Stochastic Process
Assignment - I

1) Define Stochastic Process:

Family of random variables which are function of time are known as stochastic Process.

2) Homogeneous and non-homogeneous:

The transistion Probability may or may not be independent of n. If the ~~the~~ transistion Probability P_{jk} is independent of n, the markov chain is said to be homogeneous.

If it is dependent on n, the chain is said to be non-homogeneous chains.

3) Transistion matrix:

The transistion matrix Probability P_{jk} satisfy $P_{jk} \geq 0$, $\sum_k P_{jk} = 1 \; \forall j$. These Pr may be written as matrix form

$$P = \begin{pmatrix} P_{11} & P_{12} & P_{13} & \cdots \\ P_{21} & P_{22} & P_{23} & \cdots \\ \vdots & \vdots & & \end{pmatrix}.$$

this called the transistion Probability matrix.

4) Poission Process:

Consider the Process $\{x(t), t \in T\}$ with

$Pr\{(x/t) = n\} = \frac{e^{-at}(at)^n}{n!}$ $a > 0$ $n = 1, 2 \ldots$

Mean $m(t) = at$ variant $\{x(t)\} = at$ which are function on to the Process is evalutionary

5) Random Variables :

Consider a random experiment having sample space S. A random variable x is a function that assigns a real value to each outcome in S.

6) Ergodic :

A non null Presistent and a Periodic state of a markov chain called Ergodic.

7) Martingalar :

A discreate Parameter Stochastic Process $\{X_n, n \geq 0\}$ is called a Martingular. If for all n,

i) $E\{|X_n|\} < \infty$

ii) $E\{X_{n+1} / X_n, X_{n-1}, \dots X_0\} = X_n$.

8) Left eigen Vector :

A non-zero rowl vector $\vec{x}$ which satisfy the vector equation $\vec{y}(A - t_i I) = 0$ is called a left eigen vectors of A corresponding to its eigen value.

9) Mean Recurrence Time :

When $k = j \{f_{jj}^{(n)}, n = 1, 2 \dots\}$ with out this distribution of the recurrence time of the j are $f_{jj} = 1$; $\mu_{jj} = \sum_{n=1}^{\infty} d_i = GCD\{m ; P_{jj}^{(m)} > 0\}$

$= GCD\{m ; F_{ij}^{(m)} > 0\}$

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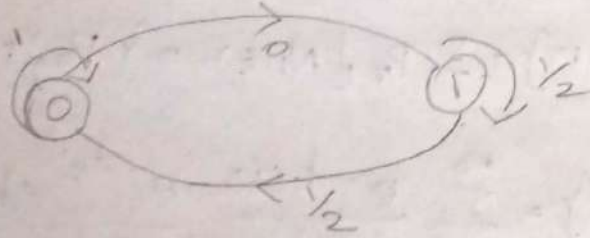
10) Consider the Markov Chain $\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0 & 0 \\ 1/2 & 1/2 \end{pmatrix} \end{array}$

i) Draw diagram

ii) If the chain irreducible

i)



ii) The state is aperiodic Period $= 1$.

11) Independents :

$N(t)$ is independents for the no. of occurrence (of the events E) in an interval Period to the interval $(0, t)$.

ie Future changes in $N(t)$ are independent of the Past changes.

12) Sum of two independent Poisson Process is a Poission Process :

Proof : Let $N_1(t)$ & $N_2(t)$ be two independent Possion with Parameter λ_1, λ_2 respectively.

Let $N(t) = N_1(t) + N_2(t)$.

P.g. function of N_1 & N_2

P.g.f of $N_2(t)$ is $E\{s^{N_2(t)}\} = e^{\lambda_2(s-1)t}$

$E\{s^{N(t)}\} = e^{[\lambda_1 + \lambda_2](s-1)t}$

$N(t)$ is a Poisson Process with Parameter $\lambda_1 + \lambda_2$.

13) Linear Combination:

Let $V_1, V_2 \cdots V_n$ certain quantities & $k_1, \cdots k_n$ constant $k_1 V_1 + \cdots + k_n V_n$ from a linear combination.

14) $P_r\{N(t)=n\} = \sum P_r\{N_1(t)=r\} P_r\{N_2(t)=n-r\}$

Proof:

$$P_r\{N(t)\} = \sum_{r=0}^{\infty} \frac{e^{-\lambda_1(t)}}{r!} \frac{e^{-\lambda_2}(\lambda_2 t)^{n-r}}{(n-r)!}$$

$$= e^{-(\lambda_1+\lambda_2)t} \sum_{n=0}^{n} \frac{\lambda_1^{r} \lambda_2^{n-r}}{n!} t^n$$

$$= e^{-(\lambda_1+\lambda_2)t} \frac{(\lambda_1+\lambda_2)^n t^n}{n!} t^n \quad ; n \geq 0.$$

$N(t)$ is a Possion Parameter $(\lambda_1+\lambda_2)$.

15) Gamma distribution:

Let x be any 2 Parameter gamma distribution with Parameter λ, t $(\lambda > 0)$ is the scale Parameter and $k > 0$ is the shape Parameter.

$$f\lambda, k(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\sqrt{k}} \quad ; x > 0.$$

16) Renewal Theorem:

Suppose that relations $b_n \geq 0$, $b = \sum b_n < \infty$ and $\{f_n\}$ is not Periodic.

a) If $f < 1$ then $V_n \to 0$ & $\sum V_n = \frac{b}{1-f}$

b) If $f = 1$ then $V_n \to b/\mu$

17) Renewal density :

Let x_n have Gamma distribution having density.

$$f(x) = \frac{a^k x^{k-1} e^{-ax}}{(k-1)!}, \quad x \geq 0$$

then $f^*(s) = \left(\frac{a}{s+a}\right)^k$ and the density $F_n'(x)$ of $S_n = x_1 + x_2 + \cdots + x_n$ has the laplace transform.

18) Renewal Period :

The interval b/w occurence of two successive Renewal is called a renewal Period of the Proces.

$f_n = P_r\{E^*$ occurs for the 1st time at the nth trivial$\}$.

19) Relation b/w F(S) and P(S) :

The event E^* success at the n^{th} trails may be a compound event r. such that E^* occurs for the 1st time out the r^{th} trail $(r < n)$ & again at the lather trail number 1.

$$f^* = F(1) = \frac{\sum P_{n-1}}{\sum P_n}$$

20) Delay or loss System :

The interval b/w consequentive arrival b/w two consecutive arrival is called the interval. This system is called delay system depending on whether a unit who on arrival find service facilities.

21) Steady State distribution:

$N(t)$ is the no. of customers in the system at time t & its Probability distribution denoted by,

$P_n(t) = P_r\{N(t) = n / N(0) = \cdot\}$ are both time dependent solution, $P_n = \lim P_n(t)$. It is said that the system has reached equlibrium (or) steady state.

22) M|G|k System:

M|G|k is mean that some system with 4th distribution R denoting the system has a limited holding capacity k.

23) Queue System:

A Queing are waiting time is formed when units are customers needing some trivial of service channel (or) counter that offers such speciality.

24) Define : Offer load:

The mean arrival rate usually denoted by λ is the mean no. of arrivals for unit time.

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It's resiprocal is the mean as inter arrival time distribution.

$\mu \Rightarrow$ service rate

$1/\mu \Rightarrow$ Mean servic time

In a single channel system the ratio

$$\rho = \frac{\lambda}{\mu} = \frac{\text{Mean service time}}{\text{Mean interval time}}$$ is called the

offer load.