Operations Research

[Computer Science - Allied III]

2 marks

1. Write the essential characteristics of decision.

(i) Objectives (ii) Alternatives (iii) Influecing factors.

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2. What are the three phases of scientific method in O.R.?

(i) Judgement phase (ii) Research phase (iii) Action phase.

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3. Give any two features of OR.

* Decision Making * Scientific Approach

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4. Define: Objective function.

Linear programming deals with the optimization (maximization or minimization) of a function of variables known as objective functions.

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5. Define Surplus variable.

Hints: $\sum_{j=1}^{n} a_{ij} x_j - S_j = b_i \quad (i=1,2,\ldots,m)$

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5 marks:

1. Describe the classifications of models in Operations Research.

Classification of models is a subjective problem. Models may be distinguished by

I-Models by Function:

These Models can further be classified as (i) Descriptive models (ii) Predictive models and Normative models.

II - Models by Structure:

(i) Iconic or physical models, (ii) Analogue models, (iii) Mathematical (or) symbolic models.

III - Models by Nature of An Environment:

These models can be classified into (i) Deterministic models and (ii) Probabilistic models.

IV - Models by the Exact of Generality:

These model can be categorized as (i) Specific models and (ii) General models

V - Models by Degree of abstraction:

Define general form of linear programming problem.

The general formulation of the L.P.P. can be stated as follows:

In order to find find the values of n decision variables $x_1, x_2 \dots x_n$ to maximize the objective function.

$$z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \rightarrow ①$$

and also satisfy m-constraints,

$$\left.\begin{array}{l} a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \cdots b_1 \\ a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \cdots b_2 \\ \vdots \\ a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \cdots b_i \\ a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n \cdots b_m \end{array}\right\} \rightarrow ②$$

where constraints may be in the form of inequality $\leq$ or $\geq$ or even in the form of an equation $(=)$ and finally satisfy the non-negative restrictions.

$$x_1 \geq 0,\ x_2 \geq 0 \ \ldots\ x_n \geq 0 \rightarrow ③.$$

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Formulation of Linear Programming Problems

The procedure for mathematical formulation of a LPP consists of the following steps:

step 1: Write down the decision variables of the problem.

step 2: Formulate the objective function to be optimized as a linear function of the decision variables.

step3: Formulate the other conditions of the problems such as resource limitation, market constraints, interrelations between variables, etc. as linear inequations or equations in terms of the decision variables.

step 4: Add the non-negative constraints from the cosiderations so that the negative values of the decision variables do not have any valid physical interpretation.

The objective function, the set of constraints and the non-negative restrictions together form a Linear Programming Problem. (LPP).

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LPP - Graphical Solution:

Graphical Solution Method:

The graphic method for solving a linear programming problem involves the following basic steps:

Step 1: Identify the problem - the decision variables, the objective function, and the constraint restrictions.

step 2: Draw a graph that includes all the constraints (or) restrictions and identify the feasible region.

step 3: The feasible region obtained in step 2 may be bounded or unbounded. Compute the coordinates of all the corner points of the feasible region.

step 4: Select the corner point obtained in step 3 that optimizes the objective function - the optimum solution.

step 5: Interpret the results.

1) Solve the following L.P.P.

Maximize $z = 4x_1 + 3x_2$

subject to the constraints
$2x_1 + x_2 \leq 1000, \rightarrow (1)$
$x_1 + x_2 \leq 800, \rightarrow (2)$
$x_1 \leq 400, \rightarrow (3)$
$x_2 \leq 700 \rightarrow (4)$
$x_1, x_2 \geq 0.$

Soln. :-

(1) $\Rightarrow 2x_1 + x_2 = 1000$

put $x_1 = 0 \Rightarrow x_2 = 1000$ $\therefore$ pt. $(0, 1000)$

put $x_2 = 0 \Rightarrow x_1 = 500$ $\therefore$ pt. $(500, 0)$

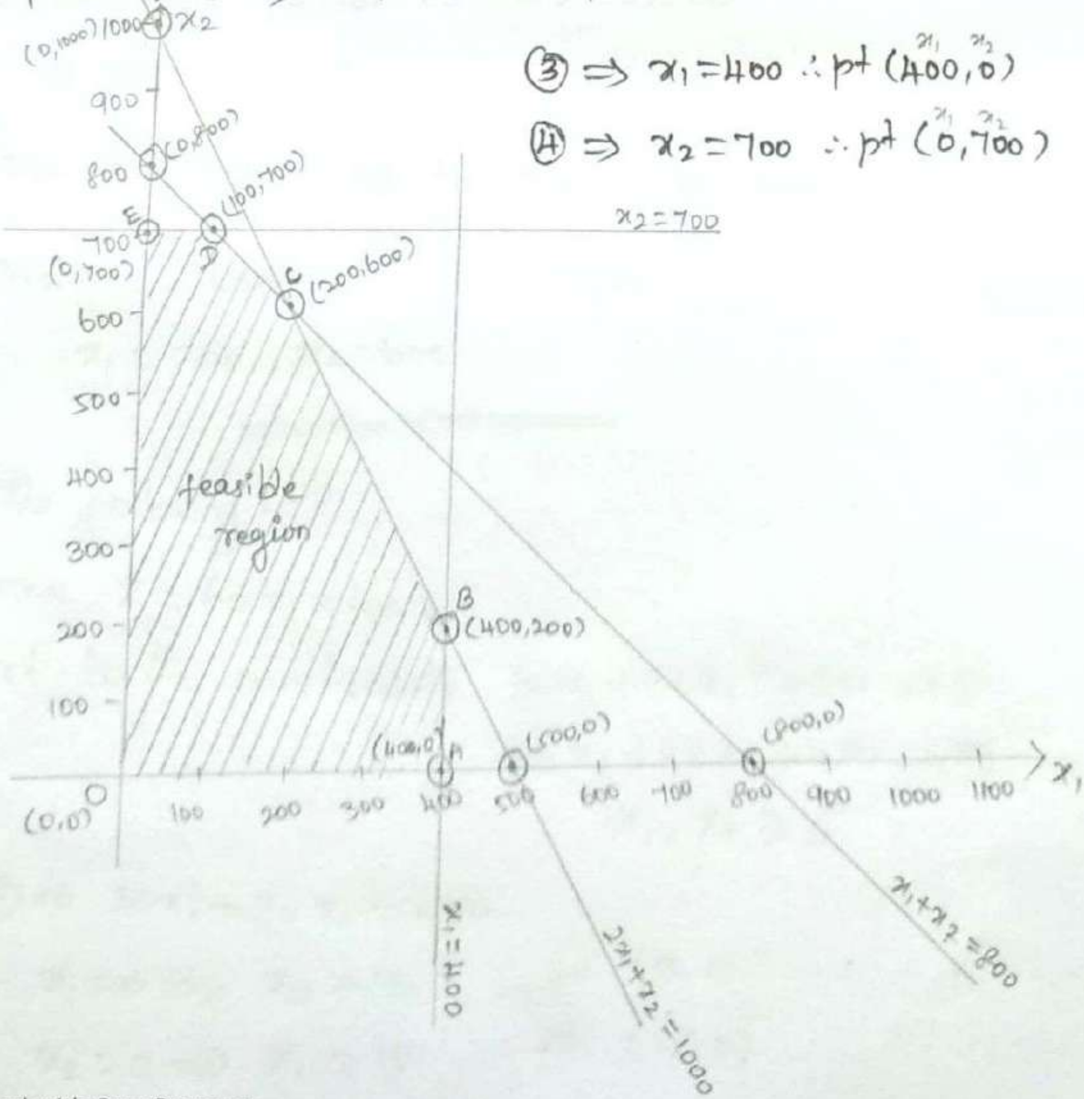
(2) $\Rightarrow x_1 + x_2 = 800$

put $x_1 = 0 \Rightarrow x_2 = 800$ $\therefore$ pt. $(0, 800)$

put $x_2 = 0 \Rightarrow x_1 = 800$ $\therefore$ pt. $(800, 0)$

(3) $\Rightarrow x_1 = 400$ $\therefore$ pt $(400, 0)$

(4) $\Rightarrow x_2 = 700$ $\therefore$ pt $(0, 700)$



From eqn: ③ & ④ $\Rightarrow$ $x_1 + x_2 = 800$

$$\begin{array}{r} x_1 + x_2 = 800 \\ \underline{(-)x_2 = 700} \\ \underline{x_1 = 100} \end{array}$$

$\therefore 100 + x_2 = 800 \Rightarrow x_2 = 700$ $\therefore$ pt $(100, 700)$

From eqn: ① & ⑤ $\Rightarrow$

$$\begin{array}{r} 2x_1 + x_2 = 1000 \\ \underline{x_1 + x_2 = 800} \\ \underline{x_1 = 200} \end{array}$$

$\therefore 200 + x_2 = 800 \Rightarrow x_2 = 600$ $\therefore$ pt $(200, 600)$

Corner Points	maxima $z = 4x_1 + 3x_2$
O (0,0)	$4(0) + 3(0) = 0$
A (400,0)	$4(400) + 3(0) = 1{,}600$
B (400,200)	$4(400) + 3(200) = 2{,}200$
C (200, 600)	$4(200) + 3(600) = \underline{\underline{2{,}600}}$
D (100, 700)	$4(100) + 3(700) = 2{,}500$
E (0,700)	$4(0) + 3(700) = 2{,}100$

Max $z = 2{,}600$ at the point $(200, 600)$.

$\therefore$ Max $z = 2{,}600$

$x_1 = 200$, $x_2 = 600$.

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② Solve the following L.P.P.

Minimize $z = 60x_1 + 90x_2$

Subject to the constraints $30x_1 + 30x_2 \geq 450 \rightarrow ①$

$24x_1 + 48x_2 \geq 480 \rightarrow ②$

$x_1, x_2 \geq 0$

Soln:-

① $\Rightarrow$ $30x_1 + 30x_2 = 450$

put $x_1 = 0 \Rightarrow x_2 = 15$ $\therefore$ pt. $(0, 15)$

$x_2 = 0 \Rightarrow x_1 = 15$ $\therefore$ pt. $(15, 0)$

② $\Rightarrow$ $24x_1 + 48x_2 = 480$

put $x_1 = 0 \Rightarrow x_2 = 10$ $\therefore$ pt. $(0,10)$

put $x_2 = 0 \Rightarrow x_1 = 20$ $\therefore$ pt. $(20,0)$

① $\times 24 \Rightarrow$

$$720x_1 + 72x_2 = 10800$$

② $\times 30 \Rightarrow$

$$720x_1 + 1440x_2 = 1440$$

$(-)$ $(-)$ $(-)$

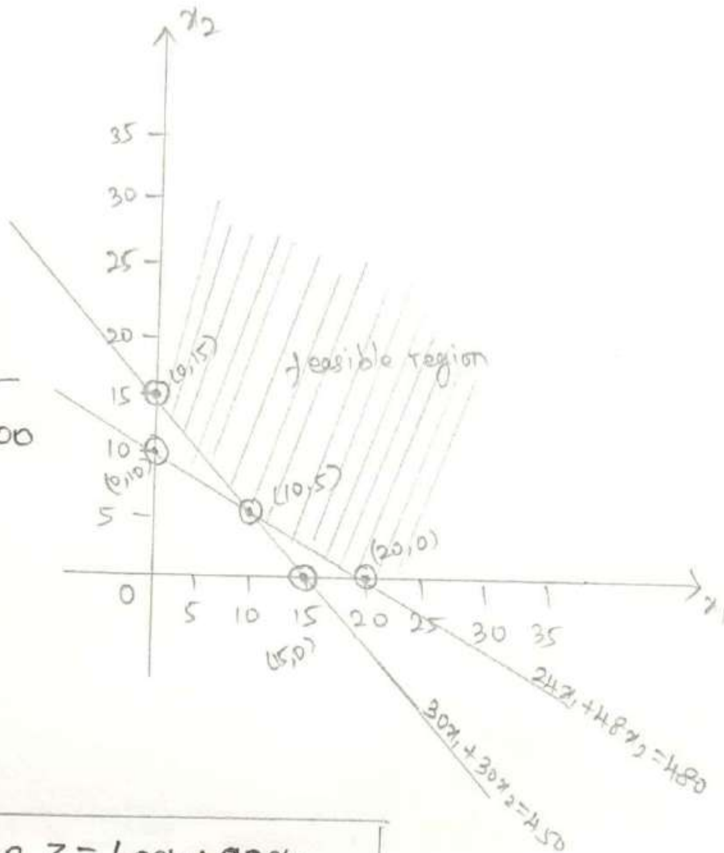
$$-720x_2 = -3600$$

$$\boxed{x_2 = 5}$$

$$30x_1 + 30(5) = 450$$

$$\boxed{x_1 = 10}$$

$\therefore$ pt. $(10,5)$



Corner point	minimize $z = 60x_1 + 90x_2$
O (0,0)	$z = 60(0) + 90(0) = 0$
A (20,0)	$z = 60(20) + 90(0) = 1200$
B (10,5)	$z = 60(10) + 90(5) = \underline{\underline{1050}}$
C (0,15)	$z = 60(0) + 90(15) = 1350$

Mini $z = 1050$ at the point $(10,5)$

$\therefore z = 1050$

$x_1 = 10$ & $x_2 = 5$.