①

Probability and Statistics.

I ⇒ Probability - Def of probability additional theories, Conditional probability, multiplication therom, Bayes theorem of probability & geometrical of problelity random variables and their properties discrete random variables, continous random, variable, probability distribution, join probability distribution and their property, transformation variables, mathematical enpotation, probability generation function.

II ⇒ Probability distribution, discrete distribution, binomid, paison, negative Binomial distribution and their properties (Def, mean, variance moment generating function) andiqua properties fiting of a distribution continous distribution: uniform, normal Emponential distribution and their properties curve fitting using principles of least square

III ⇒ Multivariate Analysis : Coralation co-relation. co-efficient, rank corelation Recuration Analysis, multiple recuration attiibuties coefficient of association χ_2-test for goodness of food test for Independence.

IV = Sample, population, Statistic, parameter Sampling. distribution Standard error, unblished efficiency maximum likely hood, estimated, Hotion and internal estimation, Test of hyportness function formulation of null hyportness critical region, level of Signifiance power of the test.

V = Queving theory: Reverse discription, characterstics of queing model, Studies state Solution, Solution of m|m|1: α model, m|m|1 : N Model.

Text Book:

T. Veerarajan. "probability statistic 4 random process."

Trail and Event :

Consider an experiment of throwing a coin when tossing a coin we make it head (H) or tail (T) a tossing of a coin head or trail. ~~and~~

Ex : 1

Throwing of a tail is a trail and getting 1 or 2 or 3 6

Ex : 2

Form a pack of cards drawing any two cards is a trail and getting a king (or) a queen are called event.

Exhaustive events :

The total no. of possible outcomes in any trail is known as exhaustive event.

Ex : 1

When tossing a coin the possible outcomes are getting head or tail. Hence are have two exhaustive events in throwing a coin.

Ex : 2

When throwing a tie the possible outcomes are getting 1 or 2 or 3 ... 6 Hence we have six exhaustive events in throwing a tie.

Ex :3 :-

The no of face of drawing any three cards from a pack is $52C_3$. Hence we have $52C_3$ exhaustive events in throwing any 3 cards from a pack.

Mutually exclusive events :

Two events are said to be mutually exclusive event when the Occurence of one affects the Occurence of others.

In Other words, If A and B are mutually exclussive events and if A happens then B will not happen.

Ex : 1

In tossing a coin the events head or tail are mutually exclusive, since both tail and head cannot appear in the same time.

Ex:2:-

In throwing a die, all the six events namely getting of 1 or 2 ... 6 are mutually exclusive events. Since the appearence of 1 influence of 2

Independent events:

Two events are said to be independent when the actual happening of one does not influence in any way the happening of the other.

Ex: 1

In tossing a coin, the event of getting a head in the first toss is independent of getting a head in the second toss, third toss etc...

Note:

If A and B are mutually exclusive, then

$P(A \cap B) = P(\phi) = 0$

If A and B are independent then

$P(A \cap B) = P(A) \cdot P(A)$

Probability

$$P(E) = \frac{\text{No. of favourable case } (m)}{\text{Total no. of exhaustive case } (n)} = \frac{m}{n}$$

where

m = No. of favourable case

n = No. of exhaustive case

Find the probability of a card drawn at random from a ordering pack is a diamond.

Soln

Total no. of ways of getting 1 card $\}$ = 52

No. of ways getting 1 diamond card is 13

$\therefore$ probability = $\dfrac{\text{No. of favarable case}}{\text{No. of Exhaustive case}}$

$= \dfrac{13}{52}$

Probability $= \dfrac{1}{4}$

Important (7)

1) A Bag Contains 7 white, 6red and 5 black balls. Two balls are drawn at random. Find the probability that the both will be white balls.

Soln:

Total no. of. balls = 7+6+5 = 18

From the 18 balls 2balls can be drawn in

$18C_2 = \dfrac{\cancel{18}^{9} \times 17}{1 \times \cancel{2}_1} = 153$

Total no. of Exhaustive case = 153

2 white balls can be drawn from 7 white ball

$$7C_2 = \frac{7\times\cancel{6}^3}{1\times\cancel{2}_1} = 21$$

Total no. of favourable case = 21

drawing two white balls = $\frac{\cancel{21}^7}{\cancel{153}_{51}} = \frac{7}{51}$.

2) Four person's are choosen at random from a group containing 3 men, 2 women and 4 children. Show that exactly two of them will be children is $\frac{10}{21}$

Soln:

Total we have 9 persons = 3+2+4 = 9

Four person can be choosen out of 9 persons in $9C_4$ $\Big\} = \frac{9\times\cancel{8}^4\times7\times\cancel{6}^2}{1\times\cancel{2}_1\times\cancel{3}_1\times4}$

= 126 way.

Total Exhaustive ways = 126 ways.

We need to choose 4 persons. Out of these 4 persons, Should be children and remaining two person's are either men (or) women.

The no. of ways choosing 2 children out of 4 children are $4c_2$ ways $= \frac{\cancel{4}^{2}\times 3}{1\times \cancel{2}_{1}}$

$= 6$ ways

∴ The remaining two person's can be chooser from 5 persons (3 men + 2 women)

$5c_2$ ways $= \frac{5\times \cancel{4}^{2}}{1\times \cancel{2}_{1}} = 10$

$= 10$ ways.

The total no. of favarable cases

6×10 ways $= 60$ ways.

Group consists of exactly two children $\Big\} = \frac{\cancel{60}^{10}}{\cancel{126}_{21}}$

$= \frac{10}{21}$

∴ The group consists of exactly two children $\Big\} = \frac{10}{21}$.

3) From a group of 3 Indians 4 Pakistans and 5 Americans, a Sub-Commitee of four people Selected by lots. Find the probability that the Sub-Commitee will consist of

i) 2 Indians & 2 Pakistans

ii) 1 Indian, 1 pakistan & 2 American's

iii) 4 Americans.

Soln:

Total no. of peoples = 3+4+5 = 12

4 people can be choosen from 12 people

$$12C_4 \text{ ways} = \frac{\overset{3}{\cancel{12}} \times 11 \times \overset{5}{\cancel{10}} \times 9}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4}}$$

$$= \frac{11 \times 5 \times 9}{1}$$

$12C_4$ ways = 459 ways.

i) 2 Indians & 2 pakistans.

2 Indians choosen from 3 Indians

$$3C_2 = \frac{3 \times \cancel{2}}{1 \times \cancel{2}} = 3$$

$$3C_2 = 3$$

2 pakistan choosen from 4 pakistans

$$4C_2 = \frac{\overset{2}{\cancel{4}} \times 3}{1 \times \cancel{2}} = 6$$

2 Indians × 2 pakistans

$3 \times 6 = 18$ ways

Total no.of favarable case = 18 ways

ii) 1 Indians, 1 Pakistan & 2 Americans

1 Indian choosen from 3 Indians

$3C_1 =$

A) A bag contain 10 white balls, 6 red balls, 4 black balls, 7 blue, 5 balls drawn at random, what is probability that 2 of them are red and 1 is black.

Soln:

Total no. of balls = 10+6+4+7 = 27 balls.

i) 5 Balls choosen from these 27 balls.

$$27C_5 = \frac{27 \times 26 \times 25 \times 24 \times 23}{1 \times 2 \times 3 \times 4 \times 5}$$

$$= 27 \times 26 \times 5 \times 23$$

$27C_5 = 80730$ ways.

ii) 2 red balls can be choosen from 6 red balls.

$$6C_2 = \frac{6 \times 5}{1 \times 2} = 15 \text{ ways.}$$

iii) 1 black ball choosen from 4 balls.

$$4C_1 = \frac{4}{1} = 4 \text{ ways.}$$

The total no. of favorable case

$15 \times 4 = 60$ ways.

$$\text{Probability} = \frac{60}{80730}.$$

5) What is the chance that a leap year (12)
Selected at random will contain 53 Sundays

Soln:

A Leap year consist of 366 days. Out of these 366 days, We have 52 full weeks and chance we have 52 sundays definitely 2 days extra these 2 days may be,

i) Monday, Tuesday
ii) Tuesday, wednesday
iii) Wednesday, Thursday
iv) Thursday, Friday
v) Friday, Saturday
vi) Saturday, Sunday
vii) Sunday, Monday.

Of the 7 cases the last 2 cases contain Sunday and hence we have 2 favarabb cases

$$\text{probability} = \frac{2}{7}$$

(4) Baye's Theorem:

Let $A_1, A_2, \dots A_n$ be 'n' mutually exclusive and Exhaustive events with $P(A_i) \neq 0$ for $i = 1, 2 \dots n$ let B be an event such that

$B \subset \bigcup_{i=1}^{n} A_i$, $P(B) = 0$, then.

$$P\left(\frac{A_i}{B}\right) = \frac{P(A_i) \cdot P\left(\frac{B}{A_i}\right)}{\sum_{i=1}^{n} P(A_i) P(B/A_i)}$$

Proof:

Given $B \subset \bigcup_{i=1}^{n} A_i$,

$$B = B \cap \left[\bigcup_{i=1}^{n} A_i\right]$$

$$B = \bigcup_{i=1}^{n} (B \cap A_i) \quad (\because \text{Distributive law})$$

$$P(B) = P\left[\bigcup_{i=1}^{n} (B \cap A_i)\right]$$

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i)$$

$[\because B \cap A_i \; (i = 1, 2 \dots n)$ are mutually disjoint events$]$

$$\therefore P(B) = \sum_{i=1}^{n} P(A_i) \cdot P\left(\frac{B}{A_i}\right)$$

also we have

$$P(B \cap A_i) = P(B) \cdot P\left(\frac{A_i}{B}\right)$$

$$P\left(\frac{A}{B}\right) = \frac{P(B \cap A_i)}{P(B)}$$

$$P\left(\frac{A_i}{B}\right) = \frac{P(A_i) \cdot P\left(\frac{B}{A_i}\right)}{\sum_{i=1}^{n} P(A_i)\, P\left(B/A_i\right)}$$

Hence proved.

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1. In a bolt factory machine A, B, C manfacturing respectively 25%, 35% and 40% of the total of their Output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. what the probabilites that it was manfactured machine's A, B and C.

A manufacturing bolt probability

B defective bolt probability.

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{i=1}^{n} P(A_i) \cdot P(B/A_i)}$$

$P(A_i)$	$P(B/A_i)$	$P(A_i) = P(A_i) \cdot P(B/A_i)$
$P(A_1) = 0.25$	0.05	$P.A_1 \cdot P(B/A_1) \cdot 0.0125$
$P(A_2) = 0.35$	0.04	$P(A_2) \cdot P(B/A_2) \cdot 0.014$
$P(A_3) = 0.40$	0.02	$P(A_3) \cdot P(B/A_3) = 0.008$
1.00	0.11	$\Sigma P(A_i) \cdot P(B/A_i) = 0.0345$

P(defective bolt manufacturing machine A1)

$$P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{\sum_{i=1}^{3} P(A_1)\,(P\,B/A_1)}$$

$$= \frac{(0.25)(0.05)}{0.0345}$$

$$= \frac{0.0125}{0.0345}$$

$$P(A_1/B) = 0.3623$$

P(defective bolt manufacturing machine A_2)

$$P(A_2/B) = \frac{P(A_2)\cdot P(B/A_2)}{\sum_{i=1}^{3} P(A_2)\,P(B/A_2)}$$

$$= \frac{(0.35)\cdot(0.04)}{0.0345} = \frac{0.014}{0.0345}$$

$$P(A_2/B) = 0.4058$$

P(defective bolt manufacturing machine A_3)

$$P\left(\frac{A_3}{B}\right) = \frac{P(A_3) \cdot P(B/A_3)}{\sum_{i=1}^{3} P(A_3)\, P(B/A_3)}$$

$$= \frac{(0.40)(0.02)}{0.0345}$$

$$= \frac{0.008}{0.0345}$$

$$P(A_3/B) = 0.2319$$

Defination of Random Variable:

A random variable x whose value is determind by the outcomes of a random experiment is called random variable.

The random variable is also called Stochastic variable.

Ex:

Consider the random experiment of throwing coin twice.

We have the result

$\{HH, HT, TH, TT\}$

The no. of heads $X = \{2, 1, 1, 0\}$

Def.

(17)

Discreate random variables:

The random variables takes on the integer value. Only on set $\{0, 1, 2, 3 \ldots n\}$

Ex:

The printing mistake in each page of a book.

Continuous random variable:

The random variable takes on the all values with in a certain interval then the random variable is called continuous random variable.

Ex:

The hight, weight, age of the individual

Distribution function of a random variable:

The distribution function of a random variable x define on the interval $(-\infty, \infty)$ is given by $f(x) = p(x \leq x)$

Note:

If $f(x)$ is a distribute function of a one dimensional random variable, then

i) $0 \leq f(x) \leq 1$

ii) $x < y$ then $f(x) \leq f(y)$

iii) $f(-\infty) = 0$, $f(\infty) = 1$

Def:

Probability mass function (or) probability function:

Let x be a one dimensional discrete random variable which takes the value $x_1, x_2 \ldots x_n$ let each possible outcomes x.

We can associate no. of P_i the (i.e)

$P_i = P(x = x_i)$ then

$P(x = x_i) = P(x_i) = P_i$ is called the $P(x_i)$

The no. of $P_i = P(x_i)$ satisfy the following condition.

i) $P(x_i) \geq 0$

ii) $\sum_{i=1}^{\infty} P(x_i) = 1$

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1. A random variable x has the following probability function.

x	0	1	2	3	4	5	6	7	8
$P(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i) Determine the value of a
ii) find $P(x<3)$
iii) $P(x \geq 3)$
iv) $P(0<X<5)$
v) find the distribution function of x.

Soln:

i) Determine the value of a.

$$\sum p(x) = 1$$

$$p(x) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$$

$$= a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a$$

$$= 81a$$

$$81a = 1$$

$$a = \frac{1}{81}$$

ii) find $p(x<3)$

$$P(x=0) + P(x=1) + P(x=2)$$

$$= a + 3a + 5a$$

$$= 9a$$

$$= \cancel{9} \times \frac{1}{\cancel{81}_{9}}$$

$$P(x<3) = \frac{1}{9}$$

iii) $P(x \geq 3)$

$P(x \geq 3) = P(x=3) + P(x=4) + P(x=5) + P(x=6)$
$+ P(x=7) + P(x=8)$

$= 7a + 9a + 11a + 13a + 15a + 17a$

$= 72a$

$= 72 \times \frac{1}{81}$

$= \frac{\cancel{72}^{8}}{\cancel{81}_{9}}$

$P(x \geq 3) = \frac{8}{9}$

iv) $P(0 < x < 5)$

$P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= 3a + 5a + 7a + 9a$

$= 24a$

$= \cancel{24}^{8} \times \frac{1}{\cancel{81}_{27}}$

$P(0 < x < 5) = \frac{8}{27}$

v) find the distribution function x.

$f(x) = P(x \leq x)$

$0 \quad P(\cancel{x}x) = 0 = a = \frac{1}{81}$

$1 \quad P(0 \leq 1) = P(0) + P(1) = a + 3a = 4a$

$P(0 \leq 1) = \frac{4}{81}$

2 $P(0 \le 2) = P(0) + P(1) + P(2) = a + 3a + 5a = 9a$

$$= \frac{9}{81} = \frac{1}{9}$$

$$P(0 \le 2) = \frac{1}{9}$$

3 $P(0 \le 3) = P(0) + P(1) + P(2) + P(3)$

$$= 16a = \frac{16}{81}$$

$$P(0 \le 3) = \frac{16}{81}$$

4 $P(0 \le 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$$= 25a = \frac{25}{81}$$

$$P(0 \le 4) = \frac{25}{81}$$

5 $P(0 \le 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$$= 36a = \frac{\cancel{36}\,4}{\cancel{81}\,9}$$

$$P(0 \le 5) = \frac{4}{9}$$

6 $P(0 \le 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$

$$= \frac{49}{81}$$

7 $P(0 \le 7) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$

$$= 64a = \frac{64}{81}$$

$$P(0 \le 7) = \frac{64}{81}$$

8 $(P0 \le 8) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$

$$= 81a = \frac{81}{81} = 1$$

$$P(0 \le 8) = 1$$

Distribution function of x :-

find the probability distribution.

Soln:

let $2p(x=1) = 3p(x=2) = p(x=3) = 5p(x=4) = k$

$2p(x=1) = k$

$p(x=1) = \frac{k}{2}$

$3p(x=2) = k.$

$p(x=2) = \frac{k}{3}$

$p(x=3) = k$

$5p(x=4) = k.$

$p(x=4) = \frac{k}{5}$

$\Sigma p_{xi} = 1$

$\frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$

$\frac{15k + 10k + 30k + 6k}{30} = 1$

$61k = 30$

$k = \frac{30}{61}$

i) $p(x=1) = \frac{k}{2}$

$= \frac{30/61}{2} = \frac{\cancel{30}^{15}}{61} \times \frac{1}{\cancel{2}_1}$

$p(x=1) = 15/61$

ii) $p(x=2) = k/3$

$$= \frac{30/61}{3} = \frac{30}{61} \times \frac{1}{3} = \frac{10}{61}$$

$$p(x=2) = \frac{10}{61}$$

iii) $p(x=3) = k$

$$p(x=3) = \frac{30}{61}$$

iv) $p(x=4) = \frac{k}{5}$

$$p(x=4) = \frac{30}{61/5} = \frac{\cancel{30}^{6}}{61} \times \frac{1}{\cancel{5}} = \frac{6}{61}$$

$$p(x=4) = \frac{6}{61}$$

x	$p(x=1)$	$p(x=2)$	$p(x=3)$	$p(x=4)$
$p(x)$	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

—— x —— x —— x ——

3. If a random variable 'x' has the p.d.f

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

find the mean and variance o

Soln:

$$\text{Mean} = \int_a^b x f(x)\, dx$$

$$= \int_{-1}^{1} x \frac{1}{2}(x+1)\, dx$$

$$= \frac{1}{2} \int_{-1}^{1} x(x+1)\,dx$$

$$= \frac{1}{2} \int_{-1}^{1} (x^2 + x)\,dx \Rightarrow \text{Integration apply}$$

$$= \frac{1}{2} \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^{1} \quad (uv - uv)$$

$$= \frac{1}{2} \left[\frac{(1)^3}{3} + \frac{(1)^2}{2} - \left(\frac{(-1)^3}{3} + \frac{(-1)^2}{2} \right) \right]$$

$$= \frac{1}{2} \left(\frac{1}{3} + \frac{1}{2} - \left(\frac{-1}{3} + \frac{1}{2} \right) \right]$$

$$= \frac{1}{2} \left(\frac{1}{3} + \cancel{\frac{1}{2}} + \frac{1}{3} - \cancel{\frac{1}{2}} \right)$$

$$= \frac{1}{2} \left(\frac{1+1}{3} \right)$$

$$= \frac{1}{\cancel{2}} \left(\frac{\cancel{2}}{3} \right)$$

$$\text{mean} = \frac{1}{3}$$

ii) $\text{Variance} = \int_a^b (x - \text{mean})^2 f(x)\,dx$

$$= \int_{-1}^{1} \left(x - \frac{1}{3}\right)^2 \frac{1}{2} (x+1)\,dx$$

$$= \frac{1}{2} \int_{-1}^{1} \underbrace{\left(x - \frac{1}{3} \right)^2}_{} (x+1)\,dx \quad (a-b)^2$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$= x^2 + \left(\frac{1}{3}\right)^2 - 2x\frac{1}{3}$$

$$= x^2 + \frac{1}{9} - \frac{2x}{3}$$

$$= \frac{1}{2}\int_{-1}^{1}\left(x^2+\frac{1}{9}-\frac{2x}{3}\right)(x+1)\,dx$$

LCM

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{9x^2+1-6x}{9}\right)(x+1)\,dx$$

$$= \frac{1}{18}\int_{-1}^{1}(9x^2+1-6x)(x+1)\,dx$$

$$= \frac{1}{18}\int_{-1}^{1}(9x^3+9x^2+x+1-6x^2-6x)\,dx$$

$$= \frac{1}{18}\int_{-1}^{1}(9x^3+3x^2-5x+1)\,dx$$

$$= \frac{1}{18}\left[\frac{9x^4}{4}+\frac{3x^3}{3}-\frac{5x^2}{2}+x\right]_{-1}^{1}$$

$$= \frac{1}{18}\left[\frac{9(1)^4}{4}+\frac{3(1)^3}{3}-\frac{5(1)^2}{2}+1 - \left(\frac{9(-1)^4}{4}+\frac{3(-1)^3}{3}-\frac{5(-1)^2}{2}-1\right)\right]$$

$$= \frac{1}{18}\left[\frac{9}{4}+\frac{3}{3}-\frac{5}{2}+1-\left(\frac{9}{4}\ \frac{3}{3}-\frac{5}{2}-1\right)\right]$$

$$= \frac{1}{18}\left[\frac{9}{4}+1-\frac{5}{2}+1-\frac{9}{4}+1+\frac{5}{2}+1\right]$$

$$= \frac{1}{18}(1+1+1+1)$$

$$= \frac{1}{\cancel{18}_{9}}(\cancel{4}^{2}) = \frac{2}{9}$$

Variance $= \frac{2}{9}$

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Probability dencity function:

26

The probability dencity function of a Continuous random variable x in the interval (a,b) is given by

$$f(x) = \begin{cases} 0 & : x < a \\ P(x) & : a \le x \le b \\ 0 & : a > b \end{cases}$$

Cumulative distributive function:

If $F(x) = P(x \le x) = \int_{-\infty}^{\infty} f(x)\,dx$ then

$f(x)$ is defined as Cumulative distributive function (or) distribution function of a Continuous distribution random variable x (c.d.f)

5m

Com. Addition theorem (expectation)

IA

5m

If x and y are two continuous random variable with pdf $f_x = (x)$ and $f_y = (y)$

then

$$E(X+Y) = E(x) + E(y)$$

Proof:

Let x and y be Continuous random variable with marginal p.d.f $f_x(x)$ and $f_y(y)$ and whose joint pdf $f_{xy}(x,y)$

Then $E(x) = \int_{-\infty}^{\infty} x \, f_x(x) \, dx$

$$E(y) = \int_{-\infty}^{\infty} y \, f_y(y) \, dx$$

Now

$$E(x+y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y) \, f_{xy}(x,y) \, dx \, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \, f_{xy}(x,y) \, dx \, dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y \, f_{xy}(x,y) \, dx \, dy$$

$$E(x+y) = \int_{-\infty}^{\infty} x \left[\int_{-\infty}^{\infty} f_{xy}(x,y) \, dy\right] dx + \left[\int_{-\infty}^{\infty} f_{xy}(x,y) \, dx\right] dy$$

$$= \int_{-\infty}^{\infty} x \, f_x(x) \, dx + \int_{-\infty}^{\infty} y \, f_y(y) \, dy$$

$$= \cancel{E(X)\,E(Y)} \quad E(X) + E(Y)$$

$$E(x+y) = E(X) + E(Y).$$

(14) **Multiplication theorem (expectation)**

5m

If x any y are independent random variable then $E(XY) = E(X) \cdot E(Y)$

Proof:

Let X and Y be continuous random variable with joint pdf's $f_{xy}(x,y)$ and pdf $f_X(x)$ and $f_Y(y)$ respectively.

$$E(x) = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

$$E(y) = \int_{-\infty}^{\infty} y f_Y(y)\, dy$$

Now

$$E(xy) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{XY}(x,y)\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_X(x) \cdot f_Y(y)\, dx\, dy$$

[Since x and y are independent]

$$= \int_{-\infty}^{\infty} x\, f_X(x)\, dx \cdot \int_{-\infty}^{\infty} y\, f_Y(y)\, dy$$

$$E(XY) = E(X) \cdot E(Y)$$

10m Prove that geomatric mean G of distribution $df = 6(2-x)(x-1)\,dx,\ 1 \le x \le 2$ is given by $6 \log(16G) = 19$.

10m

Soln:

$$\log G = \int_a^b \log x \, f(x)\, dx$$

$$= \int_1^2 \log x \; 6(2-x)(x-1)\, dx$$

$$= 6\int_1^2 \log x \,(2-x)(x-1)\, dx$$

$$= 6\int_1^2 \log x \,(2x-2-x^2+x)\, dx$$

$$= 6\int_1^2 \log x \,(-x^2+3x-2)\, dx$$

$$\log G = 6\int_1^2 \underbrace{\log x}_{u}\, \underbrace{(x^2-3x+2)}_{v}\, dx$$

$\int u\,dv = uv - \int v\,du$

$u = \log x$ | $dv = x^2 - 3x + 2$

$\frac{du}{dx} = \frac{1}{x}$ | $dv = \frac{x^3}{3} - \frac{3x^2}{2} + 2x$

$du = \frac{1}{x}\,dx$

$$= -6\left[\log x\left(\frac{x^3}{3}\right) - \frac{3x^2}{2} + 2x\right)\Big]_1^2 - \int_1^2\left[\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]\left(\frac{1}{x}\right)dx\right]$$

$$= 6\left[\log x\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_1^2 - \int_1^2\left(\frac{x^3}{3} - \frac{3x}{2} + 2\right)\right.$$

$$= -6\left[\log x\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)\right]_1^2 - \left[\frac{x^3}{3\times 3} - \frac{3x^2}{2\times 2} + 2x\right]_1^2$$

$$= -6\left[\log x\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)\right]_1^2 - \left[\frac{x^3}{9} - \frac{3x^2}{4} + 2x\right]_1^2$$

$\boxed{uv = vu}$

$$= -6\left[\log 2\left(\frac{2^3}{3} - \frac{3(2)^2}{2} + 2(2)\right) - \log(1)\right] \quad \boxed{\log 1 = 0}$$

$$- \left[\left(\frac{2^3}{9} - \frac{3(2)^2}{4} + 2(2)\right) - \left(\frac{1^3}{9} - \frac{3(1)^2}{4} + 2(1)\right)\right]$$

$$= -6\left[\log 2\left(\frac{8}{3} - 6 + 4\right) - \left(\frac{8}{9} - 3 + 4\right) - \left(\frac{1}{9} - \frac{3}{4} + 2\right)\right]$$

$$= -6\left[\log 2\left(\frac{8 - 18 + 12}{3}\right) - \left(\frac{8}{9} - 3 + 4 - \frac{1}{9} - \frac{3}{4} + 2\right)\right]$$

$$= -6\left[\log 2\left(\frac{2}{3}\right) - \left[\frac{-8}{9} - 1 - \frac{1}{9} + \frac{3}{4}\right]\right)$$

Scanned with CamScanner

$$= -6\left[\log 2\left(\frac{2}{3}\right) - \left(\frac{32-36-4+27}{36}\right)\right]$$

$$= -6\left[\log 2\left(\frac{2}{3}\right) - \left(\frac{19}{36}\right)\right]$$

$$= -\cancel{6}^{2}\log 2\left(\frac{2}{\cancel{3}_{1}}\right) + \cancel{6}^{1}\left(\frac{19}{\cancel{36}_{6}}\right)$$

$$= -2\log 2(2) + \left(\frac{19}{6}\right)$$

$$= -4\log 2 + \left(\frac{19}{6}\right)$$

$$\log G = -4\log 2 + \frac{19}{6}$$

$$\log G + 4\log 2 = \frac{19}{6} \quad [n\log m = \log m^{n}]$$

$$\log G + \log 2^{4} = \frac{19}{6}$$

$$\log G + \log 16 = \frac{19}{6} \quad [\log m + \log n = \log mn]$$

$$6(\log G + \log 16) = 19$$

$$6\log G(16) = 19.$$

$$\therefore 6\log(16G) = 19.$$

Hence its proved. $6\log(16G) = 19$.

Mathematical Expectation:

(1.) 10m

(i.) 2m [Let x be a random variable with probability density function f(x) or p(x). Then the mathematical expectation of 'x' is denoted by E(x) is given by

$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$ [for a continuous random variable]

$E(X) = \sum_{x} x\, p(x)\, dx$ [for a ~~distance~~ discrete random variable]] 2m

r^{th} moment (About Orgin)

Consider a continuous random variable 'x' with p.d.f (or) p.m.f f(x) then the r^{th} moment (about Orgin) of the probability distribution is defind as.

$$E(x^r) = \int_{-\infty}^{\infty} x^r f(x)\, dx$$

It's denoted by $\mu_r' = \int_{-\infty}^{\infty} x^r f(x)\, dx$

$\mu_1' = E(x)$ (μ_1' about Orgin)

$\mu_2' = E(x_2)$ (μ_2' about Orgin)

$\text{Mean} = \bar{X} = \mu_1' = E(x)$

Variance $= \mu_2 = \mu_2' - \mu_1'^2$

Variance $= \left[E(x^2) - E(x)\right]^2$

Now

$$E\{x - E(x)\}^r = \int_{-\infty}^{\infty} (X - E(X))^r f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x - \bar{x})^r f(x)\, dx$$

Thus μ_r $$= \int_{-\infty}^{\infty} (x - \bar{x})^r f(x)\, dx$$

Thus gives the r^{th} moment about mean and it denote by μ_r

put $r = 1$ we get,

$$\mu_1 = \int_{-\infty}^{\infty} (x - \bar{x}) f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x f(x)\, dx - \int_{-\infty}^{\infty} \bar{x} f(x)\, dx$$

$$= \bar{x} - \bar{x} \int_{-\infty}^{\infty} f(x)\, dx$$

$$= \bar{x} - \bar{x} \qquad \boxed{\therefore \int_{-\infty}^{\infty} f(x)\, dx = 1}$$

$$= 1 - 1$$

$\mu_1 = 0$

Put $r=2$ we get

$$\text{Variance} = \int_{-\infty}^{\infty} (x - E(x))^2 \, f(x)\, dx \qquad [\because E(x) = \bar{x}]$$

$$= \int_{-\infty}^{\infty} (x-\bar{x})^2 \, f(x)\, dx$$

$$\mu_2 = E\left[\{x - E(x)\}^2\right]$$

which gives the variance in terms of expectation

$$\text{Let } (g(x)) = E(k) = \int_{-\infty}^{\infty} k f(x)\, dx$$

$$= k \int_{-\infty}^{\infty} f(x)\, dx$$

$$= k \cdot 1 \qquad \left[\because \int_{-\infty}^{\infty} f(x)\, dx = 1\right]$$

$$E(k) = k$$

<u>For discrete random variable.</u>

$$E(x^r) = \sum_x x^r p(x)$$

$$\mu_r' = E(x^r) = \sum_x x^r p(x)$$

Put $r=1$ we get

$$\text{Mean} = \mu_1' = \sum x\, p(x)$$

put $r=2$ we get

$$\text{Variance} = \mu_2.$$

[illegible] the following probability distribution of X compute

i) $E(X)$ ii) $E(X^2)$

iii) $E(2X \pm 3)$ iv) Variance $(2X \pm 3)$

X	-3	-2	-1	0	1	2	3
P(x)	0.05	0.10	0.30	0	0.30	0.15	0.10

i) $E(X) = \sum_{i=1}^{n} x_i p(n)$

$= -3(0.05) - 2(0.10) - 1(0.30) + 0(0) + 1(0.30) + 2(0.15) + 3(0.10)$

$E(X) = 0.25$

ii) $E(X^2) = \sum_{i=1}^{n} x_i^2 p(n)$

$= (-3)^2(0.05) + (-2)^2(0.10) + (-1)^2(0.30) + 0(0) + (1)^2(0.30) + 2^2(0.15) + 3^2(0.10)$

$E(X^2) = 2.95$

iii) $E(2n \pm 3) = 2E(X) \pm 3$

$= 2(0.25) \pm 3$

$E(2n \pm 3) = 0.5 \pm 3$

(+, -) Values Subsitute

$0.5 + 3 = 3.5$ | $0.5 - 3 = -2.5$

iv) Variance $(2x \pm 3)$

$$\boxed{\begin{aligned} Var(ax \pm b) &= a^2 (Var(X)) \\ Variance(X) &= E(X^2) - (E(X))^2 \end{aligned}}$$

$$\begin{aligned} Var(2x \pm 3) &= 2^2\ Var(X) \\ &= 4\ E(X^2) - [E(X)]^2 \\ &= 4[(2.95) - (0.25)^2] \\ &= 4[(2.95) - (0.0625)] \\ &= 4\,(2.8875) \end{aligned}$$

$$Var(2x \pm 3) = 11.55$$

Important formulas:

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$E(y) = \int_{-\infty}^{\infty} y\, f(y)\, dx$$

$$E(ax+b) = a\,E(x) + b$$

$$Var(ax+b) = a^2\ Var\ x$$

$$Cov(xy) = E(xy) - E(x) \cdot E(y)$$

$$Var(x_1 + x_2) = Var(x_1) + Var(x_2) + \cancel{Var}\ 2\,Cov(x_1, x_2)$$

Probability Distribution:

1) Mean $= \int_a^b x f(x)\,dx$

2) Median $= \int_a^{\mu} f(x)\,dx = \frac{1}{2}$

(or)

$$\int_{\mu}^{b} f(x)\,dx = \frac{1}{2}$$

3) mode $f'(x) = 0$ and $f''(x) < 0$

4) Geometric mean $\log G = \int_a^b \log x\, f(x)\,dx$

5) Harmonic mean $\int_a^b \frac{1}{x} f(x)\,dx$

Joint probability Distribution:

⇒ Two dimensional random Variable

let S be a Sample space

⇒ let $X(s)$ and let $Y(s)$ be two function each assigning a real no's to each Outcomes s is S. Then (X, Y) is two dimension random variable

Two dimension discrete random variable (3)

The possible value of x, y are finite then x, y is called two dimensional discrete random variable

It can be represented by (x_i, y_i)

where i various from $i = 1, 2, 3 \ldots n$
j various from $j = 1, 2, 3 \ldots m$

Joint probability function of the discrete random variable X and Y :-

For two discrete random variable x & y the probability that X will takes the value x_i and y_i will be takes values y_j as

$P(X = x_i, Y = y_j)$ consequently.

2m ($f(x_i, y_i) = P(x = x_i, y = y_j)$ is called the joint probability function (or) joint probability mass function) 2m

(*)

Joint probability Distribution of xy

The set of triples $\{x_i, y_j, P_{ij}\}$ $i = 1, 2 \ldots n$ $j = 1, 2 \ldots m$ is called (J.P.D of x.y) It can be represented by from the table

X \ Y	y_1	y_2	...	y_m	$P_X(x_i)$
x_1	P_{11}	P_{12}	...	P_{1m}	$P_1 = P_1(x = x_1)$
x_2	P_{21}	P_{22}	...	P_{2m}	$P_2 = P(x = x_2)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_n	P_{n1}	P_{n2}	...	P_{nm}	$P_n = P(x = x_n)$

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Marginal probal function of X.

If the join probability distribution of Two. Random Variables x & y is given then the marginal probability function X is given by

$$P_X(x_i) = P(X = X_i)$$
$$= P(x = x_i, y = 1) + P(x = x_i, y = 2) \ldots$$
$$P(x = x_i, y = m)$$
$$= P_{i1} + P_{i2} + P_{i3} \cdots P_{im}$$
$$= \sum_{j=1}^{m} P_{ij}$$
$$P_X(x_i) = P_i$$

The set $\{x_i, P_i\}$ is called marginal distribution function X.

—— × ——

Marginal probability function of Y

Marginal probability function of Y of the join probability distribution of Two random variables X and Y given by them the marginal probability function X is given by

$$P_Y(y_j) = P(Y = y_j)$$

$$= P(x=1, y=y_j) + P(x=2, y=y_j) + \dots + (P(x=n), y=y_j)$$

$$= P_{1j} + P_{2j} + \dots P_{nj}$$

$$= \sum_{i=1}^{n} P_{ij}$$

$$P_Y(y_j) = P_{\cdot j}$$

Conditional probabilities:

The conditional probability function $x/y = y_j$ is given by

$$P\left(x = x_i / y = y_j\right) = \frac{P(X = x_i, Y = y_j)}{P(x = y_j)}$$

(or)

$$P\left(x = x_i / y = y_j\right) = \frac{P_{ij}}{P_{\cdot j}}$$

$$f(x/y) = \frac{f(x,y)}{f(y)}$$

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The conditional probability function

$x = x_i / y = y_j$ is given by

$$P(y = y_j / x = x_i) = \frac{P(x = x_i, y = y_i)}{P(x = x_i)}$$

$$P(y = y_i / x = x_i) = \frac{P_{ij}}{P_i}$$

$$f(y/x) = \frac{f(x,y)}{f(x)}$$

The two Random variable x & y are said to be independent, the

$P(x = x_i / y_j) = P_{ij}$

$$P_{ij} = P(x = x_i)\, P(y = y_j)$$

$$P_{ij} = P_i \cdot \times P_j$$

Joint probability function of two Continuous Random Variables X and Y.

if X and Y are continuous Random Variables then we shall refer to $f(x,y)$ as the joint probability

function (or) Join probability density (41)
function of these two Random Variables
if the probability that $a_1 \le x \le b_1$;
$a_2 \le x_2 \le b_2$ is given by the multiple
integral $\int_{a_1}^{b_1} \int_{a_2}^{b_2} f(x,y)\, dy\, dx$

i.e.) $P[a_1 \le x\, b_1, a_2 \le y \le b_2] = \int_{a_1}^{b_1} \int_{a_2}^{b_2} f(x,y)\, dy\, dx$

Provided i) $f(x,y) \ge 0$

ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1$

Note:

The calculate Marginal distributions when the random variable X takes Horizontal Value and y takes Vertical values.

X / y	x_1	x_2	x_3	$PY(y) = f(y)$
y_1	P_{11}	P_{12}	P_{13}	$P_{11} + P_{12} + P_{13} = P(y = y_1)$
y_2	P_{21}	P_{22}	P_{23}	$P_{21} + P_{22} + P_{23} = P(y = y_2)$
y_3	P_{31}	P_{32}	P_{33}	$P_{31} + P_{32} + P_{33} = P(y = y_3)$
$PX(x) = f(x)$	$P_{11} + P_{21} + P_{31} = P(X = x_1)$	$P_{12} + P_{22} + P_{32} = P(X = x_2)$	$P_{13} + P_{23} + P_{33} = P(X = x_3)$	

1. From the following table for bivariable distribution of (X, Y) find

i) $P(X \le 1)$ ii) $P(Y \le 3)$ iii) $P(X \le 1, Y \le 3)$

iv) $P\left(\frac{X \le 1}{Y \le 3}\right)$ v) $P\left(\frac{Y \le 3}{X \le 1}\right)$

Y	1	2	3	4	5	6
X						
0	0 ~~1/16~~	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Soln:

Y	1	2	3	4	5	6	$P_X(x)$
X							
0	0	0	1/32	1/32	2/32	3/32	8/32
1	1/16	1/16	1/8	1/8	1/8	1/8	10/16
2	1/32	1/32	1/64	1/64	0	2/64	8/64
$P_Y(y)$	3/32	3/32	11/64	13/64	6/32	16/64	1

i) $P(X \le 1) = P(X=0) + P(X=1)$

$= P(0,1) + P(0,2) + P(0,3) + P(0,4) + P(0,5)$

$+ P(0,6) + P(1,1) + P(1,2) + P(1,3) + P(1,4)$

$+ P(1,5) + P(1,6)$

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$$= \left(0+0+\frac{1}{32}+\frac{2}{32}+\frac{2}{32}+\frac{3}{32}\right)+$$

$$\left(\frac{1}{16}+\frac{1}{16}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)$$

$$= \left(\frac{8}{32}+\frac{10}{16}\right)$$

$$= \frac{\cancel{28}\,7}{\cancel{32}\,8} = \frac{7}{8}$$

$$P(x \le 1) = \frac{7}{8}$$

ii) $P(Y \le 3)$

$$P(Y \le 3) = P(Y=1)+P(Y=2)+P(Y=3)$$

$$= \left[P(0,1)+P(1,1)+P(1,2)\right]+\left[P(0,2)+P(1,2)+P(2,2)\right]+\left[P(0,3)+P(1,3)+P(2,3)\right]$$

$$= \left(0+\frac{1}{16}+\frac{1}{32}\right)+\left[0+\frac{1}{16}+\frac{1}{32}\right]+\left[\frac{1}{32}+\frac{1}{8}+\frac{1}{64}\right]$$

$$= \frac{3}{32}+\frac{3}{32}+\frac{11}{64}$$

$$P(Y \le 3) = \frac{23}{64}$$

iii) $P(x \le 1)(Y \le 3)$

$$P(x \le 1)(y \le 3) = P(0,1)+P(0,2)+P(0,3)+P(1,1)+P(1,2)+P(1,3)$$

$$= 0+0+\frac{1}{32}+\frac{1}{16}+\frac{1}{16}+\frac{1}{8}$$

$$P(x \le 1)(y \le 3) = \frac{9}{32}$$

iv) $\frac{P(x\le 1, Y\le 3)}{P(Y\le 3)}$

$$\frac{P(x\le 1, Y\le 3)}{P(Y\le 3)} = \frac{\frac{9}{32}}{\frac{23}{64}} = \frac{18}{23}$$

$$= \frac{18}{23}$$

v) $\frac{P(Y\le 3, x\le 1)}{P(x\le 1)}$

$$\frac{P(Y\le 3, x\le 1)}{P(x\le 1)} = \frac{\frac{9}{32}}{\frac{7}{8}} = \frac{9}{28}.$$

2) From the following joint distribution of X and Y find the marginal distribution.

Y \ X	0	1	2	$P_Y(y)$
0	$3/28$	$9/28$	$3/28$	$15/28$
1	$3/14$	$3/14$	0	$6/14$
2	$1/28$	0	0	$1/28$
$P_X(x)$	$10/28$	$15/28$	$3/28$	1

+ column ↓

+ Row ⟶

The marginal distribution of x

$P(x=0) = (0,0)+(0,1)+(0,2)$

$= \frac{3}{28} + \frac{3}{14} + \frac{1}{28} = \frac{10}{28}$

$P(x=0) = \frac{5}{14}$

$P(x=1) = (1,0)+(1,1)+P(1,2)$

$= \frac{9}{28} + \frac{3}{14} + 0$

$P(x=1) = \frac{15}{28}$

$P(x=2) = (2,0)+(2,1)+(2,2)$

$= \frac{3}{28}.$

The marginal distribution function: y

$P(y=0) = P(0,0)+(0,1)+(0,2)$

$= \frac{15}{28}$

$P(y=1) = (1,0)+(1,1)+P(1,2)$

$= \frac{3}{7}$

$P(y=2) = (2,0)+(2,1)+(2,2)$

$= \frac{1}{28}$

———— x ———— x ———— x ————