ANALYTICAL GEOMETRY 3D

UNIT- I

SUB CODE: 16SCCMM1

1) Find the distance between the points $(4,3,-6)$ & $(-2,1,-1)$

soln:

$$A(4,3,-6),\ B(-2,1,-1)$$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$= \sqrt{(-2-4)^2+(1-3)^2+(-1+6)^2}$$

$$= \sqrt{(-6)^2+(-2)^2+(5)^2}$$

$$= \sqrt{36+4+25}$$

$$AB = \sqrt{65}$$

2) Angle between the line whose direction cosines are $(2,1,-2)$ & $(1,1,0)$

soln:

$$(l_1,m_1,n_1) = (2,1,-2)$$

$$(l_2,m_2,n_2) = (1,1,0)$$

$$\cos\theta = l_1l_2+m_1m_2+n_1n_2$$

$$= (2)(1)+(1)(1)+(-2)(0)$$

$$\cos\theta = 3$$

3) If (3, 4, 12) of the direction ratio of the line. find its direction cosines.

<u>soln:</u>

$$(P, Q, R) = (3, 4, 12)$$

$$l = \pm \frac{P}{\sqrt{P^2+Q^2+R^2}}$$

$$= \pm \frac{3}{\sqrt{3^2+4^2+12^2}}$$

$$= \pm \frac{3}{\sqrt{9+16+144}}$$

$$= \pm \frac{3}{\sqrt{169}}$$

$$\boxed{l = \pm \frac{3}{13}}$$

$$m = \pm \frac{q}{\sqrt{P^2+q^2+R^2}}$$

$$\boxed{m = \pm \frac{4}{13}}$$

$$N = \pm \frac{R}{\sqrt{P^2+Q^2+R^2}}$$

$$\boxed{N = \pm \frac{12}{13}}$$

$\therefore$ The D.C's are $\pm\frac{3}{13}, \pm\frac{4}{13}, \pm\frac{12}{13}$

4) If α, β, γ be the angle which a line with the co-ordinated axies. then show that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma = 2$.

Soln:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

W.K.t

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$1-\sin^2\alpha + 1-\sin^2\beta + 1-\sin^2\gamma = 1$$

$$3-\sin^2\alpha - \sin^2\beta - \sin^2\gamma = 1$$

$$-(\sin^2\alpha + \sin^2\beta - \sin^2\gamma) = -2$$

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2.$$

5) Define plane.

Soln:

A plane is a surface which is such that a straight line any two points if the two lines is called plane.

UNIT - 2

1) Define straight line.

soln:

A straight line is obtained intersection of two planes. Its equation is $ax+by+cz+d=0$ and $a_1x+b_1y+c_1z+d_1=0$. It is called the non-symmetry form of line.

2) Find the equation of the straight line passing through the point $(3,0,2)$ & $(1,-2,3)$.

soln:

$$(x_1, y_1, z_1) = (3,0,2)$$

$$(x_2, y_2, z_2) = (1,-2,3)$$

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-3}{1-3} = \frac{y-0}{-2-0} = \frac{z-2}{3-2}$$

$$\frac{x-3}{-2} = \frac{y}{-2} = \frac{z-2}{1}$$

3) show that the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ and $\frac{x-1}{-1} = \frac{y-2}{1} = \frac{z-3}{-2}$ are parallel.

soln:

$(l_1, m_1, n_1) = (1, -1, 2)$

$(l_2, m_2, n_2) = (-1, 1, -2)$

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$\frac{1}{-1} = \frac{-1}{1} = \frac{2}{-2} \Rightarrow -1$$ is satisfied

the given lines are parallel.

4) Find the equation of the straight line which passes through the point $(2, 3, 4)$ which making angle $60°, 60°, 45°$ with the axies in positive direction.

soln:

$(x_1, y_1, z_1) = (2, 3, 4)$

$(\alpha, \beta, \gamma) = 60°, 60°, 45°$

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\frac{x-2}{\cos 60^\circ} = \frac{y-3}{\cos 60^\circ} = \frac{z-4}{\cos 45^\circ}$$

$$\frac{x-2}{\frac{1}{2}} = \frac{y-3}{1/2} = \frac{z-4}{\frac{1}{\sqrt{2}}}$$

$$2x-4 = 2y-6 = \sqrt{2}z - 4\sqrt{2}$$

5) Find the angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ & $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}\right)$

Soln:

$$(l_1, m_1, n_1) = \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$$

$$(l_2, m_2, n_2) = \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}\right)$$

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$= \left(\frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{4}\right) + \left(\frac{1}{4} \cdot \frac{1}{4}\right) + \left(\frac{\sqrt{3}}{2} \cdot -\frac{\sqrt{3}}{2}\right)$$

$$= \frac{3}{16} + \frac{1}{16} - \frac{12}{16}$$

$$= \frac{4}{16} - \frac{12}{16}$$

$$= -\frac{8}{16}$$

$$\cos\theta = -1/2$$

$\theta = \cos^{-1}(-1/2)$

$\boxed{\theta = 120^\circ}$

UNIT - 3

1) Find the equation of the sphere with centre $(-1, 2, -3)$ and radius 3 units.

soln:

Equation of sphere

$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

centre $= (-1, 2, -3)$

radius $= 3$

$(x+1)^2 = (y-2)^2 + (z+3)^2 = 3^2$

$x^2 + 1 + 2x + y^2 + 4 - 4y + z^2 + 9 + 6z = 9$

$x^2 + y^2 + z^2 + 2x - 4y + 6z + 5 = 0.$

2) Find the centre of the sphere $x^2 + y^2 + z^2 - 6x - 2y - 4z - 11 = 0$

soln:

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \longrightarrow (1)$

$x^2 + y^2 + z^2 - 6x - 2y - 4z - 11 = 0 \longrightarrow (2)$

$2u = -6$	$2v = -2$	$2w = -4$
$u = -3$	$v = -1$	$w = -2$

centre = $(-u, -v, -w)$

centre = $(3, 1, 2)$.

3) Define sphere.

* A sphere is the locus of a point which remains at a constant distance from fixed point.

* The constant distance is called the radius and the fixed point the centre of sphere.

4) Find the co-ordinates of centre and radius of sphere $x^2+y^2+z^2-2x+6y+4z-35=0$.

soln:

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$$x^2+y^2+z^2-2x+6y+4z-35=0$$

$2u = -2$	$2v = 6$	$2w = 4$
$u = -1$	$v = 3$	$w = 2$

centre $= (1, -3, -2)$

$$\text{radius} = \sqrt{u^2+v^2+w^2-d}$$

$$= \sqrt{1+9+4+35}$$

$$= \sqrt{14+35}$$

radius $= \sqrt{49} = 7$ units.

5) Find the radius of sphere $16x^2+16y^2+16z^2-8y-16x-55=0$.

Soln:

$$16x^2+16y^2+16z^2-8y-16x-55=0$$

($\div$ by 16)

$$x^2+y^2+z^2-\frac{1}{2}y-x-\frac{55}{16}=0 \longrightarrow (1)$$

$$x^2+y^2+z^2+2ux+2vy+2wx+d=0 \longrightarrow (2)$$

from (2) in (1)

$2u=-1$	$2v=-1/2$	$2w=-1$
$u=-1/2$	$v=-1$	$w=-1/2$

centre = $(1/2, 1, 1/2)$

radius = $\sqrt{u^2+v^2+w^2-d}$

$$=\sqrt{\frac{1}{4}+1+\frac{1}{4}+\frac{55}{16}}$$

$$=\sqrt{\frac{2}{4}+\frac{55}{16}+1}$$

$$=\sqrt{\frac{8+55+16}{16}}$$

$$=\sqrt{\frac{79}{16}}$$

radius = $\frac{\sqrt{79}}{4}$ units

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UNIT - 4

1) Define guiding curve.

Soln:

The fixed point is called the vertex A the fixed curve is called guiding curve.

2) Show that the equation of right circular cone whose vertex is o, axis ox & semi vertical α is $x^2+y^2=z^2\tan^2\alpha$.

Soln:

The eqn of the $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ the z axis is $\frac{x}{0}=\frac{y}{0}=\frac{z}{1}$

$$\cos\alpha = \frac{n}{\sqrt{l^2+m^2+n^2}}$$

$$\sqrt{l^2+m^2+n^2}\cos\alpha = n$$

squaring on both side

$$(l^2+m^2+n^2)\cos^2\alpha = n^2$$

$$(l^2+m^2)\cos^2\alpha + \cos^2\alpha\, n^2 = n^2$$

$$(l^2+m^2)\cos^2\alpha = n^2 - n^2\cos^2\alpha$$

$$(l^2+m^2)\cos^2\alpha = n^2\sin^2\alpha$$

$$(l^2+m^2) = n^2\sin^2\alpha/\cos^2\alpha$$

$$(l^2+m^2) = n^2 \tan^2\alpha$$

$$\therefore x^2+y^2 = z^2\tan^2\alpha$$

3) Define quadratic cone.

Soln:

A cone whose eqn is second degree is called quadratic cone.

4) Define right circular cone.

Soln;

A right circular cone is a surface generated by a line which of passes through a fixed point and makes a constant angle with the fixed line through the fixed point.

5) Define tangent plane.

Soln:

All tangent lies at (x_1, y_1, z_1) lie on the plane which is called the tangent plane which (x_1, y_1, z_1).

UNIT-5

1) Define central quadric.

Soln:

chords of eqn (1) which passes through O are bisect at O for this reason eqn (1) is called a central quadric.

2) Define ellipsoid:

Soln:

The surface meets the coordinates axis in the point $(\pm a, 0, 0)$ $(0, \pm b, 0)$ & $(0, 0, \pm c)$ cannot have numerical value which exceed a, b, c respectively. Hence the surface is closed. this surface is called as ellipsoid.

3) the condition for the plane $lx + my + nz = p$ to touch the conicoid $ax^2 + by^2 + cz^2 = 1$.

Soln:

Let the plane touch the conicoide at (x_1, y_1, z_1)

The eqn of the tangent plane at (x_1, y_1, z_1) is

$$axx_1 + byy_1 + czz_1 = 1 \rightarrow (2)$$

this plane is also represented by the eqn

$$lx + my + nz = p \rightarrow (2)$$

$$\frac{ax_1}{l} = \frac{by_1}{m} = \frac{cz_1}{n} = \frac{1}{p}$$

(i.e)

$$x_1 = \frac{l}{ap}, \quad y_1 = \frac{m}{bp}, \quad z_1 = \frac{n}{cp}$$

since (x_1, y_1, z_1) lies on the conicoid

$$ax_1^2 + by_1^2 + cz_1^2 = 1$$

$$\therefore ax^2 + by^2 + cz^2 = 1.$$

4) Define conicoids.

Soln:

Hence each plane section of the quadric is a conic and for this reason the quadric are called conicoids.

5) If OD is the diameter parallel to a secant APQ through A meeting the conicoid at P and Q show that $\frac{AP \cdot AQ}{OD^2}$ is constant.

Soln:

Let the conicoid be $ax^2 + by^2 + cz^2 = 1$

A be (α, β, γ) and direction cosines of the line APQ be l, m, n

the eqn APQ is $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$

the co-ordinates of a point at a distance r from A are $(\alpha + lr, \beta + mr, \gamma + nr)$. this pt lies on coincoid.

$$\therefore a(\alpha + lr)^2 + b(\beta + mr)^2 + c(\gamma + nr)^2 = 1$$

$$r^2(al^2 + bm^2 + cn^2) + 2r(a\alpha l + b\beta m + c\gamma n) + a\alpha^2 + b\beta^2 + c\gamma^2 + d = 0$$

$$\therefore AP \cdot AQ = \frac{a\alpha^2 + b\beta^2 + c\gamma^2 - 1}{al^2 + bm^2 + n^2 c}$$

the direction cosines of OP are also l, m, n. D is point (lk, mk, nk);

$$k = OD \quad al^2k^2 + bm^2k^2 + cn^2k^2 = 1$$

$$k^2 = \frac{1}{al^2 + bm^2 + cn^2} = \frac{AP \cdot AQ}{k^2} = \frac{AP \cdot AQ}{OD^2}$$

Hence

$$\frac{AP \cdot AQ}{OD^2} = a\alpha^2 + b\beta^2 + c\gamma^2 - 1 = \text{constant}.$$