Sudharsan College of Arts and Science
Perumanadu – Pudukkottai
LINEAR ALGEBRA

Class : I M.Sc Maths **Total: 75 Marks**
Sub Code: P 16 MA22 **Time: 3 Hours**

Section – A

Answer all the quetions: **(10×2=20)**

1. Define the product of two matrices.
2. Let F be a subfield of the complex numbers Prove that, in F^3, the vectors (3, 0, -3), (-1, 1, 2), (4, 2, -2) and (2, 1, 1) are linearly dependent.
3. Define the rank and nullity of a linear transformation.
4. Let V be a vector space over the field F. let U,T and T_2 be linear operators on V, then prove that $U(T_1 + T_2) = UT_1 + UT_2$.
5. When we say that two algebras are isomorphic?
6. Define an ideal in F[x].
7. Define a characteristic vector of a linear operator on a vector space over a field.
8. Give an examble for the linear operator T may not have any characteristic values.
9. Let A $=\begin{bmatrix} 3 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix}$. Then find A^3.
10. Give ann example for an invariant under transformation.

Section – B

Answer ALL the questions: **(5×5=25)**

11. a) If A & B are row – equivalent $m \times n$ matrices, then prove that the homogeneous systems of linear equations $AX = 0$ and $BX = 0$ have exactly the same solutions.

Or

b) If W is a subspace of a finite – dimensional vector space V, then prove that every linearly independent subset of W is finite and is part of a finite basis for W.

12. a) Let V and W be vector space over the field F and let T be a linear transformation from V into W. If T is invertible, then prove that the inverse function T^{-1} is a linear transformation from W and V.

Or

b) Let V be a finite – dimensional vector space over the field F, and let W be a subspace of V. Then prove that dim W+ dimW^0 = dim V.

13. a) Let f . and g be non – zero polynomials over F. then prove that fg is a non – zero polynomial.

Or

b)Prove that every n – dimensional vector space over the field F is isomorphic to the space F^n.

14. a) Suppose that Tα = cα. If f is any polynomial, then prove that f(T)α = f(c)α.

Or

b) Let K be a commutative ring with identify, let A & B be $n \times n$ matrices over K. Then prove that
det (AB) = (det A) (debB).

15. a) Let T be a linear operator on an n – dimensional vector space V. Prove that the characteristic and minimal polynomial for T have the same roots, except for multiplicities.

Or

b) Let V be a finite-dimenaional vector apace over the filed F and let T be a linear operator on V. Then prove that T in triangulable if and only if the minimal polynominl for T is a product of inear polynomiala over F.

Section –C

Answer Any THREE Questions **(3×10=30)**

16. If W_1, and W_2 are finite-dimensional subapaces of a veetor space V, then prove that W_1, + W_2, in finite- dimensional and dim W_1, +dim W_2 =dim ($W_1 \wedge W_2$) +dim($W_1 + W_2$).

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17. Let V be a n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Then prove that the space L(V, W) is finite-dimensional and had dimension mn.

18.If F is a field, then prove that a non-scalar mnonic polynomial in F(x), Can he factored as a product of monic primes in F(x) in one and, except for order, only one way.

19. State and Prove Cayley-Hamiltor theorem.

20. State and Prove Primary decomposition theorem.

☺☺ **ALL THE BEST** ☺☺