DIFFERENTIAL EQUATIONS & LAPLACE TRANSFORM

UNIT - I

SUB code: 16SCCMM2

1) Solve : $p^2 - 5p + 6 = 0$.

Soln:

$$p^2 - 5p + 6 = 0$$

$$(p-3)(p-2) = 0$$

$$p - 3 = 0 \; ; \; p - 2 = 0$$

$$p = 3 \; ; \; p = 2$$

$$dy/dx = 3 \; ; \; dy/dx = 2$$

$$dy = 3dx \; ; \; dy = 2dx$$

Integrating,

$$y = 3x + c_1 \; ; \; y = 2x + c_2$$

$\therefore$ The solutions are $(y - 3x - c);(y - 2x - c) = 0$

2) Solve : $x = y^2 + \log p$.

Soln:

$$x = y^2 + \log p$$

$$\log p = x - y^2$$

Taking exponential power both sides.

$$p = e^{x - y^2}$$

$$dy/dx = e^{x} \cdot e^{-y^2}$$

$$\frac{dy}{e^{-y^2}} = e^x \, dx$$

$$e^{y^2} dy = e^x dx$$

Integrating,

$$\int e^{y^2} dy = \int e^x dx$$

Note: The integration on left side cannot be integrated in finite terms

$$\int e^{y^2} dy = e^x + c.$$

3) Solve: $y = (x-a)p - p^2$

Soln:

$$y = (x-a)p - p^2 \longrightarrow (1)$$

$$y = xp - ap - p^2$$

$$y = px + [-(ap+p^2)] \longrightarrow (2)$$

This is an clairauts equation

put, $p = c$

$$y = cx + [-(ac+c^2)] \longrightarrow (3)$$

this is an general equation

Diff. w.r.t c

$$0 = x - (a+2c)$$

$$a+2c = x$$

$$c = \frac{x-a}{2}$$

sub in (3)

$$y = \left(\frac{x-a}{2}\right)x - a\left(\frac{x-a}{2}\right) - \left(\frac{x-a}{2}\right)^2$$

$$= \frac{x^2-ax}{2} - \frac{(ax-a^2)}{2} - \frac{(x^2+a^2-2ax)}{4}$$

$$= \frac{2x^2-2ax-2ax+2a^2-x^2-a^2+2ax}{4}$$

$$y = \frac{x^2-2ax+a^2}{4}$$

$$4y = (x-a)^2$$

4) solve: $a(xdy+2ydx) = xy\,dy$.

Soln:

$$a(xdy+2ydx) = xy\,dy$$

If we divide by xy. ($\frac{1}{xy}$ is an integrating factors)

$$a\left(\frac{x\,dy}{xy}+\frac{2y\,dx}{xy}\right)=\frac{xy\,dy}{xy}$$

$$a\left(\frac{dy}{y}+\frac{2dx}{x}\right)=dy$$

Integrating.

$$a(\log y+2\log x)=y+c$$

$$a(\log y+\log x^2)=y+c$$

$$a\log(yx^2)=y+c.$$

5) solve : $\left(\frac{dy}{dx}\right)^2-ax^2=0.$

soln:

$$\left(\frac{dy}{dx}\right)^2-ax^2=0$$

$$\left(\frac{dy}{dx}\right)^2=ax^2$$

$$\frac{dy}{dx}=\pm\sqrt{a}\,x$$

$$dy = \pm\sqrt{a}\, x\, dx$$

Integrating,

$$\int dy = \pm\sqrt{a}\int x\, dx$$

$$y = \pm\sqrt{a}\,\frac{x^2}{2} + c$$

The general solutions are

$$y = \sqrt{a}\,\frac{x^2}{2} + c_1 \; ; \; y = -\sqrt{a}\,\frac{x^2}{2} + c_2$$

UNIT-II

1) Find the C.F of $(D^2 - 6D + 13)y = 2^x$

Soln:

To find C.F :

$$D^2 - 6D + 13 = 0$$

$$m^2 - 6m + 13 = 0$$

$$m = \frac{6 \pm \sqrt{36 - 52}}{2}$$

$$m = \frac{6 \pm \sqrt{-16}}{2}$$

$$m = \frac{6 \pm 4i}{2}$$

$$m = \frac{\cancel{2}(3 \pm 2i)}{\cancel{2}}$$

$$m = 3 \pm 2i$$

$$C.F = e^{3x}(B\cos 2x + C\sin 2x)$$

2) Find the P.I of $(D^2+5D+6)y = e^{2x}$

<u>soln:</u>

$$(D^2+5D+6)y = e^{2x}$$

$$P.I = \frac{1}{D^2+5D+6} e^{2x}$$

$$= \frac{1}{(2)^2+5(2)+6} \cdot e^{2x}$$

$$= \frac{1}{4+10+6} e^{2x}$$

$$P.I = \frac{1}{20} e^{2x}$$

3) Solve : $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0$.

Soln:

The auxiliary equation is

$$m^4 - 4m^3 + 8m^2 - 8m + 4 = 0$$

(i.e) $(m^2 - 2m + 2)^2 = 0$.

$\therefore m = 1 \pm i$ twice

The solution is

$$e^{x}(A + Bx)(C\cos x + D\sin x)$$

4) Find the P.I of $(4D^2 + 12D + 9)y = 144e^{-3x}$

Soln:

$$P.I = \frac{1}{4D^2 + 12D + 9} 144e^{-3x}$$

$$= \frac{1}{4(-3)^2 + 12(-3) + 9} 144e^{-3x}$$

$$= \frac{1}{4(9) - 36 + 9} 144e^{-3x}$$

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$$= \frac{1}{36-36+9} 144 e^{-3x}$$

$$= \frac{1}{9} 144 e^{-3x}$$

$$P.I = 16 e^{-3x}$$

5) Find the P.I of $(D^3 - D^2 - D + 1)y = 1 + x^2$

Soln:

$$P.I = \frac{1}{(D-1)^2 (D+1)} (1+x^2)$$

$$= (1+D)^{-1} (1-D)^{-2} (1+x^2)$$

$$= (1-D+D^2)(1+2D+3D^2)(1+x^2)$$

Expanding as far as D^2

$$= (1+D+2D^2)(1+x^2)$$

$$P.I = 5 + 2x + x^2.$$

UNIT - III

1) Form the PDE by eliminating a and b from $z = (x+a)(y+b)$

Soln:

$$z = (x+a)(y+b) \longrightarrow (1)$$

Differentiating w.r.t x and y partially,

$$p = y+b \quad \text{and} \quad q = x+a$$

Eliminating a and by,

$$z = pq.$$

2) Find the Lagrange's auxilliary equation.

Soln:

the symtem of equations

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

is called the Langrange's auxilliary equation.

3) Define partial differential eqn.

Soln:

PDE is one which contains partial derivatives of the dependent variable w.r.t two or more independent variables.

4) Solve: $p+q=1$.

Soln:

Put, $p=a$, $q=b$

$$a+b=1$$

$$b=1-a$$

Let $z = ax+by+c$ be the trial soln

$$z = ax+(1-a)y+c \longrightarrow (1)$$

Diff. w.r.t c

$$0=1$$

There is no singular integral

Put $c=g(a)$

$$z = ax+(1-a)y+g(a) \longrightarrow (2)$$

Diff. w.r.t a

$$0 = x + (-y) + g'(a)$$

$$0 = x - y + g'(a) \longrightarrow (3)$$

Eliminate a b/w (2) and (3) we get general Integral.

5) solve: $p = y^2 q^2$

Soln:

Let, $p = a^2$, then $q = \pm \frac{a}{y}$

Hence $dz = a^2 dx \pm \frac{a}{y} dy$

$$\therefore z = a^2 x \pm a \log y + b.$$

UNIT- IV

1) Find the C.F of $(D^2 - DD')z = \sin x \sin 2y$

Soln:

$$(D^2 - DD')z = \sin x \sin 2y$$

$$m^2 - m = 0$$

$$m(m-1) = 0$$

$$m = 0, \; m = 1$$

$$C.F = \phi_1 y + \phi_2 (y+x).$$

2) solve: $(D^2 - 2DD' + D'^2)z = 0.$

soln:

$$(D^2 - 2DD' + D'^2)z = 0$$

$$(m^2 - 2m + 1) = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1, 1$$

$$C.F = \phi_1 (y+x) + x\phi_2 (y+x).$$

3) solve: $r = a^2 t.$

soln:

$$r = a^2 t$$

$$r = \frac{\partial^2 z}{\partial x^2} \; ; \; t = \frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$$

$$D^2 - a^2 D'^2 = 0$$

$$m^2 - a^2 = 0$$

$$m^2 = a^2$$

$$m = \pm a$$

$$C.F = \phi_1(y+ax) + \phi_2(y-ax).$$

4) solve : $(D^2 - 4DD' + 4D'^2)z = e^{2x-y}$

soln:

$$(D^2 - 4DD' + 4D'^2)z = e^{2x-y}$$

$$m^2 - 4m + 4 = 0.$$

$$(m-2)(m-2) = 0$$

$$m = 2, 2$$

$$C.F = \phi_1(y+2x) + x\phi_2(y+2x)$$

$$P.I = \frac{1}{D^2 - 4DD' + 4D'^2} e^{2x-y}$$

$$= \frac{1}{4+8+4} e^{2x-y}$$

$$P.I = \frac{1}{16} e^{2x-y}$$

$$z = C.F + P.I$$

$$z = \phi_1(y+2x) + x\phi_2(y+2x) + \frac{1}{16} e^{2x-y}$$

5) solve: $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$

Soln:

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$$

$$(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$$

$$m^2 - 4m + 4 = 0$$

$$(m-2)(m-2) = 0$$

$$m = 2, 2$$

$$C.F = \phi_1(y+2x) + x\,\phi_2(y+2x)$$

$$P.I = \frac{1}{D^2 - 4DD' + 4D'^2} e^{2x+y}$$

$$= \frac{1}{4-8+4} e^{2x+y}$$

$$= \frac{1}{2D-4} e^{2x+y}$$

$$P.I = \frac{1}{2} e^{2x+y}$$

$$\therefore z = \phi_1(y+2x) + x\phi_2(y+2x) + \frac{1}{2} e^{2x+y}$$

UNIT - V

1) Find $L(2t^2+3t+\cos t)$

Soln:

$$L(2t^2+3t+\cos t) = L(2t^2)+L(3t)+L(\cos t)$$

$$= 2L(t^2)+3L(t)+L(\cos t)$$

$$= 2\left(\frac{2!}{s^3}\right)+3\left(\frac{1!}{s^2}\right)+\frac{s}{s^2+1}$$

$$= \frac{4}{s^3}+\frac{3}{s^2}+\frac{s}{s^2+1}$$

2) Find $L(\sinh ax)$

Soln:

$$L(\sinh ax) = L\left(\frac{e^{ax}-e^{-ax}}{2}\right)$$

$$= \frac{1}{2}L\left(e^{ax}-e^{-ax}\right)$$

$$= \frac{1}{2}\left[\frac{1}{s-a}-\frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{s+a-s+a}{(s-a)(s+a)}\right]$$

$$= \frac{1}{2}\left[\frac{2a}{s^2-a^2}\right]$$

$$L(\sinh ax) = \frac{a}{s^2-a^2}$$

3) Find $L^{-1}[F(s+a)]$

Soln:

Let,

$$L(f(x)) = F(s)$$

Then we know that,

$$L\left(e^{-ax} f(x)\right) = F(s+a)$$

$$e^{-ax} f(x) = L^{-1}[F(s+a)]$$

$$L^{-1}[F(s+a)] = e^{-ax} L^{-1}[F(s)]$$

4) Find $L(t \sin at)$

Soln:

$$L(t f(t)) = -F'(s)$$

$$F(s) = L(\sin at)$$

$$= \frac{a}{s^2+a^2}$$

$$F'(s) = \frac{(s^2+a^2)(0) - a(2s)}{(s^2+a^2)^2}$$

$$F'(s) = \frac{-2as}{(s^2+a^2)^2}$$

$$-F'(s) = \frac{2as}{(s^2+a^2)^2}$$

$$L(t \sin at) = \frac{2as}{(s^2+a^2)^2}$$

5) Find $L^{-1}\left(\frac{s}{(s+2)^2}\right)$

soln:

$$L^{-1}(s\ F(s)) = f'(t)$$

$$F(s) = L^{-1}\left(\frac{1}{(s+2)^2}\right)$$

$$= e^{-2t} L^{-1}\left(\frac{1}{s^2}\right)$$

$$= e^{-2t}\ t$$

$$L^{-1}\left(\frac{s}{(s+2)^2}\right) = \frac{d}{dt}\ (e^{-2t} t)$$

$$= -2e^{-2t}t + e^{-2t}$$

$$= e^{-2t}(1-2t)$$

$$L^{-1}\left(\frac{s}{(s+2)^2}\right) = e^{-2t}(1-2t)$$