

Sudharsan College of Arts and Science

Perumanadu – Pudukkottai

ABSTRACT ALGEBRA

Class : III B.Sc Maths  
Sub Code: 16SCMM12

Total : 75 Marks  
Time: 3 Hours

Section – A

Answer all the questions:

(10×2=20)

1. Define Group.
2. Define Cyclic group and Given an example.
3. Define left and right cosets
4. Prove that any cyclic group is abelian
5. Define Normal subgroup and given an Example
6. If  $F$  is homomorphism and  $F$  is 1-1, then prove that  $\text{Ker } F = \{e\}$ .
7. Define quotient Rings.
8. Define a zero- divisor.
9. Define divisor.
10. Define Euclidean domain.

Section – B

Answer ALL the questions:

(5×5=25)

11. a) state and prove Euler's theorem.  
**Or**  
b) prove that a subgroup of cyclic group is cyclic
12. a) Such that the Permutation  $n \{ 1,2,3 \}$ .  
**Or**  
b) If  $R$  is a ring such that  $a^2 = a$  for all  $a \in R$ , P.T (i)  $a+a = 0$  (ii)  $a+b = 0 \rightarrow a = b$  (iii)  $ab = ba$
13. a)  $Z_n$  is an integral domain iff  $n$  is prime .  
**Or**  
b) Such that every finite cyclic group of order  $n$  is isomorphism to  $(Z_n, +)$
14. a) Let  $f: G \rightarrow G'$  be a homomorphism then the kernel  $\text{Kof } F$  is a normal subgroup of  $G$   
**Or**  
b) Every subgroup of an abelian group is a normal subgroup

15.a) Let  $G$  be a group and  $a, b \in G$  then

1.  $O(a) = O(a^{-1})$
2.  $O(a) = O(b^{-1}ab)$
3.  $O(ab) = O(ba)$

**Or**

b) If  $H$  is a subgroup of  $G$  and  $N$  is a normal subgroup of  $G$  then prove that  $HN$  is a subgroup of  $G$ .

Section – C

Answer Any THREE Questions

(3×10=30)

16.  $A$  and  $B$  are subgroup of a group  $G$ . Prove that  $AB$  is a subgroup of  $G$  iff  $AB=BA$ .

17. Prove that any permutation can be expressed as a product of disjoint cycles.

18. State and prove Cayley's theorem.

19. Let  $R$  and  $R'$  be rings and  $f: R \rightarrow R'$  be an isomorphism. Then

- (i)  $R$  is commutative  $\rightarrow R'$  is commutative.
- (ii)  $R$  is ring with identity  $\rightarrow R'$  is ring with identity.
- (iii)  $R$  is an integral domain  $\rightarrow R'$  is an integral domain.
- (iv)  $R$  is a field  $\rightarrow R'$  is a field.

20. State and prove Fundamental theorem of homomorphism on groups

☺ ☺ ALL THE BEST ☺ ☺