

## Sequencing and Replacement :

Sequencing problem :

The selection of an appropriate order for a series of jobs to be done on a finishing member of service facilities in some there assign order is called sequencing.

The procedure for a sequencing problem :

⇒ Each job once started on a machine is to be performed upto completion on that machine

⇒ The processing time on each machine is known such a time is independent of the order of the jobs in which they are to be processed.

$\Rightarrow$  The time taken by each job in changing over from one machine to another machine is negligible.

$\Rightarrow$  The job started on the machines as soon as the job and the machine both of idle and job is next to the machine and the machine is also next to job.

$\Rightarrow$  No machine may process more than one job simultaneously.

$\Rightarrow$  The order of completion of job has known significance (i.e.) No job is to be given priority. The order of completion job is independent of sequence of job

Basic terms and in sequencing:

(iii) Number of machine:

The number of service facilities which a job must pass define it is

assumed to be completed.

ii) Processing order:

The order (or sequence) in which given machines are required for completing the jobs.

iii) Processing time:

It indicates the time required by a job on each machine.

iv) Total Elapsal Time:

It is the time of interval between starting the first job and completing the last job including the idle time - In a particular order by the given set of machines.

v) Idle time on a machine:

It is the time for which a machine does not have a job to process. That is idle time from the

end of job  $(i-1)$  to the start of job  $i$ .

vii) No passing rule:

The rule of maintaining the order in which jobs are to be processed on given machine.

For examples:

If  $n$  jobs are to be processed through 3 machines  $M_1, M_2, M_3$  in the order again

Ex: 1

(i) A company has 4 jobs on hand each of these must be processed in two department. the sequential order is  $A \rightarrow B$

Department	:	A (days)	$J_1$	$J_2$	$J_3$	$J_4$
			8	6	5	4
"	:	B (days)	8	3	4	5

find the sequence in which 4 jobs should be processed so as to take minimum time to finish all the 4 jobs.

Soln

Step 1: Consider the given problem.

	J	J	J	J
Job :	1	2	3	4
$M_1$ :	8	6	5	2*
$M_2$ :	8	3	4	5

find the sequence using Johnson's rule. the processing is



$M_1$  - 1<sup>st</sup> machine in the process  $M_2$  - 2<sup>nd</sup> machine in the process

Clearly the jobs must be processed in  $M_1$  first and second to  $M_2$  Next.

Step 2:

Select the minimum processing time.

$J_4$			
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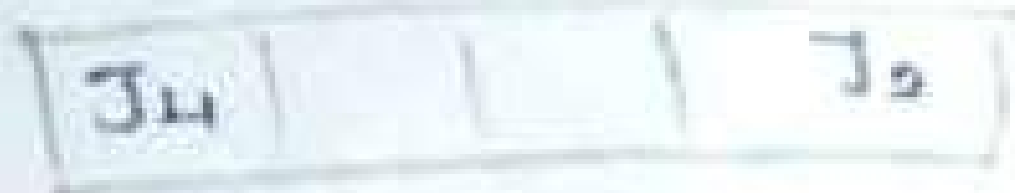
the problem reduced to

Job	$J_1$	$J_2$	$J_3$
$M_1$	8	6	5
$M_2$	8	3	4

Step 3 :

Repeat step-2 until all jobs are

considered



from the problem reduced to

Job	J <sub>1</sub>	J <sub>3</sub>
M <sub>1</sub>	8	5
M <sub>2</sub>	8	4



Hence the optimal sequence is



Consider the table

Job Sequence	Machine 1		Machine 2		Idle time for machine B
	in	out	in	out	
J <sub>4</sub>	0	1	6	6	1
J <sub>1</sub>	1	9	19	17	3
J <sub>3</sub>	9	14	21	21	-
J <sub>2</sub>	14	20	21	24	-

The minimum elapsed <sup>Time</sup> day  $\bar{T} = 24$  days hrs

Idle time for machine A =  $24 - 20 = 4$  hrs

Idle time for machine B =  $1 + 3 = 4$  hrs

assumed to be completed.

ii) Processing order:

The order (or sequence) in which given machines are required for completing the jobs.

iii) Processing time:

It indicates the time required by a job on each machine.

iv) Total Elapsal Time:

It is the time of interval between starting the first job and completing the last job including the idle time - In a particular order by the given set of machines.

v) Idle time on a machine:

It is the time for which a machine does not have a job to process. That is idle time from the

J <sub>1</sub>	J <sub>5</sub>			
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Step 3 :

Job	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>
M <sub>1</sub>	8	* 5	7
M <sub>2</sub>	10	6	5

J <sub>1</sub>	J <sub>5</sub>	J <sub>3</sub>		
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Step 4 :

Job	J <sub>2</sub>	J <sub>4</sub>
M <sub>1</sub>	8	7
M <sub>2</sub>	10	* 5

J <sub>1</sub>	J <sub>5</sub>	J <sub>3</sub>		J <sub>4</sub>
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Step 5 :

Job	J <sub>2</sub>
M <sub>1</sub>	* 8
M <sub>2</sub>	10

J <sub>1</sub>	J <sub>5</sub>	J <sub>3</sub>	J <sub>2</sub>	J <sub>4</sub>
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Job	Machine A		Machine B		Idle Time for machine B
	in	out	in	out	
J <sub>1</sub>	0	3	0	7	3
J <sub>5</sub>	3	7	7	15	1
J <sub>3</sub>	7	12	15	21	1
J <sub>2</sub>	12	20	21	31	1
J <sub>4</sub>	20	27	31	36	1



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The minimum elapsed time  $T = 36$  hrs

The idle time for machine A =  $36 - 27$   
 $= 9$  hrs.

Idle time for machine B = 3 hrs.

## Three machine problem

A company has six jobs which go through three machines,  $x$ ,  $y$  and  $z$  in the order  $xyz$ . The processing time in minutes for each job on each machine is as follows.

Job :	1	2	3	4	5	6
X :	18	12	29	36	43	37
Y :	7	12	11	2	6	12
Z :	19	12	23	47	28	36

What should be the sequence of the jobs which that the total make span is minimized

Soln

In the given problem

minimum of  $x = 17$   
 maximum of  $y = 12$   
 minimum of  $z = 10$ .

Since  $\min(x) \geq \max(y)$  for corresponding entries, we can convert three machines problem into two machines problem.

Machine G :  $x + y$

Machine H :  $y + z$

Job	1	2	3	4	5	6
	25	24	40	38	49	49
	26	24	34	49	34	48

The optimum sequence is

J <sub>2</sub>	J <sub>1</sub>	J <sub>4</sub>	J <sub>6</sub>	J <sub>3</sub>	J <sub>5</sub>
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Job machine	Machine x		Machine y		Machine z		Idle time for machine y	Idle time for machine z
	in	out	in	out	in	out		
J <sub>2</sub>	0	12	12	24	24	36	12	24
J <sub>1</sub>	12	30	24	30	37	56	6	1
J <sub>4</sub>	30	66	37+27	66	68	115	29	12
J <sub>6</sub>	66	103	113	115	115	151	35	-
J <sub>3</sub>	103	132	132	143	151	174	17	-
J <sub>5</sub>	132	175	175	181	181	209	32	7

Total elapsed time = 209 hrs

Idle time for  $x = 209 - 175$

$$x = 34 \text{ hrs}$$

Idle time for  $y = 12 + 6 + 29 + 35 + 17 + 32 +$   
 $(209 - 81)$

$$y = 131 + 28$$

$$y = 159 \text{ hrs}$$

Idle time for  $z = 24 + 1 + 12 + 7$

$$= 44 \text{ hrs}$$

5) Five jobs each of which must go through three machines A, B and C. Precedence is not allowed. Determine the order in which passing is not allowed. Determine the optimum sequence of jobs that minimize the total elapsed time when the following tables from the processing which follows.

Job	J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>	J <sub>5</sub>
M-A	8	10	6	7	11
M-B	5	6	2	3	4
M-C	4	9	8	6	5

Soln

In the given problem

minimum of M-A = 6

maximum of M-B = 6

minimum of M-C = 4

since  $\min(M-A) \geq \max(M-B)$

Job ~~J<sub>1</sub>~~ ~~J<sub>2</sub>~~ ~~J<sub>3</sub>~~ ~~J<sub>4</sub>~~ ~~J<sub>5</sub>~~ ~~J<sub>6</sub>~~

machine G = M-A + M-B

machine H = M-B + M-C

Job	J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>	J <sub>5</sub>
	13	16	*8	10	15
	*9	*15	10	*9	*9

J <sub>3</sub>	J <sub>2</sub>	J <sub>4</sub>	J <sub>1</sub>	J <sub>5</sub>
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Jobs	Machine A		Machine B		Machine C		Idle Time B	Idle Time C
	in	out	in	out	In	out		
J <sub>3</sub>	0	6	6	8	8	16	6	8
J <sub>2</sub>	6	16	16	22	22	31	8	6
J <sub>4</sub>	16	23	23	26	31	37	1	1
J <sub>1</sub>	23	31	31	36	37	41	5	1
J <sub>5</sub>	31	42	42	46	46	51	6	5

Total elapsed time = 51 hrs

Idle time for x =  $51 - 42 = 9$  hrs

Idle time for y =  $51 - 46 + 31 = 28$  hrs

Idle time for z =  $8 + 6 + 5 = 19$  hrs

Four machine problem:

- ① Solve the following sequencing problem giving an optimal solution when passing is not allowed

Job Sequence	1	2	3	4	5
M <sub>1</sub>	10	12	8	15	16
M <sub>2</sub>	3	2	4	1	5
M <sub>3</sub>	5	6	4	7	3
M <sub>4</sub>	14	7	12	8	10

elapsed time.

Soln

The given problem is

$\min M_1 = 8$

$\max M_2 = 5$

$\max M_3 = 7$

$\min M_4 = 7$

$\min (M_1) \geq \max (M_2)$

$\min (M_3) \geq \max (M_4)$

machine G =  $M_1 + M_2 + M_3$

machine H =  $M_2 + M_3 + M_4$ .

Job sequence	1	2	3	4	5
G	18	20	16	23	24
H	22	15	20	16	18

The optimum sequence is

J <sub>3</sub>	J <sub>1</sub>	J <sub>5</sub>	J <sub>4</sub>	J <sub>2</sub>
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Job	machine M <sub>1</sub>		machine M <sub>2</sub>		machine M <sub>3</sub>		machine M <sub>4</sub>		Idle time job machine M <sub>2</sub>	Idle time M <sub>3</sub>	Idle time M <sub>4</sub>
	in	out	in	out	in	out	in	out			
J <sub>3</sub>	0	8	8	12	12	16	16	28	8	12	6
J <sub>1</sub>	8	18	18	21	21	26	28	42	6	5	-
J <sub>5</sub>	18	34	34	39	39	42	42	52	13	13	-
J <sub>4</sub>	34	49	49	50	50	57	57	65	10	8	5
J <sub>2</sub>	49	61	61	63	63	69	69	76	11	6	4



$$\text{Total elapsed time} = 76 \text{ hrs}$$

$$\text{Idle time for } M_1 = 76 - 61 \\ = 15 \text{ hrs.}$$

$$\text{Idle time for } M_2 = 76 - 63 + 8 + 6 + 13 + 15 \\ + 11 \\ = 61 \text{ hrs.}$$

$$\text{Idle time for } M_3 = 76 - 69 + 12 + 5 + 13 + 8 \\ + 6 \\ = 51 \text{ hrs}$$

$$\text{Idle time for } M_4 = 76 - 16 + 5 + 4 \\ = 25 \text{ hrs.}$$

Graphical Method:

If we want to process two jobs using  $m$  machines we can make use of the graphical method using two dimension normally the  $x$  axis refers to Job 1 processing time and  $y$  axis refers to Job 2 processing time.

The Required minimum Total

elapsed time = processing time for Job 1.

Idle time of Job 1 (OR) = processing time of Job 2 + Idle time of Job 2.