① TRANSFORMATION OF EQUATION

Problems:

1) If the roots of $x^3 - 12x^2 + 23x + 36 = 0$ are $-1, 4, 9$. Find the equation whose roots are $1, -4, -9$.

Soln:- Given that $x^3 - 12x^2 + 23x + 36 = 0 \to ①$

Its roots are $-1, 4, 9$.

To find the equation whose roots of ~~are~~ the eqn. ① but the sign of changed we have to change the sign of the old powers of x. Then the required eqn. is

$$-x^3 - 12x^2 - 23x + 36 = 0$$

$$-(x^3 + 12x^2 + 23x - 36) = 0$$

$$\Rightarrow \boxed{x^3 + 12x^2 + 23x - 36 = 0}$$

2) Find the eqn. whose roots are equal in magnitude but opposite sign to the roots of the eqn. $x^{10} - 12x^8 + 40x^4 - 15x + 20 = 0$.

Soln:-

Given eqn. $x^{10} - 12x^8 + 40x^4 - 15x + 20 = 0$

change the sign of the old powers of x.

Hence the required eqn. is

$$\boxed{x^{10} - 12x^8 + 4x^4 + 15x + 20 = 0} \;//$$

3) Multiply the roots of the eqn. $x^4 + 2x^3 + 4x^2 + 6x + 8 = 0$ by $\frac{1}{2}$.

Soln:-

Given that $x^4 + 2x^3 + 4x^2 + 6x + 8 = 0 \rightarrow (1)$

Multiply the roots of eqn. (1) by $\frac{1}{2}$

$$\therefore x^4 + \left(\frac{1}{\cancel{2}}\right)\cancel{2}x^3 + \left(\frac{1}{2}\right)^2 4x^2 + \left(\frac{1}{2}\right)^3 6x + \left(\frac{1}{2}\right)^4 8 = 0$$

$$x^4 + x^3 + \frac{1}{\cancel{4}}\cancel{4}x^2 + \frac{1}{\cancel{8}_4}\cancel{6}^{3}x + \frac{1}{\cancel{16}_{2}}\cancel{8} = 0$$

$$x^4 + x^3 + x^2 + \frac{3}{4}x + \frac{1}{2} = 0$$

$$\frac{1}{4}\left(4x^4 + 4x^3 + 4x^2 + 3x + 2\right) = 0$$

$$\Rightarrow \boxed{4x^4 + 4x^3 + 4x^2 + 3x + 2 = 0}$$

5) Multiply the roots are $x^3 - 3x + 1 = 0$ by 10.

soln:-

Given that $x^3 - 3x + 1 = 0$

$$\Rightarrow x^3 + 0x^2 - 3x + 1 = 0 \rightarrow (1)$$

Multiply the root of eqn (1) by 10

$$\therefore x^3 + (10)0x^2 - (10)^2 3x + (10)^3 \cdot 1 = 0$$

$$x^3 + 0 - (100)3x + (1000)\cdot 1 = 0$$

$$\therefore \boxed{x^3 - 300x + 1000 = 0}$$

6) Remove the fractional co-efficients from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$.

soln:- Given that $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0 \rightarrow (1)$

Multiply by m.

$$\Rightarrow x^3 - \frac{m}{4}x^2 + \frac{m^2}{3}x - m^3 = 0 \rightarrow (2)$$

Here, $m = 12$. ($\because$ L.C.M $= 12$)

Sub. $m = 12$ in eqn (2) $\Rightarrow$

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$$x^2 - \frac{12}{4}x^2 + \frac{(12)^2}{3}x - (12)^3 = 0$$

$$x^2 - 3x^2 + \frac{144}{3}x - 1728 = 0$$

$$\boxed{x^2 - 3x^2 + 48x - 1728 = 0}$$

B) To transform an equation of the n^{th} degree into another whose roots are recibrocals of the roots of the given equation. Change x to $1/x$ in the given equation and multiply the resulting equation by x^n.

Problems:-

1) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$. Find the eqn. whose roots are $1/\alpha, 1/\beta, 1/\gamma, 1/\delta$.

Soln:-

Given that $x^4 + px^3 + qx^2 + rx + s = 0$

change $x \to 1/x$.

$$\Rightarrow (1/x)^4 + p(1/x)^3 + q(1/x)^2 + r(1/x) + s = 0$$

$$\Rightarrow 1/x^4 + p/x^3 + q/x^2 + r/x + s = 0$$

$$\Rightarrow 1/x^4 \left[1 + px + qx^2 + rx^3 + sx^4\right] = 0$$

(ie) $\cancel{1+px}$. $\boxed{sx^4 + rx^3 + qx^2 + px + 1 = 0}$

2) If 1, 2, 3, 6 are the roots of eqn.

$x^4 - 12x^3 + 47x^2 - 72x + 36 = 0$. Find then eqn. whose roots are 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$.

Soln:-

Given that $x^4 - 12x^3 + 47x^2 - 72x + 36 = 0$

change x to $\frac{1}{x}$.

$\therefore \left(\frac{1}{x}\right)^4 - 12\left(\frac{1}{x}\right)^3 + 47\left(\frac{1}{x}\right)^2 - 72\left(\frac{1}{x}\right) + 36 = 0$

$$\frac{1}{x^4} - \frac{12}{x^3} + \frac{47}{x^2} - \frac{72}{x} + 36 = 0$$

$$\frac{1}{x^4}\left[1 - 12x + 47x^2 - 72x^3 + 36x^4\right] = 0$$

$$\Rightarrow \boxed{36x^4 - 72x^3 + 47x^2 - 12x + 1 = 0}$$

3) Solve the equation $6x^3 - 11x^2 - 3x + 2 = 0$. Given that its roots are in H.P.

Soln:- Given that the eqn.

$$6x^3 - 11x^2 - 3x + 2 = 0$$

change x to $\frac{1}{x}$. $\Rightarrow$

$$6\left(\frac{1}{x}\right)^3 - 11\left(\frac{1}{x}\right)^2 - 3\left(\frac{1}{x}\right) + 2 = 0$$

$$\frac{6}{x^3} - \frac{11}{x^2} - \frac{3}{x} + 2 = 0$$

$$\frac{1}{x^3}\left[6 - 11x - 3x^2 + 2x^3\right] = 0$$

$$\Rightarrow 2x^3 - 3x^2 - 11x + 6 = 0$$

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Let the roots be A.P. $\alpha-d, \alpha, \alpha+d$.

Sum of the roots.

$$\alpha-\cancel{d}+\alpha+\alpha+\cancel{d} = \frac{-a_1}{a_0}$$

$$3\alpha = \frac{-(-3)}{2}$$

$$\cancel{3}\alpha = \frac{\cancel{3}}{2}$$

$$\therefore \boxed{\alpha = \frac{1}{2}}$$

Product of the root taken 3 at a time

$$(\alpha-d)(\alpha)(\alpha+d) = \frac{-a_3}{a_0}$$

$$(\alpha^2-\alpha d)(\alpha+d) = \frac{-(6)}{2}$$

$$\alpha=\tfrac{1}{2} \Rightarrow \left(\tfrac{1}{4}-\tfrac{d}{2}\right)\left(\tfrac{1}{2}+d\right) = -3$$

$$\Rightarrow \left(\frac{1-2d}{4}\right)\left(\frac{1+2d}{2}\right) = -3$$

$$\Rightarrow \frac{1}{8}\left[1+\cancel{2d}-\cancel{2d}-4d^2\right] = -3$$

$$\cancel{1-4d^2 = -24}$$

$$\cancel{-4d^2 = -24-1}$$

$$\cancel{\not{-}4d^2 = \not{-}23}$$

$$\cancel{d^2 = \frac{23}{4}}$$

$$\cancel{\frac{1}{8}-\frac{d^2}{2} = -3}$$

$$\cancel{-\frac{d^2}{2} = -3-\frac{1}{8}}$$

$$\cancel{-\frac{d^2}{2} = \frac{-24-1}{8}}$$

$$\cancel{\not{-}\frac{d^2}{2} = \frac{\not{-}25}{8}}$$

$$\cancel{d^2 = \frac{25}{8_4} \times \cancel{2}}$$

$$\cancel{d^2 = \frac{25}{4}}$$

$$1-4d^2 = -24$$

$$-4d^2 = -24-1$$

$$\not{-}4d^2 = \not{-}25$$

$$d^2 = \frac{25}{4}$$

$$\Rightarrow \boxed{d = \pm 5/2}$$

$d^2 = (5)^2$ ... $\boxed{d = 5}$

Case: 1 $\alpha = \frac{1}{2}, d = \frac{5}{2}$

$(\alpha - d), \alpha, (\alpha + d)$

$(\frac{1}{2} - \frac{5}{2}), \frac{1}{2}, (\frac{1}{2} + \frac{5}{2})$

$(-\frac{4}{2}, \frac{1}{2}, \frac{6}{2}) = (-2, \frac{1}{2}, 3)$

Case: 2 $\alpha = \frac{1}{2}, d = -\frac{5}{2}$

$(\alpha - d), \alpha, (\alpha + d)$

$(\frac{1}{2} + \frac{5}{2}), \frac{1}{2}, (\frac{1}{2} - \frac{5}{2})$

$\frac{6}{2}, \frac{1}{2}, -\frac{4}{2}$

$(3, \frac{1}{2}, -2)$

4) change the sign of the root of the equation

a) $x^7 + 4x^5 + x^3 - 2x^2 + 7x + 3 = 0$

b) $x^5 + 6x^4 + 6x^3 - 7x^2 + 2x - 1 = 0$

Soln:-

a) Given that $x^7 + 4x^5 + x^3 - 2x^2 + 7x + 3 = 0$

change the sign. of the equation

∴ Hence the required eqn:

$-x^7 + 4x^5 - x^3 - 2x^2 - 7x + 3 = 0$

(ie) $\boxed{x^7 + 4x^5 + x^3 + 2x^2 + 7x - 3 = 0}$ $\div (-)$

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b) Given that $x^5+6x^4+6x^3-7x^2+2x-1=0$

change the sign.

$\therefore -x^5+6x^4+6x^3-7x^2-2x-1=0$ $[\because \div(-)]$

(i.e) $\boxed{x^5-6x^4+6x^3+7x^2+2x+1=0}$

5) Multiply the roots of the equation $x^3-6x^2+12x-8=0$ by 10.

Soln:-

Given that $x^3-6x^2+12x-8=0$

Multiply the roots by 10.

$\therefore x^3-(10)6x^2+(10)^2\,12x-8(10)^3=0$

$x^3-60x^2+(100)12x-8(1000)=0$

(i.e) $\boxed{x^3-60x^2+1200x-8000=0}$

6) Remove the fractional co-efficient from $x^3-\frac{3}{2}x^2-\frac{1}{16}x+\frac{1}{32}=0$ such that co-efficient of the leading down remains unity.

Soln:- Given equation $x^3-\frac{3}{2}x^2-\frac{1}{16}x+\frac{1}{32}=0$

$\therefore x^3-\frac{3}{2}(m)x^2-\frac{1}{16}(m^2)x+\frac{(m)^3}{32}=0$

L.C.M. $=32$

$\therefore m=32$

$\therefore x^3 - \frac{3}{2}(\overset{16}{\cancel{32}})x^2 - \frac{1}{16}(32)^2 x + \frac{(32)^3}{32} = 0$

$\Rightarrow x^3 - 48x^2 - \frac{1024}{16}x + \frac{\cancel{32}(1024)}{\cancel{32}} = 0$

$\Rightarrow \boxed{x^3 - 48x^2 - 64x + 1024 = 0}$

7) Find the condition that the roots of the equation $x^3+3px^2+3qx+r=0$, may be in H.P.

Soln:- Given that the equation

$x^3+3px^2+3qx+r=0 \rightarrow (1)$

change the sign to x.

$(1/x)^3 + 3p(1/x)^2 + 3q(1/x) + r = 0$

$\frac{1}{x^3} + \frac{3p}{x^2} + \frac{3q}{x} + r = 0$

$\frac{1}{x^3}[1+3px+3qx^2+rx^3] = 0$

(ie) $1+3px+3qx^2+rx^3 = 0$

$\Rightarrow rx^3+3qx^2+3px+1=0 \rightarrow (2)$

Let the roots are $\alpha-d, \alpha, \alpha+d$.

Sum of the roots $\Rightarrow \alpha-\cancel{d}+\alpha+\alpha+\cancel{d} = \frac{-a_1}{a_0}$

$\Rightarrow \cancel{3}\alpha = \frac{-\cancel{3}q}{r} \Rightarrow \boxed{\alpha = -q/r}$

If $\alpha = x \Rightarrow x = -q/r$

Sub. $x = -q/r$ in eqn. (2)

$$r\left(-\frac{q}{r}\right)^3 + 3q\left(-\frac{q}{r}\right)^2 + 3p\left(-\frac{q}{r}\right) + 1 = 0$$

$$r\left(\frac{-q^3}{r^3}\right) + 3q\left(\frac{q^2}{r^2}\right) - \frac{3pq}{r} + 1 = 0$$

$$\frac{-q^3}{r^2} + \frac{3q^3}{r^2} - \frac{3pq}{r} + 1 = 0$$

$$+\frac{1}{r^2}\left[-q^3 + 3q^3 - 3pqr + r^2\right] = 0$$

(ie) $\boxed{2q^3 - 3pqr + r^2 = 0}$

This is the required condition.

(1) Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2 and find the transfored equation.

Soln:- Diminish the roots by 2.

2	1	-5	7	-4	5
	0	2	-6	2	-4
2	1	-3	1	-2	$\lfloor 1$
	0	2	-2	-2	
2	1	-1	-1	$\lfloor -4$	
	0	2	2		
2	1	1	$\lfloor 1$		
	0	2			
2	1	$\lfloor 3$			
	0				
	1				

$\therefore \boxed{x^4 \cancel{-4x^3} \cancel{+x^2} \cancel{+3x} \cancel{+1} = 0}$

$x^4 + 3x^3 + x^2 - 4x + 1 = 0$

2) Diminish the roots of $2x^5 - x^3 + 10x - 8 = 0$ by 5 and find the transformed eqn.

Soln:-

5	2	0	-1	0	10	-8
	0	10	50	245	1225	6175
5	2	10	49	245	1235	$\boxed{6167}$
	0	10	100	745	4950	
5	2	20	149	990	$\boxed{6185}$	
	0	10	150	1495		
5	2	30	299	$\boxed{2485}$		
	0	10	200			
5	2	40	$\boxed{499}$			
	0	10				
5	2	$\boxed{50}$				
	0					
	2					

$\therefore$ The equation is

$$2x^5 + 50x^4 + 499x^3 + 2485x^2 + 6185x + 6167 = 0$$

3) Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ find the transformed eqn.

-7	3	7	-15	1	-2
	0	-21	98	-581	4060
-7	3	-14	83	-580	4058
	0	-21	245	-2296	
-7	3	-35	328	-2876	
	0	-21	392		
-7	3	-56	720		
	0	-21			
-7	3	-77			
	0				
	3				

∴ The equation is

$$3x^4 - 77x^3 + 720x^2 - 2876x + 4058 = 0.$$

4) Increase by 2.

$$x^4 - x^3 - 10x^2 + 4x + 24 = 0$$

Soln:-

-2	1	-1	-10	4	24
	0	-2	6	8	-24
-2	1	-3	-4	12	0
	0	-2	10	-12	
-2	1	-5	6	0	
	0	-2	14		
-2	1	-7	20		
	0	-2			
-2	1	-9			
	0				
	1				

∴ The equation is

$$x^4 - 9x^3 + 20x^2 = 0$$

5) If α, β, γ are the roots of the equation $x^3 - 6x^2 + 12x - 8 = 0$. Find the equation whose roots are $\alpha-2, \beta-2, \gamma-2$.

Soln:-

$$
\begin{array}{c|cccc}
2 & 1 & -6 & 12 & -8 \\
 & 0 & 2 & -8 & 8 \\
\hline
2 & 1 & -4 & 4 & \lfloor 0 \\
 & 0 & 2 & -4 & \\
\hline
2 & 1 & -2 & \lfloor 0 & \\
 & 0 & 2 & & \\
\hline
 & 1 & \lfloor 0 & & \\
\end{array}
$$

$\therefore$ The transformed eqn. is

$x^3 = 0$

$\therefore$ The roots are $0, 0, 0$

$\therefore \alpha - 2 = 0, \beta - 2 = 0, \gamma - 2 = 0$

$\Rightarrow \alpha = 2, \beta = 2, \gamma = 2$.

6) If α, β, γ are the roots of $8x^3 - 4x^2 + 6x - 1 = 0$ Find the eqn. whose roots $\alpha + \frac{1}{2}, \beta + \frac{1}{2}, \gamma + \frac{1}{2}$.

Soln:-

$$
\begin{array}{c|cccc}
-\frac{1}{2} & 8 & -4 & 6 & -1 \\
 & 0 & -4 & 4 & -5 \\
\hline
-\frac{1}{2} & 8 & -8 & 10 & \lfloor -6 \\
 & 0 & -4 & 6 & \\
\hline
-\frac{1}{2} & 8 & -12 & \lfloor 16 & \\
 & 0 & -4 & & \\
\hline
-\frac{1}{2} & 8 & \lfloor -16 & & \\
 & 0 & & & \\
\hline
 & \boxed{8} & & & \\
\end{array}
$$

$\therefore$ The transformed eqn.

$8x^3 - 16x^2 + 16x - 6 = 0$