Numerical Analysis and statistics.

sub. code: 16SACMA2.

Milne's predictor - corrector method:

1) The differential eqn $dy/dx = y - x^2$ satisfies by $y(0)=1$, $y(0.2) = 1.12186$, $y(0.4) = 1.4282$, $y(0.6) = 1.7379$. compute (0.8) by Milne's predictor - corrector method.

soln

given $x_0 = 0$ $y_0 = 1$.

$x_1 = 0.2$ $y_1 = 1.12186$

$x_2 = 0.4$ $y_2 = 1.4282$

$x_3 = 0.6$ $y_3 = 1.7379$.

$h = x_1 - x_0$

$= 0.2 - 0$

$\boxed{h = 0.2}$.

Milne's predictor corrector formula:

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

$n=3$

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$y_{4,p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

Given $y' = y - x^2$

$$y'_1 = y_1 - x_1^2$$

$$= 1.12186 - (0.2)^2$$

$$\boxed{y'_1 = 1.08186}$$

$$y'_2 = y_2 - x_2^2$$

$$= 1.4282 - (0.4)^2$$

$$\boxed{y'_2 = 1.2682}$$

$$y'_3 = y_3 - x_3^2$$

$$= 1.7379 - (0.6)^2$$

$$\boxed{y'_3 = 1.3779}$$

$$y_{4,p} = 1 + \frac{4(0.2)}{3}\left[2(1.08186) - 1.2682 + 2(1.377\right.$$

$$= 1 + 0.26667\left[2.16372 - 1.2682 + 2.7558\right]$$

$$= 1 + 0.9737$$

$$\boxed{y_{4,p} = 1.9737}$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

$n=3$

$$y_{4,c} = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$y'_4 = y_4 - x_4^2$, $x_4 = 0.8$, $y_4 = 1.9737$

$= 1.9737 - (0.8)^2$

$\boxed{y'_4 = 1.3337}$

$$y_{4,c} = 1.4282 + \frac{0.2}{3}\left[1.2682 + 4(1.3779) + 1.3337\right]$$

$= 1.4282 + 0.06667\left[8.1135\right]$

$= 1.4282 + 0.5409$

$\boxed{y_{4,c} = 1.9613}$

Adam's predictor - corrector method :.

① Using Adams method determine $y(0.4)$ and $y(0.5)$ correct to 3 decimals given that $\frac{dy}{dx} = 0.5xy$ and $y(0)$, $y(0.1)$, $y(0.2)$ and $y(0.3)$ have values 1.0, 1.0025, 1.0101 and 1.0228 respectively.

Soln

Adam's predictor - corrector formula,

$$y_{n+1,p} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

$$y_{n+1,c} = y_n + \frac{h}{24}\left[9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}'\right]$$

given that,

$$y' = 0.5xy.$$

$x_0 = 0$ $\quad y_0 = 1$

$x_1 = 0.1$ $\quad y_1 = 1.0025$

$x_2 = 0.2$ $\quad y_2 = 1.0101$

$x_3 = 0.3$ $\quad y_3 = 1.0228.$

$\boxed{h = 0.1}$

$$y_{n+1,p} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

$n = 3$

$$y_{4,p} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$y' = 0.5xy$,

$y_0' = 0.5x_0y_0$

$= 0.5\,(0)(1)$

$\boxed{y_0' = 0}$

$y_1' = 0.5x_1y_1$

$= 0.5\,(0.1)\,(1.0025)$

$\boxed{y_1' = 0.050125}$

$y_2' = 0.5x_2y_2$

$= (0.5)\,(0.2)\,(1.0101)$

$\boxed{y_2' = 0.10101}$

$y_3' = 0.5x_3y_3$

$= 0.5\,(0.3)(1.0228)$

$\boxed{y_3' = 0.15342}$

$$y_{4,p} = 1.0228 + \frac{0.1}{24}\left[55\,(0.15342) - 59\,(0.10101) + 37\,(0.050125) - 9(0)\right]$$

$$= 1.0228 + 0.00417\left[8.4381 - 5.95959 + 1.854625 - 0\right]$$

$$= 1.0228 + 0.00417\left[4.333135\right]$$

$$= 1.0228 + 0.01807$$

$$\boxed{y_{4,p} = 1.04087}$$

by Adam's corrector formula,

$$y_{n+1,c} = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$$

$n=3$

$$y_{4,c} = y_3 + \frac{h}{24}\left[9y'_4 + 19y'_3 - 5y'_2 + y'_1\right]$$

$y'_4 = 0.5\,x_4 y_4$, $x_4 = 0.4$, $y_4 = 1.04087$.

$$= 0.5(0.4)(1.04087)$$

$$\boxed{y'_4 = 0.208174}$$

$$y_{4,c} = 1.0228 + \frac{0.1}{24}\left[9(0.208174) + 19(0.15342) - 5(0.101101) + 0.050125\right]$$

$$= 1.0228 + 0.00417\left[1.873566 + 2.91498 - 0.50505 + 0.050125\right]$$

$= 1.0228 + 0.00417 \quad [4.333621]$

$= 1.0228 + 0.01807$

$\boxed{y_{4,c} = 1.04087}$

To find $y(0.5)$ by Adam's method.

$n = 4$

$y_{5,p} = y_4 + \frac{h}{24}[55y_4' - 59y_3' + 37y_2' - 9y_1']$

$x_4 = 0.4, \quad y_4 = 1.04087.$

$y_4' = 0.5\, x_4 y_4$

$= 0.5\,(0.4)\,(1.04087)$

$\boxed{y_4' = 0.208174}$

$y_{5,p} = 1.04087 + \frac{0.1}{24}[55(0.208174) - 59(0.153422) + 37(0.101101) - 9(0.050125)]$

$= 1.04087 + 0.00417\,[11.44957 - 9.05178 + 3.737737 - 0.451125]$

$= 1.04087 + 0.00417\,[5.684035]$

$= 1.04087 + 0.02370$

$\boxed{y_{5,p} = 1.06457}$

$n = 4$

$$y_{5,c} = y_4 + \frac{h}{24}\left[9y_5' + 19y_4' - 5y_3' + y_2'\right]$$

$y_5 = 1.06457$, $x_5 = 0.5$

$y_5' = 0.5\, x_5 y_5$

$= 0.5\,(0.5)\,(1.06457)$

$\boxed{y_5' = 0.26614}$

$$y_{5,c} = 1.04087 + \frac{0.1}{24}\left[9(0.26614) + 19(0.208174) - 5(0.15342) + 0.10101\right]$$

$$= 1.04087 + 0.00417\left[2.39526 + 3.955306 - 0.7671 + 0.10101\right]$$

$$= 1.04087 + 0.00417\left[5.684476\right]$$

$$= 1.04087 + 0.02370$$

$\boxed{y_{5,c} = 1.06457}$