Subject : Differential Geomentry

Subject code : P16MA42

Unit - I

1. Define Normal plane

The plane through $p(x,y,z)$ and perpendicular to the tangent line at p is called the normal plane at p and hence its equation as above is

$$(R-r).t = 0 \quad (or) \quad (R-r).\dot{r} = 0$$

$$(R-r).\dot{r} = 0 .$$

2. Define curvature :

The rate of change of the direction of tangent with respect to arc length 's' as the point $P(r)$ moves along the curve is called curvature vector of the curve whose magnitude is denoted by κ (kappa) called the curvature at P.

3. Define : Torsion

The rate of change of the direction of binormal with respect to arc length as the point p(r) moves along the curve is called the torsion vector of the curve whose magnitude is denoted by τ (Tau) called the torsion at p.

4. Define : Helics

A space curve which lies on the surface of a cylinder and cuts the generator at a constant angle α is called a cyclinder helix.

5. Define : circular Helix

A helix which is described on the surface of a circular cyclinder is called circular helix or right circular helix.

Unit - II

6. Find the angle between parametric curves :

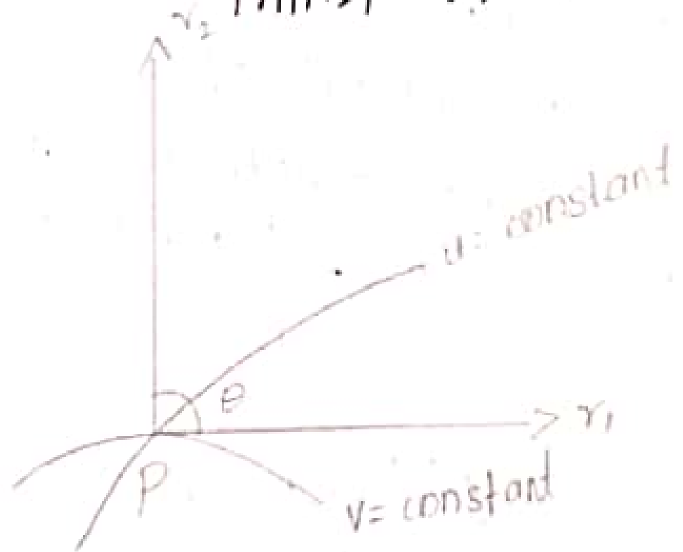
let p be the position vector of point of intersection of two parametric curves u = constant & v = constant

Then clearly as we know r_1, r_2 are along tangents of the parametric curve.

v = constant u = constant.

If θ be the angle b/w the parametric curves then

$$\cos\theta = \frac{r_1 r_2}{|r_1||r_2|} = \frac{F}{\sqrt{E(G)}} \longrightarrow (1)$$



$$\sin\theta = \frac{|r_1 \times r_2|}{|r_1||r_2|}$$

$$= \frac{H}{\sqrt{E(G)}}$$

$$= \frac{\sqrt{E(G) - F^2}}{\sqrt{E(G)}}$$

$$\tan\theta = \frac{H}{F}$$

7. Direction co-efficients :

Let p be any point on the surface $r = r(u, v)$. So that at p we have the following three linearly indepent vectors N the unit normal to the surface r_1, r_2 which are called tangential vector

u = constant and v = constant respectively

If a be any vector at p.

Then r can be uniquely expressed as a linear combination of the above vectors

$$a = a_n N + (\lambda r_1 + \mu r_2) \longrightarrow (1)$$

we shall denote the tangential part of 'a' by T called the tangential vector

$$T = \lambda r_1 + \mu r_2$$

we get, $a \cdot N = a_n N \cdot N + (\lambda r_1 + \mu r_2) \cdot N$

$$= a_n$$

$$\Rightarrow a = \lambda r_1 + \mu \lambda_2$$

$$T^2 = (\lambda r_1 + \mu r_2)^2$$

$$T = [E\lambda^2 + 2F\lambda\mu + G\mu^2]^{1/2}.$$

8. Direction ratios of a direction:

The scalar λ, μ which are d respectively to l, m the direction co-eff are called the direction ratio of that direction.

If $a = \lambda r_1 + \mu r_2$.

Then λ, μ the components of are the direction ratio of a.

9. Define a surface

The relation of the form $f(x, y, z) = 0$ b/w the co-ordinates x, y, z of a point is called a Intrinsic (or) constraint equation of the surface.

10. Types of singularity:

There are two types of singularities

i) Essential and artificial

ii) Essential singularity.

Unit - III

11. Define a Geodesic

If two points A and B on a surface S be joined by curves lying on S then the curve which possesses a sationary length (rather than strictly shortest distance) for small variations is called geodesic.

12. Define Gaussian curvature of Geodesic

The curvature vector at a point P of any curve on a surface is $r'' = t' = kn$. where k is the curvature and n the principal normal.

we can write

$$r'' = kn = k_n N + (\lambda r_1 + \mu r_2) = k_n N + k_g.$$

13. Normal property of geodesics

At every point of a geodesic the principal normal is a normal to the surface.

14. State the Gauss Bonnet theorem

A function K of u, v is called Gaussian curvature of the surface.

15. State the clairaut's theorem:

If a geodesic on a surface of revolution cuts the meridian through any point p on it any angle ψ, then ψ, then ψ is constant where u is the distance of point p from the axis.

Unit-IV

16 Define Envelope:

The locus of all characteristics of a given family of surfaces for different values of a is called the envelope of the given family and is obtained by eliminating the parameter a between the equations

$$F(a) = 0 \text{ and } \frac{\partial}{\partial a} F(a) = 0$$

17. Define a normal curvature :

Let $r = r(u, v)$ represent a surface and p be a point on it.

If $r = r(s)$ be a curve through p on this surface. Then the curvature at p is defined the component curvature vector $r'' = t' = kn$ along the normal (N) defined by k_n

hence $k_n = r'' \cdot N$

18. Define Mean curvature :

The arithmetic mean of the principal curvature k_a, k_b at a point is called the mean curvature at that point and is denoted by μ

$$\therefore \mu = \tfrac{1}{2}(k_a + k_b)$$

$$= \tfrac{1}{2}\,\frac{EN - 2FM + G^2}{EG - G^2}.$$

19. Define Anticlastic

If every point is hyperbolic

i.e] $K = k_a k_b < 0$.

i.e) k_a, k_b have opposite signs

20. Define hyperbolic point :

A point p on the surface is called hyperbolic point if the gaussion curvature at p is -ve.

i.e] $LN-M^2<0$.

since $K = K_a K_b$

$=(LN-M^2)H^2<0$.

unit - V

21. Define a Developable surfaces :

The envelope of a single parameter family of planes is called a developable surface or simply a developable.

22. Define a characteristic point

The ultimate intersection of consecutive characteristic is called characteristic points.

23. Define a Asymptotic lines :

The directions at a point p on a surface is called asymptotic if it is self-conjugate and the curve whose tangents are along asymptotic directions are called asymptotic lines.

24. Define a parallel surfaces:

A surface s^* is said to be parallel to another surface s if the points of s^* are at a constant distance along the nor-mal to surface s.

If r be the position vector of a current point p on s and N the unit normal at p. III^{ly}

$$r^* = r - aN.$$

25. Define a line of striction

The locus of central points of all generators of a ruled surface is a curve lying on the ruled surface which is called the line of striction of the ruled surface.