Numerical Analysis and statistics.

Sub. code : 16SACMA2.

class : I-BCA , I-CS.

Two Marks :.

1) State the Newton-Raphson formula.

Given an approximate value of a root of an eqn, a better and closer approximation to the root can be found by using an Iterative process called Newtons method..

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2 \ldots$$

2) Write the Newton's forward interpolation formula.

$$y_n = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

3) Write the lagranges interpolatin formula.

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1 + \cdots +$$

$$\frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} y_n.$$

4) Define Numerical Integration.

$\int_a^b f(x)\,dx$ represents the area between $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$.

This integration is possible only if the function $f(x)$ is explicitly given, and if it is integrable.

$$\int_{x_0}^{x_n} f(x)\,dx$$

5) write the Trapezoidal Rule.

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + \cdots + y_{n-1}) + y_n\right]$$

6) write simpsons $\frac{1}{3}$ rule.

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[y_0 + 2(y_2 + y_4 + y_6 + \cdots) + 4(y_1 + y_3 + y_5 + \cdots) + y_n\right]$$

7) Simpson's 3/8 Rule :

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4+y_5+\cdots+y_{n-1}) + 2(y_3+y_6+y_9+\cdots+y_{n-3}) + y_n\right]$$

8) what is the convergent condition for Gauss seidal method ?

Consider the system of equation given by

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The above system can be solved by iterative method, the convergent condition is satisfied and is given by.

$$|a_1| \geq |b_1| + |c_1|$$
$$|b_2| \geq |a_2| + |c_2|$$
$$|c_3| \geq |a_3| + |b_3|$$

9) State the procedure of Gauss - Jacobi method

Consider the system of equation.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Step 1: verify the convergent condition.

Step 2: If the convergent condition is not satisfied for the given system rearrange the eqn such that the convergent condition is satisfied.

Step 3:

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$
$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$
$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$$

Step 4:

$$x^{(n+1)} = \frac{1}{a_1}(d_1 - b_1y^{(n)} - c_1z^{(n)})$$
$$y^{(n+1)} = \frac{1}{b_2}(d_2 - a_2x^{(n)} - c_2z^{(n)})$$
$$z^{(n+1)} = \frac{1}{c_3}(d_3 - a_3x^{(n)} - b_3y^{(n)})$$

The above process is repeated until the consecutive iterative values are same.

10) State the Taylor series.

To find the numerical solution of the eqn $dy/dx = f(x, y)$

given the initial condition $y(x_0) = y_0$,

$$y_{n+1} = y_n + \frac{h}{1!} y_n' + \frac{h^2}{2!} y_n'' + \frac{h^3}{3!} y_n''' + \cdots$$

11) write the formula of second order Runge-Kutta method.

$$K_1 = h f(x_0, y_0)$$

$$K_2 = h f(x_0 + h, y_0 + K_1)$$

$$y_1 = y_0 + \tfrac{1}{2}(K_1 + K_2).$$

13) what is the disadvantage of Eulers method.

The disadvantage of this method is, if the interval increases the straight line deviates much from the actual curve and hence the accuracy cannot be obtained.

14) write the formula for milne's predictor - corrector formulae.

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})$$

15) Using Euler's method find $y(0.2)$ from $dy/dx = 1-y$ with $y(0)=0$.

Soln

Euler's formula,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$y' = 1-y$, $y(0)=0$.

$x_0 = y_0 = 0$

$h = x_1 - x_0 = 0.2 - 0 = 0.2$.

$n=0$.

$$y_1 = y(0.2) = y_0 + h f(x_0, y_0)$$

$$= 0 + (0.2)(1-0)$$

$$\boxed{y(0.2) = 0.2}$$

16) Define Median.

Median is the value of item that goes to divide the series into equal parts.

Median may be defined as the value of the item which divides the series into two equal parts, one half containing values greaterthan it and the other half containg values less than it. Arranging is necessary to compute median.

17) what is standard deviation?

There are two methods of calculating Standard deviation.

(i) Deviation taken from actual mean.

(ii) Deviation taken from Assumed mean.

$$\sigma = \sqrt{\frac{\Sigma f d'^2}{N} - \left(\frac{\Sigma f d'}{N}\right)^2} \times c.$$

18) Define geometric mean.

Individual series:

$$G.M = \text{Antilog}\left(\frac{\Sigma \log x}{N}\right)$$

Discrete series:

$$G.M = \text{Antilog}\left(\frac{\Sigma f \log x}{N}\right)$$

Continuous series:

$$G.M = \text{Antilog}\left(\frac{\Sigma f \log m}{N}\right)$$

19. What is Quartile deviation?

A measure while divides an array into four equal parts is known as quartile. Each portion contains equal number of items.

$$Q_1 = \left(\frac{N+1}{4}\right)^{th} \text{ item.}$$

$$Q_3 = \frac{3(N+1)}{4}^{th} \text{ item.}$$

$$Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

co-efficient of Q.D is Individual, Discrete and continuous series.

20. Define correlation.

The correlation refers to the relationship of two (or) more variables. we can find some relationship between two variables.

$$r = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$\sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} \qquad \sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$$cov(x,y) = \sqrt{\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}}$$

21. what is regression?

Regression means going back and it is mathematical measure showing the average relationship between two variables.

Here x is a random variable and y is a fixed variable. sometimes both the

variables may be random variables.

Regression eqn of x on y.

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

Regression eqn of y on x

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

22) what is the properties of correlation co-effitients

(i) The values of the correlation coefficient shall always lie between +1 and -1

(ii) when $r = +1$, then there is perfect positive correlation.

(iii) when $r = -1$, then there is perfect negative correlation.

(iv) when $r = 0$ then there is no relationship between the variables.