Sudharsan College of Arts and SciencePerumanadu – Pudukkottai

LINEAR ALGEBRA

Class : II B.ScMaths **Total : 75 Marks**

Sub Code: 16SCCMM8 **Time: 3 Hours**

Section – A

Answer all the quetions: **(10×2=20)**

1. Define Vector Space.
2. Define a Homomorphism.
3. Define a direct sum.
4. Define a kernel.
5. Define a Orthogonal Complement.
6. Define Inner Product.
7. Define a transpose of matrix with example.
8. Define a Hermitian matrix.
9. Define a rank matrix.
10. Find the characteristic equation of the following matrice $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

Section – B

Answer ALL the questions: **(5×5=25)**

11. a) Prove that the intersection of two sub- spaces of a vector spaces is a subspace.

(OR)

b) Let V be a vector space over a field F. Then (i) α0 = 0 for all α ϵF. (ii) 0v = 0 for all vϵ V . (iii) (-α)v = α(-v) = -(αv) for all αϵ F and vϵ V. (iv) αv = 0 ⇒ α = 0 or v = 0.

12. a) Let V be a finite – dimensional vector space over a field F. Let A and B be subspaces of V Then $\dim(A + B) = \dim A + \dim B - \dim(A \cap B)$.

(OR)

b) Let T : V → W be a linear transformation. Then $\dim V = \text{rank}\, T + \text{nullity}\, T$.

13. a) Find an orthogonal basic containing the vector (1,3,4) for $V_3(\mathbf{R})$ with the standard inner product .

(OR)

b) Let V be a vector space of polynomials with inner product given by $\langle f,g \rangle = \int_0^1 f(t)\, g(t)dt$. Let f(t) = t+2 and g(t) = t^2- 2t – 3. Find (i) ‹f,g › (ii) $\|f\|$.

14. a) Compute the inverse of the matrix A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & -1 & -5 \\ 0 & -1 & -3 \end{bmatrix}$.

(OR)

b) A square matrix A is symmetric iff A = A^T.

15.a) Using Cayley Hamilton's theorem for the matrix A = $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$.

(OR)

b) show that the equations x + y + z =6, x + 2y + 3z = 14, x + 4y + 7z = 30 are consistent and solve them.

Section –C

Answer Any THREE Questions **(3×10=30)**

16. let V be a vector space over F and W a subspace of V . Let V/W = {W+ v/vϵ V}. Then V/W is a vector space over F under the following operations. (i) $(W+v_1) + (W+v_2) = W + v_1 + v_2$. (ii) $\alpha(W + v_1) = W + \alpha v_2$

17. Let V be a finite dimensional inner product space. Let W be a subspace of V . Then V is the direct sum of W and $W^{\perp}$ (i.e) V = W $\oplus W^{\perp}$.

18. State and prove Fundamental theorem of homomorphism.

19. Find the rank of the matrix A = $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 7 \end{bmatrix}$.

20.Find the eigen values and eigen vectors of the matrix A = $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

☺☺ **ALL THE BEST** ☺☺