mu}{S.E} = \frac{67.5-65}{0.88}$$

$$= \frac{2.5}{0.88}$$

$$T = 2.84$$

---

pg.no 106

Fifty children:-

Testing of Equality of Two Sample Means :-

special diet $(N_1) = 50$ Average weight $\bar{x}_1 = 7.2$ kgs

special diet $(N_2) = 50$ Average weight $\bar{x}_2 = 5.7$ kgs

common S.D = 2kg

$H_0 \Rightarrow \bar{x}_1 = \bar{x}_2$

$H_1 \Rightarrow \bar{x}_1 \neq \bar{x}_2$

$$Z = \frac{\text{Difference between sample}}{SE}$$

$$\text{standard Error} = \sqrt{\frac{\sigma^2}{N_1} + \frac{\sigma^2}{N_2}}$$

$$= \sqrt{\frac{2^2}{50} + \frac{2^2}{50}}$$

$$= \sqrt{\frac{4}{50} + \frac{4}{50}} = \sqrt{\frac{8}{50}}$$

$$= \sqrt{0.16}$$

$$\boxed{S.E = 0.4}$$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{7.2 - 5.7}{0.4}$$

$$= \frac{1.5}{0.4}$$

$$\boxed{z = 3.75}$$

table of z test is 1.645

conclusion :-

The calculated is greater than the table value so H_0 is rejected. but H_1 is accepted.

Pg.no 109

diet $(N_1) = 7$ Average weight $(\bar{x}_1) = ?$

diet $(N_2) = 10$ Average weight $(\bar{x}_2) = ?$

common SD = ?

$H_0 \Rightarrow \bar{x}_1 = \bar{x}_2$

$H_1 \Rightarrow \bar{x}_1 \neq \bar{x}_2$

N=7 N=10

Diet X		Diet Y	
x_1	x_1^2	x_2	x_2^2
15	225	14	196
22	484	24	576
20	400	12	144
22	484	20	400
18	324	32	1024
14	196	21	441
22	484	30	900
		20	400
		22	484
		25	625
$\Sigma x = 133$	$\Sigma x^2 = 2597$	$\Sigma x = 220$	$\Sigma x^2 = 5190$

$$\bar{x}_1 = \frac{\Sigma x_1}{N_1} = \frac{133}{7} = 19$$

$$\bar{x}_2 = \frac{\Sigma x_2}{N_2} = \frac{220}{10} = 22$$

$$\sigma_1 = \sqrt{\frac{\Sigma x_1^2}{n_1} - \left(\frac{\Sigma x_1}{n_1}\right)^2} = \sqrt{\frac{2597}{7} - \left(\frac{133}{7}\right)^2}$$

$$= \sqrt{371 - (19)^2}$$

$$= \sqrt{371 - 361}$$

$$= \sqrt{10}$$

$$\sigma_1 = 3.16$$

$$\sigma_2 = \sqrt{\frac{\Sigma x_2^2}{n_2} - \left(\frac{\Sigma x_2}{n_2}\right)^2} = \sqrt{\frac{5190}{10} - \left(\frac{220}{10}\right)^2}$$

$$= \sqrt{519 - (22)^2}$$

$$= \sqrt{519 - 484}$$

$$= \sqrt{35}$$

$$\sigma_2 = 5.91$$

Standard Error $= \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$$= \sqrt{\frac{(3.16)^2}{7} + \frac{(5.91)^2}{10}}$$

$$= \sqrt{\frac{9.98}{7} + \frac{34.92}{10}}$$

$$= \sqrt{1.42 + 3.49}$$

$$= \sqrt{4.91}$$

$$SE = 2.21$$

$$T = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$$= \frac{19-22}{2.21} = \frac{3}{2.21}$$

$$\boxed{T = 1.35}$$

table value @ 5% level : 2.73

calculated value is less than table value. H_0 is accept
H_1 is reject

Theory of Probality

ABC
BCA
CAB
ACB
CBA
BAC

$$N_{P_r} = \frac{n!}{(n-r)!}$$

$N = 3$

$r = 3$

$$3C_3 = \frac{3\times2\times1}{(3-3)!} = \frac{6}{0!} = \frac{6}{1} = 6$$

$n = 3$

$r = 2$

$$nP_r = \frac{n!}{(n-r)!} = \frac{3\times2\times1}{(3-2)!}$$

$$3P_2 = \frac{6}{1!}$$

$$= \frac{6}{1}$$

$$nP_r = 6$$

$$\boxed{3P_2 = 6}$$

Find the no. of Permutations of letters is the word "COMMUNICATION".

$c = 2$; $o = 2$; $M = 2$; $U = 1$; $N = 2$; $I = 2$; $A = 1$; $T = 1$

$n = 13$

$$13P_r = \frac{13!}{2!\,2!\,2!\,1!\,2!\,1!\,1!\,2!\,1!\,1!\,1!}$$

$2!\,2!\,2!\,1!\,2!\,2!\,1!\,1!\,1!$

$$= \frac{13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2\times1\times2\times1\times2\times1\times1\times1\times2\times1\times2\times1\times1\times1\times1\times1}$$

$13P_r = 19,45,94,400$

Pg.no 52

Combinations

A Basket contains = 10 mangoes. In how many ways 4 Mangoes from the basket can be selected?

$$N_{C_r} = \frac{n!}{(n-r)!\,r!} = \frac{10!}{(10-4)!\,4!}$$

$n = 10$

$r = 4$

$$^{10}C_4 = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{6!\,4!}$$

$$= \frac{10\times\cancel{9}^{3}\times\cancel{8}^{4}\times7\times\cancel{6}\times\cancel{5}\times\cancel{4\times3\times2\times1}}{\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1\,(\cancel{4\times3\times2\times1})}$$

$^{10}C_4$ = 210 ways.

(or)

$$^{10}C_4 = \frac{10\times\cancel{9}^{3}\times\cancel{8}^{2}\times7}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$$

$^{10}C_4$ = 210 ways

Pg.no 53

How many different sets of 5 students can be chosen out of 20 qualified students to represent a school in an essay contest?

$$N_{C_r} = \frac{n!}{(n-r)!\,r!}$$

$n = 20$

$r = 5$

$$^{20}C_5 = \frac{20!}{(20-5)!\,5!} = \frac{20!}{15!\,5!}$$

$$= \frac{20\times19\times18\times17\times16}{5\times4\times3\times2\times1}$$

$$= \frac{46512}{3}$$

$^{20}C_5$ = 15504 ~~ways~~ sets

(or)

$$^{20}C_5 = \frac{\cancel{20}^{4}\times19\times18\times17\times\cancel{16}^{8}}{\cancel{5}\times\cancel{4}\times3\times\cancel{2}\times1}$$

$$= \frac{46512}{3}$$

$^{20}C_5$ = 15504 ways

Different schools of Thought on Probability

Pg.no 53. What is the chance of getting a head when a coin is tossed ?

Total no. of cases = 2

no. of favarable cases = 1

$P(H) = 1/2$

Pg.no 54. A die is thrown. Find the probability of getting.

1) A '4' 2) an even number

3) '3' or '5' 4) less than '3'

Sol :

Sample space of die is

[1, 2, 3, 4, 5, 6]

a) Probability of (4) = $1/6$

b) Probability of (an even number) = $3/6 = 1/2$

c) Probability of (3 or 5) = $2/6 = 1/3$

d) Probability of [less than '3'] = $2/6 = 1/3$

Pg.no 54. A ball is drawn from a bag containing 4 white, 6 black and 5 yellow balls. Find the probability that a ball drawn is ;

1) white 2) yellow 3) Black 4) Not yellow

5) yellow or white

Sol:

1) P(white ball) = $4/15$

2) Probability of (yellow ball) = $\frac{5}{15} = \frac{1}{3}$

3) P(Black ball) = $6/15 = \frac{2}{5}$

4) P(Not yellow) = $\frac{10}{15} = \frac{2}{3}$

5) P(yellow or white) = $\frac{9}{15} = \frac{3}{5}$

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Eg. no 54. There are 19 cards numbered 1 to 19 in a box. If a person drawn one at random, what is the probability that the no. printed on the card be an even no. greater than 10?

Sol: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [11, 12, 13, 14, 15, 16, 17, 18, 19]

P (Drawing a card with an even no. greater than 10) $= \frac{4}{9}$

Two unbiased dice are thrown. Find the probability that:

a) both the dice shown the same number
b) one die shows 6
c) ~~One~~ first dice shows 3
d) Total of the no. on the dice is 9
e) Total of the no. on the dice is greater than 8
f) A sum of 11.

Sol: when 2 dice are thrown the sample space consists of the following outcomes.

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

a) P (that Both the dice shows the same no) $= \frac{6}{36} = \frac{1}{6}$

b) P (that one dice show 6) $= \frac{10}{36} = \frac{5}{18}$

c) P (First die show 3) $= \frac{6}{36} = \frac{1}{6}$

d) P (that total of the numbers on the dice is 9) $= \frac{4}{36} = \frac{1}{9}$

e) P (that total of the no. is greater than 8) $= \frac{10}{36} = \frac{5}{18}$

f) P (that a sum of 11) $= \frac{2}{36} = \frac{1}{18}$

Problems Based on combination results :-

The box contains 6 white balls and 4 green balls. What is the Probability of drawing a green ball?

no. of Probability :> white = 6

Green = 4

10

Probable no. of cases = $4c_1$

Total no. of cases = 10_{c_1}

$$P(\text{Drawing a green ball}) = \frac{4c_1}{10_{c_1}}$$

$$= \frac{4/1}{10/1}$$

$$= \frac{4}{1} \times \frac{1}{10}$$

$$= 4/10$$

$$= 2/5 \text{ (or) } 0.4.$$

What is the Probability of getting 3 red balls in a draw of ~~3~~ 9 balls from a bag containing 5 red balls and 4 black balls?

Total no. of availability = Red + Black

= 5 + 4

= 9

Favourable no. of cases = $5c_3$

Total no. of cases = $9c_3$

$$P(\text{getting 3 white Balls}) = \frac{5c_3}{9c_3}$$

$$= \frac{\frac{5\times4\times3}{1\times2\times3}}{9\times8\times7/1\times2\times3}$$

$$= \frac{5 \times 4 \times 3}{1 \times 2 \times 3} \times \frac{1 \times 2 \times 3}{9 \times 8 \times 7}$$

$$= \frac{5}{42}$$

$$= 0.11$$

Total no. of availability = Economics + statistician

$$= 5 + 4$$

$$= 9$$

$$\left.\begin{array}{l}\text{selecting three number from} \\ \text{Total no. of availability}\end{array}\right\} = 3$$

3 ~~statis~~ economists $= \frac{5C_3}{9C_3}$

$$= \frac{\frac{5 \times 4 \times 3}{1 \times 2 \times 3}}{\frac{9 \times 8 \times 7}{1 \times 2 \times 3}} \Rightarrow \frac{5 \times 4 \times 3}{1 \times 2 \times 3} \times \frac{1 \times 2 \times 3}{9 \times 8 \times 7}$$

$$= \frac{5}{42}$$

$$= 0.11$$

3 statistician $= \frac{4C_3}{9C_3}$

$$= \frac{\frac{4 \times 3 \times 2}{1 \times 2 \times 3}}{\frac{9 \times 8 \times 7}{1 \times 2 \times 3}} = \frac{4 \times 3 \times 2}{1 \times 2 \times 3} \times \frac{1 \times 2 \times 3}{9 \times 8 \times 7}$$

$$= 1/21$$

$$= 0.04$$

2 economist and 1 statistician $\Rightarrow$

$$= \frac{5C_2 \times 4C_1}{9C_3}$$

$$= \frac{\frac{5 \times 4}{1 \times 2} \times 4/1}{\frac{9 \times 8 \times 7}{1 \times 2 \times 3}}$$

$$= \frac{5\times4}{1\times2} \times \frac{4}{1} \times \frac{1\times2\times3}{9\times8\times7}$$

$$= \frac{10}{21}$$

$$= 0.47$$

1 economist and 2 statistician

$$= \frac{5C_1 \times 4C_2}{9C_3}$$

$$= \frac{\frac{5}{1} \times \frac{4\times3}{1\times2}}{\frac{9\times8\times7}{1\times2\times3}}$$

$$= 5/1 \times \frac{4\times3}{1\times2} \times \frac{1\times2\times3}{9\times8\times7}$$

$$= 5/14$$

$$= 0.35$$

A committee

P (that committee consists of at least one girl) = boys + girls = 8 + 7 = 15

$$\Rightarrow \frac{(7C_1 \times 8C_4) + (7C_2 \times 8C_3) + (7C_3 \times 8C_2) + (7C_4 \times 8C_1) + 7C_5}{15C_5}$$

$$= \frac{\left[7 \times \frac{8\times7\times6\times5}{1\times2\times3\times4}\right] + \left[\frac{7\times6}{1\times2} \times \frac{8\times7\times6}{1\times2\times3}\right] + \left[\frac{7\times6\times5\times4}{1\times2\times3\times4} \times \frac{8}{1}\right] \cdot \left[\frac{7\times6\times5}{1\times2\times3} \times \frac{8\times7}{1\times2}\right] + \left[\frac{7\times6\times5\times4}{1\times2\times3\times4} \times \frac{8}{1}\right] + \left[\frac{7\times6\times5\times4\times3}{1\times2\times3\times4\times5}\right]}{\frac{15\times14\times13\times12\times11}{1\times2\times3\times4\times5}}$$

$$= \frac{490 + 1176 + 980 + 280 + 21}{3003}$$

$$= \frac{2947}{3003} = 0.98$$

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Theorems of Probability

Pg.no 61 What is the probability of picking a card that was red or black?

① The events are mutually exclusive.

P(picking a red card) $= \frac{26}{52}$

P(picking a black card) $= \frac{26}{52}$

P(picking a red / black card) $= \frac{26}{52} + \frac{26}{52} = \frac{26+26}{52}$

$= \frac{52}{52}$

$= 1$

② Sol

P(getting plumbing contract) $= 2/3$

P(not getting electric contract) $= 5/9$

P(getting electric contract) $= 1 - 5/9 = 4/9$

P(getting at least one contract) = P(getting electric contract) + P(getting plumbing contract) − P(getting both)

P(at least one contract) $= \frac{4}{9} + \frac{2}{3} - \frac{4}{5} = \frac{20+30-36}{45}$

$= \frac{14}{45}$

Pg.no 76. Six coins are tossed simultaneously. What is the probability of obtaining 4 heads?

Binomial distribution $P(r) = N_{C_r}\, p^r q^{n-r}$

$r = 4$, $n = 6$, $p = 1/2$, $q = 1/2$

$P(4) = {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4}$

$= {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2$

$= {}^6C_4 \left(\frac{1}{2}\right)^6$

$$= \frac{6\times5\times4\times3}{4\times3\times2\times1} \times \frac{1}{64}$$

$$= 15/64$$

$$P(4) = 0.234$$

Pg. no 76

Sachin Tendulkar :-

I) Exactly 2 Mathces :-

$P = 1/3 \quad q = 2/3$

total no. of match $(n) = 5$

$r = 2$

$$P(r) = {}^nC_r \, p^r q^{n-r}$$

$$P(2) = 5C_2 (1/3)^2 (2/3)^3 \Rightarrow \frac{5\times4}{1\times2}\left(\frac{1}{3}\times\frac{1}{3}\right)\left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right)$$

$$= \frac{5\times\cancel{4}^{2}}{1\times\cancel{2}}\left(\frac{1}{9}\right)\left(\frac{8}{27}\right)$$

$$= 10\left(\frac{8}{243}\right)$$

$$= \frac{80}{243}$$

$$P(2) = 0.329$$

(ii) No match :-

$r = 0$

$$P(0) = 5C_0 (1/3)^0 (2/3)^{5-0}$$

$$= 5C_0 (1/3)^0 (2/3)^5$$

$$= 1\times1\times\left[\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right]$$

$$= \frac{32}{243}$$

$$P(0) = 0.132$$

Mean and standard deviation of Binomial distribution

pg.no 79. For a Binomial distribution, mean $=4$, variance $=\frac{12}{9}$

Find n.

mean $= NP = 4$

variance $= NPq = 12/9$

Need $q = \frac{NPq}{NP} = \frac{12/9}{4}$

$= \frac{12}{9} \times \frac{1}{4}$

$q = 1/3$

$P = 1 - q$

$= 1 - 1/3$

$P = 2/3$

mean $= NP$

$NP = 4$

$N = 4/{2/3}$

$= 4 \times 3/2$

$N = 6$

Fit a Poisson distribution to the given data and calculate the theoretical frequencies ($e^{-0.5} = 0.6065$)

x	0	1	2	3	4
y (f)	123	59	14	3	1

x	f	fx
0	123	0
1	59	59
2	14	28
3	3	9
4	1	4
	200 $\Sigma f(x)$ N	100 Σfx

Average | mean | NP | $\bar{x}$ = $\frac{\Sigma fx}{\Sigma f}$ = $\frac{100}{200}$

$$\boxed{NP = 0.5}$$

$NP_{(0)} = Ne^{-m}$ (or) $Ne^{-0.5}$

$= 200 \times 0.6065$

$NP_{(0)} = 121.3$

Calculated the expected frequencies :-

x	frequency NP			
0	$NP_{(0)}$	$= 121.3$		121
1	$NP_{(0)} \times \frac{M}{1}$	$= \frac{121.3 \times 0.5}{1}$	$= 60.65$	61
2	$NP_{(1)} \times \frac{M}{2}$	$= \frac{60.65 \times 0.5}{2}$	$= 15.16$	15
3	$\frac{NP_{(2)} \times M}{3}$	$= \frac{15.16 \times 0.5}{3}$	$= 2.53$	3
4	$\frac{NP_{(3)} \times M}{4}$	$= \frac{2.53 \times 0.5}{4}$	$= 0.32$	0
			200.	200

A fruit seller :-

Average defective 3 %.

$m = 3$

Random variable (r) = 4.

$$P(r) = \frac{e^{-m} \cdot m^r}{r!}$$

$e = 2.7183$

$$P(r=4) = \frac{2.7183^{-3} \times 3^4}{4!}$$

$$= \frac{2.7183^{-3} \times 81}{1 \times 2 \times 3 \times 4}$$

$$= \frac{0.0497 \times 81}{24}$$

$$= \frac{4.03267}{24}$$

$$P(4) = 0.168$$

Paint the Probability at most 5 defective bottles will be found in a box of 200 bottles - if it is known that 2% such Bottles expected to be defective.

$$m = 200 \times 2/100 = 4$$

$$r = 0, 1, 2, 3, 4, 5$$

$$m = 4$$

$$P(0) = \frac{e^{-4} \cdot 4^0}{0!} = \frac{2.7183^{-4} \times 1}{1} = 0.0183$$

$$P(1) = \frac{0.0183 \times 4}{1!} = \frac{e^{-4} \cdot 4^1}{1} = 0.0732$$

$$P(2) = \frac{e^{-4} \cdot 4^2}{2!} = \frac{0.0183 \times 16}{1 \times 2} = 0.1464$$

$$P(3) = \frac{e^{-4} \cdot 4^3}{3!} = \frac{0.0183 \times 64}{1 \times 2 \times 3} = 0.1953$$

$$P(4) = \frac{e^{-4} \cdot 4^4}{4!} = \frac{0.0183 \times 256}{1 \times 2 \times 3 \times 4} = 0.1953$$

$$P(5) = \frac{e^{-4} \cdot 4^5}{5!} = \frac{0.0183 \times 1024}{1 \times 2 \times 3 \times 4 \times 5} = 0.1562$$

$$0.7847$$

Simple Correlation

$$r = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{N\Sigma x^2 - (\Sigma x)^2} \times \sqrt{N\Sigma y^2 - (\Sigma y)^2}}$$

Compute Karl Pearson co-efficient of correlation for the information given below.

X :	64	65	66	67	68	69	70
Y :	66	67	65	68	70	68	72

X	Y	XY	X^2	Y^2
64	66	4224	4096	4356
65	67	4355	4225	4489
66	65	4290	4356	4225
67	68	4556	4489	4624
68	70	4760	4624	4900
69	68	4692	4761	4624
70	72	5040	4900	5184
469	476	31917	31451	32402
ΣX	ΣY	ΣXY	ΣX^2	ΣY^2

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{N\Sigma X^2 - (\Sigma X)^2} \times \sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{7 \times 31917 - (469)(476)}{\sqrt{7 \times 31451 - (469)^2} \times \sqrt{7 \times 32402 - (476)^2}}$$

$$= \frac{223{,}419 - 223{,}244}{\sqrt{157255 - 219961} \times \sqrt{162{,}010 - 226576}}$$

$$= \frac{175}{\sqrt{-62706} \times \sqrt{-64566}}$$

$$= \frac{175}{250.41 \times 254.09}$$

$$= \frac{175}{63626.67}$$

$r = 0.00275$

calculate co-efficient of correlation from the following data.

X	12	9	8	10	11	13	7
Y	14	8	6	9	11	12	3

x	y	xy	x^2	y^2
12	14	168	144	196
9	8	72	81	64
8	6	48	64	36
10	9	90	100	81
11	11	121	121	121
13	12	156	169	144
7	3	21	49	9
70	63	676	728	651
Σx	Σy	Σxy	Σx^2	Σy^2

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{N\Sigma X^2 - (\Sigma X)^2} \times \sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{7 \times 676 - (70)(63)}{\sqrt{7 \times 728 - (70)^2} \times \sqrt{7 \times 651 - (63)^2}}$$

$$= \frac{4732 - 4410}{\sqrt{5096 - 4900} \times \sqrt{4557 - 3969}}$$

$$= \frac{322}{\sqrt{196} \times \sqrt{588}}$$

$$r = \frac{322}{14 \times 24}$$

$$= \frac{322}{336}$$

$$r = 0.9$$

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