UNIT - I

2 Marks

1. Solve. $x^2p^2 + 3xyp + 2y^2 = 0$

$$x^2p^2 + 3xyp + 2y^2 = 0$$

$$P = \frac{-3xy \pm \sqrt{9x^2y^2 - 4x^2(2y^2)}}{2x^2}$$

$$= \frac{-3xy \pm \sqrt{9x^2y^2 - 8x^2y^2}}{2x^2}$$

$$P = \frac{-3xy \pm xy}{2x^2}$$

$$P = \frac{-3xy + xy}{2x^2} \qquad P = \frac{-3xy - xy}{2x^2}$$

$$P = -y/x. \qquad P = -2y/x.$$

$$dy/dx = -y/x. \qquad dy/dx = -2y/x.$$

$$\int dy/y = -\int dx/x. \qquad \int dy/y = -2\int dx/x.$$

$$\log y = -\log x + \log c. \qquad \log y = -2\log x + \log c$$

$$xy = c \qquad x^2y = c$$

$$xy - c = 0 \qquad x^2y - c = 0.$$

The general solution $(xy-c)(x^2y-c) = 0$

Find the particular integral of $(D^2+16)\,y = \cos 4$

$$P.I = \frac{1}{D^2+16} \cos 4x.$$

$$= 1/0 \cos 4x,$$

Again diff w.r.to 'x'

$$= \frac{x}{2D} \cos 4x.$$

$$= x/_2 \; 1/_D \cos 4x.$$

$$= x/_2 \; \sin 4x/_4$$

$$P.I = \frac{x \sin 4x}{8}.$$

3. Solve $(D^3 - 3D^2 + 4)\, y = 0$

$$(D^3 - 3D^2 + 4)\, y = 0$$

Auxillary eqn

$$m^3 - 3m^2 + 4 = 0$$

Using synthetic division.

	1	-3	0	4
2	0	2	-2	-4
	1	-1	-2	0

$(m-2)$ is a factor.

$$Y_1 = Ae^{2x}.$$

$$m^2 - m - 2 = 0$$

$$(m+1)(m-2) = 0.$$

$$m = -1, \; m = 2.$$

$$Y_2 = Ae^{-x} + Be^{2x}.$$

$$Y_2 = Be^{-x} + ce^{2x}.$$

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$$Y = Y_1 + Y_2$$

$$Y = Ae^{2x} + Be^{-x} + ce^{2x}.$$

4. Solve $(D^2 - 5D + 4)y = 0$

$$D^2 - 5D + 4 = 0$$

Auxillary eqn ⇒

$$m^2 - 5m + 4 = 0$$

$$(m-1)(m-4) = 0$$

$$m_1 = 1, \quad m_2 = 4.$$

$$Y = Ae^{x} + Be^{4x}.$$

The general solution is

$$Y = Ae^{x} + Be^{4x}.$$

5. Solve $y = xp + a/p$

$$y = xp + a/p \quad \rightarrow (1)$$

It's a clairatus equation

Diff w.r. to 'x'

$$dy/dx = p + x\, dp/dx - a/p^2 \; 2p\, dp/dx$$

$$p = p + x\, dp/dx - 2a/p\, dp/dx.$$

$$0 = dp/dx\,(x - 2a/p)$$

$$\int dp = \int 0$$

$$p = c$$

Sub. $p = c$ in (1)

$y = cx + a/c.$

UNIT - II

6. Eliminate a, b from $z = (x^2+a)(y^2+b)$

$$z = (x^2+a)(y^2+b) \rightarrow (1)$$

Diff (1) w.r. to 'x'

$$\frac{\partial z}{\partial x} = 2x\,(y^2+b)$$

$$p = (y^2+b)2x$$

$$p/2x = y^2+b \rightarrow (2)$$

Diff (1) w.r. to 'y'

$$\partial z/\partial y = (x^2+a)\,2y.$$

$$q = (x^2+a)\,2y.$$

$$q/2y = x^2+a \rightarrow (3)$$

Elimination (2) & (3)

$$pq/4xy = (x^2+a)(y^2+b)$$

7. $z = f\,(x^2+y^2)$

$$z = f\,(x^2+y^2) \rightarrow (1)$$

Diff (1) w.r.to 'x'

$$p = f'(x^2+y^2)\,2x \rightarrow (2)$$

Diff (1) w.r. to 'y'

$$q = f'(x^2+y^2)2y \rightarrow (3)$$

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Using (2) & (3)

$$p/q = \frac{2\cancel{f'(x^2+y^2)}\,y}{2\cancel{f'(x^2+y^2)}\,x}.$$

$$px = qy.$$

8. Solve $p = y^2q^2$

$$p = a^2 \quad , q = \pm a/y.$$

Hence

$$dz = pdx + qdy.$$

$$dz = a^2dx \pm q\,dy.$$

$$z = a^2x + qy + b.$$

9. Solve $z = ax + by + a^2 + b^2.$

$$z = ax + by + a^2 + b^2 \rightarrow (1)$$

Diff (1) w.r. to 'x'

$$\partial z/\partial x = a.$$

$$p = a$$

$$px = ax. \rightarrow (2)$$

Diff (1) w.r. to 'y'

$$\partial z/\partial x = b$$

$$q = b$$

$$qy = by. \rightarrow (3)$$

Using (2) & (3) we get

$$px + qy = ax + by.$$

10. Solve a and b from $z = (x+a)(y+b)$

$$z = (x+a)(y+b) \rightarrow (1)$$

Diff (1) w.r.to 'x'

$$\partial z/\partial x = y+b$$

$$p = y+b \rightarrow (2)$$

Diff (1) w.r.to 'y'

$$\partial z/\partial y = x+a.$$

$$q = x+a \rightarrow (3)$$

Using (2) & (3)

$$pq = (x+a)(y+b)$$

$$pq = z.$$

UNIT - III

11 Find $L(\cosh at)$

$$\cosh at = \frac{e^{at} + e^{-at}}{2}$$

$$L(\cosh at) = L\left[\frac{e^{at} + e^{-at}}{2}\right]$$

$$= \frac{1}{2}\left[\left[L(e^{at})\right] + L\left[e^{-at}\right]\right)$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right)$$

$$= \frac{1}{2}\left[\frac{s+a+s-a}{s^2-a^2}\right]$$

$$= \frac{1}{2}\left(\frac{2s}{s^2-a^2}\right)$$

$$L(\cosh at) = \frac{s}{s^2-a^2}$$

17.

12. State final value theorem in Laplace transform.

$$L\{f'(t)\} = sL\{f(t)\} - f(0)$$

$$\lim_{s \to 0} L\{f'(t)\} = \lim_{s \to 0}\{sF(s) - f(0)\}$$

$$\int_0^\infty f'(t)\,dt = \lim_{s \to 0} sF(s) - f(0)$$

$$\lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t)$$

13. Find $L(t^2+2t+1)$

$$L(t^2+2t+1)$$

$$= L(t^2) + 2L(t) + L(1)$$

$$= 2/s^3 + 2/t^2 + 1/s.$$

$$L(t^2+2t+1) = 2/s^3 + 2/t^2 + 1/s.$$

14. Find $L(\sin^2 t)$

$$L(\sin^2 t) = L\left(\frac{1-\cos 2t}{2}\right)$$

$$= \frac{1}{2} L(1-\cos 2t)$$

$$= \frac{1}{2}\left[L(1) - L(\cos 2t)\right]$$

$$= \frac{1}{2}\left[1/s - s/s^2+2^2\right]$$

$$= \frac{1}{2}\left[1/s - s/s^2+4\right]$$

$$= \frac{1}{2}\left(\frac{s^2+4-s^2}{s(s^2+4)}\right)$$

$$= \frac{1}{2}\left(4/s(s^2+4)\right)$$

$$L(\sin^2 t) = \frac{2}{s(s^2+4)}$$

$$L\left[\frac{e^{at}+e^{-at}}{2}\right]$$

$$L\left[\frac{e^{at}+e^{-at}}{2}\right] = \frac{1}{2}\left[L\left[e^{at}\right]+L\left[e^{-at}\right]\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a}+\frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{s+a+s-a}{s^2-a^2}\right]$$

$$= \frac{1}{2}\left(\frac{2s}{s^2-a^2}\right)$$

$$= s/s^2-a^2$$

UNIT – IV

Find : $L^{-1}\left(\frac{1}{(s+a)^2}\right)$

$$L^{-1}\left[\frac{1}{(s+a)^2}\right] = e^{at}\, L^{-1}\left[1/(s+a)^2\right]$$

$$f(t) = e^{at}\, L^{-1} F(s)$$

$$= e^{at} \; L^{-1}\left(\frac{1}{t^2}\right)$$

$$= e^{at} \; \frac{2!}{s^3}$$

17. Find : $L^{-1}\left[\frac{1}{s(s+a)}\right]$

$$L^{-1}\left[\frac{1}{s(s+a)}\right] = \int_0^t L^{-1}\left(\frac{1}{s+a}\right) dt$$

$$= \int_0^t e^{-at} \, dt$$

$$= \left(+\frac{e^{-at}}{a}\right)_0^t$$

$$L^{-1}\left(\frac{1}{s(s+a)}\right) = \frac{1}{a}(1 - e^{-at})$$

18. Find $L^{-1}\left(\frac{n!}{s^{n+1}}\right)$, n is a positive integer.

$$L(f(t)) = \int_0^\infty e^{-st} f(t)\, dt.$$

$$L(t^n) = \int_0^\infty e^{-st} t^n \, dt.$$

put

$st = x$.

$s\,dt = dx$

when.

$t=0, \; x=0$

$t=\infty, \; x=\infty$

$$L(t^n) = \int_0^\infty e^{-x} (x/s)^n \, 1/s \, dx$$

$$= \frac{1}{s^{n+1}} \int_0^\infty x^n e^{-x} \, dx.$$

$$L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$$

When, n is +ve integer.

$$L^{-1}(t^n) = n!/s^{n+1}$$

19. Find $L^{-1}\left(\frac{1}{(s+2)^2+16}\right)$

$$= L^{-1}\left[\frac{1}{(s+2)^2+16}\right]$$

$$= \cancel{e^{-2t}} \; \cancel{L^{-1}\left(\frac{1}{2^2+4}\right)}$$

$$= L^{-1}\left[\frac{1}{(s+2)^2+4^2}\right] \qquad s+2 \to s$$

$$= e^{-2t} L^{-1}\left(\frac{1}{s^2+4^2}\right)$$

$$= e^{-2t} \frac{\sin 4t}{4}.$$

$$L^{-1}\left(\frac{1}{(s+2)^2+16}\right) = \frac{e^{-2t}}{4} \sin 4t$$

20. Find $L^{-1}\left(2s/s^2-16\right)$

$$L^{-1}\left(\frac{2s}{s^2-16}\right) = 2L^{-1}\left[\frac{s}{s^2-16}\right]$$

$$= 2 \, d/dt \; L^{-1}\left(\frac{1}{s^2-16}\right)$$

$$= 2\frac{d}{dt}(\cosh(4t))$$

$$= 2\sinh(4t)\,4$$

$$L^{-1}\left(\frac{2s}{s^2-16}\right) = 8\sinh(4t)$$

UNIT – V

21. Define solenoidal vector of $\vec{F}$.

If $\vec{F}$ is a vector point function such that $\nabla \cdot \vec{F} = 0$. The F is called a solenoidal vector.

22. Define an irrotational vector of $\vec{F}$

If $\vec{F}$ is a vector point function such that $\nabla \times \vec{F} = 0$. Then F is called a irrotational vector.

23. Find ϕ at $(1,1,1)$ if $\phi = x^2 - y^2 - z^2 - 2$.

$$\phi = x^2 - y^2 - z^2 - 2.$$

$$\nabla F_{(1,1,1)} = (1)^2 - (1)^2 - (1)^2 - 2.$$

$$= 1 - 1 - 1 - 2.$$

$$\nabla F = -3.$$

24. Find the unit normal to the surface $z = xy$ at $(1,1,1)$

$$z = xy$$

$$xy - z = 0.$$

$$\nabla\phi = \vec{i}y + \vec{j}x + \vec{k}(-1)$$

$$= \vec{i}y + \vec{j}x - \vec{k}$$

Point $(1,1,1)$

$$\nabla\phi = \vec{i} + \vec{j} - \vec{k}$$

25. Find $\operatorname{div} \vec{r} = 3$.

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}.$$

$$\operatorname{div} \vec{r} = \partial/\partial x\, \vec{r}\,\vec{i} + \partial/\partial y\, \vec{j}\,\vec{r} + \partial/\partial z\, \vec{r}\,\vec{k}$$

$$= \vec{i}(i) + (j)\vec{j} + \vec{k}(\vec{k})$$

$$= 1+1+1$$

$$= 3$$

$$\operatorname{div} \vec{r} = 3.$$